

# The story of a simpler universe

Enrico Pajer



**Universiteit Utrecht**



# Outline

- Introduction: how the universe just got simpler
- The conformal limit of inflation
- The origin of the adiabatic mode
- Conclusion

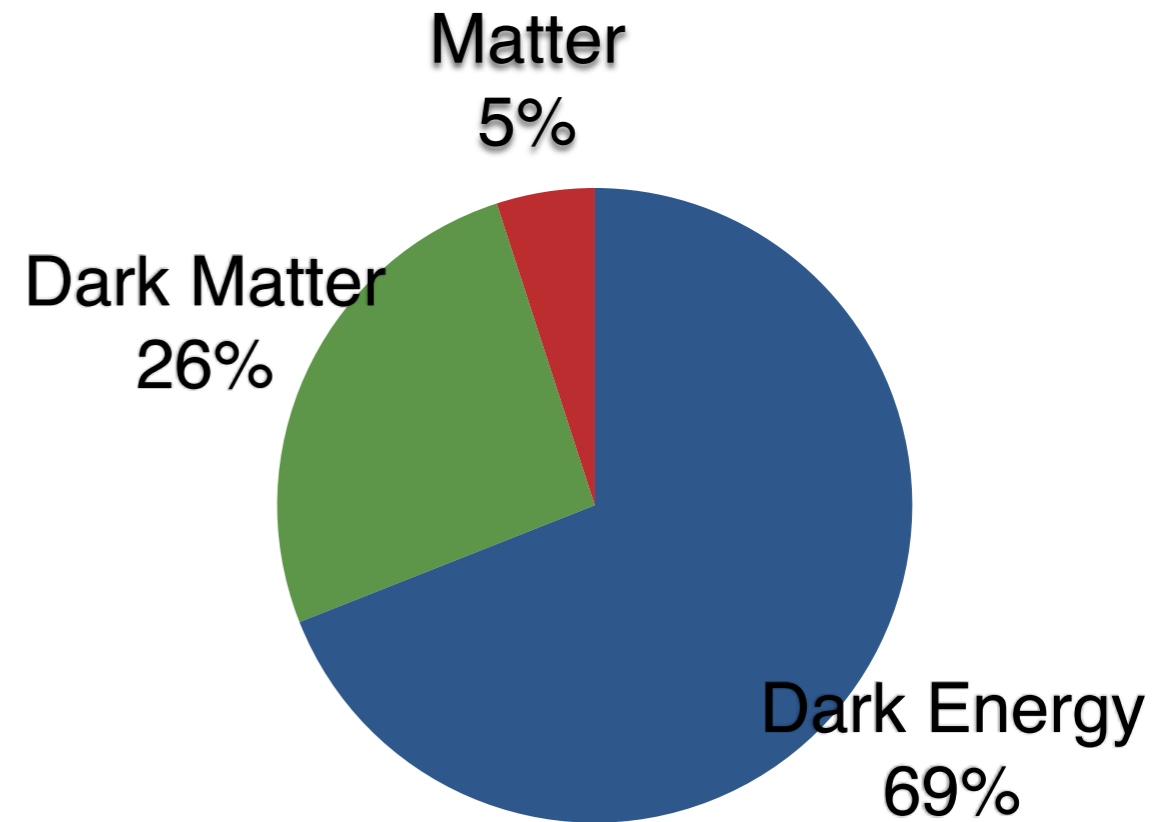


# Introduction

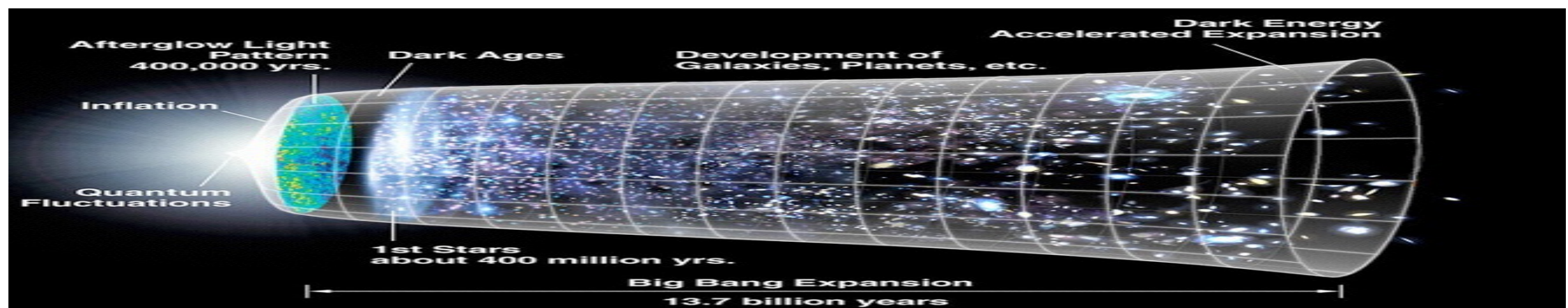
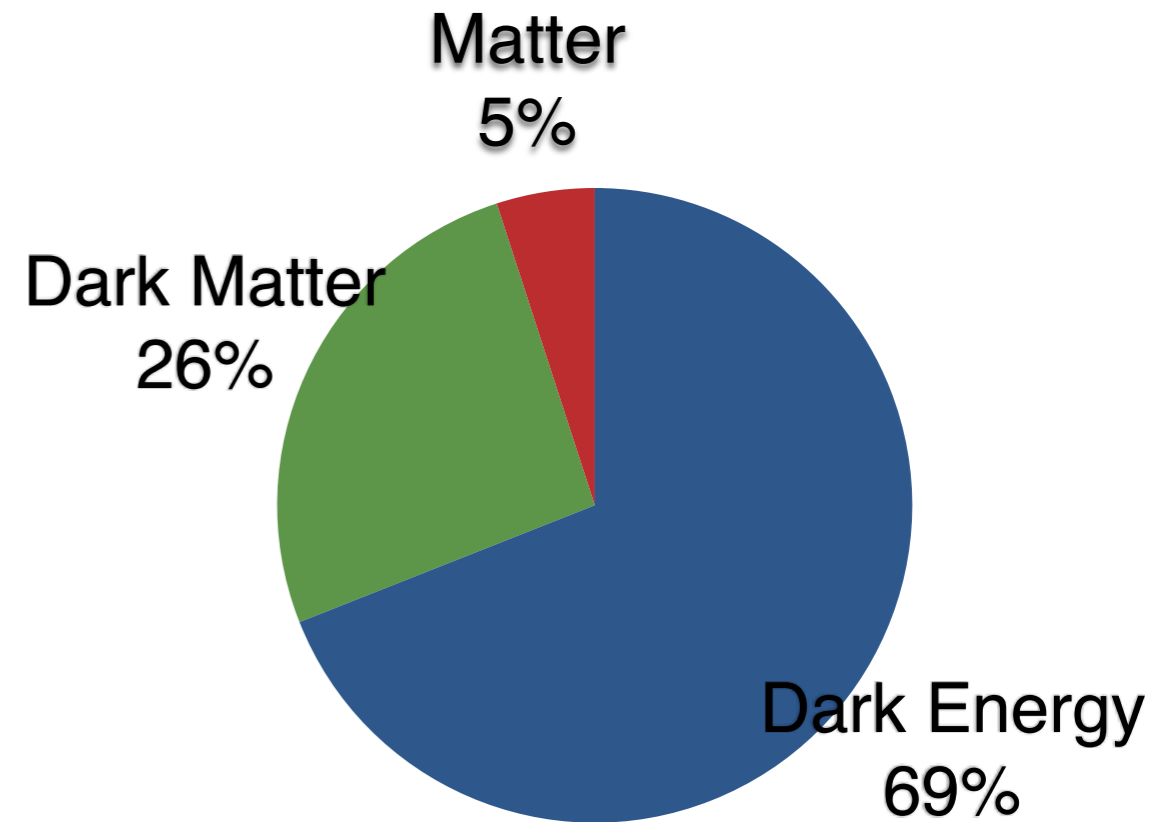
- Cosmic pies
- Cosmic pies recipe book
- No news is interesting news



