Framing and localization in Chern-Simons theories with matter

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Motivations and summary

We study expectation values of susy Wilson Loops in 3d realizations of AdS/CFT correspondence

BPS Wilson loops

- gauge invariant non–protected observables. Their value non-trivially interpolates between the weak and strong coupling descriptions
- they can often be computed exactly by using localization techniques

3d CSM models

- imaginary contributions in the localization results
- indications pointing to the a role played by framing regularization, but still poorly understood

Main Goals

- understand imaginary contributions in terms of framing in non–topological CSM models
- give a prescription to identify framing in the localization results
- a physical interpretation of framing?

Outline

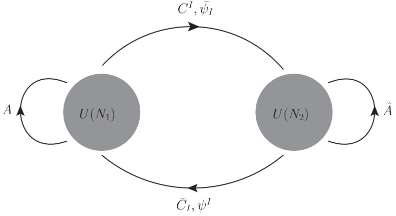
- The model, the observable and the localization result
- Framing in pure CS models
- Adding matter, a revision of framing
- Conclusions and outlook

The model, the observable and the localization result

1/6 BPS Wilson loop in ABJM theory

ABJ(M) theory

- $\mathcal{N} = 6$ superconformal 3d model
- $U(N_1)_k \times U(N_2)_{-k}$ Chern-Simons coupled to bifundamental matter
- in the perturbative regime, dual to type IIA strings on $AdS_4 \times \mathbb{CP}_3$



1/6 BPS Wilson loop

[Drukker, Plefka, Young 08]
[Chen, Wu 08]
[Rey,Suyama,Yamaguchi 08]

$$W_{1/6} = \mathrm{Tr}_{\Box} \mathcal{P} \exp\left(-i \oint (A_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| M_J^I C_I \bar{C}^J) \right)$$

$$M_J^I = \mathrm{diag}(1, 1, -1, -1) \qquad x^\mu = (0, \cos\tau, \sin\tau)$$

Localization in ABJ models

[Kapustin, Willett, Yaakov 09]

Partition function of ABJ localizes to a non–gaussian matrix model

$$Z_{ABJ} = \mathcal{N} \int \prod_i^{N_1} \frac{d\mu_i}{2\pi} \prod_j^{N_2} \frac{d\nu_j}{2\pi} e^{\frac{ik}{4\pi}(\sum_i \mu_i^2 - \sum_j \nu_j^2)} \frac{\prod_{i<j} \left(2\sinh\left(\frac{\mu_i-\mu_j}{2}\right)\right)^2 \left(2\sinh\left(\frac{\nu_i-\nu_j}{2}\right)\right)^2}{\prod_{i,j}\left(2\cosh\left(\frac{\mu_i-\nu_j}{2}\right)\right)^2}$$

1/6 BPS Wilson loop can be localized

$$\langle W_{1/6} \rangle = \langle \mathrm{Tr}\, e^{\mu_i} \rangle_{MM}$$

The Wilson Loop matrix integral can be solved exactly (in a implicit form)

[Drukker, Marino, Putrov 11]

Matrix Model solution can be expanded at weak/strong regimes

$\updownarrow$

Weak/strong coupling computation of the field/string theory description

Weak coupling expansion

The matrix model expansion at weak coupling: [Marino, Putrov 09]

$$\langle W_{1/6}\rangle_{MM} = e^{i\pi\lambda_1}\left(1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) - i\frac{\pi^3}{2}\lambda_1\lambda_2^2 + \mathcal{O}(\lambda^4)\right)$$

QFT perturbative computation: [Rey, Suyama, Yamaguchi 09]

$$\langle W_{1/6}\rangle = 1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) + \mathcal{O}(\lambda^4)$$

How can we interpret the mismatch?

Partial explanation: $e^{i\pi\lambda_1}$ is the framing factor of topological CS theory

[Kapustin, Willett, Yaakov 09]

No attempt to explain the other immaginary terms.

