Hidden Gauge Structure of Supersymmetric Free Differential Algebras

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Based on the work with Laura Andrianopoli and Riccardo D'Auria

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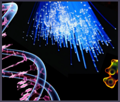

Introduction

Supergravity theories in $4 \leq D \leq 11$

$\downarrow$

Metric + 1-forms + $(p+1)$-form gauge potentials, $p \leq 9$

Free Differential Algebras (FDA's)

FDA's can be described in terms of hidden superalgebras
(SUSY $\rightarrow$ Subalgebra)

- Also required from Superstring theories $\rightarrow$ Higher form potentials are related to the NS-NS and R-R sectors of the different Superstring theories

Introduction

$D = 11$ SUGRA

[E. Cremmer, B. Julia and J. Scherk, Phys. Lett. B **76** (1978) 409]

- Metric $g_{\mu\nu}$
- AS tensor $A_{\mu\nu\rho}$, $\mu, \nu, ... = 0, 1, ..., D-1$
- Single Majorana gravitino Ψ_μ

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[R. D'Auria and P. Fré, "Geometric Supergravity in $d = 11$ and Its Hidden Supergroup", Nucl. Phys. B **201** (1982)]

Non-trivial SUSY FDA

- Supervielbein (V^a, Ψ)
- 3-form potential $A^{(3)}$ $\rightarrow$ Field-strength $F^{(4)} = dA^{(3)} + ...$
- Hodge-dual $F^{(7)} =^* F^{(4)} \rightarrow 6$-form potential $A^{(6)}$
  SUSY closure $\Rightarrow F^{(7)} = dA^{(6)} - 15 A^{(3)} \wedge F^{(4)} + ...$

[R. D'Auria and P. Fré, "Geometric Supergravity in $d = 11$ and Its Hidden Supergroup", Nucl. Phys. B **201** (1982)]

- Geometric approach, FDA investigated
- Existence of a hidden superalgebra $\rightarrow$ (almost-central) bosonic generators Z_{ab}, $Z_{a_1 \dots a_5}$, $a, b, \dots = 0, 1, \dots, 10$, nilpotent fermionic generator Q'

$$\{Q, Q\} = iC\Gamma^a P_a + iC\Gamma^{ab} Z_{ab} + iC\Gamma^{a_1 \dots a_5} Z_{a_1 \dots a_5}$$
$$\{Q', Q'\} = 0$$
$$[Q, P_a] = i\Gamma_a Q'$$
$$[Q, Z_{ab}] \propto \Gamma_{ab} Q'$$
$$[Q, Z_{a_1 \dots a_5}] \propto \Gamma_{a_1 \dots a_5} Q'$$

Introduction

[C. M. Hull and P. K. Townsend, "Unity of superstring dualities", Nucl. Phys. B **438** (1995)]
[P. K. Townsend, "P-brane democracy", hep-th/9507048]

- Z_{ab}, $Z_{a_1 \dots a_5}$ understood as p-brane charges, sources of dual potentials $A^{(3)}$, $A^{(6)}$

$$\{Q, Q\} = iC\Gamma^a P_a + iC\Gamma^{ab} Z_{ab} + iC\Gamma^{a_1 \dots a_5} Z_{a_1 \dots a_5}$$

- Generalization of SUSY algebra in higher dimensions in the presence of non-trivial topological extended sources $\rightarrow$ black p-branes

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But... What about Q'?

Introduction

We argue that...

Q' is a nilpotent topological generator required for the closure of the SUSY FDA written in terms of 1-forms

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We develop our proposal through...

