Spin degeneracy of Hadronic molecules in the heavy quark region

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Hadronic systems containing a Heavy Quark

1. Introduction
   - Hadronic molecule
   - Heavy Quark Spin Symmetry and one pion exchange potential

2. Meson-Nucleon molecules: $\bar{D}N$ and $BN$

3. Heavy quark mass limit ($m_Q \to \infty$)

4. Summary
Hadrons in the heavy quark region

- Hadron: Composite particle of **Quarks** and **Gluons**

- Constituent quark model (Baryon($qqq$) and Meson $q\bar{q}$) has been successfully applied to the hadron spectra!

![Diagram showing quarks and gluons in a baryon and a meson](image)

Baryon (proton, nucleon, ...)  Meson ($\pi$, $K$, ...)
Exotic hadrons in the heavy quark region

Introduction

Observation of the Exotic Hadron in the heavy quark (c, b) sectors!
Observation of the Exotic Hadron in the heavy quark (c, b) sectors!

e.g. Spectra of Charmonia

Charmonium $c\bar{c}$

- $\psi(4415)$
- $\psi(4160)$, $\psi(4040)$
- $\chi_{c2}(2P)$
- $\eta_c(2S)$, $\psi(3770)$, $\psi(2S)$
- $\chi_{c0}(1P)$, $\chi_{c1}(1P)$, $\chi_{c2}(1P)$, $h_c(1P)$
- $J/\psi(1S)$

S. Godfrey and N. Isgur, PRD32(1985)189
Observation of **the Exotic Hadron in the heavy quark (c, b) sectors!**

**e.g.** Spectra of Charmonia

<table>
<thead>
<tr>
<th>Mass (GeV/c^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
</tr>
<tr>
<td>Y(4664)</td>
</tr>
<tr>
<td>Y(4630)</td>
</tr>
<tr>
<td>ψ(4415)</td>
</tr>
<tr>
<td>Y(4360)</td>
</tr>
<tr>
<td>Y(4260)</td>
</tr>
<tr>
<td>ψ(4160)</td>
</tr>
<tr>
<td>ψ(4040)</td>
</tr>
<tr>
<td>Y(4008)</td>
</tr>
<tr>
<td>G(3900)</td>
</tr>
<tr>
<td>ψ(3770)</td>
</tr>
<tr>
<td>ψ(2S)</td>
</tr>
<tr>
<td>4.0</td>
</tr>
<tr>
<td>X(3872)</td>
</tr>
<tr>
<td>X_{c2}(2P)</td>
</tr>
<tr>
<td>3.5</td>
</tr>
<tr>
<td>η_{c}(2S)</td>
</tr>
<tr>
<td>J/ψ(1S)</td>
</tr>
<tr>
<td>3.0</td>
</tr>
<tr>
<td>η_{c}(1S)</td>
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Charmonium c̅c

**Exotic hadrons (≠ c̅c)**

\[ X, Y, Z \]


S. Godfrey and N. Isgur, PRD**32**(1985)189

What is **the structure of exotic hadrons?**

Why are many exotic hadrons found in **the heavy quark region?**
Mass degeneracy of heavy hadrons

Introduction

- Mass difference between vector and pseudoscalar mesons. 
  \((Q\bar{q}, \, q = u, \, d)\)

\[\Delta m\]
\begin{itemize}
  \item \(\Delta m\) decreases when the quark mass increases.
  \item Masses of \(\{B, B^*\}\) (\(\{D, D^*\}\)) are almost degenerate.
\end{itemize}
Mass difference between vector and pseudoscalar mesons. \((Q\bar{q}, q = u, d)\)

- \(\Delta m\) decreases when the quark mass increases.
- Masses of \(\{B, B^*\}\) (\(\{D, D^*\}\)) are almost degenerate.

→ Heavy Quark Spin Symmetry!
Heavy Quark Spin Symmetry (HQS)  

- **Suppression of Spin-spin force** in $m_Q \to \infty$.
- Decomposition of **Heavy quark spin** and **Light components**
  $$\vec{J} = \vec{L} + \vec{S} = \vec{S}_Q + \vec{j}$$

$\Rightarrow$ **Mass degeneracy** of hadrons with the different spins.

- Mass degeneracy of $\{D, D^*\}(Q\bar{q})$, $\{\eta_c, J/\psi\}(Q\bar{Q})$, $\{\Sigma_c, \Sigma^*_c\}(Qqq)$ (baryons)…
Interaction between $K$ (light meson) and $N$
⇒ Short range force ($\rho$, $\omega$ exchanges...) dominates.

