Wilson loops, Scattering amplitudes and Integrability in N=4 SUSY gauge theory

Author: Bonini Alfredo

Based on: D. Fioravanti, S.Piscaglia, M. Rossi arXiv:1503.0879
A.Bonini, D. Fioravanti, S.Piscaglia, M. Rossi arXiv:1511.05851

Universitá di Bologna

20 May 2016

Scattering amplitudes

- Gluons ($SU(N)$ with $N \to \infty$) $\to$ Helicity, color and momentum
- The color can be disentangled from the rest $\to$ Color ordered amplitudes
- Maximum helicity Violation (MHV) $\to$ $n-2$ with positive (negative) helicity
- Dependence on the momenta and the coupling constant $\mathcal{A}_n^{MHV}(p_1, \cdots, p_n; g)$
- We define the dual coordinates

$$p_i \equiv x_{i+1} - x_i$$

- The loop integrands exhibit dual conformal symmetry
- BDS ansatz, works for $n \leq 5$ at any coupling (Bern, Dixon, Smirnov)
- What is yet to be determined is the remainder function R_n for $n \geq 6$

Wilson loops

- Bosonic Wilson loops, with a polygonal contour
- Defined by its vertices $x_{i=1,\cdots,n}$
- $(x_{i+1} - x_i)^2 = 0$ null sides
- Conformal symmetry $\rightarrow$ Up to $n = 5$ is fixed
- $n > 5 \rightarrow 3n - 15$ conformal ratios τ_i,σ_i,ϕ_i

$$\mathcal{W}_n(\tau_i, \sigma_i, \phi_i; g), \quad n > 5$$

Classical string results

- $\mathcal{N} = 4$ SYM $\overset{AdS/CFT}{\longleftrightarrow}$ IIB String theory in $AdS_5 \times S^5$
- Planar limit ($N \to \infty$) with $\lambda = g^2 N$ (t'Hooft coupling) fixed
- Strong coupling in gauge theory $\to$ Classical string theory
- String moving in AdS_5 attached to a polygonal contour on its boundary $\to$ Wilson loop and Scattering amplitude (Alday, Maldacena)
- Classical string solution $\to$ Minimal area A_n
- The problem is recast into a set of TBA-like equations (free energy $\to$ area)

$$\mathcal{W}_n = \mathcal{A}_n^{MHV} = e^{-\frac{\sqrt{\lambda}}{2\pi} A_n + \cdots}$$

Duality Wilson loops/amplitudes

- Identification $p_i \equiv x_{i+1} - x_i \rightarrow$ conjecture: (Alday, Maldacena, Korchemsky, Drummond, Sokatchev)

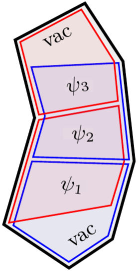
$$\mathcal{W}_n = \mathcal{A}_n^{MHV}$$

- Strong coupling equivalence from AdS/CFT
- Loop computations $\rightarrow$ They are the same at weak coupling as well
- Conformal symmetry $\leftrightarrow$ Dual conformal symmetry (up to $n = 5$)
- Non-trivial correspondece for $n > 5$
- It can be extended to the NMHV amplitudes (SUSY Wilson loops)

Non perturbative approach to the Wilson loops

- Conformal Field Theory $\rightarrow$ An Operator Product Expansion for the Null Polygonal Wilson loops (Alday, Gaiotto, Maldacena, Sever, Vieira)
- Wilson loop $\rightarrow$ Evolution of a flux-tube from the bottop to the top
- The flux-tube is sustained by two Wilson lines
- OPE $\rightarrow$ Pentagonal approach (Basso, Sever, Vieira)
- The method gives the remainder function

Pentagonal approach



- Cusp $\to$ Transition
- $n < 6$ is trivial in this picture
- Propagation phases $e^{-E_{\psi_i}\tau_i + ip_{\psi_i}\sigma_i + im_{\psi_i}\phi_i}$
- Transition amplitudes $P(\psi_i|\psi_j)$
- ψ are the flux-tube excitations

- The Wilson loops is written as a (OPE) series

$$\mathcal{W}_n = \sum_{\{\psi\}} \prod_{i=1}^{n-5} e^{-E_{\psi_i}\tau_i + ip_{\psi_i}\sigma_i + im_{\psi_i}\phi_i} P(0|\psi_1)P(\psi_1|\psi_2)\cdots P(\psi_{n-5}|0)$$

2D World:Integrability

- Flux-tube states: GKP string excitations in string theory, or large spin operator $TrZD_+^s Z$ in gauge theory ($S \to \infty$)
- Gauge theory picture: operator insertion $\to$ Excitation of the flux-tube

$$TrZD_+^{s-s_1}\hat{O}D_+^{s_1}Z, \qquad \hat{O} = F, \bar{F}, \Psi^a, \bar{\Psi}^{\bar{a}}, \Phi^i$$

