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Luca Santoni[a]

Inflation with Weakly Broken Galileon Symmetry

in collaboration with
D. Pirtskhalava[b], E. Trincherini[a], F. Vernizzi[c]

[a] Scuola Normale Superiore e INFN, Pisa
[b] Institut de Théorie des Phénomènes Physiques, EPFL Lausanne
[c] Institut de Physique Théorique, Université Paris Saclay, CEA, CNRS

[arXiv:1505.00007], [arXiv:1506.06750], [arXiv:1511.01817].

Outline

The standard Big Bang model and inflation

The Galileon symmetry

The notion of Weakly Broken Galileon (WBG) symmetry

The Effective Field Theory of inflation

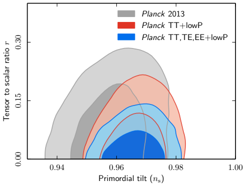
Inflation with WBG symmetry

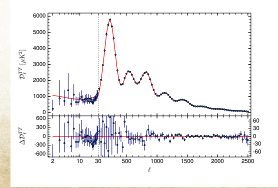
Conclusions

The standard Big Bang model and inflation

The Big Bang model. In the last decades, experiments have provided the following picture of the Universe:

- it is expanding and, in particular, accelerating;
- it appears very homogeneous and isotropic;
- it is extremely flat;
- primordial perturbations, imprinted on the CMB ($\delta T/T \sim 10^{-5}$), are *almost*
  - scale-invariant ($n_s = 0.968 \pm 0.006$),
  - adiabatic ($|\alpha_{\text{non-adi}}| \lesssim 10^{-2}$),
  - Gaussian ($f_{\text{NL}}^{\text{equil}} = -4 \pm 43$);
- no tensor modes ($r < 0.07$, 95% CL).



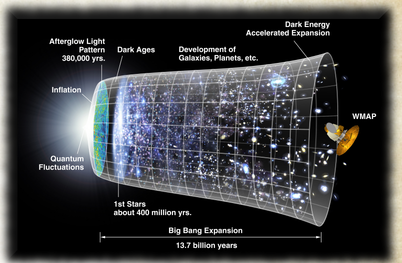


The standard Big Bang model and inflation

Inflation consists in an accelerated expansion at early times,

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad \ddot{a} > 0,$$

and provides an explanation of the observed features and of the origin of the density perturbations, as seeds for the large-scale structure formation in the Universe.



Many models of inflation. In terms of a single scalar field:

- $\mathcal{L} = (\partial\phi)^2 - V(\phi)$, with a flat $V(\phi)$ (slow-roll inflation);
- $\mathcal{L} = P\left((\partial\phi)^2, \phi\right)$ (DBI, *k*-inflation);
- …
- …

The Galileon symmetry

What about higher derivative operators? Higher derivative operators are not easy to deal with. Usually, either

- they are irrelevant (for instance, in the effective field theory of inflation[†], which can parametrize all single-field models in a single framework, they are generically expected to be negligible with respect to lower derivative operators)

or

- lead to instabilities,

unless some symmetry protects the theory.
This is the case of the Galileon symmetry:

$$\phi \to \phi + c + b_\mu x^\mu .$$

Irrespective of inflation, this is of some theoretical interest. Before considering its consequences in cosmology, let's study its main properties.

[†] See next slides.

Galileons in flat space-time

[Nicolis, Rattazzi, Trincherini '09]

What are the lowest order operators in a 4-*d* flat space-time invariant under $\phi \to \phi + c + b_\mu x^\mu$? Apart from constant terms and total derivatives, the kinetic operator

$$\mathcal{L}_2 = (\partial\phi)^2$$

is clearly invariant. Moreover, one finds for instance

$$\mathcal{L}_3 = (\partial\phi)^2 \Box\phi \,.$$

A generic Galileon theory

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + c_3 \frac{(\partial\phi)^2 \Box\phi}{\Lambda_3^3} + \ldots$$

satisfies some remarkable properties:

- the coefficient c_3 is not renormalized (non-renormalization theorem) [Luty, Porrati, Rattazzi '03];
- second order equations of motion.

Weakly Broken Galileon symmetry: a simple example in flat space-time

It turns out to be interesting to break slightly† the Galileon symmetry. Let's introduce a *small* breaking ($\Lambda_2 \gg \Lambda_3$) of the form

$$\mathcal{L}' = -\frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^2\Box\phi}{\Lambda_3^3} + \frac{(\partial\phi)^4}{\Lambda_2^4}$$

- We expect symmetry breaking operators of the form $(\partial\phi)^{2n}$ to be generated by quantum corrections.
- All the symmetry breaking operators are generated at a scale which is parametrically higher than Λ_2.
  $\Rightarrow$ *The operator* $(\partial\phi)^4$ *gets only small corrections through loop effects*.
  This is the remnant of the non-renormalization theorem.

†This is not a mere exercise, but it is necessary once gravity is turned on. Indeed, the Galileon symmetry can only be defined in a flat space-time, being explicitly broken by gravity.

