

# Cubic vertices for Maxwell-like higher spins

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# Plan of the talk

- Higher Spins & the interaction problem
  - Construction of consistent vertices
  - The Maxwell-Like case
  - Conclusions
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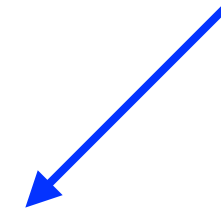
# Higher Spins & the interaction problem

# MODERN FIELD THEORIES

Main theories of modern physics can be identified according to the spin of the involved particles:

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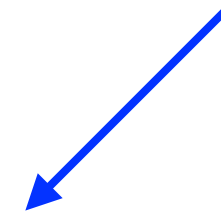
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Standard model :  
spin 0, spin 1/2, spin 1

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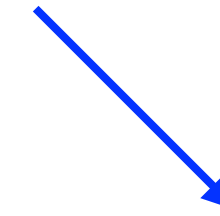
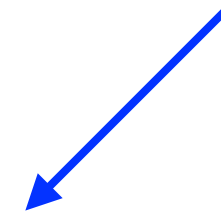
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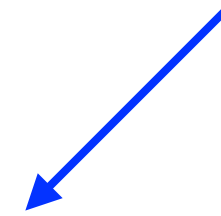
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Supergravity:  
spin 3/2 and spin 2

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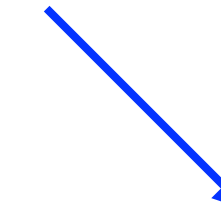
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## WHY NOT HIGHER SPINS (HS: SPIN $S > 2$ )?



# Higher spins

- Unitary irreps of the Poincaré group (covariantly, via symmetric tensors,  $\varphi_{\mu_1 \dots \mu_s}$ )
- Predicted by (super-)strings
- Consistent free Lagrangian theories (Fronsdal Lagrangian, [1978]):

$$\mathcal{L} = \frac{1}{2} \left\{ \varphi \mathcal{F} - \frac{s(s-1)}{2} \varphi' \mathcal{F}' \right\}, \quad \mathcal{F} = \square \varphi - \partial \partial \cdot \varphi + \partial^2 \varphi$$

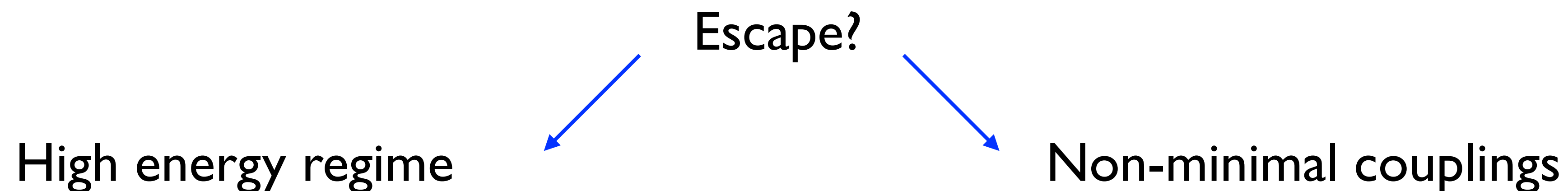
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# (Some) No-Go Theorems

- Weinberg theorem (1964): studying **soft**-particle emission, forbids couplings  $s - s - s'$  with  $s' > 2$ .
  - Weinberg-Witten-Porrati theorem [1980-2008]: forbids massless higher spins interacting with ordinary gravity ( $s - s - 2$ ).
  - Coleman-Mandula theorem [1967]: admits at most a susy extension of the Poincaré algebra.  
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# Cubic vertices [Bengtsson-Bengtsson-Brink 1983 and several more]

- Every no-go theorem rests on (strong) hypothesis that might be relaxed
- Interaction vertices: may still be possible if they are subleading at low energy (*to address Weinberg's theorem*)
- At any rate, knowledge of HS interactions relevant to study mechanisms for HS symmetry breaking
- An important result: the Metsaev bound [1997-2006] constrains the overall number of derivatives in a *CUBIC* vertex

$$s_1 + s_2 - s_3 \leq \#\partial \leq s_1 + s_2 + s_3 ,$$

$$s_3 \leq s_2 \leq s_1$$



Higher-derivative vertices!

