Quantum Electrodynamics in the background of evaporating black holes

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Overview

Quantum vacuum in local quantum field theory

QFT starts with a specification of a set of local field operators $\mathcal{A}$. This set builds the so-called $*$-algebra. Elements of this algebra being invariant under the $*$-involution are physical observables.

With each spacetime domain O one associates a local algebra $\mathcal{A}(O)$ built from all operators having non-vanishing support within O. This net of algebras satisfies certain reasonable properties: isotony, locality and covariance.

A state $|\omega\rangle$ on $\mathcal{A}$ is defined as a linear, normalised and positive functional on elements of the algebra $\mathcal{A}$.

One can assign a Hilbert space $\mathcal{H}_\omega$ of the algebra $\mathcal{A}$ to a state $|\omega\rangle$. This is achieved through the Gelfand-Naimark-Segal construction. If the state is physical, then particles $\{|p\rangle\}$ are elements of $\mathcal{H}_\omega$.

More in R. Haag "Local Quantum Physics. Fields, Particles, Algebras" (1996)

Quantum vacuum and its macroscopic manifestations

▷ **Minkowski space: Physical vacuum**

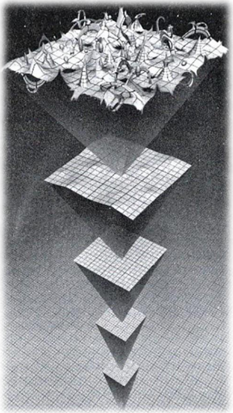
In Minkowski space one can introduce a preferred state - the vacuum state $|\Omega\rangle$, - which is characterised by its invariance under the Poincaré group $\mathcal{P}_{+}^{\uparrow}$ and the spectrum condition, i.e. the spectrum of the momentum operator $\hat{P}_{\mu}$ lies in the forward light-cone.

According to Wigner, one associates particle states with an irreducible representations of the Poincaré group. Thus, particles states ($|\text{electron}\rangle, |\text{positron}\rangle$ and so on) are elements of the (Minkowski) Hilbert space $\mathcal{H}_{\Omega}$.

Quantum vacuum and its macroscopic manifestations

▷ **Minkowski space: Vacuum fluctuations**

Quantum vacuum is full of **quantum or vacuum fluctuations**: This immediately follows from the commutation relation imposed on the quantum field operator at different spacetime points. Thus, the field fluctuates and has a non-vanishing value even in a state with no physical particles, i.e. in the vacuum.



Quantum vacuum and its macroscopic manifestations

▷ **Minkowski space: Vacuum fluctuations**

This is fully consistent with **the Reeh-Schlieder property** of the vacuum, of course: there are no local field operators $\mathcal{A}(O)$ with a non-vanishing support within O which annihilate the vacuum $|\Omega\rangle$, i.e. for any $\hat{\mathcal{O}} \in \mathcal{A}(O)$ one has $\langle\Omega|\hat{\mathcal{O}}|\Omega\rangle \neq 0$, unless $\hat{\mathcal{O}} = 0$.

An example of a local observable being one of the elements of $\mathcal{A}(O)$ is a thermometer. Thus, thermometers must measure fluctuations of non-zero temperature even in the vacuum!

Quantum vacuum and its macroscopic manifestations

▷ **Minkowski space: Vacuum fluctuations**

The spectrum of quantum fluctuations is defined via the mean square fluctuation of the field which reads for a massive scalar field

$$\boxed{\delta(\mathbf{k}) \;\propto\; |\mathbf{k}| \times \begin{cases} 1\,, & |\mathbf{k}| \gg m\,, \\ \left(|\mathbf{k}|/m\right)^{\frac{1}{2}}\,, & |\mathbf{k}| \ll m\,. \end{cases}}$$

V. Mukhanov, S. Winitzki, "Introduction to quantum effects in gravity" (2007)

The smaller detector, the larger influence of quantum fluctuations.

Quantum vacuum and its macroscopic manifestations

▷ **Minkowski space: Vacuum fluctuations**

It follows from **the Tomita-Takesaki theorem**: Whenever one treats a local quantum field operators $\mathcal{A}(O)$ having a non-vanishing support in a given domain O of space, these operators satisfy the so-called KMS condition with respect to a one parametric group of automorphism α_t of the set $\mathcal{A}(O)$, i.e. $\alpha_t\colon \mathcal{A}(O) \to \mathcal{A}(O)$.

