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# Large-N $CP^{N-1}$ sigma model on a finite interval: Physical Boundary Effect


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based on arXiv:1604.05630
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## Introduction

- $\mathbb{C}P^{N-1} \simeq \{ n^i \in \mathbb{C},\ i=1,\cdots,N \mid \sum_{i=1}^{N} |n^i|^2 = r \} / U(1)$
- Many common properties between 4d QCD and 2d $\mathbb{C}P^{N-1}$ model

  asymptotic freedom, mass gap and confinement.....

- 2d $\mathbb{C}P^{N-1}$ sigma model

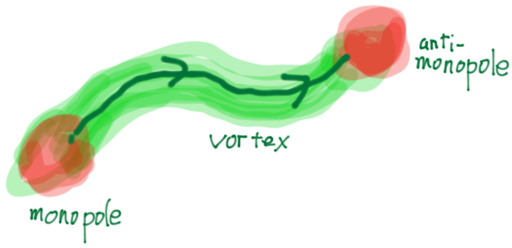
  $\Leftarrow$ Low energy effective action on nonAbelian vortex

  nonAbelian vortex

  string like topological soliton in the Higgs vacuum

  internal moduli space : $\dfrac{SU(N)}{U(1)\times SU(N-1)} \simeq \mathbb{C}P^{N-1}$

A. Hanany and D. Tong, JHEP 0307 (2003) 037,
L. Auzzi, S. Bolognesi, J. Evslin, K. Konishi and A. Yung, Nucl. Phys. B 673 (2003) 187,
M. Shifman and A. Yung, Phys. Rev. D 70, 045004 (2004)




$\Longleftrightarrow$ $\mathbb{C}P^{N-1}$ sigma model on a finite interval

## • Large-N $\mathbb{C}P^{N-1}$ sigma model on an infinite interval

• action for linearized $\mathbb{C}P^{N-1}$ sigma model

Lagrange multiplier

$$S = \int dx\,dt \left[ (D_\mu n^i)^* D^\mu n^i - \lambda (|n^i|^2 - r) \right]$$

U(1) cov. derivative

with $n^i = n^i(x,t) \in \mathbb{C}$, $i = 1, 2, \cdots, N$

• the classical defining condition for $\mathbb{C}P^{N-1}$

$$\frac{\delta S}{\delta \lambda} = 0 \Rightarrow \boxed{|n^i(x,t)|^2 = r}$$

size of $\mathbb{C}P^{N-1}$

$\Downarrow$ quantization

• the gap equation

UV cut off

$$\boxed{r = \langle (n^i)^* n^i \rangle} = N \int_0^{\Lambda_{UV}} \frac{k\,dk}{2\pi} \frac{1}{k^2 + \lambda} \simeq \frac{N}{2\pi} \log \frac{\Lambda_{UV}}{\sqrt{\lambda}}$$

in the "confinement phase"

$$\lambda = \Lambda^2 \neq 0 \qquad \langle n^i \rangle = 0$$

const.

the well-known scale-dependent renormalized coupling

$$\Rightarrow \quad r(\mu) = \frac{4\pi}{g^2(\mu)} \simeq \frac{N}{2\pi} \log \frac{\mu}{\Lambda}$$

4dim gauge coupling

- **Two phases of the model on a finite interval with translational invariance Ansatz**

Equation of motion

$$0 = \partial_\mu^2 \sigma + \lambda \sigma = \lambda \sigma$$

for classical configuration $\sigma \equiv \langle n^{i=1} \rangle$,

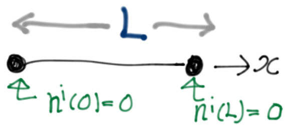
with $\langle n^i \rangle = 0$ for $i = 2, 3, \cdots, N$

two phases

$$\Rightarrow \begin{cases} \lambda \neq 0,\ \sigma = 0 : & \text{the "confinement phase"} \quad E \simeq \frac{N\Lambda^2}{4\pi} L \\ \lambda = 0,\ \sigma \neq 0 : & \text{the "Higgs phase"} \quad E = -\frac{N\pi}{12L} \end{cases}$$

