# Exact results from the Quench Action Method for a certain class of initial states

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**arXiv:1404.1319 [cond-mat.stat-mech],10.1103/PhysRevA.91.021603 , Andrea De Luca, G.M., Jacopo Viti**
**Exact results from the Quench Action Method for a certain class of initial states, Andrea De Luca, G.M., Jacopo Viti, to appear**

# Summary

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1. Quantum Quench and Motivations

2. State of art of transport properties

3. XX chains: NESS, GGE and exact results

# Quantum Quench

- we select an initial state
$$\rho_0 = |\psi_0\rangle\langle\psi_0|$$
- $H_0 \to H$: change of a parameter (global quench), change of the geometry of the problem (local quench)
- Unitary evolution
$$\rho(t) = e^{-iHt}\rho_0 e^{iHt}$$
- pure state $\to$ pure state...stationary state or statistical ensemble only in the thermodynamic limit (TL)
- $|\psi(t)\rangle = \sum_n e^{-iE_n t}|n\rangle\langle n|\psi_0\rangle$, the importance of the overlaps
- technical difficult: the double sum in EV of an observable $\mathcal{O}$
$$\langle\psi(t)|\mathcal{O}|\psi(t)\rangle = \sum_n \sum_m e^{-i(E_n - E_m)t}\langle m|\mathcal{O}|n\rangle\langle n|\psi_0\rangle\langle m|\psi_0\rangle$$
- we don't solve exactly the dynamics, but we can compute the expectation value of observables in the limit $t \to \infty$
- it's possible to obtain many results for integrable system, question integrable systems equilibrates to a particular ensemble?

# GGE and NESS states

- GGE conjecture: integrable systems does not relax to a thermal state, but the equilibrium is described by a Generalized Gibbs Ensemble M. Rigol et all., Phys. Rev. Lett. 98, 050405 (2007)
- I need to described the equilibrium states with all the local conserved charges of the theory
- Attention!!! If I use all the conserved charges we have a tautology: GGE or diagonal ensemble is only a change of basis!!!!

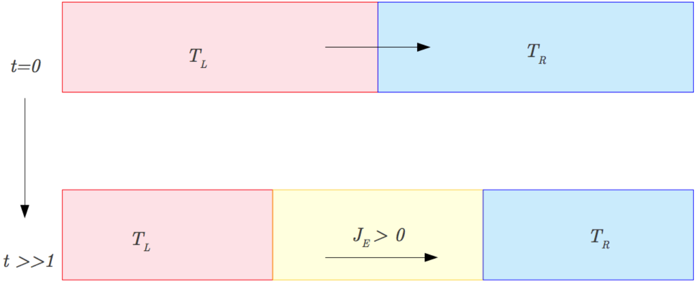
$$[I_n, I_m] = 0$$

$$\rho_{GGE} = \frac{1}{Z_{GGE}} \exp(-\sum_n \lambda_n I_n), \qquad Tr\, I_n \rho_{GGE} = \langle I_n \rangle_0$$

- failure of the GGE (Wouters et al., Pozsgay et al. 2014 for XXZ model) in the sense of local charges
- Quench in XXZ from Néel state: success of the Quench Action Method (QAM) (Caux, Essler2013)
- GGE in the sense of local charges is always valid in interacting to free Quantum Quench (Sotiriadis, Calabrese 2014, Sotiriadis, G.M. 2016)
- unbalanced of energy: NESS, persistent current in the TL

# Summary

# Framework



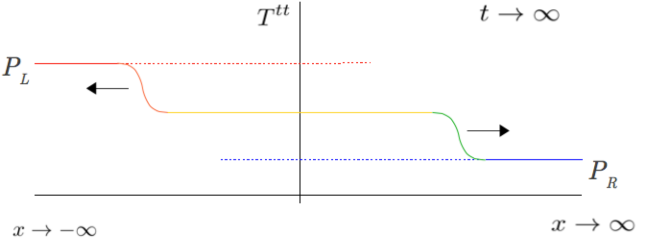

- Universal properties in 1+1dimensions (Bernard,Doyon, 2012) (Karrasch et al., 2012), experimentally (Brantut et al, 2013) (Schmidutz et al., 2013)

$$T^{tx} = \frac{\pi c}{12}\left(\frac{1}{\beta_{\mathcal{L}}^2} - \frac{1}{\beta_{\mathcal{R}}^2}\right)$$