Coupling to gravity

As anticipated, gravity explicitly breaks the Galileon symmetry:

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^2\Box\phi}{\Lambda_3^3}\right], \qquad \Box = \nabla_\mu\nabla^\mu .$$

- As in the previous example, the symmetry breaking operators $(\partial\phi)^{2n}$ are quantum mechanically generated.
- In particular, *the smallest scale by which the operators* $(\partial\phi)^{2n}$ *are suppressed is* $(M_{\rm Pl}\Lambda_3^3)^{1/4}$.

Therefore, we define a WBG theory to be of the form

$$\mathcal{L}^{\rm WBG} = \sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 + \frac{(\partial\phi)^2\Box\phi}{\Lambda_3^3} + \frac{(\partial\phi)^4}{\Lambda_2^4}\right],$$

where $\Lambda_2 \equiv (M_{\rm Pl}\Lambda_3^3)^{1/4}$.
Non-renormalization theorem. *Loop corrections are suppressed by powers of* $\frac{\Lambda_3}{M_{\rm Pl}}$.

The most general WBG theory

Introducing the other Galileon interactions, a further generalization (in Horndeski class), that does not spoil the properties, is still possible:

$$\mathcal{L}_2^{\rm WBG} = \Lambda_2^4 G_2(X)$$

$$\mathcal{L}_3^{\rm WBG} = \frac{\Lambda_2^4}{\Lambda_3^3} G_3(X)\Box\phi$$

$$\mathcal{L}_4^{\rm WBG} = \frac{\Lambda_2^8}{\Lambda_3^6} G_4(X) R + 2\frac{\Lambda_2^4}{\Lambda_3^6} G_{4X}(X)\left([\Phi]^2 - [\Phi^2]\right)$$

$$\mathcal{L}_5^{\rm WBG} = \frac{\Lambda_2^8}{\Lambda_3^9} G_5(X) G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{\Lambda_2^4}{3\Lambda_3^9} G_{5X}\left([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3]\right)$$

where $X \equiv -\frac{(\partial\phi)^2}{\Lambda_2^4}$, $[\Phi^2] = \nabla^\mu\nabla_\nu\phi\nabla^\nu\nabla_\mu\phi$, ...

Remark. The coefficients of the polynomials $G_i(X)$ are not renormalized up to some powers of $\frac{\Lambda_3}{M_{\rm Pl}}$ (WBG symmetry).

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

WBG theories in Cosmology: de Sitter solutions

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

Let's study the consequences of the (approximate) Galileon symmetry for the cosmological scalar fields:

$$\phi \to \phi + c + b_\mu x^\mu .$$

An inflationary theory with WBG symmetry can be written as

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{1}{2} M_{\mathrm{Pl}}^2 R - V(\phi) + \sum_{i=2}^{5} \mathcal{L}_i^{\mathrm{WBG}} \right] .$$

From the equations of motion one infers that:

- if $V \equiv 0$, exact linear solution $\phi \sim t$: the acceleration is driven by the derivative operators;
- if $V \neq 0$, slight deviation from de Sitter ($|\dot{H}| \ll H^2$): the evolution is of *slow-roll* type.

Remark. In both cases *quantum corrections are fully under control*.

The Effective Field Theory of inflation

[Creminelli, Luty, Nicolis, Senatore '06], [Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore, '08]

Parametrizing all single-field models, irrespective of the underlying microscopic theory, the effective theory of inflation turns out to be a useful framework to study cosmological perturbations.

General idea. The inflationary phase has to be connected to a standard decelerated evolution, inducing a privileged slicing. In ADM variables:

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + \gamma_{ij}(N^i \mathrm{d}t + \mathrm{d}x^i)(N^j \mathrm{d}t + \mathrm{d}x^j)\,.$$

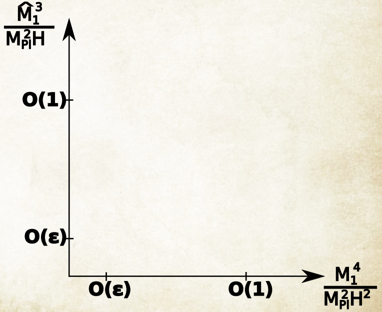
Fixing the gauge $\delta\phi(t,\vec{x}) = 0$, we write down the most general Lagrangian compatible with the residual symmetries, $x^i \to x^i + \xi^i(t,\vec{x})$:

$$S = \int \mathrm{d}^4x \sqrt{-g}\Big[\mathcal{L}_{\mathrm{EH}} - \frac{M_{\mathrm{Pl}}^2 \dot{H}}{N^2} - M_{\mathrm{Pl}}^2(3H^2 + \dot{H}) + M_1^4 \delta N^2 - \hat{M}_1^3 \delta K \delta N + \ldots\Big]$$

The Effective Field Theory of inflation

$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_{\text{back}} + M_1^4 \delta N^2 - \hat{M}_1^3 \delta K \delta N + \dots \right].$$

- One expects that physical observables are predominantly determined by the operators with the least number of derivatives.
- How can a theory with equally dominant δN^2 and $\delta K \delta N$ be consistent, without exiting the regime of validity of the effective theory?