(the deconfinement phase)

- A. Milekhin, Phys. Lev. D 86(2012) 105002




with the **Dirichlet** boundary conditions

→ the 2nd order phase transition

- S. Monin, M. Shifman and A. Yung, Phys.Lev. D 92 (2015) 2, 025011

with a **periodic** boundary condition


$n^i(x+2L) = n^i(x)$

→ the 1st order phase transition

They conclude **existence of phase transition** at $L \sim \Lambda^{-1}$

But,

Dirichlet b.c. $\Rightarrow$ ~~translational invariance~~

Does translational-invariance Ansatz give good approximation ?

Does the phase transition really occur?

Summary of this talk ….. **No !!**

For the model with the Dirichlet boundary conditions,

The translational inv. Ansatz can not give a true vacuum.

The phase transition **never** occur.

## The (full) gap equation with an arbitraly $\lambda(x)$

After integlation of quantum fluctuation $\delta n^i = n^i - \langle n^i \rangle$ and setting $A_\mu = 0$.

Effective energy

$$E = E(\lambda, \sigma) = N \sum_n \omega_n + \int dx \left[ \sigma'(x)^2 + \lambda(x)(\sigma(x)^2 - r) \right]$$

where the eigensystem $\{\omega_n, f_n(x)\}$ is defined by, with given $\lambda$

$$-f_n''(x) + \lambda(x) f_n(x) = \omega_n^2 f_n(x)$$

with given boundary conditions.

and orthogonal relation $\int dx\, f_n(x) f_m(x') = \delta_{n,m}$

In a vacuum, $E$ is extreamized with $\lambda, \sigma$:

$$\boxed{\begin{aligned} 0 = \frac{\delta E}{\delta \lambda(x)} &= \langle n^i(x)^* n^i(x) \rangle - r \\ &= \underbrace{\langle (\delta n^i(x))^* \delta n^i(x) \rangle}_{\text{quantum correction}} + \underbrace{\sigma(x)^2 - r}_{\text{classical part.}} \\ 0 = \frac{\delta E}{\delta \sigma(x)} &= -\sigma''(x) + \lambda(x) \sigma(x) \end{aligned}}$$

$$N \sum_n \frac{\delta \omega_n}{\delta \lambda(x)} = N \sum_n \frac{1}{2\omega_n} f_n(x)^2$$

## Does translational invariance Ansatz satisfy the full gap equation ?

With a constant potential $\lambda = m^2$

the full gap equation $\sigma^2 + \langle |\delta n|^2 \rangle = r$ gives

$$\sigma^2 = r - \langle (\delta n(x))^* \delta n(x) \rangle$$

boundary effect

$$= \underbrace{\frac{N}{2\pi} \log \frac{m}{\Lambda} - \frac{N}{\pi} \sum_{n=1}^{\infty} K_0(2nmL)}_{\text{constant}} + \frac{\eta}{2\pi} \underbrace{\sum_{n \in \mathbb{Z}} K_0(2m|x - nL|)}_{\Rightarrow \cancel{\text{translational inv}}}$$

$$\eta = \begin{cases} 1 & : \text{the Dirichlet b.c.} \quad \delta n^i(0) = \delta n^i(L) = 0 \\ -1 & : \text{the Neumann b.c.} \quad \delta n^{i\prime}(0) = \delta n^{i\prime}(L) = 0 \\ 0 & : \text{a periodic b.c. of a period } 2L \end{cases}$$

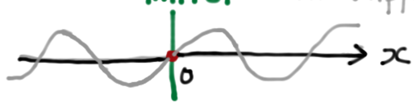
**The Dirichlet b.c. (and the Neumann b.c.) never allow translational invariant solutions**