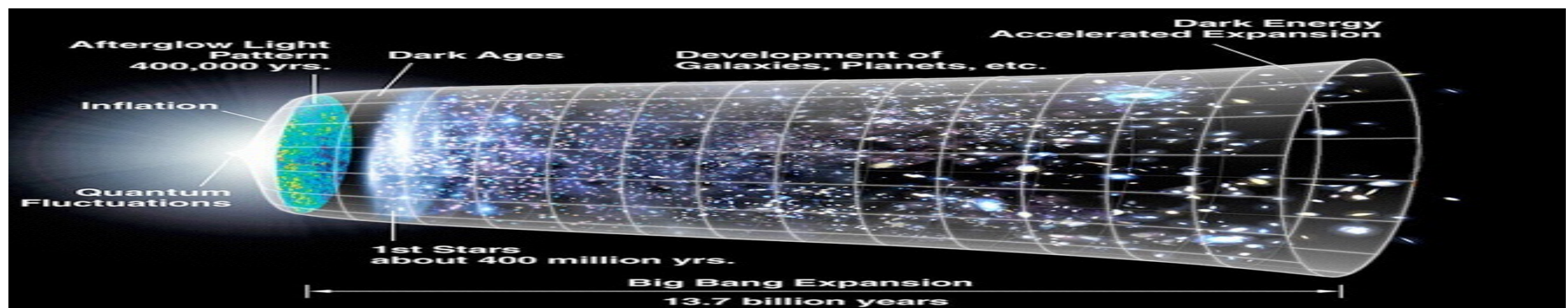
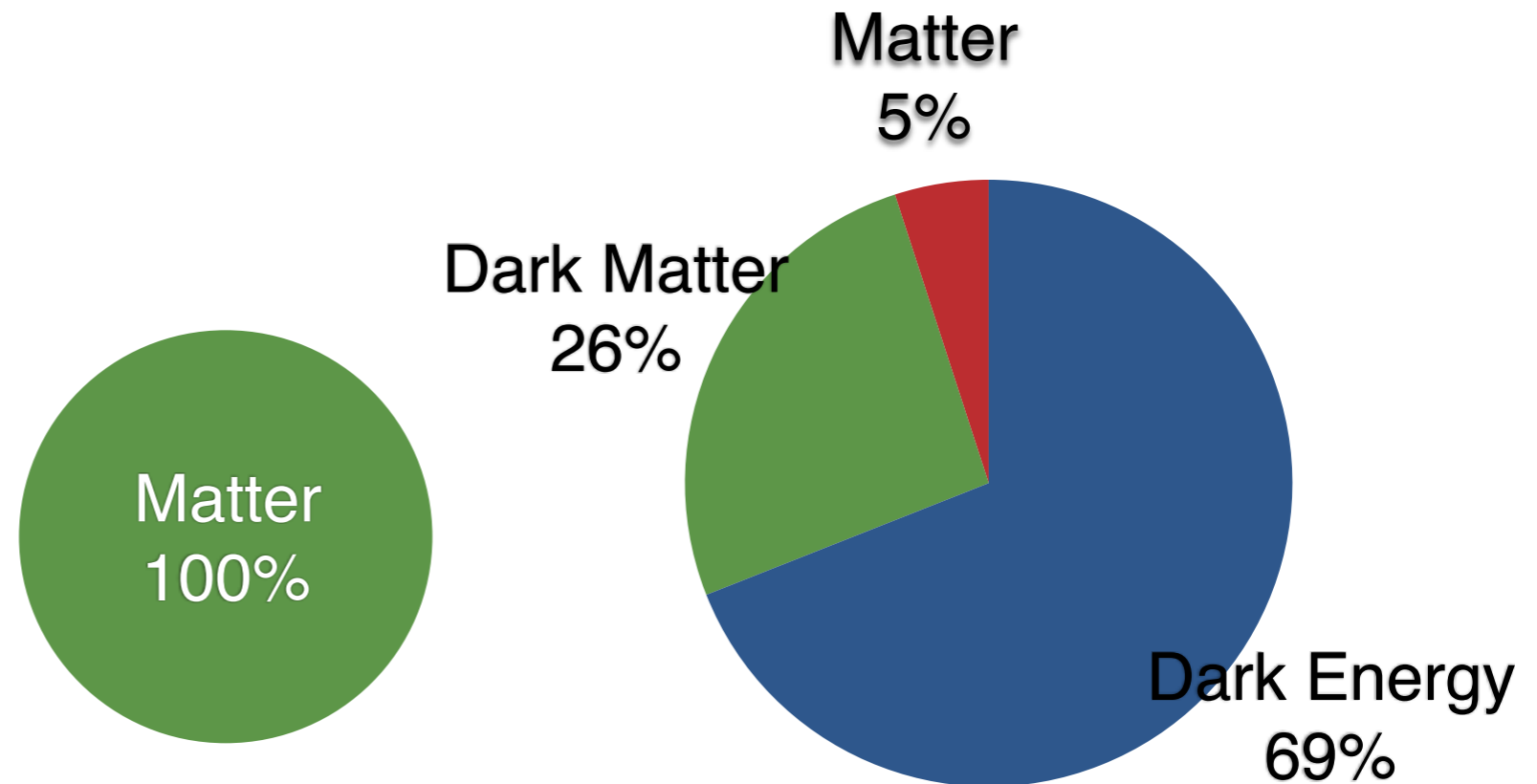
# Cosmic pies