Minimal $D = 7$, $\mathcal{N} = 2$ SUGRA $\rightarrow$ FDA, 1-forms

- Rich gauge structure
- 2 extra nilpotent fermionic generators required in the fully extended hidden superalgebra
- Vacuum $\rightarrow$ Topological structure

Outlines

- Review of $D = 11$ hidden superalgebra
- The hidden gauge algebra of minimal $D = 7$ FDA
- Relation with $D = 11$ Supergravity
- Gauge symmetries $\rightarrow$ Physical interpretation of ξ_μ and η_μ
- Conclusions and Outlook

$D = 11$ hidden superalgebra

SUSY FDA

SUSY FDA which defines the ground state of the theory

$$R^{ab} \equiv d\omega^{ab} - \frac{1}{2}\omega^{ac} \wedge \omega^{bd}\eta_{cd} = 0$$

$$T^a \equiv DV^a - \frac{i}{2}\overline{\psi} \wedge \Gamma^a \psi = 0$$

$$\rho \equiv D\Psi = 0$$

$$F^{(4)} \equiv dA^{(3)} - \frac{1}{2}\overline{\psi} \wedge \Gamma^{ab}\psi \wedge V_a \wedge V_b = 0$$

$$F^{(7)} \equiv dB^{(6)} - 15A^{(3)} \wedge dA^{(3)} - \frac{i}{2}\overline{\psi} \wedge \Gamma_{a_1 \dots a_5}\psi \wedge V^{a_1} \dots V^{a_5} = 0$$

FDA invariant under two gauge transformations

$$\begin{cases} \delta A^{(3)} = d\Lambda^{(2)} \\ \delta B^{(6)} = 15\,\Lambda^{(2)} \wedge dA^{(3)} \end{cases} \qquad \left\{ \; \delta B^{(6)} = d\Lambda^{(5)} \right.$$

$D = 11$ hidden superalgebra

[R. D'Auria and P. Fré, "Geometric Supergravity in $d = 11$ and Its Hidden Supergroup", Nucl. Phys. B **201** (1982)]

- Trade the FDA with an ordinary super Lie algebra $\rightarrow$ Cartan form in terms of 1-form gauge fields
- Disclose the extended superalgebra hidden in the FDA
- $A^{(3)} \rightarrow B^{ab}$, $B^{(6)} \rightarrow B^{a_1 \dots a_5}$, in the antisymmetric representations of $SO(1,10)$
- Extra spinor 1-form η

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Conditions

$$DB^{ab} = \frac{1}{2} \overline{\psi} \wedge \Gamma^{ab} \psi$$

$$DB^{a_1 \ldots a_5} = \frac{i}{2} \overline{\psi} \wedge \Gamma^{a_1 \ldots a_5} \psi$$

$$D\eta = iE_1 \Gamma^a \psi \wedge V_a + E_2 \Gamma_{ab} \psi \wedge B^{ab} + iE_3 \Gamma_{a_1 \ldots a_5} \psi \wedge B^{a_1 \ldots a_5}$$

$D = 11$ hidden superalgebra

Ansatz for the 3-form in terms of 1-forms

$$A^{(3)} = T_0 B^{ab} \wedge V_a \wedge V_b + T_1 B_{ab} \wedge B^b{}_c \wedge B^{ca} +$$
$$+ T_2 B_{b_1 a_1 \dots a_4} \wedge B^{b_1}{}_{b_2} \wedge B^{b_2 a_1 \dots a_4} +$$
$$+ T_3 \epsilon_{a_1 \dots a_5 b_1 \dots b_5 m} B^{a_1 \dots a_5} \wedge B^{b_1 \dots b_5} \wedge V^m +$$
$$+ T_4 \epsilon_{m_1 \dots m_6 n_1 \dots n_5} B^{m_1 m_2 m_3 p_1 p_2} \wedge B^{m_4 m_5 m_6 p_1 p_2} \wedge B^{n_1 \dots n_5} +$$
$$+ i S_1 \overline{\psi} \wedge \Gamma^a \eta \wedge V_a + S_2 \overline{\psi} \wedge \Gamma^{ab} \eta \wedge B_{ab} +$$
$$+ i S_3 \overline{\psi} \wedge \Gamma^{a_1 \dots a_5} \eta \wedge B_{a_1 \dots a_5}$$

