Does Analog Computation Exist?

M. Cristina Diamantini

Nips Laboratory, INFN and Department of Physics
University of Perugia

Coll: L. Gammaitoni and C.A. Trugenberger

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“Operating ICT basic switches below the Landauer limit”

Partners:
NiPS Laboratory, Università degli Studi di Perugia, Italy. Coordinator.
GHOST Group, Università degli Studi di Perugia, Italy.
Julius-Maximilians Universität Würzburg, Würzburg, Germany.
Physics of Information Group, Instituto de Telecomunicações, Lisbon, Portugal.
Technische Universiteit Delft, Delft, Netherlands

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Landauer's Principle

- Landauer principle: information is physical
  →information theory
  →thermodynamics (second law)

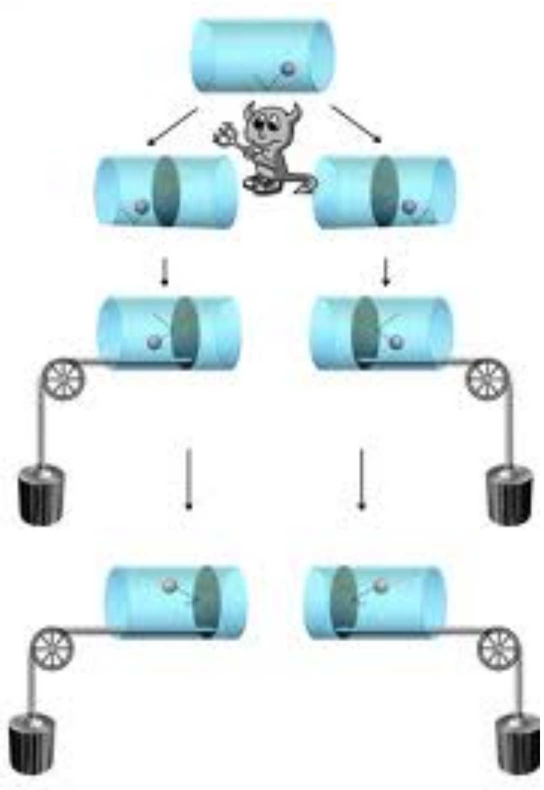
- Relevant for the engineering of devices used to process information:
  Landauer principle= fundamental bound on energy dissipation ⇒
  energy consumption of a device is:
  Landauer bound + how good we are in engineering devices

  ~~Landauer~~ bound + how good we are in engineering devices?

  (at least in principle)

Szilard engine

- One-particle gas in a partitioned box (Szilard 1929)



Initial probability ½ of the atom being on either side of the barrier

The demon performs a measure to determine the position of the particle (no heat production): information is gained

The piston is moved to one side or the other: extract work $W = kT \ln 2$ that can be used to lift a weight

Apparent violation of the II law

Resolution of the puzzle:

forgetting is costly !

(Landauer 1961)

- Szilard engine: the Demon has a single memory register (0,1), initially 0; after measure it represents the location of the particle, 0 or 1

- To complete the cycle the memory of the Demon must be erased (information lost)

- Landauer principle: erasure of information requires a minimun heat production: $\langle Q \rangle \geq \langle Q_{Landauer} >= kT \ \ln2$ per bit $\Rightarrow$ the conversion of kTln2 work into heat compensates the work extracted!

- Information is physical

- information is physical ⇒ system with $i= 1.....M$ possible states

  initial state: systems can be in any one of the possible states with probability p_i: $S_S = - \sum_{i\in M} p_i \ln p_i$

  final state: system is in a specific state with probability 1, $S_S = 0$

  variation of Shannon entropy

  $\Delta S_S = \sum_{i\in M} p_i \ln p_i \equiv$ Boltzmann (thermodynamic) entropy ($k=1$)

  since information can only be processed by physical systems (computers)

- $-\Delta S \geq k \ln 2$ per bit for perfect erasure, what happens if we admit errors?

  p = error probability $0 \leq p \leq ½$

  Landauer bound: $-\Delta S/k \geq \ln 2 + p \ln p + (1-p) \ln (1-p)$

  $-\Delta S/k = \ln 2 + p \ln p + (1-p) \ln (1-p)$ if the erasure is efficient

  for $p=0$ $-\Delta S \geq k \ln 2$; for $p= ½$ $-\Delta S \geq 0$

Landauer principle for analog computing systems:

we want to show that the continuous generalization

$\Delta S_S = \int_{x \in M} p(x) \ln p(x)$ ≡ Boltzmann (thermodynamic) entropy generated

when the erasure is realized in a physical system

⚠ dimensionless probability density p(x) cannot be defined by dimensionful physical degrees of freedom without an appropriate regularisation

- define a density (note: not probability density) that regularizes the entropy (Jaynes)

Landauer's principle and phase transitions

Idea: continuous phase transitions with order parameter $m \equiv$ information erasure by resetting to standard value $\Rightarrow$ entropy change must satisfy Landauer bound

$0 \leq m \leq 1$ plays the role of an error probability p

- SSB: degenerate vacua, the system "chooses" a state ≡ reset
- $T = 0 \Longrightarrow m = 1$ perfectly ordered phase ≡ reset with no errors
- $0 < T < T_C \Longrightarrow 0 < m < 1$ partially ordered phase ≡ reset with errors
- $T = T_C \Longrightarrow m = 0$ (disordering procedure efficient) →

  → Landauer bound saturated
- phase transition at $T = T_C$

How do we prove this conjecture for classical systems?

