(Effective actions for) Fluids from black holes

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with *Jan de Boer* and *Michal P. Heller*

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Fluid dynamics - The phenomenological description

Fluid dynamics:

As an **effective description** of a system valid when fluctuations around thermal equilibrium are sufficiently long-wavelength

$$\lambda \gg l_{\mathrm{mfp}}$$

Phenomenological description:

- E.o.m: $\nabla_\mu T^{\mu\nu} = 0, \quad \nabla_\mu J^\mu = 0$
- Constitutive relations: $T_{\mu\nu} \sim T^{(0)}_{\mu\nu}(u_\mu, T) + T^{(1)}_{\mu\nu}(\partial u, \partial T) + \ldots$
- Local form of second law of thermodynamics: $\nabla_\mu J^\mu_S \geq 0$
- Onsanger relations
- Stochastic contributions: $\nabla_\mu T^{\mu\nu} = \xi^\nu$

Fluid dynamics - First principles formulation

Need a **first principles description** of hydrodynamics

- Based on symmetries & E.o.m. from a variational principle

  [G. Herzlog (1911); A. H. Taub (1954); B. Carter (1973); S. Dubovsky, T. Gregoire, A. Nicolis, R. Rattazzi (2006); N. Andresson, G. Comer (2007)]

- To systematically account for the dissipationless transport and the entropy current constraint

  [N. Banerjee, J. Bhattacharya, S. Bhattacharyya, S. Jain, S. Minwalla, T. Sharma (2012); K. Jensen, M. Kaminski, A. Yarom (2012), F. Becattini, L. Bucciantini, E. Grossi, L. Tinti (2014)]

- For a systematical treatment of stochastic noise

  [P. Kovtun, G. D. Moore, P. Romatschke (2014); M. Harder, P. Kovtun, A. Ritz (2015)]

- Interesting problem on its own (underlying Schwinger-Keldysh structure, Goldstones,. . .). New constraints?

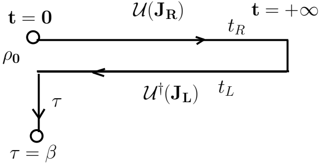
  [F. Haehl, R. Loganayagam, M. Rangamani (2015); M. Crossley, P. Glorioso, H. Liu (2015); + work in progress with K. Jensen & A. Yarom (2016)]

Fluid dynamics - The underlying Schwinger-Keldysh formalism

A thermal state perturbed out of equilibrium is naturally described in the Schwinger-Keldysh formalism

The **SK partition function**:

$$Z_{\rm SK}[J_R, J_L] = {\rm Tr}[\mathcal{U}(J_R)\,\rho_0\,\mathcal{U}^\dagger(J_L)\,]$$



$$\delta S_{\rm SK} = \int d^{d+1}x\,(J_R\mathcal{O}_R - J_L\mathcal{O}_L) = \int d^{d+1}x\,(J_r\mathcal{O}_a + J_a\mathcal{O}_r)$$

with $\quad \mathcal{O}_r = \frac{1}{2}(\mathcal{O}_R + \mathcal{O}_L) \quad$ and $\quad \mathcal{O}_a = \mathcal{O}_R - \mathcal{O}_L$

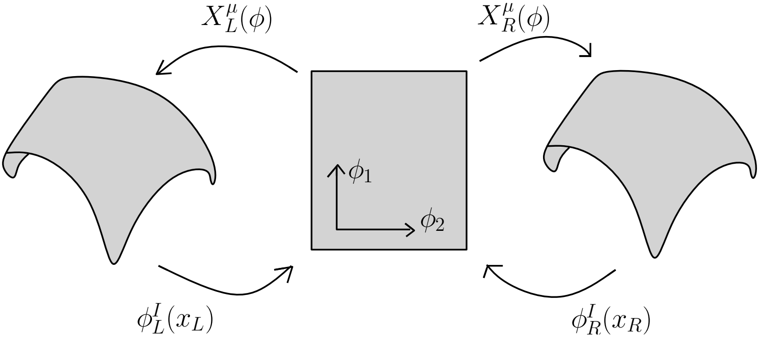
The **correlators**:

$$G_{\alpha_1,\ldots\alpha_n}(x_1,\ldots x_n) \sim \frac{\delta \log Z_{\rm SK}}{\delta J_{\bar\alpha_1}(x_1)\ldots\delta J_{\bar\alpha_n}(x_n)}\Big|_{J_a=J_r=0}$$

E.g. $\qquad G_{ra} = G_R \qquad G_{ar} = G_A \qquad G_{rr} = G_S, \qquad G_{aa} = 0$

Fluid dynamics - The Goldstones

- Dynamics of light (massless) degrees of freedom
- Two copies of the physical Eulerian spacetime and one Lagrangian frame of the fluid elements ϕ^I



- Symmetry principles to be imposed on the Fluid variables ϕ^I

Fluid dynamics - Example: Effective action for diffusion problem

The diffusion equation + stochastic noise contribution:

$$\partial_t n - D\,\partial_i^2 n - \xi = 0$$

Implement the e.o.m. @ the level of an effective action using the trick:

$$\mathcal{O}(n_\xi) = \int \mathcal{D}n\,\delta(n_\xi - n)\,\mathcal{O}(n) = \int \mathcal{D}n\,\delta(e.o.m._\xi)\,J_\xi\,\mathcal{O}(n)$$