# Cosmic pies

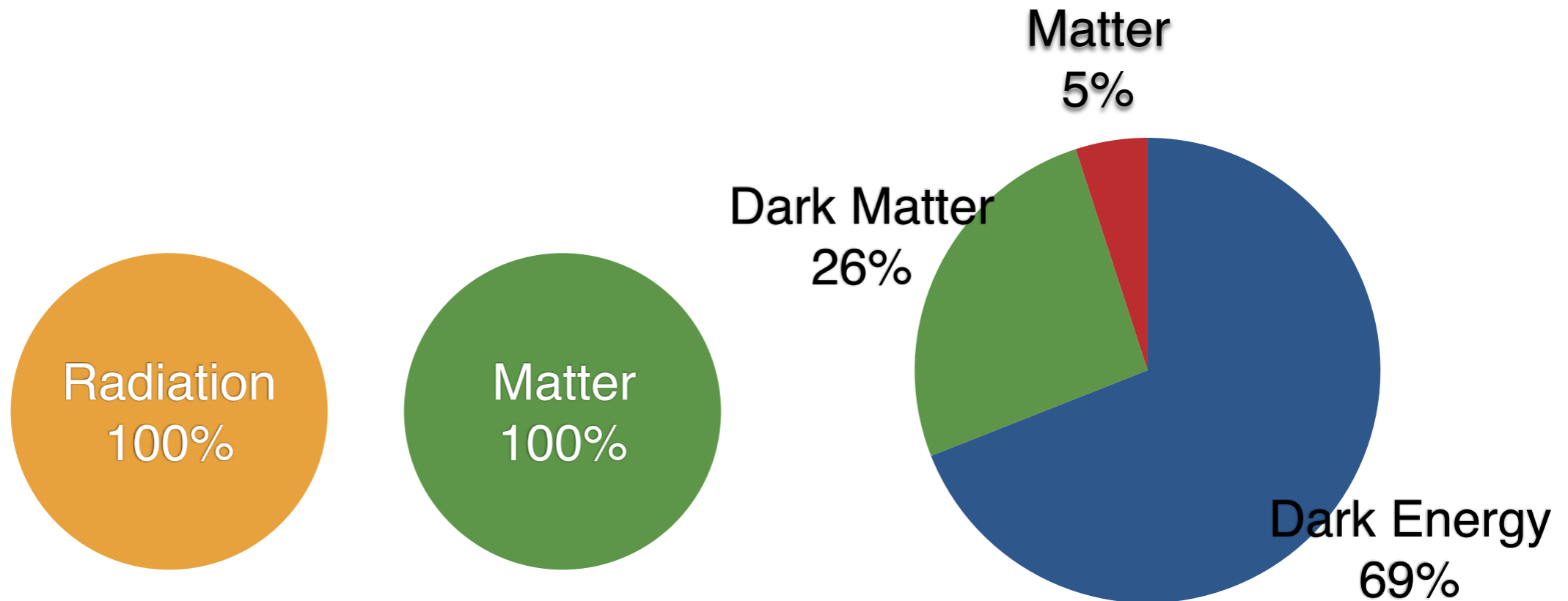


# Cosmic pies

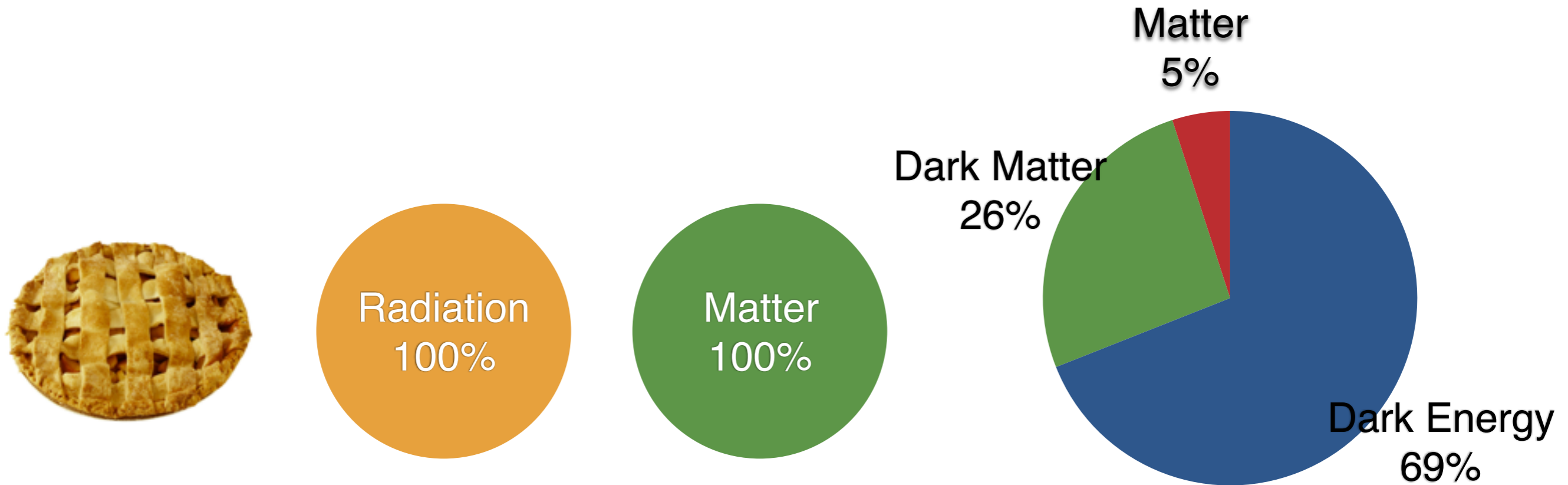