$D = 11$ hidden superalgebra

Ansatz for the 3-form in terms of 1-forms

$$
\begin{aligned}
A^{(3)} = & T_0 B^{ab} \wedge V_a \wedge V_b + T_1 B_{ab} \wedge B^b{}_c \wedge B^{ca} + \\
& + T_2 B_{b_1 a_1 \dots a_4} \wedge B^{b_1}{}_{b_2} \wedge B^{b_2 a_1 \dots a_4} + \\
& + T_3 \epsilon_{a_1 \dots a_5 b_1 \dots b_5 m} B^{a_1 \dots a_5} \wedge B^{b_1 \dots b_5} \wedge V^m + \\
& + T_4 \epsilon_{m_1 \dots m_6 n_1 \dots n_5} B^{m_1 m_2 m_3 p_1 p_2} \wedge B^{m_4 m_5 m_6 p_1 p_2} \wedge B^{n_1 \dots n_5} + \\
& + i S_1 \overline{\psi} \wedge \Gamma^a \eta \wedge V_a + S_2 \overline{\psi} \wedge \Gamma^{ab} \eta \wedge B_{ab} + \\
& + i S_3 \overline{\psi} \wedge \Gamma^{a_1 \dots a_5} \eta \wedge B_{a_1 \dots a_5}
\end{aligned}
$$

- The requirement $dA_{(3)} = \frac{1}{2} \overline{\psi} \Gamma^{ab} \psi V_a V_b$ (at zero curvatures) fixes T_i, S_j in terms of E_1, E_2, E_3
- The consistency of the theory (closure of theory) also required the d^2 closure (trivial for the bosonic 1-forms) $\Rightarrow$ On η: $E_1 + 10 E_2 - 720 E_3 = 0$

Solutions depending on one parameter E

Map between 1-forms and generators in $D = 11$

$$V^a(P_b) = \delta^a_b, \quad \Psi(Q) = \mathbb{I}, \quad B^{ab}(Z_{cd}) = \frac{1}{2}\delta^{ab}_{cd}$$
$$B^{a_1\dots a_5}(Z_{b_1\dots b_5}) = \frac{1}{5!}\delta^{a_1\dots a_5}_{b_1\dots b_5}, \quad \eta(Q') = \mathbb{I}$$

$D = 11$ hidden superalgebra

$$\{Q, Q\} = -i\Gamma^a P_a - \frac{1}{2}\Gamma^{ab} Z_{ab} - \frac{i}{5!}\Gamma^{a_1\dots a_5} Z_{a_1\dots a_5}$$
$$\{Q', Q'\} = 0$$
$$[Q, P^a] = 20i(1 - 72E)\Gamma^a Q'$$
$$[Q, Z^{ab}] = -4\Gamma^{ab} Q'$$
$$[Q, Z^{a_1\dots a_5}] = -2(5!)iE\Gamma^{a_1\dots a_5} Q'$$

Hidden superalgebra of $D = 7, \mathcal{N} = 2$

Minimal $D = 7$ SUGRA $\rightarrow$ Rich structure

Physical content

- V^a
- Triplet of vectors A^x ($x = 1, 2, 3$)
- 2-form $B^{(2)}$
- Gravitino field $\rightarrow$ pseudo-Majorana fields $\psi_{A\mu}$ ($A = 1, 2$), where $\overline{\psi}^A = \epsilon^{AB}(\psi_B)^T$
- 3-form $B^{(3)}$ (field strengths Hodge-dual to that of $B^{(2)}$)
- Triplet of 4-forms $A^{x|(4)}$ (field strengths Hodge-dual to the gauge vectors A^x)

Hidden superalgebra of $D = 7, \mathcal{N} = 2$

FDA

$$
\begin{aligned}
& d\omega^{ab} = \omega^{ac} \wedge \omega^{bd} \eta_{cd} \\
& DV^a = \frac{i}{2} \overline{\psi}^A \wedge \Gamma^a \psi_A \\
& D\psi = 0 \\
& dA^x = \frac{i}{2} \sigma^{x|B}{}_A \overline{\psi}^A \wedge \psi_B \\
& dB^{(2)} = -dA^x A^x + \frac{i}{2} \overline{\psi}^A \wedge \Gamma_a \psi_A \wedge V^a \\
& dB^{(3)} = \frac{1}{2} \overline{\psi}^A \wedge \Gamma_{ab} \psi_A \wedge V^a \wedge V^b \\
& dA^{x|(4)} = -\frac{1}{2} \left(dA^x B^{(3)} + A^x \wedge dB^{(3)} \right) + \\
& \quad + \frac{1}{6} \sigma^{x|B}{}_A \overline{\psi}^A \wedge \Gamma_{abc} \psi_B V^a \wedge V^b \wedge V^c
\end{aligned}
$$