The EFT of inflation with WBG symmetry

[Pirtskhalava, L.S., Trincherini '15]

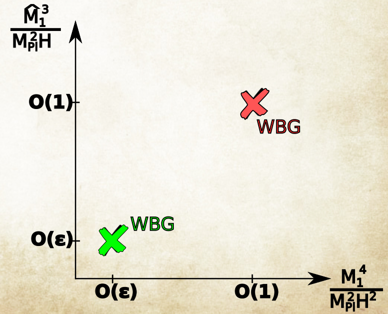
$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_{\rm back} + M_1^4 \delta N^2 - \hat{M}_1^3 \delta K \delta N + \ldots \right].$$

$$M_1^4 = \Lambda_2^4 (2X^2 G_{2XX} + \ldots) \qquad \hat{M}_1^3 H = \Lambda_2^4 (-2XZG_{3X} + \ldots)$$

- In the case of theories with WBG symmetry one can explore a wider region beyond $\hat{M}_1^3 H \ll M_1^4$. Indeed,

$$|M_1^4| \sim |\hat{M}_1^3 H| \lesssim M_{\rm Pl}^2 H^2 \,.$$

- $\delta K \delta N$ is relevant for non-Gaussianity.
- Various regimes show up.



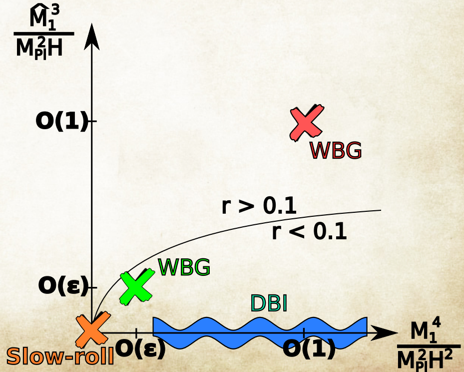
Experimental constraints on single-field inflation

[Pirtskhalava, L.S., Trincherini ’15]

A qualitative sketch of the experimental constraints on the effective parameter space and examples of fundamental theories:

$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_{\text{back}} + M_1^4 \delta N^2 - \hat{M}_1^3 \delta K \delta N + \dots \right]$$

- The bound on r alone puts strong and robust constraints on the parameter space of the effective theory.
- Further constraints are imposed by f_{NL}.



Conclusions

- The consequences of an approximate $\phi \to \phi + c + b_\mu x^\mu$ symmetry have been studied.
- We have established the notion of *weakly broken Galileon invariance*, introducing a particular non-minimal coupling to gravity and a higher scale $\Lambda_2 \equiv (M_{\rm Pl}\Lambda_3^3)^{1/4}$.
- The resulting effective theory is characterized by the technically natural hierarchy $\Lambda_2 \gg \Lambda_3$, allowing to retain the quantum non-renormalization properties of the Galileon for a broad range of physical backgrounds.
- The de Sitter solutions that these theories possess are insensitive to loop corrections.
- The accelerate expansion can be sustained by the potential or the derivative operators, leading to potentially and kinetically driven phases.
- On slowly-rolling backgrounds large non-Gaussian signals are in principle allowed and detectable, well within the validity of the effective theory.

Thank you

Backup slides

Galileon transformations

General idea. In presence of gravity we want to study the properties of the following (necessarily approximate) symmetry of some scalar fields:

$$\phi \to \phi + c + b_\mu x^\mu .$$

Why *approximate*? Couplings to gravity break the Galileon symmetry explicitly.
Purpose. Studying theories which retain as much as possible the Galileon symmetry ⇒ *Weakly Broken Galileon invariance*.

Weakly Broken Galileon symmetry: a simple example

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda_3^3}(\partial\phi)^2\Box\phi$$

- Exactly invariant under $\phi \to \phi + c + b_\mu x^\mu$.
- Λ_3 is the cutoff.

Let's introduce a *small* breaking ($\Lambda_2 \gg \Lambda_3$):

$$\mathcal{L}' = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda_3^3}(\partial\phi)^2\Box\phi + \frac{1}{\Lambda_2^4}(\partial\phi)^4$$

- We expect symmetry breaking operators of the form $(\partial\phi)^{2n}$ to be generated by quantum corrections.
- All the symmetry breaking operators are generated at a scale which is parametrically higher than Λ_2.
  $\Rightarrow$ *The operator* $(\partial\phi)^4$ *gets only small corrections through loop effects*.

Weakly Broken Galileon symmetry

What happens if we introduce gravity?
Let's generalize the simple example

$$\mathcal{L}' = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda_3^3}(\partial\phi)^2\Box\phi + \frac{1}{\Lambda_2^4}(\partial\phi)^4$$

following this guide principle:

- preserve the quantum properties (small quantum corrections).

We will find a class of theories with *Weakly Broken Galileon* (WBG) invariance. As a consequence, they will have second order equations of motion.

Foretaste. Applied to inflation, this will provide a *radiatively stable* cosmological evolution with interesting phenomenological properties.