- Gluons, fermions (antifermions) and scalars; they are representation of the $SU(4)$ R-symmetry, respectively 1,4,$\bar{4}$,6
- $\mathcal{N} = 4$ is asymptotically integrable $\to$ Asymptotic Bethe Ansatz equations (Beisert, Staudacher)
- The ABA equations are useful for the excitations on the BMN vacuum (ferromagnetic)
- They have been derived for a different vacuum, GKP $\leftrightarrow$ antiferromagnetic (Piscaglia, Rossi, Fioravanti, Basso)
- ψ are then multiparticle states (gluons and bound states, fermions, scalars)
- Integrability $\to$ Scattering matrices $S_{a,b}(u,v)$, pentagonal transitions P and dispersion relations $E_a(u)$, $p_a(u)$

Strong coupling limit

- Two different contributions, perturbative and non-perturbative (string theory picture)
- The perturbative is related to the classical string regime, the non-perturbative is given by the string dynamics on S^5
- In the OPE series, the perturbative comes from the gluons and fermions, by rescaling the rapidities with the coupling constant
- The non perturbative is due to the scalars (Basso, Sever, Vieira)

Perturbative regime

- Fermions contribute through bound states $\psi\bar{\psi}$ ($SU(4)$ singlet)
- They are not real bound states, but effective excitations at $\lambda = \infty$
- Relativistic dispersion $E = m\cosh\theta$, $p = m\sinh\theta$
- Gluons, mesons are singlet under $SU(4)$ $\rightarrow$ The transitions are just product of two particles terms

$$P(0|u_1, \cdots, u_n)P(u_1, \cdots, u_n|0) = \prod_{i<j} \frac{1}{P(u_i|u_j)P(u_j|u_i)}$$

- $P_{a,b}(u|v)$ are known at strong coupling for any particles a, b

Saddle-point analysis

- Consider for simplicity the hexagonal Wilson loop $\mathcal{W}_6$
- We define two quantum fields X_a ($a =$ gluon, meson) by their propagators

$$\frac{1}{P_{a,b}(u|v)P_{b,a}(v|u)} = e^{\langle X_a(u)X_b(v)\rangle}$$

- Gaussian integrals identity applied to the path integrals $\rightarrow$ Hubbard-Stratonovich transformation

$$\mathcal{W}_6 = \int DX_1 DX_2 e^{-S[X]}$$

- The action is an integral over the rapidity u (one dimensional) $\rightarrow$ Like a (non-local) quantum mechanics
- S is proportional to $\sqrt{\lambda} \rightarrow$ very large
- Saddle point equations $\rightarrow$ The same TBA-like equations and free energy
- The generalization to $n > 6$ is similar, and shows agreement with the classical string result

Non-perturbative regime

- The scalars acquire a non-perturbatively generated mass $m \sim \lambda^{1/8} e^{-\sqrt{\lambda}}$
- Their dynamics is encoded in the $O(6)$ sigma-model
- For the hexagon:

$$\mathcal{W}_6(z) \simeq \langle \hat{\phi}_p(z) \hat{\phi}_p(0) \rangle_{O(6)} \simeq C z^A (\log 1/z)^B, \quad z \to 0$$

- Relativistic invariance of the $O(6)$: depends only on the combination $\to$ $z = m\sqrt{\tau^2 + \sigma^2}$
- $\hat{\phi}_p \to$ Pentagonal operator (twist field), which satisfies $\langle 0 | \hat{\phi}_p(0) | \theta_1, \cdots, \theta_n \rangle \equiv P(0|\theta_1, \cdots, \theta_n)$
- Physical consideration on the twist fields lead to a conjecture for the coefficients A, B, and for the Wilson loop we have $\mathcal{W}_6 \simeq f_6 \lambda^{-\frac{7}{288}} e^{\frac{\sqrt{\lambda}}{144}}$ (Basso,Sever,Vieira)
- Contribution of the same order of the classical one, and similar considerations for $n > 6$

Form factors approach

- In the strong coupling limit, the scalars contribution enjoys the Form factors expansion

$$\mathcal{W}_6 = \sum_n \frac{1}{n!} \int \prod_i d\theta_i |P(\theta_1, \cdots, \theta_n)|^2 e^{-z \sum_k \cosh \theta_k}$$

- The transition is factorizable in two parts

$$|P(\theta_1, \cdots, \theta_n)|^2 = \Pi_{dyn}(\theta_1, \cdots, \theta_n) \Pi_{mat}(\theta_1, \cdots, \theta_n)$$

- In this limit $|P|^2$ depends only on the differences $\theta_i - \theta_j$, and we can capture the divergence for $z \to 0$ from the series
- The coefficient A enjoys a series expansion (analytical formula)
- Some analytical results about A,B,C

$SU(4)$ Matrix part

- $\Pi_{mat} \rightarrow$ is represented as a multiple integral over auxiliary variables a, b, c
- It does not depend on λ and on the dynamics of the theory, encodes the $SU(4)$ symmetry
- Therefore it is a very general object
- Some similarities with the Nekrasov Partition Function in $\mathcal{N} = 2$ appear
- Sum over Young Tableaux configurations for $\mathcal{N} = 2$ (Nekrasov, Okunkov)
- This expansion seems possible for the Π_{mat} too

Conclusions

- Integrability, along with the pentagonal approach, allowed us to find some results in the strong coupling limit for the Null Polygonal Wilson loops
- These results are the same obtained from the classical string theory
- From the OPE series, a correction to the minimal area result is predicted
- We are finding some analytical results to confirm this contribution
- Some interesting analogies with $\mathcal{N} = 2$ should be analized deeper