# Cosmic pies



# Cosmic pies

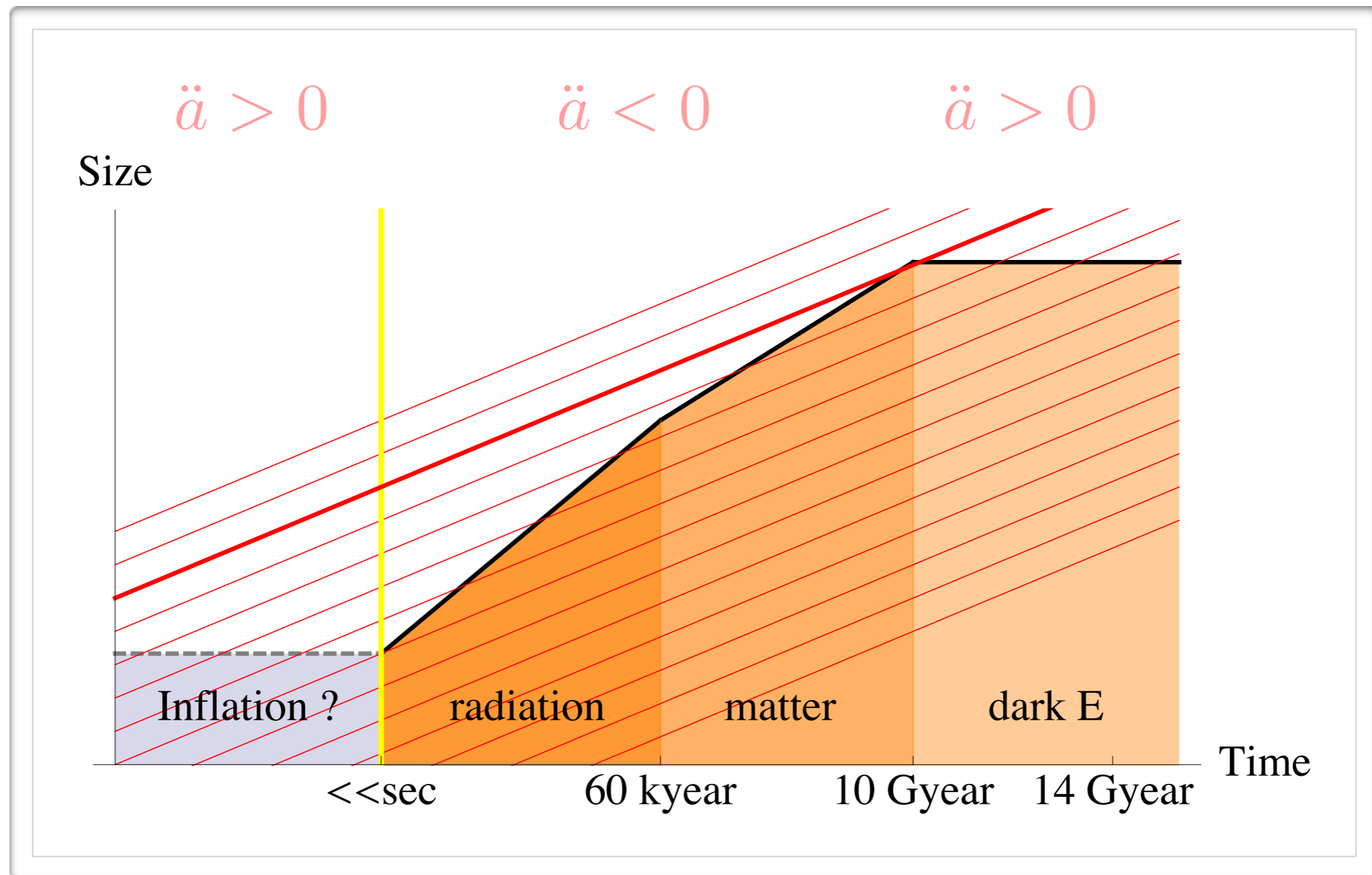




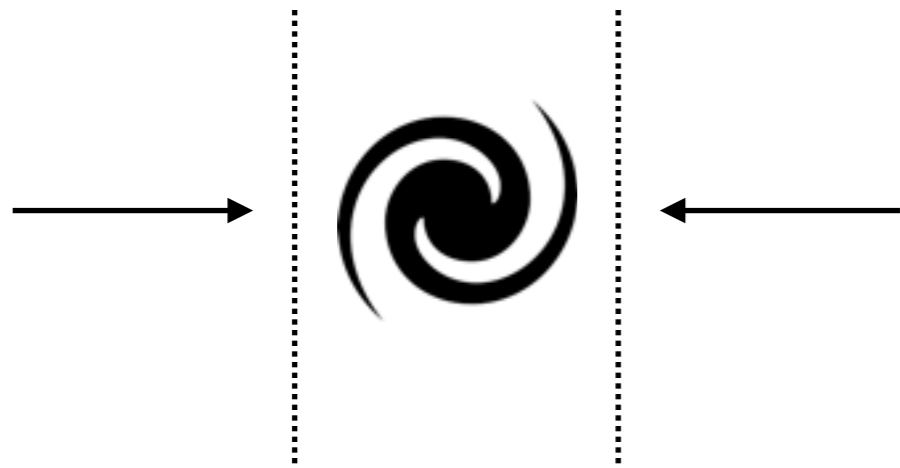
