# Cubic vertices for Maxwell-like higher spins

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# Plan of the talk

- Higher Spins & the interaction problem
- Construction of consistent vertices
- The Maxwell-Like case
- Conclusions

# Higher Spins & the interaction problem

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WHY NOT HIGHER SPINS (HS: SPIN S>2)?

# Higher spins

- Unitary irreps of the Poincaré group (covariantly, via symmetric tensors, $\varphi_{\mu_1 \ldots \mu_s}$ )
- Predicted by (super-)strings
- Consistent free Lagrangian theories (Fronsdal Lagrangian, [1978]):

$$\mathcal{L} = \frac{1}{2}\left\{\varphi\mathcal{F} - \frac{s(s-1)}{2}\varphi'\mathcal{F}'\right\}\ , \qquad \mathcal{F} = \Box\varphi - \partial\partial\cdot\varphi + \partial^2\varphi$$

# (Some) No-Go Theorems

- Weinberg theorem (1964): studying soft-particle emission, forbids couplings $s-s-s'$ with $s'>2$.
- Weinberg-Witten-Porrati theorem [1980-2008]: forbids massless higher spins interacting with ordinary gravity ( $s-s-2$ ).
- Coleman-Mandula theorem [1967]: admits at most a susy extension of the Poincaré algebra. *(Maldacena-Zhiboedov [2011-2012]: sort of Coleman-Mandula for AdS/CFT)*

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Escape?

High energy regime

Non-minimal couplings

# Cubic vertices [Bengtsson-Bengtsson-Brink 1983 and several more]

- Every no-go theorem rests on (strong) hypothesis that might be relaxed
- Interaction vertices: may still be possible if they are subleading at low energy *(to address Weinberg's theorem)*
- At any rate, knowledge of HS interactions relevant to study mechanisms for HS symmetry breaking
- An important result: the Metsaev bound [1997-2006] constrains the overall number of derivatives in a *CUBIC* vertex

$$s_1 + s_2 - s_3 \leq \#\partial \leq s_1 + s_2 + s_3 \,,$$

$$s_3 \leq s_2 \leq s_1$$

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Higher-derivative vertices!

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Example: for couplings $s - s - 2$
we get $\#\partial_{\text{min}} = 2(s-1)$ :
No Minimal coupling for $s \geq 3$

# Construction of consistent vertices

# Noether procedure [Berends-Burgers-Van Dam 1985]

- If we want to build up interactions for massless particles, we need to keep gauge invariance
- Starting point: free theory (action $S_0$ and free transformation $\delta_0$)
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Cubic Step

# Cubic interactions

- We are looking for a Lagrangian deformation involving arbitrary spins:

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On-shell terms: define the structure of the transformation

Off-shell terms: define the structure of the vertex

# The Maxwell-Like case

# Maxwell-Like Higher spins [Campoleoni-Francia 2013]

- Free Lagrangian: $\mathcal{L} = \frac{1}{2}\varphi\mathcal{M}(\varphi)\,, \quad \mathcal{M}(\varphi) = \Box\varphi - \partial\partial\cdot\varphi$

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**New features**

Simpler Lagrangian w.r.t. Fronsdal

Reducible spectrum: spin $s, s-2, \ldots, 1 \; or \; 0$

# Building Maxwell-like cubic vertices

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On-shell terms; but it is impossible to extract in a local way the equations of motion

Is it impossible to build up a local vertex?

# Modified Noether procedure

- Quadratic step: $$\frac{\delta S_0}{\delta \varphi_j}\delta_0\varphi_j \sim \int \partial\cdot\epsilon_j\,\partial\cdot\partial\cdot\varphi_j$$ ⟵ The same double-divergence term!

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Deformation of the constraint:

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- Example: 2-2-2 vertex

$$\partial\cdot\epsilon = 0 \quad \rightarrow \quad \underbrace{(\eta_{\mu\nu}+h_{\mu\nu})}_{g_{\mu\nu}}\partial^\mu\epsilon^\nu = 0$$

Constraint of unimodular gravity [e.g. Alvarez-Boas-Garriga-Veldaguer 2006]:
information about the geometry

# The general result

- The vertex: $\mathcal{L}_1 = \mathcal{L}^{TT} + \mathcal{L}^{\mathcal{D}} + \mathcal{L}^{\mathcal{D}\mathcal{D}} + \mathcal{L}^{\mathcal{D}\mathcal{D}\mathcal{D}}$

$$\mathcal{L}^{TT} = \sum_{n_i} K_{n_i} \int d^D\mu \; T(n_1, n_2, n_3|Q_{12}, Q_{23}, Q_{31})\varphi_1(a, x_1)\varphi_2(b, x_2)\varphi_3(c, x_3)$$