FRAMING IN PURE CS MODELS

Chern-Simons $U(N)_k$ models

[Witten 89]

$$Z_{CS} = \int \mathcal{D}A \, \exp\left(\frac{ik}{4\pi} \int_M \mathrm{Tr}\left(A \wedge dA + \tfrac{2}{3} A \wedge A \wedge A \right) \right)$$

$$\langle W_{\mathrm{CS}} \rangle = \left\langle \mathrm{Tr}_{\square} \, \mathcal{P} \exp\left(-i \int_\Gamma dx^\mu A_\mu(x) \right) \right\rangle$$

The action and WL definitions are topological (crucial to make a connection to 2d CFT and knot invariants) . . .

. . . *but* general covariance is broken at the quantum level

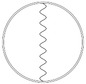
Can we choose a regularization preserving metric invariance?

This can be done introducing a framing $\rightsquigarrow$ exact and topological results for the framed Z_{CS} and $\langle W_{CS} \rangle$

How does framing work?

Framing in perturbation theory [Guadagnini, Martellini, Mintchev 90]

$$\langle W_{\rm CS}\rangle = \sum_{n=0}^{\infty} {\rm Tr}\, \mathcal{P} \int_\Gamma dx_1^{\mu_1} \dots dx_n^{\mu_n} \langle A_{\mu_1}(x_1) \dots A_{\mu_n}(x_n)\rangle$$

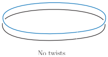
One loop: 

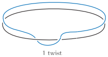
Landau gauge: $\langle A_\mu(x) A_\nu(y)\rangle \sim \frac{\varepsilon_{\mu\nu\rho}(x-y)^\rho}{[(x-y)^2]^{\frac{3}{2}}}$

The result is metric and Γ dependent

Introducing the frame, point splitting regularization of singularities

$$\Gamma_f: \quad y^\mu(\tau) \to y^\mu(\tau) + \alpha\, n^\mu(\tau)$$
$$|n(\tau)| = 1 \qquad n \cdot \dot{y} = 0$$





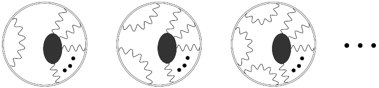
taking $\alpha \to 0$ one gets a topological result, Gauss integral

$$\langle W_{\rm CS}\rangle^{(1)} = -i\pi\lambda\,\chi(\Gamma, \Gamma_f) \qquad \lambda = \frac{N}{k}$$

$$\chi(\Gamma, \Gamma_f) = \frac{1}{4\pi} \oint_\Gamma dx^\mu \oint_{\Gamma_f} dy^\nu\, \varepsilon_{\mu\nu\rho} \frac{(x-y)^\rho}{|x-y|^3} = \text{linking } \#$$

Higher loops: [Alvarez, Labastida 93]

- collapsible propagators are the only source of framing dependence



- gauge propagator does not receive quantum corrections

⇒ framing dependent contributions factorizes and exponentiates

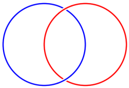
$$\langle W_{\text{CS}} \rangle = \underbrace{e^{-i\pi\lambda\,\chi(\Gamma,\Gamma_f)}}_{\text{framing phase}} \cdot \overbrace{\rho(\Gamma,\lambda)}^{\text{topological series}}$$

Framing and localization in ABJ model

$$\langle W_{1/6}\rangle_{MM} = \overbrace{e^{i\pi\lambda_1}}^{\text{pure CS framing}} \left(1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) \underbrace{-i\frac{\pi^3}{2}\lambda_1\lambda_2^2}_{???} + \mathcal{O}(\lambda^4)\right)$$

Susy localization is sensible to framing! It gives the result at $\chi(\Gamma, \Gamma_f) = -1$

Susy forces the contour and its frame to wrap two different Hopf fibers of S^3



Additional terms? They are not present in the standard field theory result . . .

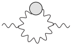
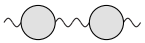
⇒ framing analysis should be revised in CSM theories

Guess: gauge propagator gets quantum corrections due to matter . . .