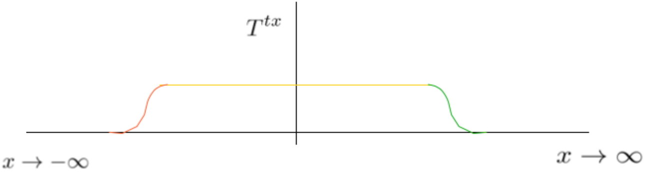
- no additivity in d>1 ansatz by (Bhaseen,Doyon,Lucas,Schalm,2013), (Chang,Karch,Yarom,2013) and (Amado,Yarom,2015)

$$T^{tx} = a\left(\frac{T_{\mathcal{L}}^{d+1} - T_{\mathcal{R}}^{d+1}}{u_{\mathcal{L}} + u_{\mathcal{R}}}\right)$$

# D=1

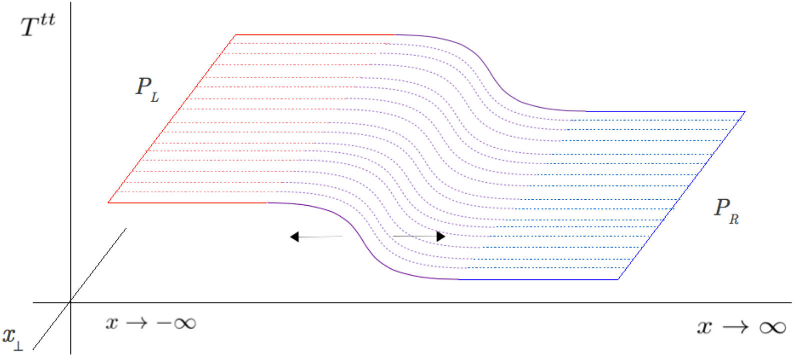



$$T^{tt} = \frac{\pi c}{12}\left(T_{\mathcal{L}}^2 + T_{\mathcal{R}}^2\right)$$




$$T^{tx} = \frac{\pi c}{12}\left(T_{\mathcal{L}}^2 - T_{\mathcal{R}}^2\right)$$

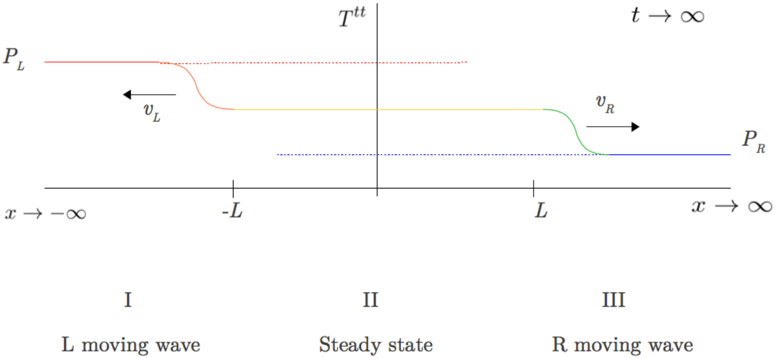
# D>1(1)




- Universal heat flow and energy density determined imposing only $\nabla_\mu T^{\mu\nu} = 0$

# D>1(2)

- Assumption: same structure of L and R moving waves describes the system




- this solution is correct if and only if $T_{\mathcal{L}} \simeq T_{\mathcal{R}}$, the left-moving shock violates the second law of thermodynamics (Lucas,Schalm, Doyon, Bhaseen, 2015)