$$\mathcal{L}^{\mathcal{D}} = \sum_{n_i} K_{n_i} \int d^D\mu \; \Big\{\frac{s_1 n_1}{2} T(n_1 - 1, n_2, n_3|Q_{ij})\mathcal{D}_1(a, x_1)\varphi_2(b, x_2)\varphi_3(c, x_3)$$
$$\frac{s_2 n_2}{2} T(n_1, n_2 - 1, n_3|Q_{ij})\varphi_1(a, x_1)\mathcal{D}_2(b, x_2)\varphi_3(c, x_3)$$
$$\frac{s_3 n_3}{2} T(n_1, n_2, n_3 - 1|Q_{ij})\varphi_1(a, x_1)\varphi_2(b, x_2)\mathcal{D}_3(c, x_3)$$

$$\mathcal{L}^{\mathcal{D}\mathcal{D}} = \sum_{n_i} K_{n_i} \int d^D\mu \; \Big\{\frac{s_1 s_2 n_1 n_2}{2} T(n_1 - 1, n_2 - 1, n_3|Q_{ij})\mathcal{D}_1(a, x_1)\mathcal{D}_2(b, x_2)\varphi_3(c, x_3) +$$
$$\frac{s_2 s_3 n_2 n_3}{2} T(n_1, n_2 - 1, n_3 - 1|Q_{ij})\varphi_1(a, x_1)\mathcal{D}_2(b, x_2)\mathcal{D}_3(c, x_3) +$$
$$\frac{s_3 s_1 n_3 n_1}{2} T(n_1 - 1, n_2, n_3 - 1|Q_{ij})\mathcal{D}_1(a, x_1)\varphi_2(b, x_2)\mathcal{D}_3(c, x_3)\Big\}$$

$$\mathcal{L}^{\mathcal{D}\mathcal{D}\mathcal{D}} = \sum_{n_i} K_{n_i} \int d^D\mu \; \frac{s_1 s_2 s_3 n_1 n_2 n_3}{2} T(n_k - 1|Q_{ij})\mathcal{D}_1(a, x_1)\mathcal{D}_2(b, x_2)\mathcal{D}_3(c, x_3)$$

# The general result

- The deformed transformation:

$$\delta\varphi_1(a,x_1)=\sum_{n_i}\frac{ks_1!}{2Q_{23}!}\Big\{+n_2s_2\left[(\partial_b\nabla_3)^{n_2-1}(a\nabla_2)^{n_1}(\partial_c\nabla_1)^{n_3}(\partial_b\partial_c)^{Q_{23}}\epsilon_2(b,x_2)\varphi(c,x_3)\right]\big|_{b,c=a}+$$
$$-n_3s_3\left[(\partial_b\nabla_3)^{n_2}(a\nabla_2)^{n_1}(\partial_c\nabla_1)^{n_3-1}(\partial_b\partial_c)^{Q_{23}}\varphi_2(b,x_2)\epsilon(c,x_3)\right]\big|_{b,c=a}+$$
$$+n_2s_2n_3s_3\left[(\partial_b\nabla_3)^{n_2-1}(a\nabla_2)^{n_1}(\partial_c\nabla_1)^{n_3-1}(\partial_b\partial_c)^{Q_{23}}\epsilon_2(b,x_2)\mathcal{D}_3(c,x_3)\right]\big|_{b,c=a}\Big\}$$

- Deformation of the constraint:

$$\partial\cdot\epsilon_1(a,x_1)=\sum_{n_i}\frac{2kn_1s_3}{(s_1-2)!Q_{23}!}\Big\{-\left[(\partial_b\nabla_3)^{n_2}(a\nabla_2)^{n_1-1}(\partial_c\nabla_1)^{n_3}(\partial_b\partial_c)^{Q_{23}}\varphi_2(b,x_2)\epsilon_3(c,x_3)\right]\big|_{b,c=a}+$$
$$+\frac{n_2s_2}{2}\left[(\partial_b\nabla_3)^{n_2-1}(a\nabla_2)^{n_1-1}(\partial_c\nabla_1)^{n_3-1}(\partial_b\partial_c)^{Q_{23}}\mathcal{D}_2(b,x_2)\epsilon_3(c,x_3)\right]\big|_{b,c=a}\Big\}$$

# Vertices and spectrum

Two possibilities

Diagonalization of the spectrum:

Possibility to truncate to the irreducible case: $\varphi' = 0\,,\ \epsilon' = 0$

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Relation to Fronsdal and its algebraic constraint

# Conclusions

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- Maxwell-like HS alternative (simpler?) to Fronsdal: propagate a *reducible* spectrum. Possibility to truncate and get a single propagating particle
- The construction of consistent vertices needs a modified Noether procedure: the fundamental feature is the deformation of the differential constraint. It may help to unconver the underlying geometry
- The reducible spectrum allows to deal simultaneously with more vertices, with fixed relative coefficients
- The spectrum of the would-be full theory would not obviously match with that of known Vasiliev's theories
- As an exercise, why not investigating deformations of Fronsdal'a algebraic constraints?

Thank you for your kind attention