**Strange (Light)**

<table>
<thead>
<tr>
<th>$K$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$, $\omega$, ...</td>
<td></td>
</tr>
</tbody>
</table>

| $K$ | $N$ |

**Charm (Heavy)**

<table>
<thead>
<tr>
<th>$\bar{D}$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
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<td>$\rho$, $\omega$, ...</td>
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| $\bar{D}$ | $N$ |
Interaction between $K$ (light meson) and $N$
$\Rightarrow$ Short range force ($\rho$, $\omega$ exchanges...) dominates.

**Strange (Light)** ($KK\pi \times$)  

**Charm (Heavy)**

In the heavy ($c$, $b$) sector, **the Heavy Quark Spin Symmetry** induces the $\bar{D} - \bar{D}^*$ mixing.

$m_{K^*} - m_K \sim 400$ MeV $\Leftrightarrow m_{D^*} - m_D \sim 140$ MeV

The mixing enhances **the one $\pi$ exchange potential (OPEP)**.
Introduction

- OPEP is important to bind atomic nuclei.
- **Tensor force** of the OPEP generates a strong attraction.

\[ \vec{s} \cdot \vec{q} \]

**Deuteron**

\[ ^3S_1 \]

\[ ^3D_1 \]

\[ \pi \]

\[ \vec{s} \cdot \vec{q} \]

\[ ^3S_1 \]

\[ ^3D_1 \]

**P\(^(*\)\) − N**

\[ ^2S_{1/2} \]

\[ ^4D_{1/2} \]

\[ ^2S_{1/2} \]

\[ \vec{s} \cdot \vec{q} \]

**Tensor force** \(\Rightarrow\) \( ^3S_1 \rightarrow ^3D_1 \)

\( \text{PN}(^2S_{1/2}) \rightarrow \text{P}^*\text{N}(^4D_{1/2}) \)
OPEP is important to bind atomic nuclei.

Tensor force of the OPEP generates a strong attraction.

\[ \vec{s} \cdot \vec{q} \]

\[ N \]

\[ 3S_1 \]

\[ \pi \]

\[ 3D_1 \]

\[ N \]

\[ \vec{s} \cdot \vec{q} \]

\[ N \]

\[ 3S_1 \]

\[ \pi \]

\[ 3D_1 \]

\[ N \]

\[ \vec{s} \cdot \vec{q} \]

\[ P \]

\[ 2S_{1/2} \]

\[ \pi \]

\[ 4D_{1/2} \]

\[ N \]

\[ \vec{s} \cdot \vec{q} \]

\[ P^* \]

\[ 2S_{1/2} \]

\[ \pi \]

\[ \vec{s} \cdot \vec{q} \]

\[ N \]

\[ PN(2S_{1/2}) - P^*N(4D_{1/2}) \]

\[\pi\] exchange \[\Rightarrow\] Nucleus-like state?
Hadronic molecules in the heavy quark region

Introduction

- Hadronic molecules (Hadron composite systems)
  → Appearing near the thresholds (M-M, M-B,...)

Exotic hadrons ⇒ Hadronic molecules?

Meson-Meson (X, Y, Z?)

- X(3872), Z_b

Meson-Baryon

- \( \Lambda_c^* \), Pentaquark???

Theoretical researches

- \( X(3872) \) as \( D \bar{D}^* \),

- \( Z_b \) as \( B \bar{B}^* \), J. R. Zhang, et al., PLB704(2011)312, S. Ohkoda, et al., PRD86(2012)014004

Main Subject: Heavy meson in nuclei

- Hadronic molecules formed by Heavy meson-Nucleon with the $\pi$ exchange potential.
- Nature of the states containing heavy quarks

- New exotic states containing $\bar{Q}qqqq$
- **Strong attractions** of $\pi$ exchange potential from the HQS.
**P(*)N Interaction** (P(*) = D(*), B(*)): OPEP

\[ P(*) \rightarrow N \]

\[ \mathcal{L}_{\pi P(*)P(*)} - \mathcal{L}_{\pi NN} \]

**P**: Pseudoscalar meson

**P***: Vector meson

**N**: Nucleon

Fig: Meson exchange diagram

\[
V_{PN-P*N}^{\pi} = -\frac{g_{\pi} g_{\pi NN}}{\sqrt{2m_N f_\pi}} \frac{1}{3} \left[ \vec{\epsilon}^\dagger \cdot \vec{\sigma} C(r) + S_\epsilon T(r) \right] \vec{\tau}_P \cdot \vec{\tau}_N
\]

\[
V_{P* N-P*N}^{\pi} = \frac{g_{\pi} g_{\pi NN}}{\sqrt{2m_N f_\pi}} \frac{1}{3} \left[ \vec{T} \cdot \vec{\sigma} C(r) + S_T T(r) \right] \vec{\tau}_P \cdot \vec{\tau}_N
\]

