Symmetries on Every Black Hole Horizon

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Presentation Plan

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Introduction

Symmetries on Every Black Hole Horizon

Non Stationary Case

Conserved charges

Classical Central Extensions

Conclusions and Future Developments

The BH Information Paradox

It is known that

- In $D = 4$ there are *no-hair theorems* for the BH. (S. Hawking, G. Ellis, 1973)
- Every BH emits Hawking radiation. (V. Frolov, I. Novikov, 1998)

We have the

Information Paradox

A pure quantum state $|\Psi\rangle$ defined before the formation of the BH evolves in a termal quantum state $|X\rangle$ violating unitarity.

The computation is *semiclassical* but the paradox is well defined in such regime. (S. Mathur, 2009)

If there is an infinite dimensional asymptotic symmetry group at the horizon with generators ξ on $\mathcal{H}$ with associated charge $Q_\xi = 0$ classically, we can have at a quantum level

$$Q_\xi |M\rangle = |M'\rangle \neq |M\rangle$$

because

$$0 = \langle M|Q_\xi|M\rangle \Rightarrow 0 = \langle M|M'\rangle \Rightarrow |M\rangle \neq |M'\rangle$$

So with infinite asymptotic symmetries on $\mathcal{H}$ we can distinguish between classically degenerate quantum states.

Soft BH Hair

The No-Hair theorems are evaded at a quantum level.

The HPS proposal is formulated only for Schwarzschild BH in $D = 4$ in an asymptotically flat spacetime (S. Hawking, M. j. Perry, A. Strominger, 2016).

The HPS proposal is based on the existence of ASG on both the horizon and the null infinity $\mathscr{I}^+$ of the asymptotically flat spacetimes.
The symmetry group of $\mathscr{I}^+$ is called BMS by Bondi, van Burg, Metzner and Sachs (H. Bondi, M. v. d. Burg, A. Metzner, 1961; R. Sachs. 1961). Its peculiarity is to be infinite dimensional with non trivial generator called supertranslation given by

$$\xi = f(\{\theta\})\frac{\partial}{\partial u} + \dots$$

It was recently associated to the pure general relativistic effect called Gravitational Memory Effect and to the pure quantum mechanical theorem as the Weinberg's soft graviton theorem (A. Strominger, A. Zhiboedov, 2014).
It is also important for the HPS proposal described previously.

What we Know

About the Asymptotic Symmetries at the Horizon

Carlip first (S. Carlip,1998, 1999) and Koga then (J.i. Koga, 2001) studied the near horizon symmetries of Black Holes in order to compute by a micro-state counting the Entropy.

It is known that Schwarzschild BH possess a BMS - like ASG on the Horizon generated by the so called *supertranslations* whose classical central extension is vanishing.

No micro-state counting is possible using Cardy's formula.

Donnay *et al.* (L. Donnay, G. Giribet, H. A. Gonzalez and M. Pino, 2015), extend such result for BTZ and Kerr, weakening also the boundary conditions in order to get also the *superrotations* at the horizon.

A General Statement about the Existence of ASG

It is easy to find ASG on Every BH; Defining also the *Strong* and the *Weak* Condition we can also define the ASG and the modified $\widetilde{\text{ASG}}$.

- Strong:

$$\delta g_{vv} = \mathcal{O}(r),\ \delta g_{vA} = \mathcal{O}(r),\ \delta g_{rr} = 0,$$
$$\delta g_{rA} = 0, \qquad \delta g_{vr} = 0, \quad \delta g_{AB} = \mathcal{O}(r).$$

- Weak:

$$\delta g_{vv} = \mathcal{O}(r),\ \delta g_{vA} = \mathcal{O}(r),\ \delta g_{rr} = 0,$$
$$\delta g_{rA} = 0, \qquad \delta g_{vr} = 0, \quad \delta g_{AB} = \mathcal{O}(1).$$

We firstly need the introduction of Gaussian Null Coordinates for non extremal BH by which the Near Horizon Metric is (H. Friedrich, I. Racz, R. Wald, 1998; I. Booth, 2012)

$$ds^2 = +2dvdr + q_{AB}d\theta^A d\theta^B$$
$$-2\kappa(\{\theta\})rdv^2 + 2rh_A(\{\theta\})dvd\theta^A + \mathcal{O}(r^2),$$