This group of "time" translations has a geometrical meaning in certain cases, i.e. its generator can be assigned to one of the Killing vectors of spacetime. Examples:

- Bisognano&Wichmann 75': Boost generator in Rindler patch
- Buchholz 78': Dilatation generator in Milne patch

Quantum vacuum and its macroscopic manifestations

- **Minkowski space: Electromagnetic properties**

  The electromagnetic properties of the vacuum are described by electric permittivity ε and magnetic permeability μ appearing in

$$\mathcal{L}_{\mathrm{QED}} \;=\; \frac{1}{2}\left(\varepsilon E_i E_i - \frac{1}{\mu} B_i B_i\right) + \text{interaction terms}$$

  Integrating the fermion d.o.f. out one obtains an effective action $\mathcal{L}_{\mathrm{QED,eff}}$ for slowly varying field in terms of ε_{ij} and μ_{ij} of the vacuum

$$\begin{aligned}
\varepsilon_{ij} \;&=\; \delta_{ij} + \frac{e^4}{45\pi m_e^4}\Big(2\big(\mathbf{E}^2 - \mathbf{B}^2\big)\delta_{ij} + 7B_iB_j\Big) + \cdots \\
\mu_{ij} \;&=\; \delta_{ij} - \frac{e^4}{45\pi m_e^4}\Big(2\big(\mathbf{E}^2 - \mathbf{B}^2\big)\delta_{ij} - 7E_iE_j\Big) + \cdots
\end{aligned}$$

H. Euler, B. Kockel, Naturwiss. **23**, 246 (1935)

W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936)

Quantum vacuum and its macroscopic manifestations

▷ **Minkowski space: Electromagnetic properties**

In absence of the background electromagnetic field, the non-linear terms do not influence propagation of photons in the leading order approximation of the perturbation theory.

To have a non-trivial effect, one needs either choose a non-vacuum state (assigned to a medium, e.g. plasma) or change somehow the electromagnetic field operators, i.e. $\hat{E}_i$ and $\hat{B}_i$, structurally.

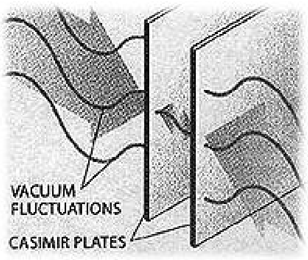
The latter can be achieved, for instance, in-between conducting plates in the Casimir set-up.

Quantum vacuum and its macroscopic manifestations

▷ **Minkowski space: Electromagnetic properties**

Due to boundary conditions satisfied by the electromagnetic field the field operators $\hat{\mathcal{E}}_i$ and $\hat{\mathcal{B}}_i$ differ from the operators $\hat{E}_i$ and $\hat{B}_i$ defined in Minkowski space!

$$\boxed{\langle \hat{\mathcal{E}}_i \hat{\mathcal{E}}_j \rangle \neq 0 \quad \& \quad \langle \hat{\mathcal{B}}_i \hat{\mathcal{B}}_j \rangle \neq 0}$$



The non-vanishing vacuum expectation value of $\hat{\mathcal{E}}_i \hat{\mathcal{E}}_j$ and $\hat{\mathcal{B}}_i \hat{\mathcal{B}}_j$ entails $\varepsilon\mu \neq 1$, i.e. the electromagnetic characteristics of the vacuum modify: Photons propagating perpendicularly to the conducting plates are superluminal. This is **the Scharnhorst effect**.

K. Scharnhorst, Phys. Lett. B**236**, 354 (1990)
G. Barton, Phys. Lett. B**237**, 559 (1990)

Quantum vacuum and its macroscopic manifestations

- **Schwarzschild space: Physical vacuum**

We await that very, very far away from black holes there should not be any significant imprints of black holes in local experiments performed, e.g., on earth.

We await further that **the physical vacuum** in a black-hole background is that which "goes over" to the ordinary Minkowskian one very, very far from the hole. Therefore, we denote that by the same symbol $|\Omega\rangle$. In other words, the Hilbert space representation $\mathcal{H}_\Omega$ does not change when a black hole forms.

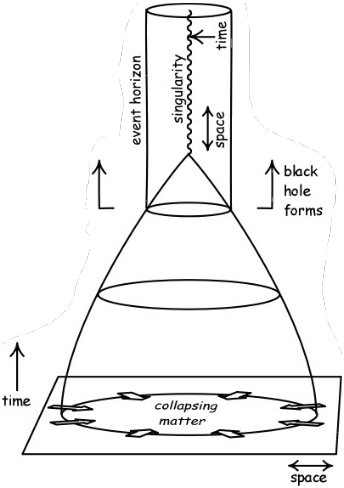
We expect that the notion of a particle should be unchanged independent on the state of motion of an observer, because the Hilbert space representation $\mathcal{H}_\Omega$ of the operator algebra is independent on that.