S. Yasui and K. Sudoh PRD80(2009)034008

**C(r)**: Central force, **T(r)**: Tensor force

\( \triangleright \) **T(r)** generates a strong attraction! ⇔ Deuteron
\( P(\ast) N \) Interaction \((P(\ast) = D(\ast), B(\ast))\): OPEP

\[
V_{PN-P*N}^\pi = -\frac{g_\pi g_{\pi NN}}{\sqrt{2m_Nf_\pi}} \frac{1}{3} \left[ \vec{\sigma} \cdot \vec{C}(r) + S_\varepsilon \mathbf{T}(r) \right] \vec{\tau}_P \cdot \vec{\tau}_N
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S. Yasui and K. Sudoh PRD 80(2009)034008

\( C(r) \): Central force, \( \mathbf{T}(r) \): Tensor force

\( \triangleright \) \( \mathbf{T}(r) \) generates a strong attraction! ⇔ Deuteron
Results of $P^{(*)}N$ states (2-body)

$\bar{D}^{(*)}$ or $B^{(*)}$

$N$

$\bar{Q}q$

$\pi$

$\bar{D}N$, $BN$

Exotic states ($\bar{Q}q + qqq$)

Bound state and Resonance

- We solve the coupled-channel Schrödinger equations for $PN$ and $P^*N$ channels.
- Interaction: $\pi$, $\rho$, $\omega$ exchange potentials.
$J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm$ with $I = 0$

Unit: MeV

\( J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm \) with \( I = 0 \)

- One bound state

\[ \bar{D}N \]

\[ \bar{D}^*N \]

- \( \bar{D}N \) threshold
- \( \bar{D}^*N \) threshold

\[ P = - \quad P = + \]

\[ E \]

Unit: MeV

$D\bar{N}$ and $B\bar{N}$ for $l = 0$ (2-body)

- $J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm$ with $l = 0$
- One bound state, and resonances in charm

![Diagram](image)

$176.0 - i87.4$ \(5/2^+\)

$148.2 - i5.1$ \(3/2^+\)

$26.8 - i65.7$ \(1/2^+\)

---

$J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm$ with $I = 0$

- One bound state, and resonances in charm and bottom sectors!

Many states near the thresholds. $\Leftrightarrow$ No KN bound state

Expectation values in Bound state of $J^P = 1/2^-$

$\bar{D}N$ and $BN$

- Expectation values of OPEP in $\bar{D}N$

<table>
<thead>
<tr>
<th>$\bar{D}N$</th>
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<td>Central</td>
<td>-2.5</td>
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- The tensor force of $\pi$ exchange potential generates the strong attraction. Especially, $\bar{D}N - \bar{D}^*N$ mixing is important.
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- The tensor force of $\pi$ exchange potential generates **the strong attraction**. Especially, $\bar{D}N - \bar{D}^*N$ mixing is important.
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- The tensor force of $\pi$ exchange potential generates the strong attraction. Especially, $\bar{D}N - \bar{D}^*N$ mixing is important.

<table>
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<tr>
<th>$BN$</th>
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<th>$\langle V_{B^*N-B^*N} \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td>$-8.2$</td>
<td>$1.3$</td>
</tr>
<tr>
<td>Tensor</td>
<td>$-90.2$</td>
<td>$-8.3$</td>
</tr>
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- Mixing effects are enhanced in $BN$ due to small $\Delta m_{BB^*}$.  

19 May 2016

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New Frontiers in Theoretical Physics@GGI, Firenze
Results of $P_Q N$ states ($m_Q \rightarrow \infty$)

$P_Q^{(*)} N$ ($m_{P_Q^*} - m_{P_Q} = 0$)

Heavy quark mass limit
Mass degeneracy should appear not only in the ordinary states but also in the hadronic molecules!
Mass degeneracy can be seen by introducing

**New basis** \( (\bar{Q} - [qN]) \).


\[
\begin{align*}
\text{Hadron basis } (P - N) & \Leftrightarrow \text{Brown muck basis } (\bar{Q} - [qN]) \\
|[[\ell, [S_P, S_N]_{S_{PN}}]_J] \rangle & \Leftrightarrow |[S_Q, [\ell, [S_q, S_N]_{S_{qN}}]_J]_J \rangle \\
\end{align*}
\]

**Diagram:**

- **Heavy Hadron**
  - \( J \)
  - \( Q \)
- **Light quarks + Gluons**
  - \( \bar{S}_Q \)
  - \( j \)

Light quarks + Gluons (Brown muck)
Mass degeneracy can be seen by introducing

**New basis** \((\bar{Q} - [qN])\).


\[|P_{QN}\rangle = U|\bar{Q} [qN]\rangle\]