A General Statement about the Existence of ASG

Existence of ASG

Every stationary spacetime $(\mathcal{M}, g_{\mu\nu})$ with a null Horizon $\mathcal{H}$ generated by the vector k whose topology is a sphere, has:

1. *An* $\mathrm{ASG} = SO(3,1) \ltimes ST$ *if we impose the Strong asymptotic conditions with generator*

$$\xi^\mu = f k^\mu - r g^{\mu\nu} \nabla_\nu f + \mathcal{O}\left(r^2\right),$$

2. *An* $\widetilde{\mathrm{ASG}} = SR \ltimes ST$ *if we impose the Weak asymptotic conditions with additional generator*

$$\zeta = R(\{\theta\}).$$

A General Statement about the Existence of ASG

Differences with the previous results

1. We found that every Horizon in any dimension has ASG, not only BHs. In the weak sense, we obtain a larger group $\widetilde{\mathrm{ASG}}$ which includes superrotations at the Horizon.
2. Also the function defining the supertranslation has a $v-$dependence:
$$f = f(v, \theta^A).$$
3. The result is obtained in every spacetime dimension $D > 3$.
4. Also there is no need to have precise Equations of Motion; ASG is a property of the Horizon no matter the theory is.

The ASG algebra

We can compute the algebra as

$$\{V, W\} = \mathcal{L}_V W.$$

We obtain that

Asymptotic Symmetry Algebra

$$\{\xi_1, \xi_2\} = \xi[f_{12}]$$
$$\{\xi_1, \zeta_{\tilde{2}}\} = -\xi[F_{1\tilde{2}}]$$
$$\{\zeta_1, \zeta_2\} = \zeta[\tilde{f}_{\tilde{1}\tilde{2}}]$$

where

$$f_{12} = f_1 \partial_v f_2 - f_2 \partial_v f_1,\ F_{1\tilde{2}} = R^a \partial_a f,\ \tilde{f}_{\tilde{1}\tilde{2}} = R_1^b \partial_b R_2^a - R_2^b \partial_b R_1^a$$

The peculiarities of Non-Stationary Spacetimes

The Vaidya Spacetime

The Gaussian Null coordinates expansion could be done also for non-stationary spacetime. In that case we have:

$$\begin{aligned} ds^2 = & 2dvdr + q_{AB}(v,\{\theta\})d\theta^A d\theta^B - 2\kappa(v,\{\theta\})r dv^2 \\ & + 2r h_A(v,\{\theta\})dv d\theta^A + r p_{AB}(v,\{\theta\})d\theta^A d\theta^B + \mathcal{O}(r^2) \end{aligned}$$

We of course obtain the same $\widetilde{\mathrm{ASG}}$ but we must be careful:
In non-stationary spacetimes the usual Horizon is expanding (or contracting). e.g: Vaidya Spacetime.

In the Vaidya Spacetime

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dvdr + r^2 d^2\Omega$$

the Event Horizon in not located at $r = 2m(v)$ (which is not a null surface, but a FOTH). We are in presence of Dynamical Horizon (and not an Isolated Horizon)

Of course we can implement the Gaussian null coordinates set; but around which surface are we expanding? We can perform a coordinate transformation $r \to r - f(v)$ and solve perturbatively in terms of $\dot{m}$ which gives

$$r = 2m(v)\left(1 + 4\dot{m} + \mathcal{O}(\dot{m}^2)\right)$$

the same result order $\mathcal{O}(\dot{m})$ of a constant area change expanding null surface (A. Nielsen, 2014)

Conserved charges and Complementarity

Derivation

We can use both *Wald covariant formulation of the conserved charges* (Iyer, Wald, 1994; Wald, Zoupas, 1999) and the *Brown and York quasi-local stress energy tensor formalism* (Brown, York, 1992) to compute the conserved charges at the horizon.

We advocate here some *complementarity* principle. The conserved charges at the horizon are related to the ones at infinity.