Correlation functions as averages over the noise contribution

$$\langle \mathcal{O}\,\mathcal{O}\ldots\rangle = \int \mathcal{D}n\,\mathcal{D}\xi\,\delta(e.o.m._\xi)\,J_\xi\,e^{-S[\xi]}\,\mathcal{O}\,\mathcal{O}\ldots$$

The partition function is then:

$$Z[J_a, J_r] = \int \mathcal{D}n\,\mathcal{D}\xi\,\mathcal{D}\tilde{n}\,J_\xi\,e^{i\int(\partial_t n - D\,\partial_i^2 n - \xi)\,\tilde{n} - S[\xi] + i\int J_r\,\tilde{n} + i\int J_a\,n}$$

Integrate out the noise, reabsorb J_ξ in the integration measure and choose the noise probability distribution such that the fluctuation-dissipation theorem is satisfied: $\langle \xi\,\xi\rangle \sim T\,D\,\partial_i^2\delta(x' - x)$

$$Z[J_a, J_r] = \int \mathcal{D}n\,\mathcal{D}\tilde{n}\,e^{i\int(\partial_t n - D\,\partial_i^2 n)\,\tilde{n} - \frac{1}{2}\,T\,D\int(\partial_i\tilde{n})^2 + i\int J_r\,\tilde{n} + i\int J_a\,n}$$

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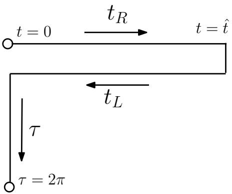
Fluid dynamics from black holes - Motivation

Holography = QFT + its RG flow

- Fluid/gravity duality is a very established subject within holography which gives a natural geometrical embedding where these ideas can be tested
- Relevant for questions in gravity: *e.g.* how to describe the interior of a black hole from the boundary point of view?

Goal: derive the effective action for the diffusion problem from holography

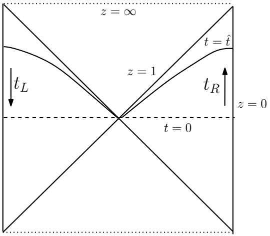
Fluid dynamics from black holes - The set-up



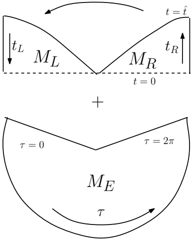
Real time holography: [B. Van Rees, K. Skenderis (2008)]

- Real time segment $\rightarrow$ Lorentzian spacetime
- Imaginary time segment $\rightarrow$ Euclidean spacetime
- Smoothly glue together the manifolds along finite time slices

- E.g. thermal $\mathrm{CFT}_2 \rightarrow \mathrm{BTZ}_3$ Lorentzian + Euclidean black holes



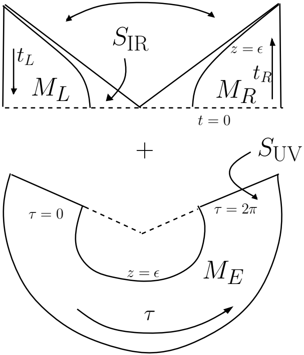
$\Rightarrow$



The bulk solution is fully determined by J_R and J_L

$\Rightarrow$ non local $S_{\text{on-shell}}[J_a, J_r] \sim \int (J_a G_{ra} J_r + J_r G_{ar} J_a + J_a G_{rr} J_a)$

Fluid dynamics from black holes - The set-up



- Divide $S = S_{\rm UV} + S_{\rm IR}$
- $S_{\rm UV}$ = **double-Dirichlet** problem x3
- **Goldstones** as **non-trivial boundary conditions** on the second boundary in a double-Dirichlet problem
- $S_{\rm IR}$ = **simple** non-local contribution with rescaled SK correlators
- Match the two regions by integrating out the Dirichlet values along the cutoff

$$\frac{\delta S_{\rm IR}}{\delta \Phi_\epsilon} + \frac{\delta S_{\rm UV}}{\delta \Phi_\epsilon} = 0$$

- Perform the hydrodynamic and near-horizon limit

Fluid dynamics from black holes - E.g. Probe gauge field

The Goldstone: $\phi \sim \int_0^\epsilon A_z dz$

The SK combinations: $\phi_r = \frac{1}{2}(\phi_R + \phi_L), \quad \phi_a = \phi_R - \phi_L$

The **local** effective action for **diffusion** in AdS_5:

$$i\, S_{\mathrm{SK}}[\phi_r, \phi_a] = i \int d^4x \left(-\partial_t^2 \phi_r + \partial_x^2 \partial_t \phi_r + \frac{i}{2\pi} \partial_x^2 \phi_a \right) \phi_a$$

The effective action for diffusion problem with

$$n = \partial_t \phi_r, \quad \tilde{n} = \phi_a \quad \text{and} \quad D = \frac{1}{2\pi T}$$

The fluctuation-dissipation theorem is automatically satisfied!

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- Effective action for fluids: Goldstones + underlying Schwinger-Keldysh structure
- Explicit **derivation** of the effective action for **diffusion** problem in AdS_5 at **first order** in a derivative expansion
- In holography Goldstones as Wilson-line like objects extending between two boundaries
- Effective action for *dissipative (fluids)* from holography
  [J. de Boer, M. P. Heller, N. P. F. work in progress (2016)]
- What is the dual gravity principle that gives the fluctuation-dissipation theorem at the boundary?

Thank you!