Unitary transformation
Comparing the Hamiltonians with different $J^P$

- Hamiltonian $H_{JP}(= K + V_\pi)$ of Hadron basis ($P - N$) in the heavy quark mass limit ($m_Q \to \infty$)

\[
H_{1/2^-} = \begin{pmatrix}
K_0 & \sqrt{3}C & -\sqrt{6}T \\
\sqrt{3}C & K_0 - 2C & -\sqrt{2}T \\
-\sqrt{6}T & -\sqrt{2}T & K_2 + C - 2T
\end{pmatrix}
\]

\[
H_{3/2^-} = \begin{pmatrix}
K_2 & \sqrt{3}T & -\sqrt{3}T & \sqrt{3}C \\
\sqrt{3}T & K_0 + C & 2T & T \\
-\sqrt{3}T & 2T & K_2 + C & -T \\
\sqrt{3}C & T & -T & K_2 - 2C
\end{pmatrix}
\]

* $K_i$: Kinetic term, $C$: Central force, $T$: Tensor force

- Can you expect they are degenerate?
Comparing the Hamiltonians with different $J^P$

- Hamiltonian $H_{JP}(= K + V_\pi)$ of Brown muck basis ($\bar{Q}[qN]$) in the heavy quark mass limit ($m_Q \to \infty$)

$$H_{BM}^{1/2^-} = \begin{pmatrix} K_0 - 3C & 0 & 0 \\ 0 & K_0 + C & 2\sqrt{2}T \\ 0 & 2\sqrt{2}T & K_2 + C - 2T \end{pmatrix}$$

$$H_{BM}^{3/2^-} = \begin{pmatrix} K_0 + C & 2\sqrt{2}T & 0 & 0 \\ 2\sqrt{2}T & K_2 + C - 2T & 0 & 0 \\ 0 & 0 & K_2 - 3C & 0 \\ 0 & 0 & 0 & K_2 + C + 2T \end{pmatrix}$$

* $K_i$: Kinetic term, $C$: Central force, $T$: Tensor force

- In the Brawn muck basis, the Hamiltonians are **block diagonalized** in the heavy quark limit ($m_Q \to \infty$)!
Comparing the Hamiltonians with different $J^P$

- Hamiltonian $H_{J^P} (= K + V_\pi)$ of **Brown muck basis ($\bar{Q}[qN]$)** in the heavy quark mass limit ($m_Q \to \infty$)

\[
H_{1/2^-}^{BM} = \begin{pmatrix}
K_0 - 3C & 0 & 0 \\
0 & K_0 + C & 2\sqrt{2}T \\
0 & 2\sqrt{2}T & K_2 + C - 2T
\end{pmatrix}
\]

\[
H_{3/2^-}^{BM} = \begin{pmatrix}
K_0 + C & 2\sqrt{2}T & 0 & 0 \\
2\sqrt{2}T & K_2 + C - 2T & 0 & 0 \\
0 & 0 & K_2 - 3C & 0 \\
0 & 0 & 0 & K_2 + C + 2T
\end{pmatrix}
\]

* $K_i$: Kinetic term, $C$: Central force, $T$: Tensor force

- In the Brawn muck basis, the Hamiltonians are **block diagonalized** in the heavy quark limit ($m_Q \to \infty$)!
- **Blue components** produce the degenerate states!
In the $\bar{D}N$ and $BN$ sectors (with finite heavy quark mass), bound states ($J^P = 1/2^-$) and resonances ($3/2^-$) were found.

![Energy spectrum diagram]

Degenerate states are found! ($1/2^-$ and $3/2^-$)
In the $\bar{D}N$ and $BN$ sectors (with finite heavy quark mass), Bound states ($J^P = 1/2^-$) and resonances ($3/2^-$) were found.

Degenerate states are found! ($1/2^-$ and $3/2^-$)

In the $\bar{D}N$ and $BN$ sectors (with finite heavy quark mass), Bound states ($J^P = 1/2^-$) and resonances ($3/2^-$) were found.

Degenerate states are found! ($1/2^-$ and $3/2^-$)  
$\Rightarrow$ The molecules belong to the HQS doublet.

Summary

Subject: Hadronic molecules $P^{(*)}N$
by introducing Heavy quark symmetry and OPEP

New Bound states and Resonances are found in $P^{(*)}N$ in the heavy quark sectors.

The Heavy quark symmetry enhances the OPEP between the heavy meson $P$ and the nucleon $N$.

**Tensor force of OPEP in PN — P*N mixing** plays a crucial role to produce the **New Exotic states**.

In $m_Q \to \infty$, we have obtained the degenerate states in the hadronic molecule.

Thank you for your kind attention.