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Gravitational properties**

Very far away from a hole, one cannot tell probing the quantum vacuum, whether there is a hole, because the effect is too tiny. It implies gravitational effects due to vacuum fluctuations should be negligible far from the holes.

Indeed, the stress tensor of quantum fields is non-vanishing in the background of black holes, i.e. $\langle\Omega|\hat{T}^{\mu}_{\nu}|\Omega\rangle \neq 0$ and vanishes as $(2M/r)^2$ at spatial infinity, where M is the black-hole mass.



Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Modified propagator**

A black-hole horizon entails the modification of quantum field operators. This leads in turn to the modification of the correlation function of the field, e.g. **the propagator of a massless scalar** is

$$\boxed{G_\Omega(k,k') \approx \left(\frac{i}{k^2 + i\varepsilon} + \left[\frac{27R_H^2}{16R^2}\right] \frac{2\pi\delta(k^2)}{e^{\beta_H k_0} - 1}\right)\delta(k-k'),}$$

where $\beta_H^{-1} = T_H = \frac{M_{\rm Pl}^2}{8\pi M}$ is the Hawking temperature, $R_H = 2M$, if

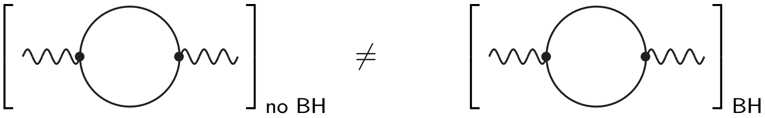
$R \gg R_H$	$-$	weak gravity approximation
$\lvert x - x' \rvert \ll R\,(R/R_H)^{\frac{1}{2}}$	$-$	local flatness of spacetime

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Main observation**

Fermion propagator also changes whenever the event horizon is present. This entails



Note that in the case of the Scharnhorst effect, the non-trivial contribution to the photon self-energy appears only at two-loop level, because the fermion propagator does not change in-between the conducting plates - one needs to have internal photon propagator.

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Small black holes**

Analytic approximation of the propagator is hard to derive if the fermion is massive. One can "overcome" this technical difficulty by considering small black holes, i.e. $T_H \gg m_e$. This implies

$$\boxed{M_{\mathrm{Pl}} \ll M \ll 10^{21} M_{\mathrm{pl}} \, (10^{-17} M_{\odot}) \quad - \quad \text{small black holes}}$$

If so, we can neglect m_e with respect to T_H. In other words, fermions are effectively massless (the hard thermal loop approximation).

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: New temperature parameter**

The correction to the fermion propagator induced by a small BH provides with two poles in the photon propagator. This is rather analogous to what happens in the hot physical plasma.

For a basic reference, see: M. Le Bellac, "Thermal Field Theory" (1996)

The hot plasma is however characterised by two parameters, namely the coupling α and temperature T, whereas the black-hole environment is described by one more temperature parameter $\mathcal{T} \ll T_H$:

$$\boxed{\mathcal{T} \approx \frac{3\sqrt{3}}{16\pi} M_{\text{Pl}} \frac{L_{\text{Pl}}}{R} = 2.34\times 10^{-7} \left(\frac{1\,\text{km}}{R}\right) \text{K}}$$

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Transverse pole – photon**

Long-wavelength regime:

$$k \ll e\mathcal{T}: \quad \omega_t^2 \approx \omega_p^2 + \frac{6}{5}k^2\,, \qquad \text{where} \qquad \omega_p^2 \equiv \frac{1}{9}\alpha\mathcal{T}^2$$

Short-wavelength regime:

$$k \gg e\mathcal{T}: \quad \omega_t^2 \approx m_\gamma^2 + k^2\,, \qquad \text{where} \qquad m_\gamma^2 \equiv \frac{1}{6}\alpha\mathcal{T}^2$$

ω_p – plasma-like frequency

m_γ – gauge invariant photon mass

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Longitudinal pole – plasmon**

Long-wavelength regime:

$$k \ll e\mathcal{T}: \quad \omega_l^2 \approx \omega_p^2 + \frac{3}{5}k^2 \quad - \quad \text{propagating mode}$$

Short-wavelength regime:

$$k \gg e\mathcal{T}: \quad \omega_l^2 \approx k^2\left(1 + 4e^{-k^2/m_\gamma^2}\right) - \text{non-propagating}$$

ω_p – plasma-like frequency
m_γ – gauge invariant photon mass

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Effective photon mass m_γ**

Thus, the effective photon mass due to virtual $e^- e^+$ pair is

$$\boxed{m_\gamma^2 \approx \frac{27\alpha}{384\pi} M_{\mathrm{Pl}}^2 \left(\frac{L_{\mathrm{Pl}}}{R}\right)^2}$$

According to Coulomb-law tests m_γ cannot be larger than 10^{-14} eV.