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

Coupling to gravity

How can we couple our scalar theory to gravity? There are many ways to couple the theory to gravity...
This necessarily breaks Galileon invariance.
Let's concentrate on quantum generated operators of the form

$$(\partial\phi)^{2n}.$$

We will see that *coupling to gravity defines the sense in which the breaking can be considered weak*.

Coupling to gravity: minimal coupling

The minimally coupled Galileon theory is obtained by replacing

$$\partial_\mu \to \nabla_\mu .$$

Operators of the form

$$\frac{(\partial\phi)^{2n}}{M_{\rm Pl}^k \Lambda_3^{4n-k-4}}, \qquad k, n > 0,$$

are generated.
One can show that *the smallest scale by which such operators are suppressed is* $(M_{\rm Pl}\Lambda_3^5)^{1/6}$.

Can one do better? Can one enhance such a scale?

Coupling to gravity: non-minimal coupling

Let's consider the following non-minimally coupled Galileon theory:

$$\mathcal{L}_3^{\rm nm} = \sqrt{-g}(\partial\phi)^2[\Phi]\,,$$
$$\mathcal{L}_4^{\rm nm} = \sqrt{-g}\left[(\partial\phi)^4 R - 4(\partial\phi)^2\left([\Phi]^2 - [\Phi^2]\right)\right],$$
$$\mathcal{L}_5^{\rm nm} = \sqrt{-g}\left[(\partial\phi)^4 G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{2}{3}(\partial\phi)^2\left([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3]\right)\right].$$

One can prove that in this case only operators of the form

$$\frac{(\partial\phi)^{2n}}{M_{\rm Pl}^n \Lambda_3^{3n-4}}$$

are generated.

Result. *The smallest scale by which the operators $(\partial\phi)^{2n}$ are suppressed is $\Lambda_2 \equiv (M_{\rm Pl}\Lambda_3^3)^{1/4}$* $\Rightarrow$ WBG symmetry.

Remark. The theory turns out to have 2nd order e.o.m. also for the metric: *Covariant Galileon*. [Deffayet, Esposito-Farese, Vikman '09]

WBG symmetry in presence of a scalar potential

Let's introduce a *flat* ($\epsilon_{\rm v}, \eta_{\rm v} \ll 1$) potential $V(\phi)$:

$$\int \mathrm{d}^4x\sqrt{-g}\left[\mathcal{L}^{\rm WBG} - V(\phi)\right], \qquad V(\phi) \sim \phi^m \,.$$

Warning. Do quantum corrections to the vertices $V(\phi)\frac{h_c^n}{M_{\rm Pl}^n}$ spoil the notion of WBG symmetry of the theory? No. Indeed, for instance,

- only internal graviton lines: $\sim \left(\frac{\Lambda_3}{M_{\rm Pl}}\right)^n$;
- one internal scalar field: $\sim \frac{V'\Lambda_3^2}{VM_{\rm Pl}} \sim \sqrt{\epsilon_{\rm v}}\left(\frac{\Lambda_3}{M_{\rm Pl}}\right)^2$;
- …

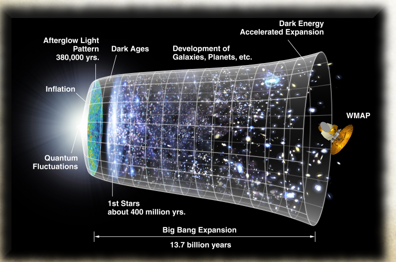
Result. These loop corrections are consistent with the notion of WBG symmetry.

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

The inflationary epoch

Big Bang puzzles:

- horizon problem;
- flatness;
- generation of density perturbations as seeds for Universe large-scale structure formation.



What we observe:

(almost)

- scale-invariant ($|n_s - 1| \sim 10^{-2}$),
- adiabatic ($|\alpha_{\text{non-adi}}| \lesssim 10^{-2}$),
- Gaussian ($f_{\text{NL}}^{\text{equil}} = -4 \pm 43$)

spectrum for density perturbations;

- no tensor modes ($r < 10^{-1}$).

Inflation, as a possible solution, with SEC violation:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad \ddot{a} > 0,$$

with approximate shift symmetry, $\phi \to \phi + c$, of the scalar field driving the background evolution, with degree of breaking $\varepsilon \equiv -\frac{\dot{H}}{H^2}$.

The inflationary epoch

Contact with observations

Experimental constraints (Planck2015+BICEP2+*Keck Array*):

- tensor-to-scalar ratio, $r \propto \frac{P_\gamma(k)}{P_s(k)}$:

$$r < 0.07 \quad (95\%\ \text{CL}) \quad ;$$

- amplitude of scalar modes, $\mathcal{A}_s$:

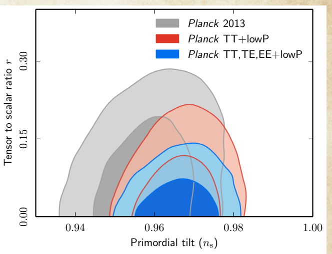
$$\delta T/T \sim 10^{-5} \ ;$$

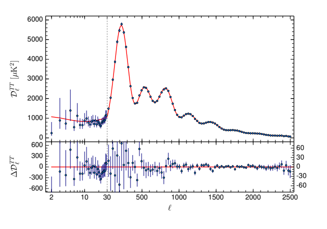
- scalar spectral index, n_s:

$$n_s = 0.968 \pm 0.006 \ ;$$

- non-Gaussianity, $f_{\rm NL}^{\rm equil} \propto \frac{B(k,k,k)}{P_s(k)^2}$:

$$f_{\rm NL}^{\rm equil} = -4 \pm 43 \ .$$





The inflationary phase

A simple action for the *inflaton*:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\rm Pl}^2 R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right].$$

Inflaton fluctuations:

$$\phi(t, \vec{x}) = \phi_0(t) + \delta\phi(t, \vec{x}).$$

Equations of motions:

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V'(\phi_0) = 0, \qquad H(t) \equiv \frac{\dot{a}(t)}{a(t)}.$$

In order to have a sufficiently long inflationary expansion, the inflaton field ϕ is assumed to slowly roll down a very flat potential:

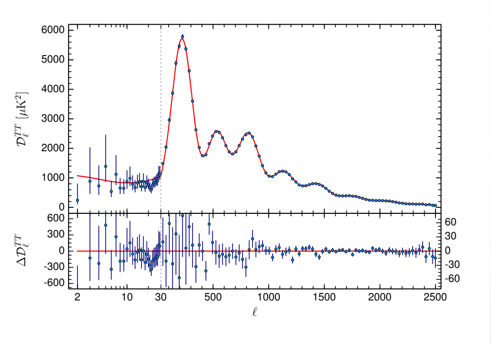
$$|\dot{H}| \ll H^2, \qquad M_{\rm Pl}|V'| \ll |V|.$$

Contact with observations: scalar modes

The inflaton fluctuations $\delta\phi$ are responsible for the CMB anisotropies. The distribution function of the scalar fluctuations is parametrized in terms of

- *power spectrum*:

$$\langle \delta\phi_{\vec{k}} \delta\phi_{\vec{k}'} \rangle = (2\pi)^3 P_s(k) \delta(\vec{k} + \vec{k}'), \quad P_s(k) k^3 \equiv \mathcal{A}_s k^{n_s - 1} \, ;$$



Contact with observations: scalar modes

- *bispectrum*:

$$\langle \delta\phi_{\vec{k}_1}\delta\phi_{\vec{k}_2}\delta\phi_{\vec{k}_3}\rangle = (2\pi)^3 B(k_1,k_2,k_3)\delta(\vec{k}_1+\vec{k}_2+\vec{k}_1)\,.$$

The bispectrum is related to the field's self-interactions. A possible detection could discriminate among different models.

Simple parametrization:

$$B(k_1,k_2,k_3) \propto f_{\rm NL}\Big[P_s(k_1)P_s(k_2) + P_s(k_1)P_s(k_3) + P_s(k_2)P_s(k_3)\Big]\,.$$

$f_{\rm NL}$ is highly suppressed in *slow-roll* models: the extreme flatness of the potential gives rise to small non-Gaussianity.

What about derivative interactions?

Observable non-Gaussianity

In a generic low-energy EFT

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{(\partial\phi)^4}{\Lambda^4} + \dots\right],$$

what is the impact of derivative contributions at the level of non-Gaussianity? One finds [Creminelli '03]

$$f_{\rm NL} \sim \frac{\dot{\phi}_0^2}{\Lambda^4}.$$

Any amount of $f_{\rm NL} \gtrsim 1$ would be out of the regime of validity of the effective theory.
It could not be trusted, unless the infinite series of derivative operators can be re-summed because of some symmetry protecting the theory against large quantum corrections, *e.g.* in DBI inflation which predicts $f_{\rm NL}^{\rm equil} \sim \frac{1}{c_s^2}$, $c_s^2 \ll 1$.

Observable non-Gaussianity

Message. Theories with weakly broken Galileon symmetry admit sub-luminal scalar perturbations within a well defined low-energy EFT, resulting in possibly large non-Gaussian deviations, $f_{\rm NL} \gg 1$.

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

Contact with observations: tensor modes

Fluctuations of the metric tensor, $g_{\mu\nu} = \bar{g}_{\mu\nu} + \gamma_{\mu\nu}$, give rise to

$$\langle \gamma^{s}_{\vec{k}} \gamma^{s'}_{\vec{k}'} \rangle = (2\pi)^3 P_\gamma(k) \delta^{ss'} \delta(\vec{k} + \vec{k}') , \qquad P_\gamma(k) = \frac{2H^2}{M_{\rm Pl}^2 k^3} .$$

- The tensor modes are much more *model independent* and *robust*. [Creminelli, Gleyzes, Noreña, Vernizzi '14]
  Conversely, scalar modes can be adjusted in many ways (shape of the potential, many scalars, speed of sound c_s, ...).
- The amplitude fixes the energy scale of inflation.