The charges are

$$ST: \quad H[g-\bar{g};\xi] = -\frac{\kappa_0}{2\pi}\frac{1}{4}\int d\Omega\, f(v,\theta,\varphi).$$

$$SR: \quad H[g-\bar{g};\zeta] = +\frac{1}{16\pi}\int d\Omega\, h_A(\{\theta\})\zeta^A(\{\theta\})$$

We can expand (in $D = 4$ for convenience) the functions defining the ASG generators and obtain

$$T_{lm} \equiv H[g - \bar{g}; \xi_{lm}] = \delta_{l,0}\delta_{m,0} \, \frac{\kappa_0}{2\pi} \frac{\mathcal{A}}{4} f_{lm}(v)$$

$$J_n \equiv H[g - \bar{g}; \zeta_n] = \delta_{n,0} \, \mathcal{J}$$

We have re-obtained the usual conserved charges associated to the generators, enforcing the Complementarity assumption.

We can use the Wald formalism as well the Barnich - Brandt formalism (G. Barnich, F. Brandt, 2001) in order to compute the central extension for the General Relativity.

The computation done in all formalism gives of course the same results; No background central extension is found for the BMS-like subgroup of ASG.

This means that

No CC for BMS-like ASG

$$K[\xi, \eta] = 0, \quad \forall \xi, \eta.$$

Which is the same result found previously in the literature.

Conclusions

Our Results

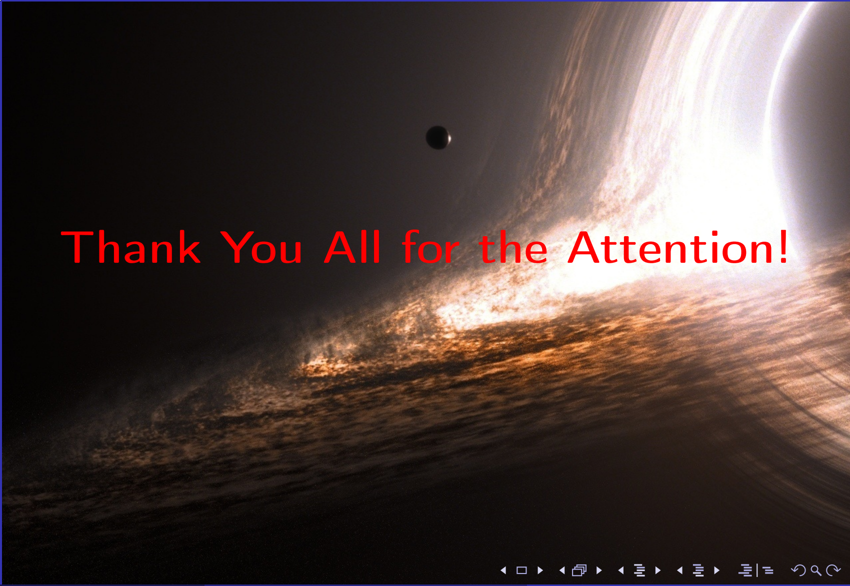
1. We found that every Null Horizon which admits a near-horizon expansion in Gaussian Null Coordinates possess an Asymptotically Symmetry Group which is infinite - dimensional and shares some similarities with the BMS group of the Null Infinity, but have also differences, for example the $v-$dependence of the supertranslation.
2. The computed conserved charges reproduce the usual charges associated to BHs and we proposed a Complementarity interpretation.
3. There are no central extensions for the BMS-like subgroup of the ASG.