# What actually happens around the boundaries?

the Dirichlet b.c. $\delta n^i(0)=0$


$\Leftrightarrow$

mirror with flipped sign




$\delta n^i(-x) = -\delta n^i(x)$

Using the mirror method, we find around the boundary $x \approx 0$

$$\langle (\delta n^i(x))^* \delta n^i(x) \rangle = -\langle \delta n^{i*}(x)\, \delta n^i(-x) \rangle \approx \frac{N}{2\pi} \log \frac{2x}{\epsilon}$$

UV cut off

the mass term can be negligible in UV region $\Rightarrow$ 2dim massless prop.

$$\lim_{x \to 0} x^2 \lambda(x) = 0$$

Therefore we obtain.

$$\sigma(x)^2 = r - \langle |\delta n(x)|^2 \rangle \approx \frac{N}{2\pi} \operatorname{Log} \frac{1}{\Lambda x}$$

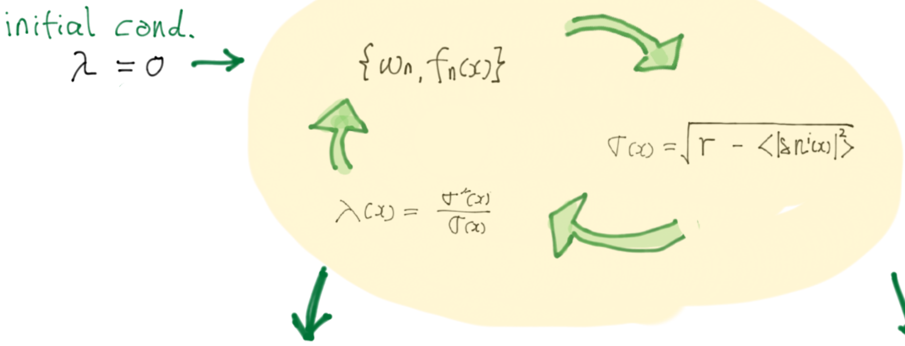
$$\lambda(x) = \frac{\sigma''(x)}{\sigma(x)} \approx \frac{1}{2x^2 \operatorname{Log} \frac{1}{\Lambda x}} \qquad \text{for } x \approx 0$$

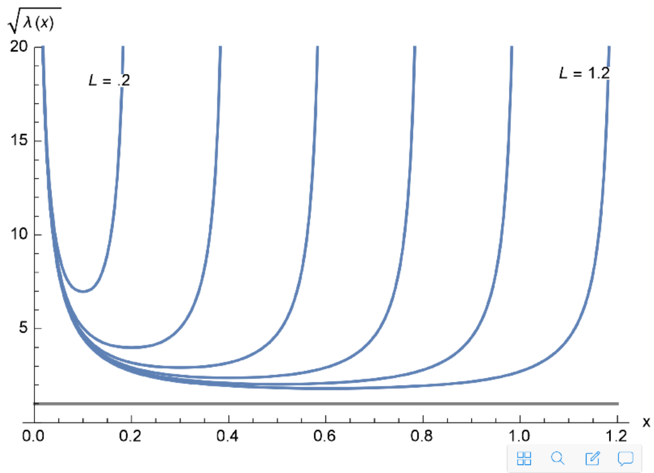
"$x$-dependent renormarized coupling"