# Cosmic (pie) evolution



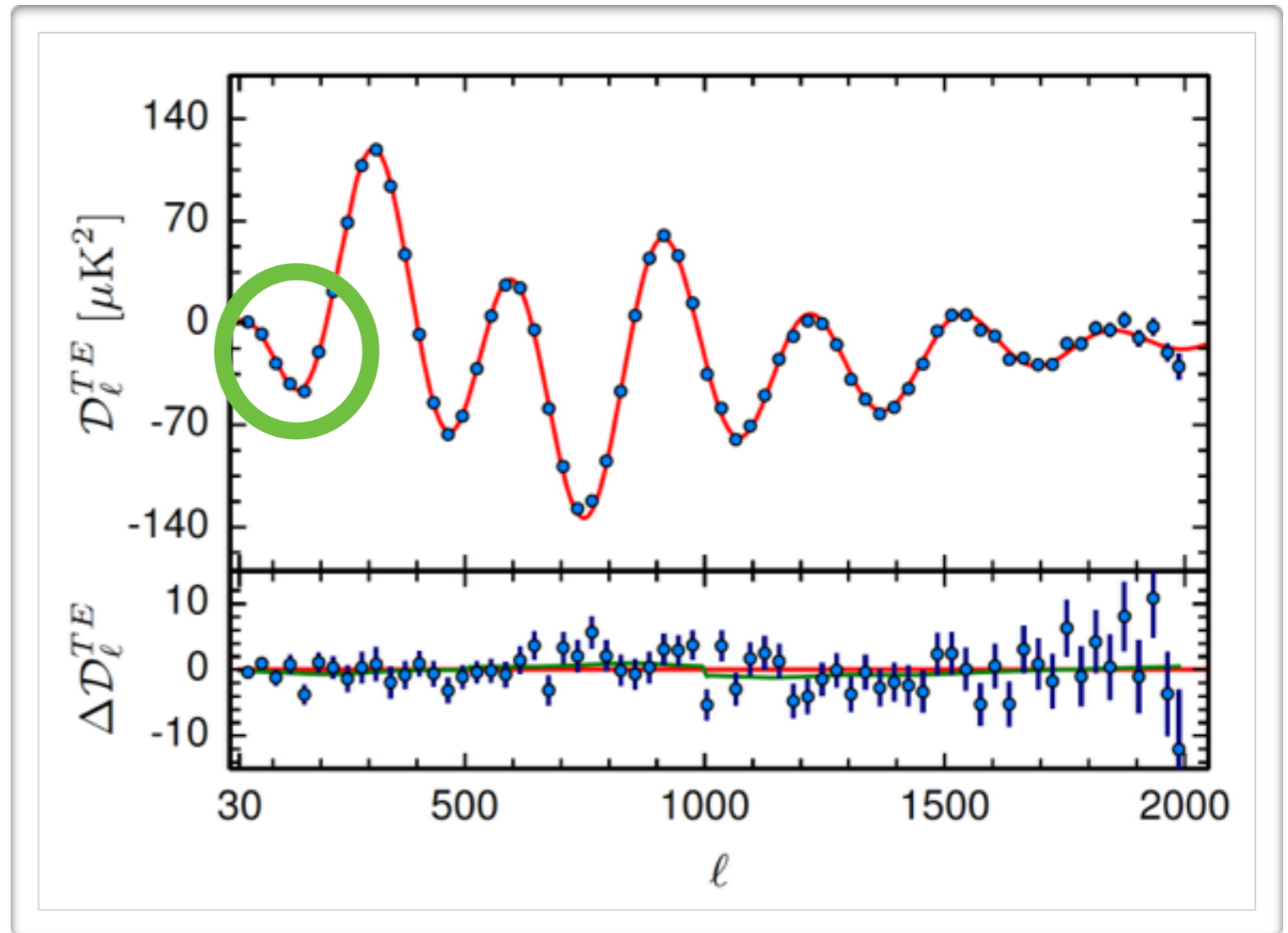
# Cosmic (pie) evolution



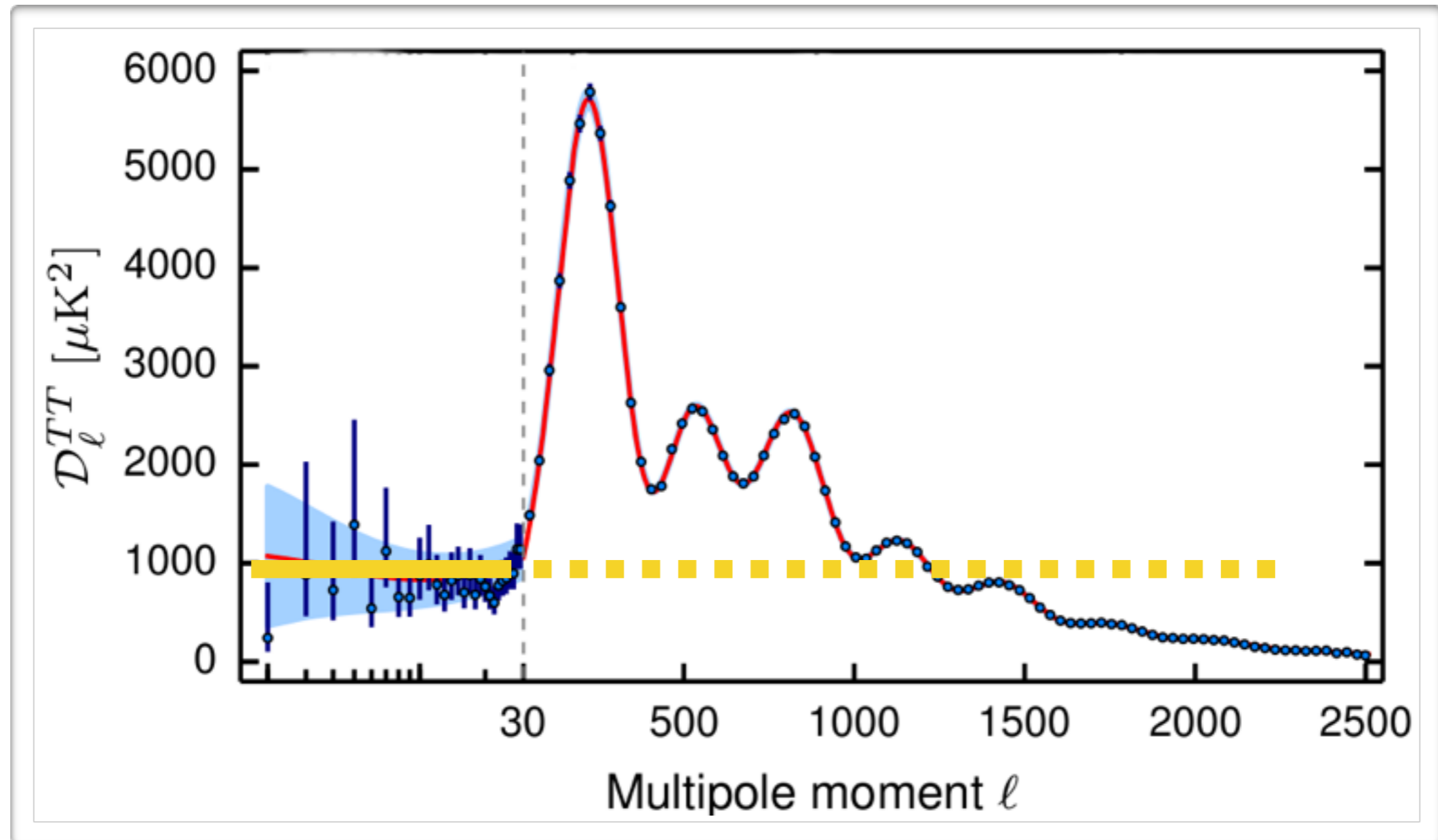
# Cosmic recipe book, or how we learned to love inflation



Out-of-phase,  
coherent waves.  
Primordial  
explanation!