(C.A. Trugenberger, MCD PhysRevE.89 (2014) 052138)

- associative memory: (Hopfield model) bits are stochastic neurons $s_i = \pm 1$, mean field
- phase transition from a disordered phase to an ordered phase with order parameter increasing from the value 0 at $T= T_C$ to 1 at $T=0$

$P= ½ (1-m)$;

$$\frac{-\Delta S(J,T)}{kN} = \frac{1}{kN}\left(S_{T_c} - S_T\right)$$

$$\frac{-\Delta S}{kN} = \ln 2 + p \ln(p) + (1-p)\ln(1-p),$$

thermodynamic entropy completely scales in terms of this error probability alone

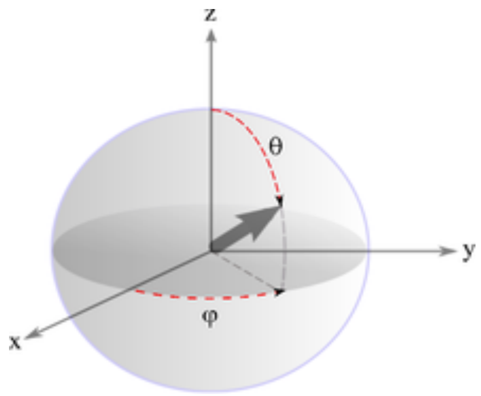
- Z_2 , easy generalization to $Z_{(2n+1)}$ n=1/2, 1,3/2,……..

Continuous Symmetries

discrete → continuous symmetry

digital →analog

- O(3)→ O(2), Heisenberg ferromagnet $S^2 = 1$



infinite many possible orientation

- mean field: classical limit of the discrete case

$$\frac{S(T,J)}{kN} = \ln\left[\frac{4\pi \sinh \beta m J}{(\beta m J)}\right] - (\beta m J) L(\beta m J)$$

- $L(x) = \coth x - 1/x$ Langevin function,

  $L(x) \to 1$ $x \to \infty$; $L(x) \to 0$ $x=0$

- $T = T_C$ $m \to 0$ $\Longrightarrow$ $S(T_C)/kN = \ln 4\pi$ = volume of the phase space ≡ area of the 2-sphere

- What happen when $T \to 0$?

$$\lim_{T\to 0} \frac{S(T,J)}{kN} = \ln(0) \to -\infty$$

- why the entropy is negative and diverges?

classical systems e.g. system of N classical harmonic oscillator:

$$\frac{S_{\mathrm{ho}}}{kN} = \left[\ln \frac{kT}{\hbar\omega} + 1\right] \to -\infty \quad \text{for} \quad T \to 0$$

- discrete case (quantum): entropy not divergent, can we use this to regularize the entropy for the continuous symmetry?

- Spin s, (2s +1) components ≡ representation of O(3); $s \to \infty$ classical spin on the unit sphere

(Millard and Leff, J. Math Phys 12 (1971), 1000;
Lieb, Commun. Math Phys 31 (1973) 327)

- classical limit: $s \to s/s_{max}$ $\Delta(2s_{max}+1) \to 4\Pi$ for $s_{max} \to \infty$, $\Delta \to 0$

- Heisenberg principle:

$$\Delta = \frac{1}{2}\hbar^2 s_{\max} \; ; \hbar s_{\max} \to s_{\text{clas}} \text{ for } s_{\max} \to \infty \, , \hbar \to 0$$

$S_{clas} = 1$

$$\frac{S}{kN} = \ln \frac{4\pi}{(2s_{\max}+1)} \left[\frac{\sinh(1+\frac{1}{2s_{\max}})\beta m J}{\sinh \frac{\beta m J}{2s_{\max}}} \right] - \beta m J B_{s_{\max}}(\beta m J)$$

- $S(T = T_C)/kN = \ln 4\Pi$ for $s_{max} \to \infty$
- $S(T \to 0)/kN = \ln [4\Pi /(2s_{max} + 1)] = \ln \frac{1}{2} \hbar^2 s_{max} = \ln \frac{1}{2} \hbar$

- $\Delta S / kN = (S(T_C) - S(T=0))/kN = \ln (8\Pi / \hbar)$

- Analog Landauer bound: entropy production during erasure process = available configuration volume measured in units of the minimum quantum of configuration volume ⇒ regularization

- even if we start with continuous, analog information, only a finite countably amount of information can be encoded in a physical system (Shannon, Bekenstein)

Grazie!