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Example: for couplings  $s - s - 2$

we get  $\#\partial_{\min} = 2(s - 1)$  :

No Minimal coupling for  $s \geq 3$

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# Construction of consistent vertices

# Noether procedure [Berends-Burgers-Van Dam 1985]

- If we want to build up interactions for massless particles, we need to keep gauge invariance
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# Cubic interactions

- We are looking for a Lagrangian deformation involving arbitrary spins:

$$\mathcal{V}_{cubic} = \mathcal{L}_1(\varphi_1, \varphi_2, \varphi_3)$$

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Off-shell terms: define the structure of the vertex

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# The Maxwell-Like case

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# Maxwell-Like Higher spins [Campoleoni-Francia 2013]

- Free Lagrangian:  $\mathcal{L} = \frac{1}{2}\varphi\mathcal{M}(\varphi), \quad \mathcal{M}(\varphi) = \square\varphi - \partial\partial\cdot\varphi$
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On-shell terms; but it is impossible to extract in a local way the equations of motion

Is it impossible to build up a local vertex?

# Modified Noether procedure

- Quadratic step:

$$\frac{\delta S_0}{\delta \varphi_j} \delta_0 \varphi_j \sim \int \partial \cdot \epsilon_j \partial \cdot \partial \cdot \varphi_j \quad \leftarrow$$

The same double-divergence term!

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Deformation of the constraint:

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Link with the cubic step:  
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- Example: 2-2-2 vertex  $\partial \cdot \epsilon = 0 \rightarrow \underbrace{(\eta_{\mu\nu} + h_{\mu\nu})}_{g_{\mu\nu}} \partial^\mu \epsilon^\nu = 0$

Constraint of unimodular gravity [e.g. Alvarez-Boas-Garriga-Veldaguer 2006]:  
information about the geometry

# The general result

◦ The vertex:

$$\mathcal{L}_1 = \mathcal{L}^{TT} + \mathcal{L}^{\mathcal{D}} + \mathcal{L}^{\mathcal{D}\mathcal{D}} + \mathcal{L}^{\mathcal{D}\mathcal{D}\mathcal{D}}$$

$$\mathcal{L}^{TT} = \sum_{n_i} K_{n_i} \int d^D \mu T(n_1, n_2, n_3 | Q_{12}, Q_{23}, Q_{31}) \varphi_1(a, x_1) \varphi_2(b, x_2) \varphi_3(c, x_3)$$

$$\mathcal{L}^{\mathcal{D}} = \sum_{n_i} K_{n_i} \int d^D \mu \left\{ \frac{s_1 n_1}{2} T(n_1 - 1, n_2, n_3 | Q_{ij}) \mathcal{D}_1(a, x_1) \varphi_2(b, x_2) \varphi_3(c, x_3) \right. \\ \left. \frac{s_2 n_2}{2} T(n_1, n_2 - 1, n_3 | Q_{ij}) \varphi_1(a, x_1) \mathcal{D}_2(b, x_2) \varphi_3(c, x_3) \cdot \right. \\ \left. \frac{s_3 n_3}{2} T(n_1, n_2, n_3 - 1 | Q_{ij}) \varphi_1(a, x_1) \varphi_2(b, x_2) \mathcal{D}_3(c, x_3) \right\}$$

$$\mathcal{L}^{\mathcal{D}\mathcal{D}} = \sum_{n_i} K_{n_i} \int d^D \mu \left\{ \frac{s_1 s_2 n_1 n_2}{2} T(n_1 - 1, n_2 - 1, n_3 | Q_{ij}) \mathcal{D}_1(a, x_1) \mathcal{D}_2(b, x_2) \varphi_3(c, x_3) + \right. \\ \left. \frac{s_2 s_3 n_2 n_3}{2} T(n_1, n_2 - 1, n_3 - 1 | Q_{ij}) \varphi_1(a, x_1) \mathcal{D}_2(b, x_2) \mathcal{D}_3(c, x_3) + \right. \\ \left. \frac{s_3 s_1 n_3 n_1}{2} T(n_1 - 1, n_2, n_3 - 1 | Q_{ij}) \mathcal{D}_1(a, x_1) \varphi_2(b, x_2) \mathcal{D}_3(c, x_3) \right\}$$