Future Developments

Open questions

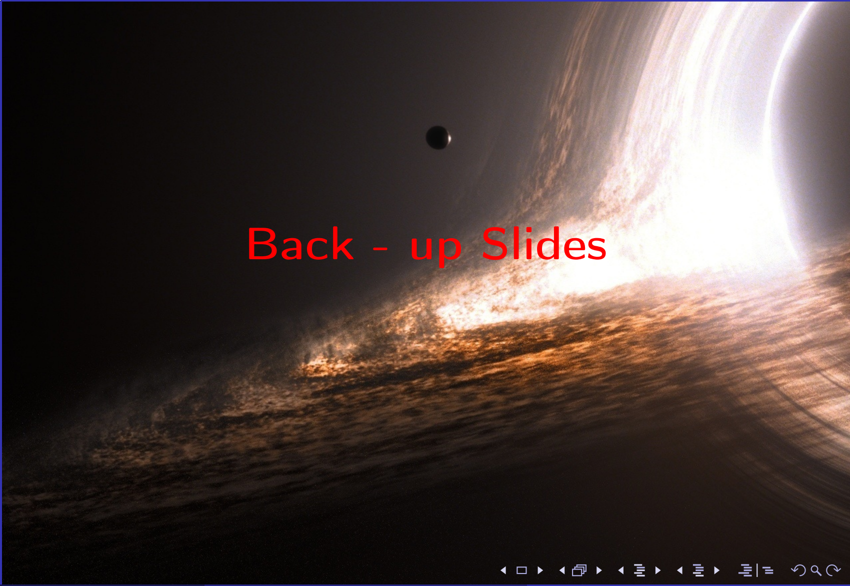
- Why vanishing c but non-vanishing S?
- Different allowed horizon deformations brings non-vanishing c?
- What happens in different topologies?
- We need a stronger infinity - horizon relation? (HPS)
- What happens in BPS solutions?
- Has consequences in the membrane paradigm formalism for the BH Horizon? (C. Eling, Y. Oz, 2016)
- Has consequences in the Large D expansion?

The End.



Thank You All for the Attention!

The End.



The ASG algebra

We can compute the algebra as

$$\{V, W\} = \mathcal{L}_V W.$$

We can expand the generators in Fourier mode by expanding the functions defining them We can expand (in $D = 3$ for convenience) the functions

$$f(v, \varphi) = \sum_{l=0}^{\infty} f_m(v)\, e^{i\tilde{n}\varphi}, \quad \tilde{f}(\varphi) = \sum_{n=-\infty}^{+\infty} \tilde{f}_n\, e^{i\tilde{n}\varphi}$$

and we found two classical Virasoro's algebras entwined

$$\begin{aligned} i\{t_{n,\tilde{n}}, t_{m,\tilde{m}}\} &= (n-m) t_{n+m,\tilde{n}+\tilde{m}}, \\ i\{y_{\tilde{n}}, y_{\tilde{m}}\} &= (\tilde{n}-\tilde{m}) y_{\tilde{n}+\tilde{m}}, \\ i\{t_{n,\tilde{m}}, y_{\tilde{m}}\} &= -\tilde{n}\, t_{n,\tilde{n}+\tilde{m}}. \end{aligned}$$

From the Brown - York charge we can compute the central extensions by means of the variation

$$\{H[\bar{g};\xi], H[\bar{g};\eta]\} + K[\xi,\eta] = \delta_\xi H^{BY}[\bar{g};\eta] - \delta_\eta H^{BY}[\bar{g};\xi] \quad (1)$$

by this the expression for the central extension is

$$K[\xi,\eta] = \int d\Omega \, \left[\left(\bar{\nabla}_\sigma \xi^\sigma\right) \tau_{\mu\nu} n^\mu \eta^\nu + \left(\mathcal{L}_\xi \tau_{\mu\nu}\right) n^\mu \eta^\nu + \tau_{\mu\nu} \left(\mathcal{L}_\xi n^\mu\right) \eta^\nu - (\xi \leftrightarrow \eta) \right] \quad (2)$$

We can do the same in the Wald formalism and obtain the expression for the central charge found by Koga

$$K[\xi_1,\xi_2] = -\frac{1}{8\pi}\int_{\partial\mathcal{H}} \sqrt{-g}\,\varepsilon_{\mu_1\ldots\mu_{D_2}\alpha\beta}\,C^{\alpha\beta}[\xi_1,\xi_2] \tag{3}$$

with

$$\begin{aligned} C^{\alpha\beta}[\xi_1,\xi_2] = \bar{R}^{\alpha\beta}{}_{\rho\sigma}\xi_1^\rho\xi_2^\sigma & \\ -\bar{g}^{\rho\sigma}(\bar{\nabla}_\rho\xi_1^\alpha\bar{\nabla}_\sigma\xi_2^\beta - \bar{\nabla}_\rho\xi_2^\alpha\bar{\nabla}_\sigma\xi_1^\beta) & \\ +(\bar{\nabla}^\alpha\xi_2^\beta\,\bar{\nabla}_\sigma\xi_1^\sigma - \bar{\nabla}^\alpha\xi_1^\beta\,\bar{\nabla}_\sigma\xi_2^\sigma) & \end{aligned} \tag{4}$$