One more brief comment…

Because of the breaking of time diffeomorphisms, the unitary gauge action describes 3 d.o.f. : the 2 graviton helicities + 1 scalar mode.
(In analogy, the same happens for a spin-1 massive particle: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}A_\mu^2$)

This mode can be made explicit by performing a broken gauge transformation, $t \to t + \pi(t, \vec{x})$ (Stückelberg trick).
(For a non-Abelian gauge group: $A_\mu \to U A_\mu U^\dagger - \frac{i}{g} U \partial_\mu U^\dagger, \quad U \equiv e^{i\pi^a T^a}$)

In some cases the physics of the Goldstone decouples from the two graviton helicities at short distances, when the mixing with gravity can be neglected (decoupling limit).
(In analogy with the *equivalence theorem* for the longitudinal components of the massive gauge boson: $m \ll E \ll 4\pi m/g$)

Inflation with WBG symmetry

[Pirtskhalava, L.S., Trincherini, Vernizzi ’15]

The Friedmann equations:

$$3M_{\rm Pl}^2H^2 = V + \Lambda_2^4 X\left[\frac{1}{2} - \frac{G_2}{X} + 2G_{2X} - 6ZG_{3X} + \dots\right],$$

$$2M_{\rm Pl}^2\dot{H} = -\frac{\Lambda_2^4 X\left[1 + 2G_{2X} - 3ZG_{3X} + \dots\right]}{1 + 2G_4 - 4XG_{4X} - 2ZXG_{5X}}.$$

Background quantum stability: $X = \frac{\dot{\phi}_0^2}{\Lambda_2^4} \lesssim 1, \qquad Z \equiv \frac{H\dot{\phi}_0}{\Lambda_3^3} \lesssim 1.$

Depending on the values of X and Z, two phenomenologically distinct regimes:

- kinetically driven phase ($X \sim Z \sim 1$) $\Rightarrow$ mixing with gravity is order-one important at all scales (*i.e.* decoupling limit does not apply);
- potentially driven phase ($X \sim \sqrt{\varepsilon}$, $Z \sim 1$) with $M_{\rm Pl}^2H^2 \sim V \gg (\partial\phi)^2 \Rightarrow$ decoupling limit applies at the Hubble scale and possible large non-Gaussianity.

The Effective Field Theory of inflation

[Creminelli, Luty, Nicolis, Senatore '06], [Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore, '08]

The inflationary phase has to be connected to a standard decelerated evolution, inducing a privileged slicing. In ADM variables:

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + \gamma_{ij}(N^i \mathrm{d}t + \mathrm{d}x^i)(N^j \mathrm{d}t + \mathrm{d}x^j)\,.$$

In unitary gauge, *i.e.* $\delta\phi(t,\vec{x}) = 0$, the most general perturbative Lagrangian, invariant under $x^i \to x^i + \xi^i(t,\vec{x})$, is

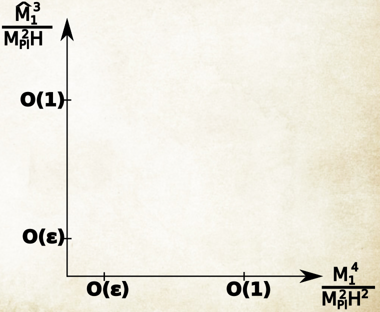
$$\begin{aligned} S = \int \mathrm{d}^4x \sqrt{\gamma} N \Bigg[& \frac{M_{\rm Pl}^2}{2}\left({}^{(3)}R + K_{\mu\nu}K^{\mu\nu} - K^2\right) - \frac{M_{\rm Pl}^2 \dot{H}}{N^2} - M_{\rm Pl}^2(3H^2 + \dot{H}) \\ & + M_1^4 \delta N^2 + M_2^4 \delta N^3 + \ldots \\ & - \hat{M}_1^3 \delta K \delta N + \hat{M}_2^3 \delta K \delta N^2 + \ldots \\ & - \bar{M}_1^2 \left(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu} - {}^{(3)}R\,\delta N\right) + \ldots \Bigg] \end{aligned}$$

The Effective Field Theory of inflation

$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_{\rm back} + M_1^4 \delta N^2 - \hat{M}_1^3 \delta K \delta N + \dots \right].$$

In other words:

- One expects to have control only over the region $\hat{M}_1^3 H \ll M_1^4$.
- Are we able to explore a wider region of the parameter space? WBG symmetry allows this…



The EFT of inflation with WBG symmetry

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

$\mathcal{L}_i^{\rm WBG}$	$M_1^4\Lambda_2^{-4}$	$\hat{M}_1^3 H\Lambda_2^{-4}$	...
$i=2$	$2X^2G_{2XX}$	$\times$	...
$i=3$	$-3XZ(G_{3X}+2XG_{3XX})$	$-2XZG_{3X}$	...
$i=4$	$12XZ^2(3G_{4XX}+\dots)$	$8XZ^2\left(\frac{G_{4X}}{X}+\dots\right)$	...
$i=5$	$XZ^3\left(3\frac{G_{5X}}{X}+\dots\right)$	$2XZ^3\left(3\frac{G_{5X}}{X}+\dots\right)$	...