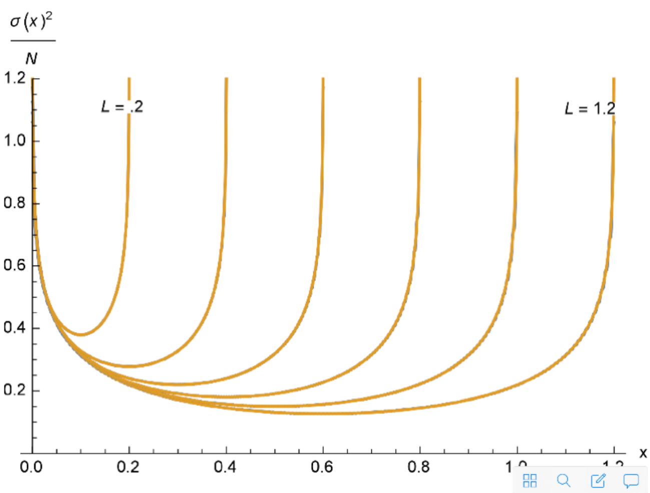
energy scale

$$\frac{4\pi}{g^2(x)} \equiv r(x) \equiv \sigma^2(x) \equiv \frac{N}{2\pi} \log \frac{2\mu(x)}{\Lambda} \quad \text{with} \quad \mu(x) \approx \frac{1}{x}$$

# Neumerical calculations using recursive eqs.



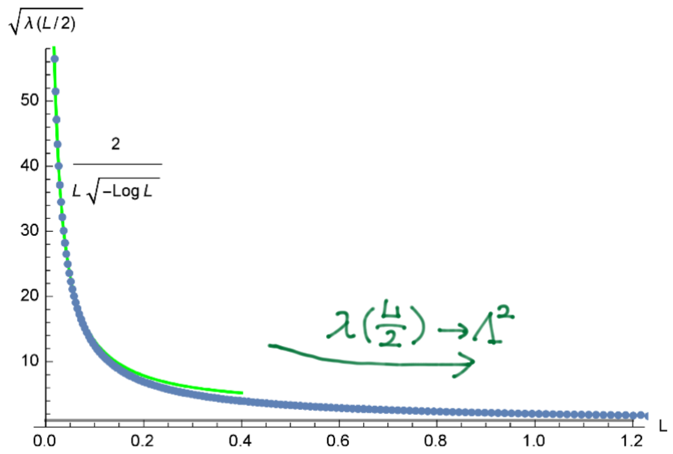

with $\Lambda = 1$.

We find

$\sigma(x) \neq 0, \quad \lambda(x) \neq 0$

$\Rightarrow$ $\boxed{\text{The Higgs phase } (\lambda=0) \text{ never appear. The phase transition never occur.}}$

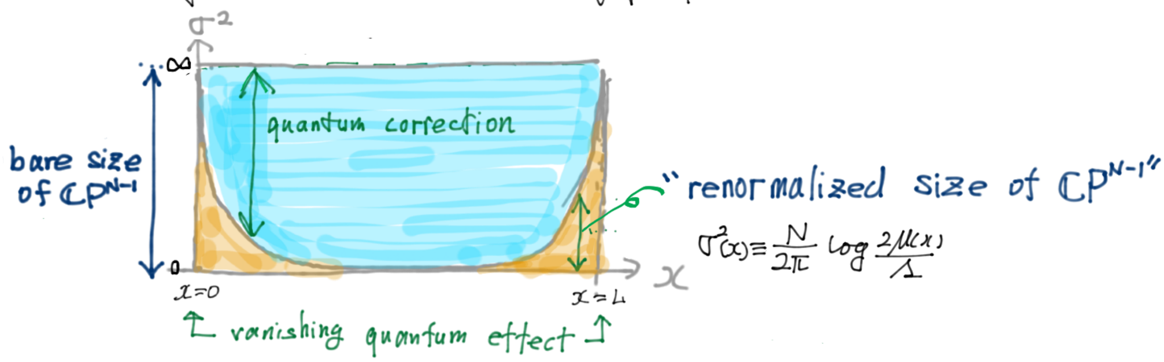
We expect $\lim_{L\to\infty} \lambda\left(\frac{L}{2}\right) = \Lambda^2$ the confinement phase




$\Lambda = 1$

# Summary

quantized defining cond. of $\mathbb{C}P^{N-1}$ = the full gap equation




The phase transition never occur.

## Comments

- Due to $\lambda(x) \sim \frac{1}{2x^2 (\log \frac{1}{x})}$, the Dirichlet b.c. = the Neumann b.c.
- L dependence of the total energy.... work in progress.