Surprises from the resummation of ladders for the ABJ(M) cusp anomalous dimension

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Outline

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Wilson loop

In any gauge theory

$$W_{\mathcal{R}}[\mathcal{C}] = \frac{1}{\dim_{\mathcal{R}}} \mathrm{Tr}_{\mathcal{R}} \mathcal{P} e^{\left(i \oint_{\mathcal{C}} A_\mu \dot{x}^\mu d\tau\right)}$$

measures the phase of an external particle.

For a given theory the Wilson loop depends on

- The trajectory of the particle $\mathcal{C} \rightarrow x^\mu(\tau)$
- The flavor/charge of the particle $\mathcal{R} \rightarrow$ gauge group repr.

There are a lot of interesting implications...

Wilson loop in $\mathcal{N}=4$ SYM

- Vector supermultiplet $(A_\mu, \lambda^a_\alpha, \Phi^I)$;
- Free parameters: g_{YM}, $N \to$ 't Hooft coupling $\lambda = g^2_{\mathrm{YM}} N$.

The loops couple also the scalars:

$$W_{\mathcal{R}}[\mathcal{C}] = \frac{1}{\dim_{\mathcal{R}}} \mathrm{Tr}_{\mathcal{R}} \mathcal{P} \exp \oint_{\mathcal{C}} d\tau \, \left(i A_\mu(x) \dot{x}^\mu(\tau) + \dot{y}_I(\tau) \Phi^I \right)$$

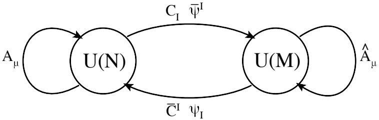
The scalar coupling must satisfy $\dot{x}^2 = \dot{y}^2 \to$ local SUSY
With a suitable choice of scalar coupling $\to$ global SUSY

- Zarembo $\dot{y}_I = M^\mu_I \dot{x}_\mu \to \langle W \rangle = 1$ [K.Zarembo '02]
- DGRT $\dot{y}_I = -\dot{x}_\mu \sigma^{\mu\nu}_i x_\nu M^i_I \to$ loops on S^3
  [N.Drukker, S.Giombi, R.Ricci, D.Trancanelli '07]

The number of preserved supercharges depends on the path $x_\mu(\tau)$

Wilson loop ABJ(M)

- (A_μ, ψ^I, C^I) in the bifundamental of the group $U(N)\times U(M)$;
- Free parameters: $\kappa, N, M \to$ 't Hooft coupling $\lambda_1 = \frac{N}{\kappa}$, $\lambda_2 = \frac{M}{\kappa}$.



The contours couple also the scalars and the fermions:

$$W_{\mathcal{R}}[\mathcal{C}] = \frac{1}{\dim_{\mathcal{R}}} \mathrm{Tr}_{\mathcal{R}}\left[\mathcal{P}\exp\left(-i\int_{\mathcal{C}} d\tau \mathcal{L}(\tau)\right)\right]$$

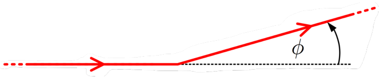
the $U(N)\times U(M)$ gauge connection with the super-connection

$$\mathcal{L}(\tau) \equiv -i \begin{pmatrix} i\mathcal{A} & \sqrt{\frac{2\pi}{\kappa}}|\dot{x}|\eta_I\bar{\psi}^I \\ \sqrt{\frac{2\pi}{\kappa}}|\dot{x}|\psi_I\bar{\eta}^I & i\hat{\mathcal{A}} \end{pmatrix} \quad \text{with} \quad \begin{cases} \mathcal{A} \equiv A_\mu \dot{x}^\mu - \frac{2\pi i}{\kappa}|\dot{x}|M_J{}^I C_I \bar{C}^J \\ \hat{\mathcal{A}} \equiv \hat{A}_\mu \dot{x}^\mu - \frac{2\pi i}{\kappa}|\dot{x}|\hat{M}_J{}^I \bar{C}^J C_I, \end{cases}$$

belonging to the super-algebra of $U(N|M)$.

[N.Drukker, D.Trancanelli '10]

Cusp anomalous dimension Γ_{CUSP}



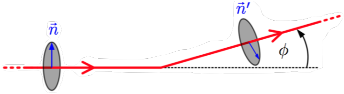
$$W \sim e^{-\log\left(\frac{L}{\epsilon}\right)\Gamma_{CUSP}(\lambda,\varphi)}$$

with L and ϵ the IR and UV cut-off

Supersymmetric configuration: $\varphi = 0$

- Universal cusp anomaly $\Gamma_{CUSP}(\lambda,\varphi) \xrightarrow[\varphi\to\infty]{\varphi\to i\varphi} \varphi\,\gamma_{CUSP}(\lambda)$
- $Q\bar{Q}$-potential $\Gamma_{CUSP}(\lambda,\varphi) \xrightarrow{\varphi\to\pi} \frac{V(\lambda)}{\pi-\varphi}$
- Bremsstrahlung function $\Gamma_{CUSP}(\lambda,\varphi) \xrightarrow{\varphi\to 0} -\varphi^2\mathcal{B}(\lambda)$