⇒ quantum correction to framing factor

ADDING MATTER, A REVISION OF FRAMING

Gauge propagator: two–loop corrections

	Diagram	Color		Diagram	Color
(a)		λ_2^2	(e)		$2\lambda_2^2 - \lambda_1\lambda_2$
(b)		λ_2^2	(f)		$\lambda_1\lambda_2$
(c)		$\lambda_1\lambda_2$	(g)		$\lambda_1\lambda_2 - \lambda_2^2$
(d)		$\lambda_1\lambda_2$	(h)		λ_2^2

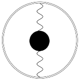
Planar perturbative computation using DRED scheme

The two–loop correction is finite (non–trivial cancelation of divergences)

$$= -\left(\frac{i\pi^2}{4k}\right)\left[\lambda_2^2 + \lambda_1\lambda_2\left(\frac{1}{4} + \frac{2}{\pi^2}\right)\right]\varepsilon_{\mu\nu\rho}\,\frac{(x-y)^\rho}{\left[(x-y)^2\right]^{\frac{3}{2}}}$$

This has to be inserted into the WL

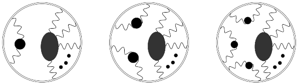
Color sector $\lambda_1\lambda_2^2$



$$\longrightarrow \quad \langle W_{1/6}\rangle^{(3)}\big|_{\lambda_1\lambda_2^2} = i\frac{\pi^3}{2}\lambda_1\lambda_2^2\,\chi(\Gamma,\Gamma_f)$$

$$\langle W_{1/6}\rangle_{MM} = \overbrace{e^{\pi i\lambda_1}}^{\text{pure CS framing}} \left(1-\frac{\pi^2}{6}(\lambda_1^2-6\lambda_1\lambda_2)\underbrace{-i\frac{\pi^3}{2}\lambda_1\lambda_2^2}_{\text{matter framing}}+\mathcal{O}(\lambda^4)\right)$$

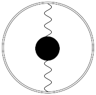
- ✓ the correction is a framing $\chi(\Gamma,\Gamma_f)=-1$ contribution due to matter!
- ✓ factorization and exponentiation still work for corrected propagators



$\cdots$

$$\langle W_{1/6}\rangle_{MM} = e^{i\pi\overbrace{\left(\lambda_1-\frac{\pi^2}{2}\lambda_1\lambda_2^2+\mathcal{O}(\lambda^5)\right)}^{\text{framing function}}\chi(\Gamma,\Gamma_f)}\left(1-\frac{\pi^2}{6}(\lambda_1^2-6\lambda_1\lambda_2)+\mathcal{O}(\lambda^4)\right)$$

Color sector $\lambda_1^2\lambda_2$



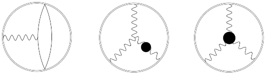
$$\longrightarrow \quad \langle \mathcal{W} \rangle^{(3)}\big|_{\lambda_1^2\lambda_2} = i\frac{\pi^3}{2}\lambda_1^2\lambda_2\left(\frac{1}{4}+\frac{2}{\pi^2}\right)\chi(\Gamma,\Gamma_f)$$

✗ it has a lower transcendentality term

✗ it is not present in the matrix model expansion

$$\langle \mathcal{W} \rangle = e^{\pi i \lambda_1}\left(1-\frac{\pi^2}{6}(\lambda_1^2-6\lambda_1\lambda_2)-i\frac{\pi^3}{2}\lambda_1\lambda_2^2+\mathcal{O}(\lambda^4)\right)$$

... but we have non vanishing contributions from vertex type diagrams which should cancel it



Remark: Assuming cancelation, vertex type diagrams must give framing dependent results

✓ Numerical check of linear dependence on the linking number

CONCLUSIONS AND OUTLOOK

Conclusions and outlook

We explained the framing origin of imaginary terms in the expansion of the Matrix Model result for the 1/6 BPS WL operator in ABJ theory

Novel features of framing in CSM models

- the need to consider a non–trivial framing function
- vertex diagrams can be framing dependent
- a conjecture for ABJM [Bianchi, Griguolo, Leoni, Penati, Seminara 14]

$$B_{1/2}(\lambda) = \frac{1}{8\pi} \tan \Phi_{1/6}(\lambda) \qquad \langle W_{1/6} \rangle = e^{i\pi \Phi_{1/6}(\lambda)} \rho(\lambda)$$

following our analysis $\Phi_{1/6}(\lambda)$ is the framing function!
Physical interpretation of the framing phase

Open problems:

- analytical study of vertex diagrams
- framing for fermionic 1/2 BPS Wilson loops
- framing in $\mathcal{N} = 4$ CSM theories (in progress)
- framing at strong coupling?