- As anticipated, a *radiatively stable* hierarchy follows:

$$M_1^4 \sim \hat{M}_1^3 H.$$

- kinetically driven phase ($X \sim Z \sim 1$):

$$M_1^4 \sim M_{\rm Pl}^2 H^2, \qquad \hat{M}_1^3 \sim M_{\rm Pl}^2 H, \qquad \dots$$

- potentially driven phase ($X \sim \sqrt{\varepsilon}$, $Z \sim 1$):

$$M_1^4 \sim \varepsilon M_{\rm Pl}^2 H^2, \qquad \hat{M}_1^3 \sim \varepsilon M_{\rm Pl}^2 H, \qquad \dots$$

A more quantitative analysis

[Maldacena ’05], [Chen, Huang, Kachru, Shiu ’08], [Pirtskhalava, L.S., Trincherini ’15]

Let’s consider the effective theory

$$S = \int d^4x \sqrt{\gamma} N \left[\mathcal{L}_{\rm back} + M_1^4 \delta N^2 + M_2^4 \delta N^3 - \hat{M}_1^3 \delta K \delta N + \hat{M}_2^3 \delta K \delta N^2 \right].$$

We want to explore configurations in which the decoupling limit does not apply and the full computation is required.

- Defining the gauge $\gamma_{ij} = a(t)^2 \, e^{2\zeta(t,\vec{x})} \, \delta_{ij}$,
- solving linearly the Hamiltonian constraints,
- expanding up to the 3$^{\rm rd}$ order

yields the action for the scalar mode

$$S = \frac{1}{2} \int d^4x \, a^3 \left[\dot{\zeta}^2 - c_s^2 \frac{(\vec{\nabla}\zeta)^2}{a^2} + \mathcal{L}^{(3)} \right].$$

Constraints on single-field inflation

[Pirtskhalava, L.S., Trincherini ’15]

In terms of the dimensionless parameters

$$\alpha \equiv \frac{\hat{M}_1^3}{M_{\rm Pl}^2 H}, \quad \beta \equiv \frac{M_1^4}{M_{\rm Pl}^2 H^2}, \quad \gamma \equiv \frac{\hat{M}_2^3}{M_{\rm Pl}^2 H}, \quad \delta \equiv \frac{M_2^4}{M_{\rm Pl}^2 H^2},$$

the speed of sound is

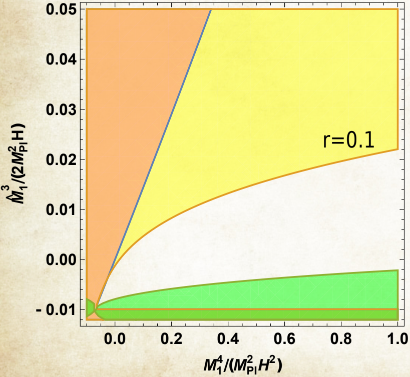
$$c_s^2 = \frac{(\varepsilon + \alpha)(1-\alpha) + \frac{\dot{\alpha}}{H}}{\varepsilon + \beta - 3\alpha(2-\alpha)}\,.$$

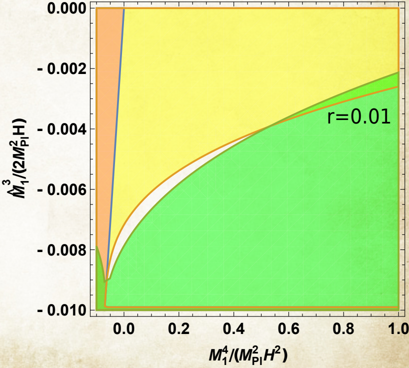
Different choices for the values of the parameters can yield a *small* speed of sound.

Constraints on single-field inflation: summary

[Pirtskhalava, L.S., Trincherini '15]

Parameters: $\varepsilon = 10^{-2}, \qquad \gamma \simeq \delta \simeq 1$





Constraints on single-field inflation: summary

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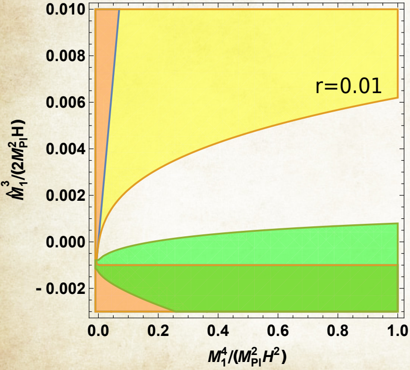

Remarks.

- DBI allowed.
- WBG inflation is allowed as well with the (stable) tuning $\alpha \lesssim 10^{-2}$, while $\beta, \gamma, \delta \lesssim 1$.
- In particular, slow-roll-WBG inflation is allowed.