Deforming the observable



R-symmetry deformation $\vec{n}\cdot\vec{n}' = \cos\theta$

$$W \sim e^{-\log\left(\frac{L}{\epsilon}\right)\Gamma_{CUSP}(\lambda,\theta,\varphi)}$$

with $\Gamma_{CUSP}(\lambda,\theta,\varphi)$ the generalized cusp anomalous dimension

[N.Drukker, V.Forini '11] [D.Correa, J.Henn, J.Maldacena, A.Sever '12]

Supersymmetric configuration: $\varphi = \pm\theta$

It is possible to have exact result for the generalized cusp?

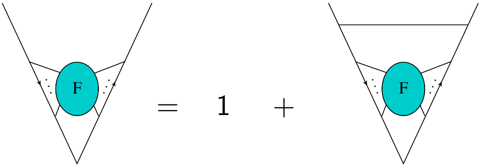
Outline

Bethe-Salpeter equation for generalized cusp in $\mathcal{N}=4$ SYM

We study the scaling limit which selects only ladder diagrams

$$i\theta \to \infty, \quad \lambda \to 0 \quad \text{with} \quad \hat{\lambda} = \frac{\lambda e^{i\theta}}{4} \quad \text{finite.}$$

[Correa, Henn, Maldacena, Sever '12]



Ladder diagram resummation $\rightarrow$ Bethe-Salpeter equation

$$F(S,T) = 1 + \int_0^S ds \int_0^T dt\, F(s,t)\, P(s,t)$$

Integral equation $\rightarrow$ Schroedinger equation with potential depending on the integral kernel $P(s,t)$

The 1-d Schroedinger problem

It is convenient to think about the problem on the sphere ($s = e^{\sigma}$, $t = e^{\tau}$)

$$\partial_{\sigma'}\partial_{\tau'}F(\sigma',\tau') = P(\sigma',\tau')F(\sigma',\tau')$$

Defining $x = \tau' - \sigma'$ and $y = \tau' + \sigma'/2$ and making the ansatz $F = \sum_n e^{-E_n y}\Psi_n(x)$

$$\left[-\partial_x^2 - P(x,y)\right]\Psi(x) = -\frac{E^2}{4}\Psi(x)$$

when $T = S \to \infty$, F is governed by the lowest eigenvalue then

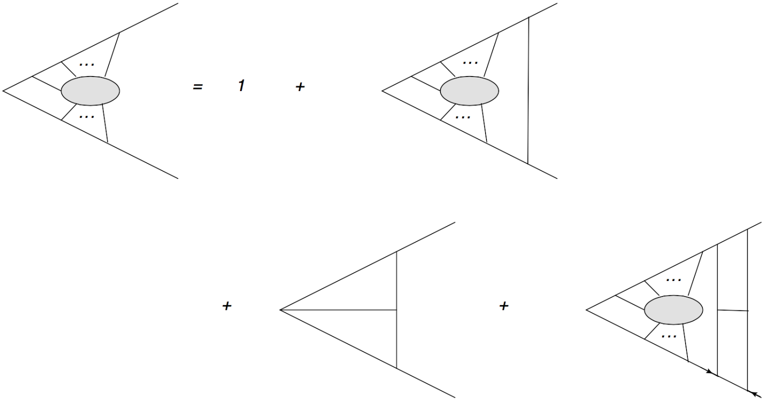
$$E_0 = -\Gamma_{\text{cusp}}$$

- The energy can be computed exactly for $\varphi = 0$
- For arbitrary φ the Schroedinger problem is not exactly solvable

$$\Gamma_{\text{cusp}} = -\frac{\sqrt{\hat{\lambda}}}{2\pi\cos\varphi/2} \qquad \hat{\lambda} >> 1 \qquad \text{in agreement with AdS/CFT}$$

[Correa, Henn, Maldacena, Sever '12] [Henn, Huber '12]

A sketch of the NLO



[Henn, Huber '12]

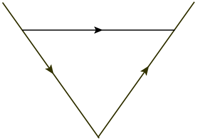
Outline

The ABJ(M) kernels

The ABJ scaling limit

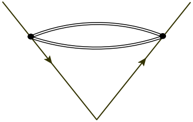
$$i\theta \to \infty, \quad \lambda_{1,2} \to 0 \quad \text{with} \quad \hat{\lambda}_{1,2} = \frac{\lambda_{1,2} e^{i\theta}}{2} \quad \text{finite.}$$