# Summary

## The Quench Action Method(QAM)(1)

- GOAL: we want to use the QAM to obtain the NESS for the two temperatures case

$$\mathcal{O}(t) = \sum_{\{\lambda\}} \sum_{\{\mu\}} e^{-S^*_{\{\lambda\}} - S_{\{\mu\}}} e^{i(\omega_{\{\lambda\}} - \omega_{\{\mu\}})t} \langle \{\lambda\} | \mathcal{O} | \{\mu\} \rangle.$$

- we go in the continuum limit

$$\mathcal{O}(t) = \int \mathcal{D}[\rho] e^{S^{YY}_\rho} \sum_{\{\lambda\}} \Big( e^{-S^*_{\{\lambda\}} - S_\rho} e^{i(\omega_{\{\lambda\}} - \omega_\rho)t} \frac{\langle \{\lambda\} | \mathcal{O} | \rho \rangle}{2} + \lambda \leftrightarrow \rho \Big).$$

- Using a saddle point approximation (stationary phase) we obtain a new free energy $\mathcal{F}_\rho = 2\mathrm{Re}S_\rho - S^{YY}_\rho$

$$\frac{\partial \mathcal{F}_\rho}{\partial \rho} |_{\rho_s} = 0.$$

- this equation is coupled with $\rho(\lambda) + \rho^h(\lambda) = \frac{1}{2\pi} + \int_{-\infty}^{+\infty} d\lambda' K(\lambda - \lambda') \rho(\lambda')$

$$\lim_{t \to \infty} \langle \mathcal{O}(t) \rangle = \langle \rho_s | \mathcal{O} | \rho_s \rangle.$$

- the question is : in the case of the problem of the two temperature $|\rho_s\rangle$ is the NESS or the GGE or into the overlaps are present both the two states?

# The Quench Action Method(QAM)(2)

- the QAM works very well for translationally invariant quench (De Nardis et al., 2014 for LL model);
- how to treat a non translationally invariant quench?
- we study a very well known problem in literature: the NESS in the XX chain (free fermions) starting from two chain at different temperatures
- RESULTS: a persistent energy current when ((De Luca, Viti, Bernard, Doyon, 2013) and (Collura, Karevski, 2014))

$$\rho_S(\phi) = \frac{1}{\pi}[\Theta(\phi) f_l(\phi) + \Theta(-\phi) f_r(\phi)]$$

- absence of an energy current when (Collura, Karevski, 2014)

$$\rho_S(\phi) = \frac{1}{2\pi}(f_l(\phi) + f_r(\phi))$$

- The linear size of the system L is sent to infinity before the observation time T, phisically $T << L/v_{\text{max}}$ (fastest mode of the system) $\rightarrow$ NESS
- For much longer times, boundaries start to be relevant and a complete time-reversal symmetric state is restored, phisically $L/v_{\text{max}} << T << T_{\text{rev}} \propto L^2$ $\rightarrow$ GGE

## XX Chain

- We consider two disconnected spin-$1/2$ XX chains with Hamiltonian $H_0 = H_l + H_r$

$$\hat{H}_r = \frac{1}{2}\sum_{n=1}^{L}(\hat{\sigma}^x_n\hat{\sigma}^x_{n+1} + \hat{\sigma}^y_n\hat{\sigma}^y_{n+1})$$

$$\hat{H}_l = \frac{1}{2}\sum_{n=1}^{L-1}(\hat{\sigma}^x_{-n}\hat{\sigma}^x_{-n+1} + \hat{\sigma}^y_{-n}\hat{\sigma}^y_{-n+1})$$

- The model is equivalent to a free fermionic chain after the Jordan-Wigner transformation

$$H_0 = \sum_{\lambda=l,r}\int_0^{\pi} d\theta\ \varepsilon(\theta)\hat{\psi}^\dagger_\lambda(\theta)\hat{\psi}_\lambda(\theta),$$

- with dispersion relations $\varepsilon(\theta) = -2\cos\theta$
- the initial density matrix

$$\hat{\rho}_0 = Z^{-1}e^{-\beta_l H_l} \otimes e^{-\beta_r H_r}$$

## QAM for Free Fermions: 2 Temperatures

- master equation

$$\rho_(k) = \int_{-\pi}^{\pi} dk' \; f(k',k) e^{i(\varepsilon(k')-\varepsilon(k))t}$$