$$\mathcal{L}^{\mathcal{D}\mathcal{D}\mathcal{D}} = \sum_{n_i} K_{n_i} \int d^D \mu \frac{s_1 s_2 s_3 n_1 n_2 n_3}{2} T(n_k - 1 | Q_{ij}) \mathcal{D}_1(a, x_1) \mathcal{D}_2(b, x_2) \mathcal{D}_3(c, x_3)$$



# The general result

- The deformed transformation:

$$\delta\varphi_1(a, x_1) = \sum_{n_i} \frac{ks_1!}{2Q_{23}!} \left\{ +n_2s_2 [(\partial_b \nabla_3)^{n_2-1} (a \nabla_2)^{n_1} (\partial_c \nabla_1)^{n_3} (\partial_b \partial_c)^{Q_{23}} \epsilon_2(b, x_2) \varphi(c, x_3)] \Big|_{b,c=a} + \right. \\ \left. -n_3s_3 [(\partial_b \nabla_3)^{n_2} (a \nabla_2)^{n_1} (\partial_c \nabla_1)^{n_3-1} (\partial_b \partial_c)^{Q_{23}} \varphi_2(b, x_2) \epsilon(c, x_3)] \Big|_{b,c=a} + \right. \\ \left. +n_2s_2n_3s_3 [(\partial_b \nabla_3)^{n_2-1} (a \nabla_2)^{n_1} (\partial_c \nabla_1)^{n_3-1} (\partial_b \partial_c)^{Q_{23}} \epsilon_2(b, x_2) \mathcal{D}_3(c, x_3)] \Big|_{b,c=a} \right\}$$

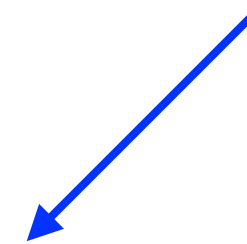
- Deformation of the constraint:

$$\partial \cdot \epsilon_1(a, x_1) = \sum_{n_i} \frac{2kn_1s_3}{(s_1-2)!Q_{23}!} \left\{ - [(\partial_b \nabla_3)^{n_2} (a \nabla_2)^{n_1-1} (\partial_c \nabla_1)^{n_3} (\partial_b \partial_c)^{Q_{23}} \varphi_2(b, x_2) \epsilon_3(c, x_3)] \Big|_{b,c=a} + \right. \\ \left. + \frac{n_2s_2}{2} [(\partial_b \nabla_3)^{n_2-1} (a \nabla_2)^{n_1-1} (\partial_c \nabla_1)^{n_3-1} (\partial_b \partial_c)^{Q_{23}} \mathcal{D}_2(b, x_2) \epsilon_3(c, x_3)] \Big|_{b,c=a} \right\}$$

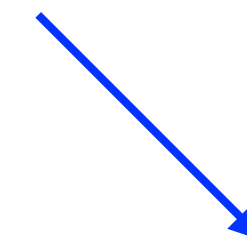
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# Vertices and spectrum

Two possibilities



Diagonalization of the spectrum:



Possibility to truncate to the irreducible case:  $\varphi' = 0, \epsilon' = 0$



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Simultaneous study of more vertices:  
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Relation to Fronsdal and  
its algebraic constraint

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# Conclusions

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- Maxwell-like HS alternative (simpler?) to Fronsdal: propagate a *reducible* spectrum. Possibility to truncate and get a single propagating particle
  - The construction of consistent vertices needs a modified Noether procedure: the fundamental feature is the deformation of the differential constraint. It may help to uncover the underlying geometry
  - The reducible spectrum allows to deal simultaneously with more vertices, with fixed relative coefficients
  - The spectrum of the would-be full theory would not obviously match with that of known Vasiliev's theories
  - As an exercise, why not investigating deformations of Fronsdal's algebraic constraints?
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Thank you for your kind attention

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