At leading order the relevant contributions are



$$= \int ds\, dt\; P^{(F)}(s,t)$$

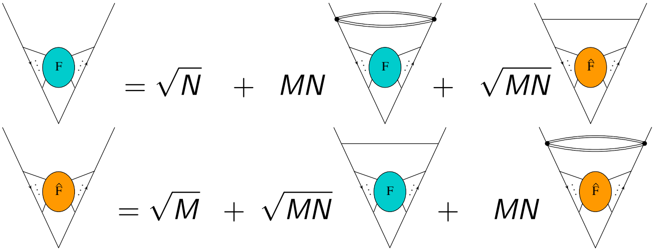
$$= \left(\frac{2\pi}{\kappa}\right) MN \frac{\Gamma\left(\frac{1}{2}-\epsilon\right)}{4\pi^{3/2-\epsilon}} (\mu L)^{2\epsilon} \frac{1}{\epsilon} \frac{\cos\frac{\theta}{2}}{\cos\frac{\varphi}{2}} .$$



$$= \int ds\, dt\; P^{(B)}(s,t)$$

$$= -\left(\frac{2\pi}{\kappa}\right)^2 MN^2 \frac{\Gamma^2\left(\frac{1}{2}-\epsilon\right)}{16\pi^{3-2\epsilon}} (\mu L)^{4\epsilon} \cos^2\frac{\theta}{2} \frac{1}{\epsilon} \frac{\varphi}{\sin\varphi} .$$

Bethe-Salpeter equation for generalized cusp in ABJ(M) (I)



$$F(S,T) = \sqrt{N} + \int_0^S ds \int_0^T dt\; MNF(s,t)P^{(B)}(s,t) + \sqrt{MN}\hat{F}(s,t)P^{(F)}(s,t)$$

$$\hat{F}(S,T) = \sqrt{M} + \int_0^S ds \int_0^T dt\; \sqrt{MN}F(s,t)P^{(F)}(s,t) + MN\hat{F}(s,t)P^{(B)}(s,t).$$

$$\langle \mathcal{W}_\pm \rangle = \frac{\langle \mathcal{W}_\uparrow \rangle \pm \langle \mathcal{W}_\downarrow \rangle}{N \pm M} \quad \Rightarrow \quad \langle \mathcal{W}_\pm \rangle = \frac{\sqrt{N}F \pm \sqrt{M}\hat{F}}{N \pm M}$$

Bethe-Salpeter equation for generalized cusp in ABJ(M) (II)

We decouple the integral equations defining

$$\mathcal{H}(x,y) = F + \hat{F} = h(y)\psi_+(x) \qquad \mathcal{K}(x,y) = F - \hat{F} = k(y)\psi_-(x)$$

Two SUSY-QM Schroedinger equations

$$\left(-\partial_x^2 + a^2 W^2(x) - aW'(x)\right)\psi_+(x) = E\psi_+(x)$$
$$\left(-\partial_x^2 + a^2 W^2(x) + aW'(x)\right)\psi_-(x) = \tilde{E}\psi_-(x)$$
$$h(y) = C_1 e^{2\sqrt{-E+\frac{a^2}{2}}y} + C_2 e^{-2\sqrt{-E+\frac{a^2}{2}}y}$$
$$k(y) = C_3 e^{2\sqrt{-\tilde{E}+\frac{a^2}{2}}y} + C_4 e^{-2\sqrt{-\tilde{E}+\frac{a^2}{2}}y}$$

Solving the equations we get

$$\langle \mathcal{W}_+ \rangle = \frac{(\sqrt{M}+\sqrt{N})^2}{2(M+N)} e^{\sqrt{2}a\log\frac{\Lambda_{UV}}{\Lambda_{IR}}} + \frac{(\sqrt{M}-\sqrt{N})^2}{2(M+N)} e^{-\sqrt{2}a\log\frac{\Lambda_{UV}}{\Lambda_{IR}}},$$
$$\langle \mathcal{W}_- \rangle = \frac{1}{2} e^{\sqrt{2}a\log\frac{\Lambda_{UV}}{\Lambda_{IR}}} + \frac{1}{2} e^{-\sqrt{2}a\log\frac{\Lambda_{UV}}{\Lambda_{IR}}}.$$

[M.Bonini, L.Griguolo, M.P., D.Seminara '16]

Bethe-Salpeter equation for generalized cusp in ABJ(M) (III)