Hidden superalgebra of $D=7, \mathcal{N}=2$

1-forms introduced for finding the hidden superalgebra

1-forms

- $B^a \to B^{(2)}$
- $B^{ab} \to B^{(3)}$
- $A^x{}_{abc} \to A^{x|(4)}$

Conditions

$$DB^{ab} = \frac{1}{2}\overline{\psi}^A \wedge \Gamma^{ab}\psi_A$$

$$DB^a = \frac{i}{2}\overline{\psi}^A \wedge \Gamma^a \psi_A$$

$$DA^{x|abc} = \frac{1}{6}\sigma^{x|B}{}_A \overline{\psi}^A \wedge \Gamma^{abc}\psi_B$$

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$$DA^{x|abc} = \frac{1}{6}\sigma^{x|B}{}_A\overline{\psi}^A \wedge \Gamma^{abc}\psi_B$$

- The integrability ($\to$ closure) of the parametrizations of $B^{(2)}$ and $B^{(3)}$ will also require 2 fermionic 1-forms, η_A and ξ_A

$$D\eta_A = l_1\Gamma_a\psi_A \wedge V^a + l_2\Gamma_a\psi_A \wedge B^a + l_3\Gamma_{ab}\psi_A \wedge B^{ab} + \\ + l_4\psi_B\sigma^{x|B}{}_A \wedge A^x + l_5\Gamma_{abc}\psi_B\sigma^{x|B}{}_A \wedge A^{x|abc}$$

$$D\xi_A = e_1\Gamma_a\psi_A \wedge V^a + e_2\Gamma_a\psi_A \wedge B^a + e_3\Gamma_{ab}\psi_A \wedge B^{ab} + \\ + e_4\psi_B\sigma^{x|B}{}_A \wedge A^x + e_5\Gamma_{abc}\psi_B\sigma^{x|B}{}_A \wedge A^{x|abc}$$

Hidden superalgebra of $D=7, \mathcal{N}=2$

Ansatz for the 2 and 3-forms in terms of 1-forms

$$
\begin{aligned}
B^{(2)} =& \sigma B_a \wedge V^a + \tau \overline{\psi}^A \wedge \eta_A \\
B^{(3)} =& \tau_0 B_{ab} \wedge V^a \wedge V^b + \tau_1 B_{ab} \wedge B^a V^b + \tau_2 B_{ab} \wedge B^a B^b + \\
& + \tau_3 B_{ab} \wedge B^{bc} \wedge B_c{}^a + \\
& + \epsilon_{ab_1 \dots b_3 c_1 \dots c_3} (\tau_4 V^a + \tau_5 B^a) \wedge A^{x|b_1 \dots b_3} \wedge A^x{}_{c_1 \dots c_3} + \\
& + \tau_6 B_{ab} \wedge A^x{}_{acd} \wedge A^{x|bcd} + \tau_7 \epsilon_{xyz} A^x \wedge A^y{}_{abc} \wedge A^{z|abc} + \\
& + \tau_8 \epsilon_{xyz} A^x \wedge A^y \wedge A^z + \\
& + \tau_9 \epsilon_{xyz} \epsilon_{abcdlmn} A^{x|abc} \wedge A^{y|dlp} \wedge A^{z|mn}{}_p + \\
& + \sigma_1 \overline{\psi}^A \wedge \Gamma_a \xi_A \wedge V^a + \sigma_2 \overline{\psi}^A \wedge \Gamma_a \xi_A \wedge B^a + \\
& + \sigma_3 \overline{\psi}^A \wedge \Gamma_{ab} \xi_A \wedge B^{ab} + \\
& + \sigma_4 \overline{\psi}^A \wedge \xi_B \sigma^{x|B}{}_A \wedge A^x + \sigma_5 \overline{\psi}^A \wedge \Gamma_{abc} \xi_B \sigma^{x|B}{}_A \wedge A^{x|abc}
\end{aligned}
$$