Inflationary models

$$\alpha \equiv \frac{\hat{M}_1^3}{M_{\rm Pl}^2 H}, \quad \beta \equiv \frac{M_1^4}{M_{\rm Pl}^2 H^2}, \quad \gamma \equiv \frac{\hat{M}_2^3}{M_{\rm Pl}^2 H}, \quad \delta \equiv \frac{M_2^4}{M_{\rm Pl}^2 H^2}.$$

Inflationary models	Parameter hierarchy	c_s^2	Non-Gaussianity amplitude
Canonical slow-roll	$\varepsilon \ll 1; \ \ \alpha, \beta, \gamma, \delta = 0$	1	$f_{\rm NL} \sim \varepsilon$
DBI-like theories	$\alpha, \gamma = 0; \ \ \varepsilon \ll \beta \ll \delta$	$\frac{\varepsilon}{\beta}$	$f_{\rm NL} \sim \frac{1}{c_s^2}$
Galileon inflation	$\varepsilon \ll \alpha, \beta, \gamma, \delta$	$\frac{\alpha(1-\alpha)}{\beta-3\alpha(2-\alpha)}$	$f_{\rm NL} \sim \frac{1}{c_s^2}$ (or $\sim \frac{1}{c_s^4}$)
Kinetically driven WBG	$\varepsilon \ll \alpha, \beta, \gamma, \delta \sim 1$	$\frac{\alpha(1-\alpha)}{\beta-3\alpha(2-\alpha)}$	$f_{\rm NL} \sim \frac{1}{c_s^6}$ †
Potentially driven WBG	$\varepsilon \sim \alpha, \beta, \gamma, \delta$	$\frac{\varepsilon+\alpha}{\varepsilon+\beta-6\alpha}$	$f_{\rm NL} \sim \frac{1}{c_s^4}$ ‡

† Such a steep growth is ruled out by experimental constraints.
‡ Slight deformation of slow-roll inflation, but with possible large non-Gaussianity.

de Sitter space and symmetries

Inflation takes place in (an approximate) de Sitter space,

$$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec{x}^2\right) = -dt^2 + e^{2Ht}d\vec{x}^2\,,$$

whose isometry group is SO(4,1). Spatial translations and rotations are exact symmetries because of homogeneity and isotropy of the background evolution. The other de Sitter isometries are

- dilation: $\eta \to \lambda\eta, \quad \vec{x} \to \lambda\vec{x}$,
- $\eta \to \eta - 2\eta(\vec{b}\cdot\vec{x}), \quad x^i \to x^i + b^i(-\eta^2 + \vec{x}^2) - 2x^i(\vec{b}\cdot\vec{x})$.

In general, these are not symmetries of the background. However, invariance under dilation, which guarantees a scale-invariant power spectrum and constraints the 2-point function to be

$$\left\langle \delta\phi_{\vec{k}}\delta\phi_{\vec{k}'}\right\rangle = (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{F(k\eta)}{k^3}\,,$$

can be recovered by an additional $\phi \to \phi + c$ invariance. [Creminelli '12]

Potentially driven WBG inflation

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

The de Sitter phase is sustained by the potential, that satisfies the conditions

$$\varepsilon_{\rm v} \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 , \qquad |\eta_{\rm v}| \equiv M_{\rm Pl}^2\left|\frac{V''}{V}\right| ,$$

as in standard slow-roll inflation. The equations of motion are

$$3H\dot{\phi}_0 F(X,Z) \simeq -V'(\phi_0) , \qquad X = \frac{\dot{\phi}_0^2}{\Lambda_2^4} \sim \sqrt{\varepsilon} , \quad Z \sim 1 .$$

Moreover, $c_s^2 \propto \varepsilon + \alpha$: then, $\alpha \simeq -\varepsilon \Rightarrow$ strongly sub-luminal scalar perturbations. The WBG symmetry guarantees that this is respected by loop corrections.

$$\mathcal{L}^{(3)} \supset (\partial\zeta)^2\partial^2\zeta \quad \Rightarrow \quad f_{\rm NL}^{\rm equil} \sim \frac{1}{c_s^4} .$$

Summary of Part I

[Pirtskhalava, L.S., Trincherini, Vernizzi '15]

- The consequences of a theory with an approximate Galileon symmetry ($\phi \to \phi + c + b_\mu x^\mu$) in presence of gravity have been studied.
- We have introduced the notion of WBG invariance, which characterizes the unique class of couplings of such a theory to gravity that maximally retain the defining symmetry: it is the curved-space remnant of the Galileon's non-renormalization properties.

Conclusions (1)

- The consequences of an approximate $\phi \to \phi + b_\mu x^\mu$ symmetry have been studied.
- Among all possible couplings to gravity, the “more invariant” one under Galileon transformations has been chosen: this has provided the notion of *weakly broken Galileon invariance*.
- The resulting action defines a sub-class of Horndeski theories (with second order equations of motion), that one with fine quantum properties.
- The resulting effective theory is characterized by two scales (with a technically natural hierarchy $\Lambda_2 \gg \Lambda_3$), allowing to retain the quantum non-renormalization properties of the Galileon for a broad range of physical backgrounds.

Conclusions (2)

- The de Sitter solutions that these theories possess are insensitive to loop corrections.
- The accelerate expansion can be sustained by the potential or the derivative operators, leading to potentially and kinetically driven phases.
- On slowly-rolling backgrounds large non-Gaussian signals are in principle allowed and detectable, well within the validity of the effective theory.
- The theories characterized by WBG symmetry can also be applied in the context of late-time cosmic acceleration.