The straight-line case ($\varphi = 0$) is exactly solvable with $\epsilon \neq 0$

$$\Box\mathcal{H}(x',y') = \left[\vec{\nabla}W(x',y')\cdot\vec{\nabla}W(x',y') - \Box W(x',y')\right]\mathcal{H}(x',y')\,,$$
$$\Box\mathcal{K}(x',y') = \left[\vec{\nabla}W(x',y')\cdot\vec{\nabla}W(x',y') + \Box W(x',y')\right]\mathcal{K}(x',y')\,,$$

with

$$W(x',y') = \frac{2^{\epsilon-1/2}a_\epsilon}{\epsilon}e^{\epsilon y'}\cos^{2\epsilon}\frac{x'}{2}\,, \qquad a_\epsilon = \sqrt{\hat{\lambda}_1\hat{\lambda}_2}\,\frac{\Gamma(1/2-\epsilon)(\mu L)^{2\epsilon}}{(2\pi)^{1/2-\epsilon}}\,.$$

Solving the equations we get

$$\langle\mathcal{W}_+^{\varphi=0}\rangle = \frac{(\sqrt{M}+\sqrt{N})^2}{2(M+N)}e^{-\frac{2^{\epsilon-1/2}}{\epsilon}a_\epsilon} + \frac{(\sqrt{M}-\sqrt{N})^2}{2(M+N)}e^{\frac{2^{\epsilon-1/2}}{\epsilon}a_\epsilon}\,,$$
$$\langle\mathcal{W}_-^{\varphi=0}\rangle = \frac{1}{2}e^{-\frac{2^{\epsilon-1/2}}{\epsilon}a_\epsilon} + \frac{1}{2}e^{\frac{2^{\epsilon-1/2}}{\epsilon}a_\epsilon}\,.$$

[M.Bonini, L.Griguolo, M.P., D.Seminara '16]

The mixing matrix and the cusp anomaly

The two operators mix $(a, b = \pm)$

$$\mathcal{W}_a^B = \tilde{Z}_{ab}\,\mathcal{W}_b^R\,, \qquad \tilde{Z}_{ab} = (Z_{\text{open}}\tilde{Z}_{\text{cusp}})_{ab} \xrightarrow{i\theta\to\infty} (\tilde{Z}_{\text{cusp}})_{ab}$$

the cusp anomalous dimension is defined by

$$(\Gamma_{\text{cusp}})_{ab} = \left[\mu\frac{\partial}{\partial\mu}\log\tilde{Z}_{\text{cusp}}\right]_{ab}\,.$$

Thus we have 2 cusp anomalous dimensions for $\varphi = 0$

$$\Gamma^{(1)}_{\text{cusp}} = -\Gamma^{(2)}_{\text{cusp}} = -\sqrt{\hat{\lambda}_1\hat{\lambda}_2}$$

and the same for $\varphi \neq 0$

$$\Gamma^{(1)}_{\text{cusp}}(\varphi) = -\Gamma^{(2)}_{\text{cusp}}(\varphi) = -\frac{\sqrt{\hat{\lambda}_1\hat{\lambda}_2}}{\cos\frac{\varphi}{2}}$$

In the "strong coupling" limit $\Rightarrow$ no match with the $\sqrt{\hat{\lambda}}$ behavior of ST

The limits do not commute

$$\lambda\to 0\,,\; i\theta\to\infty \,+\, \hat{\lambda}\to\infty \;\neq\; \lambda\to\infty \,+\, i\theta\to\infty$$

Outline

Conclusions and Outlook

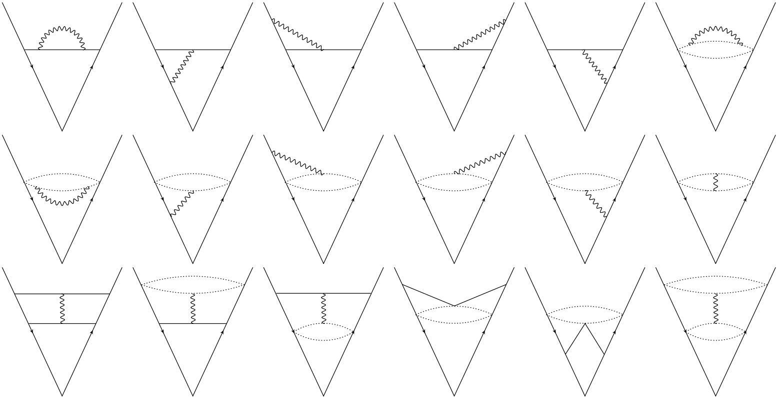
Conclusions

- Using the BS resummation approach we have computed the VEV's for the traced and supertraced operator;
- We have extracted the two cusp anomalous dimensions from the VEV's noticing that the two operators mix;
- We propose a new double exponentiation for the cusped WL;
- We argue that the disagreement with the AdS/CFT prediction is due to the fact that the scaling limit does not commute with the strong-coupling limit.

Outlook

- To check the double exponentiation using PT;
- Study the mixing problem relaxing the scaling limit;
- Compute the NLO kernels and perform Bethe-Salpeter resummation.

NLO kernel



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Attention