Higgs regulated amplitudes in $\mathcal{N} = 4$ SYM, the generalized cusp anomalous dimension and bound states of W-bosons

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Short outline

Main focus: calculation of the cusp anomalous dimension Γ_{cusp} in $\mathcal{N} = 4$ Super Yang-Mills

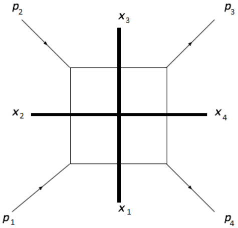
- weak coupling: from scattering amplitudes, with a general symmetry breaking structure

- strong coupling: from string calculations, exploiting AdS/CFT correspondence

- from Γ_{cusp} to bound states energy spectrum

$\mathcal{N} = 4$ Super Yang-Mills theory

- Dual conformal symmetry: intrinsic property of massless MHV amplitudes under dimensional regularization (*Drummond, Henn, Korchemsky, Sokatchev;* 2008)



after the following change of variables

$$p_i = x_i - x_{i+1} ,$$

it's invariant under conformal transformations in x space.

Dual conformal symmetry constrains the form of the amplitudes.

Higgsed $\mathcal{N} = 4$ SYM

- massless amplitudes $\rightarrow$ IR divergencies

- alternative to dimensional regularization $\rightarrow$ spontaneous symmetry breaking (*Alday, Henn, Plefka, Schuster; 2010*)

$$U(N+M) \rightarrow U(N) \times U(1)^M$$

- gauge invariance preserved

- turning on a vacuum expectation value (VEV) m_j , with $(j = N+1, \ldots, N+M)$, for scalar fields along one particular scalar direction

$$\hat{\Phi} = \Phi + \delta_{I9} \langle \Phi_9 \rangle$$

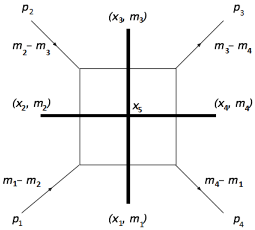
Preserved dual conformal symmetry

Defining enhanced momenta

$$\hat{p}_k = \left(p_k, m_{i_k} - m_{i_{k+1}}\right) \qquad \hat{l}_k = \left(l_k, m_{i_k}\right)$$

amplitudes become $5 - d$ integrals.

From now on, we are considering this box amplitude



Dual conformal symmetry is preserved $\rightarrow \mathcal{M} = \mathcal{M}\,(u, v)$, where

$$u = \frac{m_1 m_3}{s+(m_1-m_3)^2} \qquad v = \frac{m_2 m_4}{t+(m_2-m_4)^2}$$

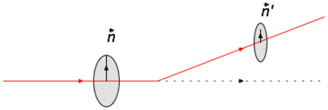
Cusp anomalous dimension Γ_{cusp}

Γ_{cusp} has been introduced as an UV divergence of a Wilson loop with cusp.

In $\mathcal{N} = 4$ SYM, a second angle is introduced for the cusp, linked to the coupling with scalars

$$W \sim \mathrm{Tr}\left[P e^{i \oint A \cdot dx + \oint |dx| \vec{n} \cdot \vec{\Phi}} \right] \qquad \cos\theta = \vec{n} \cdot \vec{n}'$$

where $\vec{n}$ and $\vec{n}'$ are the directions before and after the turn.



Γ_{cusp} from scattering amplitudes

Γ_{cusp} characterizes also IR divergences from scattering massive W bosons on the Coulomb branch of $\mathcal{N} = 4$ SYM.

The cusp angle can be related to amplitude's momenta by

$$\cos\phi = \frac{p_2 \cdot p_3}{\sqrt{(p_2)^2\,(p_3)^2}}$$

We choose a two-mass configuration ($m_1 = m_3 = m$, $m_2 = m_4 = M$), with $s, t \sim M^2$

$\rightarrow$ external massive particles
$\rightarrow$ m as an IR cutoff

Γ_{cusp} can be extracted from amplitudes (*Correa, Henn, Maldacena, Sever; 2012*)

$$\lim_{u\to 0} \ln\left(\mathcal{M}(u,v)\right) = (\ln u)\,\Gamma_{cusp}(\lambda,\varphi) + \mathcal{O}\left(u^0\right)$$

More general Higgsing

We give a VEV along different scalar directions

$$\hat{\Phi}_I = \sum_{J=4}^{9} \delta_{IJ} \langle \Phi_J \rangle + \Phi_I$$

and promote them to momenta variables according to

$$\hat{p}_k = \left(p_k, m^4_{i_k} - m^4_{i_{k+1}}, \ldots, m^9_{i_k} - m^9_{i_{k+1}}\right) \qquad \hat{l}_k = (l_k, m^4_{i_k}, \ldots, m^9_{i_k})$$