- $f(k',k) = \langle \psi_0 | c^\dagger_{k'} c_k | \psi_0 \rangle$ essentially the Wick Theorem
- two different limit : $t \to \infty$ and $\delta \to 0$, the other limit $\delta \to 0$ and $t \to \infty$

$$\rho_S(\phi) = \frac{1}{\pi}[\Theta(\phi) f_l(\phi) + \Theta(-\phi) f_r(\phi)]$$

$$\rho_S(\phi) = \frac{1}{2\pi}(f_l(\phi) + f_r(\phi))$$

- at equilibrium we have that any observables have expectation value(EV) as the average of EV on the single half
- for the NESS we recover a sort of Landauer's formula

$$\mathcal{J}_{NESS} = 2\int_0^{\pi} \frac{d\phi}{2\pi} \sin(2\phi)(n_l(\phi) - n_r(\phi)).$$

## QAM for Free Fermions: Neel, DWp and Domain Wall (1)

- master equation

$$\boxed{\rho_(k) = \int_{-\pi}^{\pi} dk' \; f(k',k) e^{i(\varepsilon(k')-\varepsilon(k))t}}$$

$$|DW\rangle_p = |\overbrace{1\dots11}^{p}\overbrace{0\dots00}^{p}\overbrace{1\dots11}^{p}\dots\rangle,$$

- p=1 Neel state

$$\langle c^\dagger(x,t)c(y,t)\rangle = \frac{1}{2}\left(\delta_{x,y} + (-1)^y (i)^{x-y} J_{y-x}(2t)\right),$$

$$G^{zz}(t) = \langle N|S^z(x,t)S^z(y,t)|N\rangle = \frac{1}{4}(\delta_{x,y} - J^2_{y-x}(2t) + (-1)^{x-y} J^2_0(2t)).$$

- we recover the results of (Mazza et al., 2015)

## QAM for Free Fermions: Neel, DWp and Domain Wall (2)

- master equation

$$\boxed{\rho_(k) = \int_{-\pi}^{\pi} dk' \; f(k', k) e^{i(\varepsilon(k') - \varepsilon(k))t}}$$

- p=L/2 Domain Wall: we recover the well known result of (Antal et al., 1999)

$$\rho_S(k) = \lim_{t \to \infty} \int_{-\pi}^{\pi} dk' \; f(k', k) e^{i(\varepsilon(k') - \varepsilon(k))t} = \Theta(k)$$

- and using the p-states

$$\langle c^\dagger(x, t) c(y, t) \rangle = -it(J_1^2(t) + J_0^2(t)).$$

- we recover the results of (Viti et al., 2015)
- for p finite the current is always zero and only for $p \to \infty$ we have current different from zero

## Conclusions

### Results

- we find the NESS and GGE states with the QAM for the XX chains
- we find a master equation from QAM to evaluate the whole time evolution starting from any initial free fermionic states

### Work in Progress

- we will use the QAM to study the transport properties of integrable interacting systems
- we can recover the Sabetta-Misguich conjecture with the DWp states for DW in XXZ?

**THANKS**

## Brief Review of Integrable Systems

- What is a Quantum integrable system?
- the dynamics are two-body irreducible $\rightarrow$ Yang-Baxter equation
- Bethe Ansatz works
- infinite set of conserved charges
- more simple words: an integrable system is a free system with an interaction Kernel
- Lieb-liniger model

$$H = \int dx(\partial_x \phi^\dagger(x)\partial_x \phi(x) + c\phi^\dagger(x)\phi^\dagger(x)\phi(x)\phi(x))$$

- Bethe Ansatz wave function as a superposition of N plane waves with momenta $\lambda_i$
- $E = \sum_i^N \lambda_i^2$ and $P = \sum_i^N \lambda_i$

$$\lambda_j + \sum_{i \neq j}^{N} K(\lambda_j - \lambda_i) = \frac{2\pi n_j}{L}$$