E.R. Williams, J.E. Faller, H.A. Hill, Phys. Rev. Lett. **26**, 721 (1971)

Exploiting this result as the upper bound on m_γ, one has

$$\boxed{m_\gamma \lesssim 10^{-14}\,\mathrm{eV} \quad \Longrightarrow \quad R \gtrsim 252\,\mathrm{km}}$$

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: No plasmon excitations**

The Compton wavelength λ_γ of the photon of mass $m_\gamma \lesssim 10^{-14}\,\text{eV}$:

$$\boxed{\lambda_\gamma = \frac{h}{m_\gamma c} \gtrsim 1.24 \times 10^5\,\text{km}}$$

This is much bigger than $R \gtrsim 252\,\text{km}$. In the hot physical plasma, plasmons propagate if $\omega_l \sim \omega_p$ or, in other words, $k \ll e\mathcal{T}$. Hence, there are no plasmon-like excitations in the black-hole environment.

This is consistent with the fact that there is no mediator whose collective excitation could be interpreted as a plasmon.

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

▷▷ **One-loop approximation: Debye-like screening**

The black-hole environment can mimic a certain property of the physical plasma, specifically **the Debye screening** of charges.

The potential of a point-like charge q located at the distance R from the small black hole is given by

$$\boxed{V(r) \;=\; \int \frac{d^3p}{(2\pi)^3} \frac{q\, e^{-i\mathbf{px}}}{\mathbf{p}^2 + \pi_L(p_0 = 0, \mathbf{p})} \;=\; \frac{q}{r}\, e^{-r/r_D}\,,}$$

where

$$\boxed{r_D \;=\; \frac{1}{\sqrt{2}m_\gamma} \quad - \quad \text{Debye radius}}$$

Quantum vacuum and its macroscopic manifestations

▷ **Schwarzschild space: Electromagnetic properties**

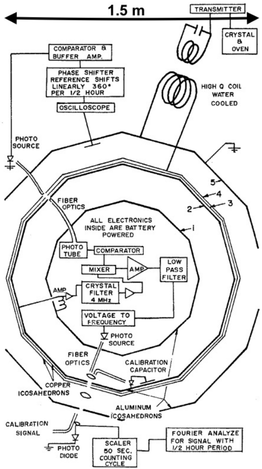
▷▷ **One-loop approximation: Debye-like screening**

Above we have found that for a photon of mass $m_\gamma \lesssim 10^{-14}$ eV generated by the small black hole, this hole must be at a distance $R \gtrsim 252$ km.

For the same upper bound on the photon mass m_γ, one has

$$\boxed{r_D \gtrsim 8.77\times10^4\,\text{km} \gg R}$$

Hence, the point-like charge q can be **only partially screened** due to the vacuum polarization effect.



E.R. Williams et.al., PRL**26**, 721 (1971)

Concluding remarks

- The electromagnetic properties of the quantum vacuum changes in the presence of black holes. This is a consequence of the modification of the quantum field operators, rather than the vacuum itself.
- Photon acquires an effective mass m_γ in the background of black holes which is vanishing far from the small black holes. For large black holes, i.e. $M \gg 10^{13}$ kg, this effect is exponentially suppressed as e^{-m_e/T_H}.
- The collective excitations known from hot plasma physics are impossible in the black-hole environment. This is due to the absence of mediators in the quantum vacuum (no-particle state). In plasma that is provided by the plasma particles.

Concluding remarks

- However, a point-like charge can be partially screened in the black-hole environment. In plasma that is due to collective response of plasma particles to the static electric field of the probe charge. Near small black holes the Debye-like screening is a vacuum polarization effect.
- Electrons get a one-loop correction to their mass which is of no significance at the distances $R \gg 4.4 \times 10^{-15}$ m, whereas the classical estimate of the electron size is about 2.8×10^{-15} m.
- The one-loop correction to the fine structure constant α near the small black holes is also negligible: $\delta\alpha/\alpha \ll 10^{-23}$ at a distance 1 m from the hole.