We get 10−dimensional integrals. A dual conformal shift is still possible, defining 10−dimensional coordinates

$$\hat{x}^\mu_i := x^\mu_i \qquad \hat{x}^J_i := m^J_i$$

→ preserved dual conformal symmetry

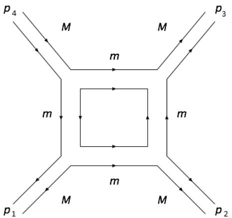
Mass configuration

Defining a mass vector $\vec{\alpha}_i = \left(m_i^4, \ldots, m_i^9\right) = \|\vec{\alpha}_i\| \, \vec{n}_i$, VEV can be written as

$$\left\langle \vec{\Phi} \right\rangle = diag\left(\|\vec{\alpha}_1\| \, \vec{n}_1, \|\vec{\alpha}_2\| \, \vec{n}_2, \ldots\right)$$

Internal propagators have squared mass $\|\vec{\alpha}_i\|^2$, while external legs have a $(\vec{\alpha}_i - \vec{\alpha}_{i-1})^2$ squared mass. Let's impose a two mass configuration by

$$\begin{cases} \|\vec{\alpha}_1\|^2 = \|\vec{\alpha}_3\|^2 = m^2 \\ \|\vec{\alpha}_2\|^2 = \|\vec{\alpha}_4\|^2 = M^2 \end{cases} \qquad \text{with } M^2 \gg m^2 \qquad s, t \sim M^2$$



Extracting $\Gamma_{cusp}(\phi, \theta)$

$\Gamma_{cusp}(\phi)$ can be also expressed as a

$$\Gamma_{cusp}(x, \xi) \quad \text{where} \;\; x = e^{i\phi} \;\; \text{and} \;\; \xi = \frac{\cos\phi - 1}{\sin\phi}$$

$\Gamma_{cusp}(\phi, \theta)$ can be obtained from $\Gamma_{cusp}(\phi)$ with this substitution

$$\xi \to \frac{\cos\phi - \cos\theta}{\sin\phi}$$

Example (1 loop)

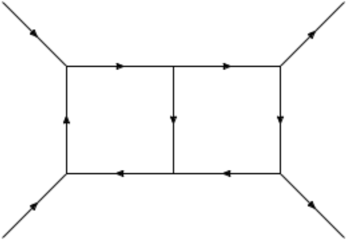
$$I(s, t, m, M) = \left[1 + \frac{2M^2}{t}(1 - \cos\theta)\right] I(s, t)$$

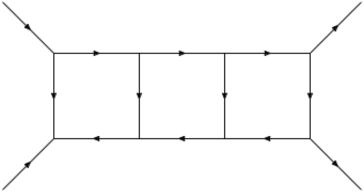
Now, $\cos\phi = \frac{p_2 \cdot p_3}{\sqrt{(p_2)^2 (p_3)^2}} \to t \sim -2M^2(1 - \cos\phi)$. Moreover,

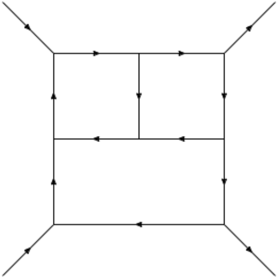
it's known that one loop $\Gamma^{(1)}_{cusp}(\phi) \sim \ln u \, \xi \ln x$, so

$$\Gamma^{(1)}_{cusp}(\phi, \theta) = \frac{\cos\phi - \cos\theta}{\sin\phi} \ln u \ln x$$

Beyond one loop







Physical meaning

Higgsed $\mathcal{N} = 4$ Super Yang-Mills theory describes massive W-bosons interacting by exchange of massless gauge fields from the unbroken part of the gauge group.

Interaction is attractive $\rightarrow$ bound states

At weak coupling, hydrogen-like states (Wick-Cutkowski $SO(4,2)$ hydrogen atom model)

Dual conformal symmetry $\rightarrow$ spectrum organizes into $SO(4,2)$ multiplets

Exploiting Regge theory, spectrum can be calculated (*Caron-Huot, Henn; 2015*)

From scattering amplitudes to bound states

Regge theory connects bound states energy and spin according to

$$j(s_n) + 1 = n \quad \text{for} \;\; s = E_n^2$$

where n is the principal quantum number $(n = 1, 2, \ldots)$ and $j(s)$ describes spin.

$j(s)$ is linked to a scattering amplitude $\mathcal{M}$ (when $t \to \infty$ at $s < 0$ fixed) according to

$$\mathcal{M} \sim t^{j(s)+1}$$

$\Gamma_{cusp}(\phi)$ and bound states

Due to dual conformal symmetry ($\mathcal{M}$ in function of t only through $v = \frac{M^2}{t}$), $t \to \infty$ limit is equivalent to $M \to 0$. In this limit

$$\mathcal{M} \sim M^{\Gamma_{cusp}}$$

Linking the two results

$$j(s) + 1 = -\Gamma_{cusp}(\phi) \qquad \text{with} \;\; s = 4m^2 sin^2 \frac{\phi}{2}$$