Hidden superalgebra of $D = 7, \mathcal{N} = 2$

- Set of 1-forms: $\sigma^{\Lambda} = \{V^a,\ \psi_A,\ B^a,\ B^{ab},\ A^{x|abc},\ \xi_A,\ \eta_A\}$
- Set of generators: $T_{\Lambda} = \{P_a,\ Q^A,\ Z_a,\ Z_{ab},\ T_{x|abc},\ Q'^A,\ Q''^A\}$

Hidden contributions to the superalgebra (we need 2 spinors)

$$\{Q^A, \overline{Q}_B\} = -i\Gamma_a \left(P^a + Z^a\right)\delta^A_B - \frac{1}{2}\Gamma_{ab}Z^{ab}\delta^A_B + - \sigma^{x|A}{}_B\left(iT^x + \frac{1}{18}\Gamma^{abc}T^x_{abc}\right)$$

$$[Q_A, P^a] = -2\Gamma^a(e_1 Q'_A + l_1 Q''_A)$$

$$[Q_A, Z^a] = -2\Gamma^a(e_2 Q'_A + l_2 Q''_A)$$

$$[Q_A, Z^{ab}] = -4e_3\Gamma^a Q'_A$$

$$[Q_A, T^x] = -2\sigma^{x|B}{}_A(e_4 Q'_B + l_4 Q''_B)$$

$$[Q_A, T^{x|abc}] = -12 e_5 \Gamma^{abc}\sigma^{x|B}{}_A Q'_B$$

Hidden superalgebra of $D = 7, \mathcal{N} = 2$

Lagrangian subalgebras

$\exists$ subalgebras that we can define with 1 spinor

- $Q'_A = Q''_A = \frac{1}{2}\hat{Q}_A$ ($l_3 = l_5 = 0 \Rightarrow e_3 = e_5 = 0$), NOT $B^{(3)}$
- $Q'_A \to 0$ (vanishing of $\{l_i\}$), NOT $B^{(2)}$, we can set $e_2 = 0$

Hidden contributions to the Lagrangian subalgebra
($Q'_A = Q''_A = \frac{1}{2}\hat{Q}_A$, $\eta_A = \xi_A$)

$$\{Q^A, \overline{Q}_B\} = -i\Gamma_a \left(P^a + Z^a\right) \delta^A_B - i\sigma^{x|A}{}_B T^x$$

$$[Q_A, P^a] = -2\Gamma^a e_1 \hat{Q}_A$$

$$[Q_A, Z^a] = -2\Gamma^a e_2 \hat{Q}_A$$

$$[Q_A, T^x] = -2e_4 \sigma^{x|B}{}_A \hat{Q}_B$$

Relation with $D = 11$ SUGRA

- If we add $B^{a_1 \dots a_5}$ $\Rightarrow$ Same procedure and similar results (2 spinors required)

BUT

- Now we are also able to link $11D$ and $7D$ performing

DIMENSIONAL REDUCTION $11D \to 7D$

$$
\begin{aligned}
A^{(3)} &\to B^{(3)} + A^x \wedge J^x_{ij} + V^i \wedge V^j \\
B^{(6)} &\to B^{(6)} + A^x_{(4)} \wedge J^x_{ij} + V^i \wedge V^j - 8B^{(2)} \wedge \Omega^{(4)}
\end{aligned}
$$