In the work by Barnich and Brandt (Barnich, Brandt, 2001) appears the expression for the Classical Central Extension in GR

$$K[\xi,\eta] = \frac{1}{8\pi}\int k_\xi[\bar{g};\mathcal{L}_\eta\bar{g}] \tag{5}$$

where

$$k_\xi[\bar{g};h] = \left(d^{D-2}x\right)_{\nu\mu}\sqrt{-\bar{g}}\Big[\xi^\nu\bar{\nabla}^\mu h - \xi^\nu\bar{\nabla}_\sigma h^{\sigma\mu} + \frac{1}{2}h\bar{\nabla}^\nu\xi^\mu - h^{\nu\sigma}\bar{\nabla}_\sigma\xi^\mu\Big] \tag{6}$$

But if we consider the non-BMS like part of the group, i.e. supertranslations can depends on the $v-$coordinate, we have a non-vanishing central charge

$$K[\xi_1, \zeta_2] = -\frac{1}{2G}\int_{B_{v_0}} d\Omega\, \partial_v f_1(v, \{\theta\})\partial_A R^A(\{\theta\}).$$

which cannot be re-absorbed in a redefinition of the conserved charges by a shift since the ASG algebra is non-abelian;

But, if we limit the ASG group to the surface gravity-preserving subgroup, we have to solve a differential equation $\delta_\xi g_{vv} = \mathcal{O}(r^2)$ (instead of $\mathcal{O}(r)$) which impose the f to be $f = f(\{\theta\})e^{-\kappa_0 v}$ and in this case $\{\xi_1, \xi_2\} = 0$, so the cc can be re-absorbed.

The Classical Central Extensions in GR

A not-well-understood preliminary result

Unless we consider possible $v-$deformations of the algebra, in the sense that $\partial_v f \neq 0$ so in that case, we can compute, after the Background shift, two non vanishing central extensions

$$
\begin{aligned}
K[\xi_1, \xi_2] &= 0, \\
K[\xi_1, \zeta_2] &= -\frac{\kappa_0}{2G} \int d\varphi \left(f_1(v,\varphi) \partial_\varphi \tilde{f}_2(\varphi) \right), \\
K[\zeta_1, \zeta_2] &= -\frac{J}{4G} \int d\varphi \left(\tilde{f}_1 \partial_\varphi \tilde{f}_2 - \tilde{f}_2 \partial_\varphi \tilde{f}_1 \right).
\end{aligned}
$$

where we have shown the results for the BTZ case, the simplest. In FT the extended algebra becomes

$$
\begin{aligned}
i\{T_{n,\tilde{n}}, T_{m,\tilde{m}}\} &= (n-m) T_{n+m,\tilde{n}+\tilde{m}}, \\
i\{T_{n,\tilde{m}}, Y_{\tilde{m}}\} &= -\tilde{n}\, T_{n,\tilde{n}+\tilde{m}} \qquad -\frac{\kappa}{2G} \tilde{n}\, \delta_{\tilde{m}+\tilde{n},0}, \\
i\{Y_{\tilde{n}}, Y_{\tilde{m}}\} &= (\tilde{n}-\tilde{m}) Y_{\tilde{n}+\tilde{m}} \qquad -\frac{J}{2G} \tilde{n}\, \delta_{\tilde{m}+\tilde{n},0}.
\end{aligned}
$$

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Algebra, again

Central Extension, again

Central Extension: computations

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Symmetries on Every Black Hole Horizon

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Algebra, again

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Central Extension: computations

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