so, to get the spectrum, we have to solve

$$-\Gamma_{cusp}(\phi) = n$$

with

$$E_n^2 = 4m^2 sin^2 \frac{\phi}{2}$$

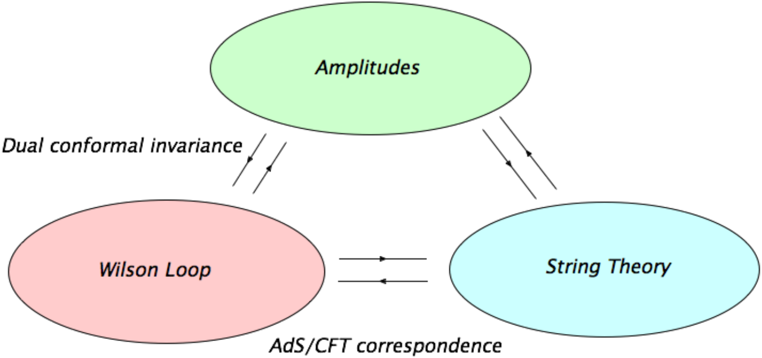
Low energy spectrum for $\Gamma_{cusp}(\phi, \theta)$

From previous amplitudes calculations, the spectrum of the low energy theory can be derived

$$E_n = 2M - \frac{M\lambda^2}{64\pi^2 n^2}(1 - \cos\theta)^2 + O\left(\lambda^2\right)$$

We notice that the effect produces a splitting of the energy levels.

Exploiting AdS/CFT correspondence



Strong coupling spectrum

In order to get the spectrum, we have to consider

$$\Gamma_{cusp}(\phi, \theta) \sim \sqrt{\lambda} F(\phi, \theta) \to -F(\phi, \theta) = -\frac{n}{\sqrt{\lambda}}$$

Two different limits can be taken

$$\begin{cases} \sqrt{\lambda} \gg n & \text{(small quantum number)} \\ \sqrt{\lambda} \ll n & \text{(large quantum number)} \end{cases}$$

Small quantum number limit

Near BPS limit of Γ_{cusp} $(\theta = \pm\phi)$ (Drukker, Forini; 2011 & Correa, Henn, Maldacena, Sever; 2012)

$$\Gamma_{cusp}\left(\phi,\theta\right) = -\frac{\sqrt{\lambda}}{4\pi^2}\frac{\phi^2-\theta^2}{\sqrt{1-\frac{\phi^2}{\pi^2}}} + O\left(\left(\phi^2-\theta^2\right)^2\right)$$

For the spectrum, we obtain

$$E \simeq 2m\sin\frac{\theta}{2} + 2m\frac{n\pi}{\sqrt{\lambda}}\sqrt{\frac{\pi^2}{\theta^2}-1}\cos\frac{\theta}{2}$$

with $\theta \neq 0, \pi$.

Consistency check

For BPS limit, we have ($\theta \to 0$)

$$\text{from } J + 1 = n \to 1 \ll J \ll \sqrt{\lambda}$$

so

$$\begin{cases} \phi \approx 2\pi \frac{\sqrt{J}}{\lambda^{1/4}} \\ E \sim 2\pi m \frac{\sqrt{J}}{\lambda^{1/4}} \end{cases}$$

$$E \simeq \frac{2\pi m}{\lambda^{1/4}} \sqrt{J} + \theta^2 \frac{m}{2\pi} \frac{\lambda^{1/4}}{\sqrt{J}}$$

The first term is consistent with holographic description of semi-classical rotating strings (Kruczenski, Mateos, Myers, Winters; 2003).

Large quantum number limit

"Antiparallel lines" limit of Γ_{cusp} ($\phi \sim \pi$ and $\theta \neq \pi$)

$$\Gamma_{cusp}\left(\phi, \theta\right) = -\frac{2}{\pi\left(\pi - \phi\right)} \frac{\left(\mathbb{E}\left(k^2\right) - \left(1 - k^2\right)\mathbb{K}\left(k^2\right)\right)^2}{k\sqrt{1 - k^2}}$$

(Drukker, Forini; 2011) where $\mathbb{E}$ and $\mathbb{K}$ are elliptic integrals of the first and second kind and

$$\theta = 2\sqrt{1 - 2k^2\mathbb{K}\left(k^2\right)}$$

$$E \simeq 2m - \frac{\lambda}{n^2}\frac{m\gamma^2\left(\theta\right)}{4}$$

where

$$\gamma\left(\theta\right) = \frac{2}{\pi}\frac{\left(\mathbb{E}\left(k^2\right) - \left(1 - k^2\right)\mathbb{K}\left(k^2\right)\right)^2}{k\sqrt{1 - k^2}}$$

Conclusions

- these results have also been checked numerically

- results for $\theta = 0$ are consistent with already known AdS/CFT calculations of rotating strings

- there are no results for a model at $\theta \neq 0$: a rotation not only in AdS but also in the S^5 may be useful

- general symmetry breaking regularization may be extended to other conformal theories (for instance, ABJ(M))