- We get simple algebraic relations between $\{e_i\}$ and $\{E_j\}$

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- We get simple algebraic relations between $\{e_i\}$ and $\{E_j\}$
- If $B^{a_1 \dots a_5} = \frac{1}{2} B^{ab} \epsilon_{a_1 \dots a_5 ab}$

$$E_1 = e_1 = 0$$

is the only solution that survives

Gauge symmetries in $D = 7$

$D = 7$ FDA is invariant under the gauge transformations

$$\delta_0 \quad : \quad \begin{cases} \delta_0 A^x = d\Lambda^x \\ \delta_0 B^{(2)} = -\Lambda^x dA^x \\ \delta_0 A^{x|(4)} = -\frac{1}{2}\Lambda^x dB^{(3)} \end{cases}$$

$$\delta_1 \quad : \quad \begin{cases} \delta_1 B^{(2)} = d\Lambda^{(1)} \end{cases}$$

$$\delta_2 \quad : \quad \begin{cases} \delta_2 B^{(3)} = d\Lambda^{(2)} \\ \delta_2 A^{x|(4)} = -\frac{1}{2}\Lambda^{(2)} \wedge dA^x \\ \delta_2 A^{(6)} = 15\Lambda^{(2)} \wedge dB^{(3)} \end{cases}$$

$$\delta_3 \quad : \quad \begin{cases} \delta_3 A^{x|(4)} = d\Lambda^{x|(3)} \end{cases}$$

$$\delta_5 \quad : \quad \begin{cases} \delta_5 A^{(6)} = d\Lambda^{(5)} \end{cases}$$

We need to project η_A and ξ_A on the fermionic direction of superspace in order to trivialize the gauge structure of the FDA

Gauge symmetries in $D = 7$

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$$\delta_1 \quad : \quad \left\{ \delta_1 B^{(2)} = d\Lambda^{(1)} \right.$$

$$\delta_2 \quad : \quad \begin{cases} \delta_2 B^{(3)} = d\Lambda^{(2)} \\ \delta_2 A^{x|(4)} = -\frac{1}{2}\Lambda^{(2)} \wedge dA^x \\ \delta_2 A^{(6)} = 15\Lambda^{(2)} \wedge dB^{(3)} \end{cases}$$

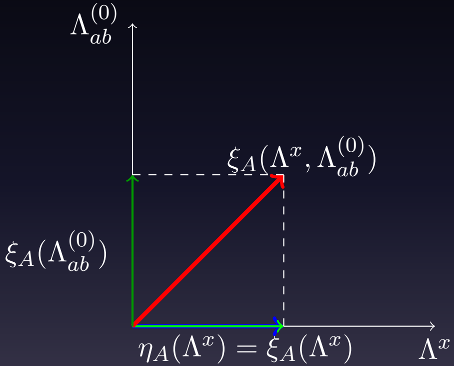
$$\delta_3 \quad : \quad \left\{ \delta_3 A^{x|(4)} = d\Lambda^{x|(3)} \right.$$

$$\delta_5 \quad : \quad \left\{ \delta_5 A^{(6)} = d\Lambda^{(5)} \right.$$

Ansatz: $\Lambda^{(2)} = \Lambda^{(0)}_{ab} V^a \wedge V^b, \quad \delta B_{ab} = d\Lambda^{(0)}_{ab}$

Gauge symmetries in $D = 7$

Without including $B^{a_1 \dots a_5}$ (with $B^{a_1 \dots a_5} \to \eta_A(\Lambda^{(0)}_{ab}) \neq 0$)



$$\eta_A(\Lambda^{(0)}_{ab}) = 0$$

$$\delta\eta_A = -l_4 \sigma^{x|B}{}_A \Lambda^x \psi_B, \; \delta\xi_A = -e_4 \sigma^{x|B}{}_A \Lambda^x \psi_B - e_3 \Lambda^{(0)}_{ab} \Gamma^{ab} \psi_A$$

Conclusions and Outlook

Physical interpretation of ξ_μ and η_μ (Q'_A and Q''_A) in $D = 7$

- Nilpotent topological generators $\rightarrow$ Necessary when the algebra includes 1-forms associated to extended forms with non-trivial topology

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- Required for the closure of the FDA when the gauge invariances involve non-trivial transformations

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- We need them in order to write the FDA in terms of 1-forms $\rightsquigarrow$ Hidden superalgebra

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Physical interpretation of ξ_μ and η_μ (Q'_A and Q''_A) in $D = 7$

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Future work

- How many spinors in $D = 11$? (We argue 1)
- What happens outside the vacuum (interaction theory)?
- What happens with cosmological constant?

Thank you!