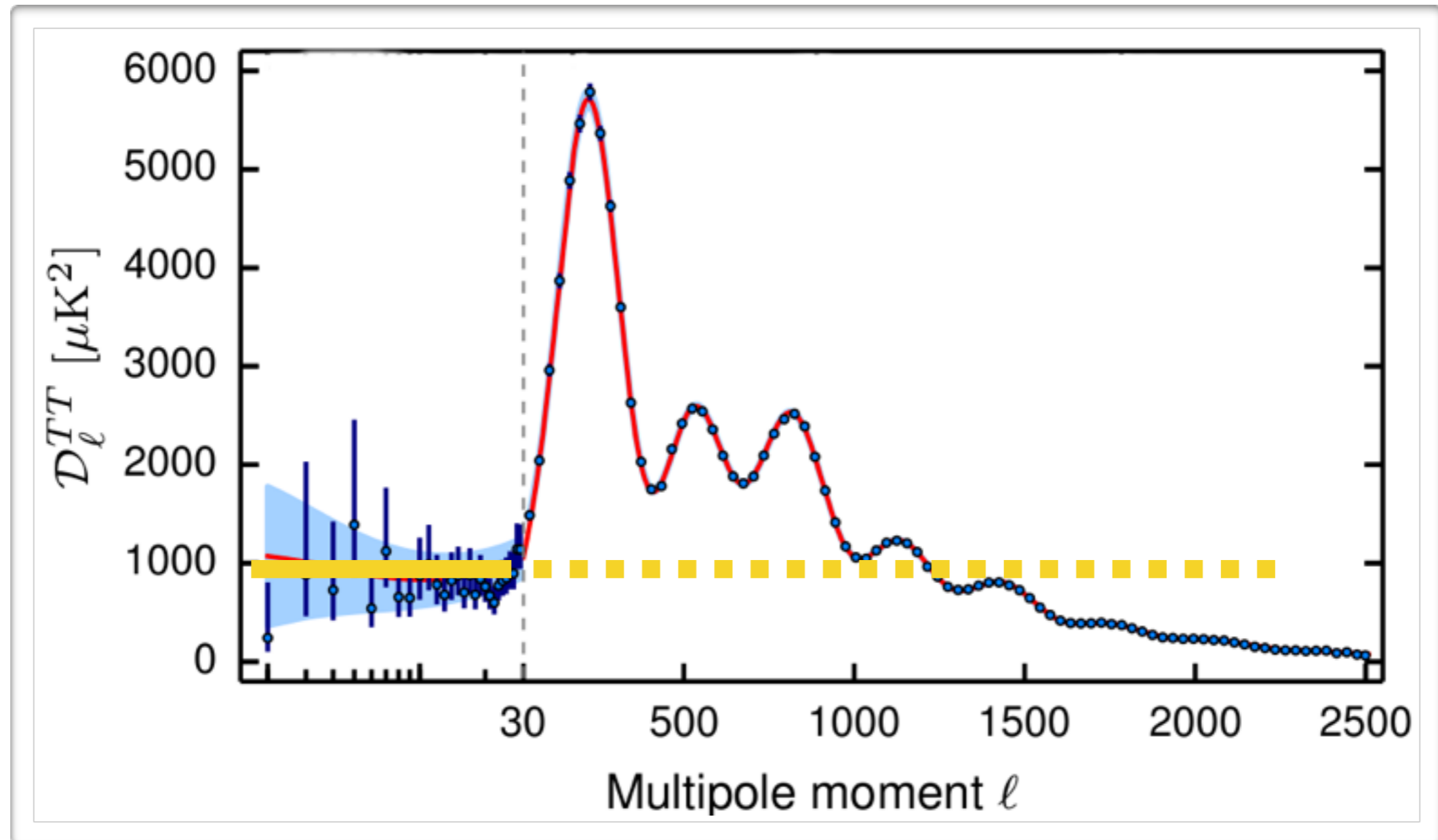
# Scale invariance



Scale invariance emerges naturally from *de Sitter isometries*



# Scale invariance



Scale invariance emerges naturally from *de Sitter isometries*





# Inflation

- A phase of quasi-de Sitter (accelerated expansion) in the first fraction of a second
- At least one (new) field, the *inflaton*
- At least three *new* (independent?) scales
  - Hubble Rate of expansion:  $H$
  - Rate of deceleration:  $\epsilon \equiv \frac{\dot{H}}{H^2}$
  - Rate of deceleration or inflaton mass:  $\eta \equiv \frac{\dot{\epsilon}}{\epsilon H}$



# Primordial power spectra

- Describe the free theory on time-dep background
- Scalars, a.k.a. *curvature* perturbation

$$\langle g_{ii}(k)g_{ii}(k) \rangle \sim \frac{H^2}{\epsilon M_{Pl}^2} \frac{1}{k^{3+(1-n_s)}}$$

- Tensors: aka *gravitational waves*

$$\langle \gamma_{ij}(k)\gamma_{ij}(k) \rangle \sim \frac{H^2}{M_{Pl}^2} \frac{1}{k^{3+8\epsilon}}$$



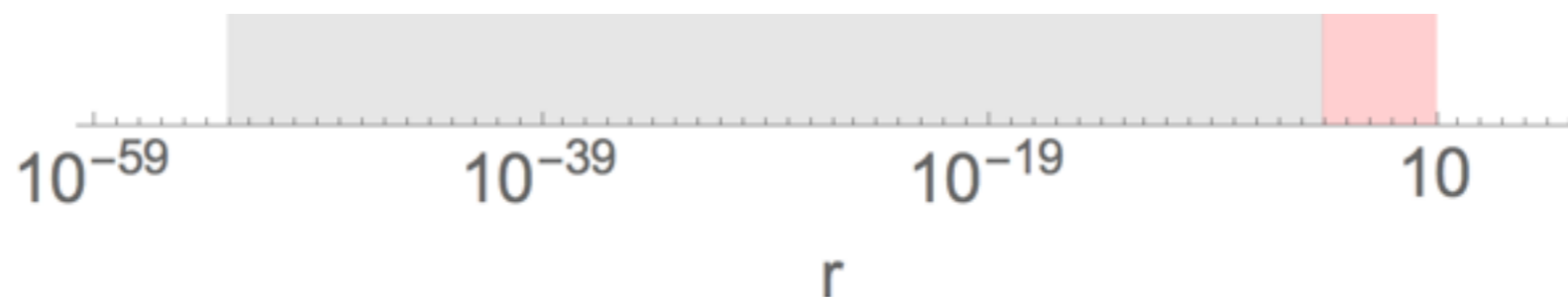
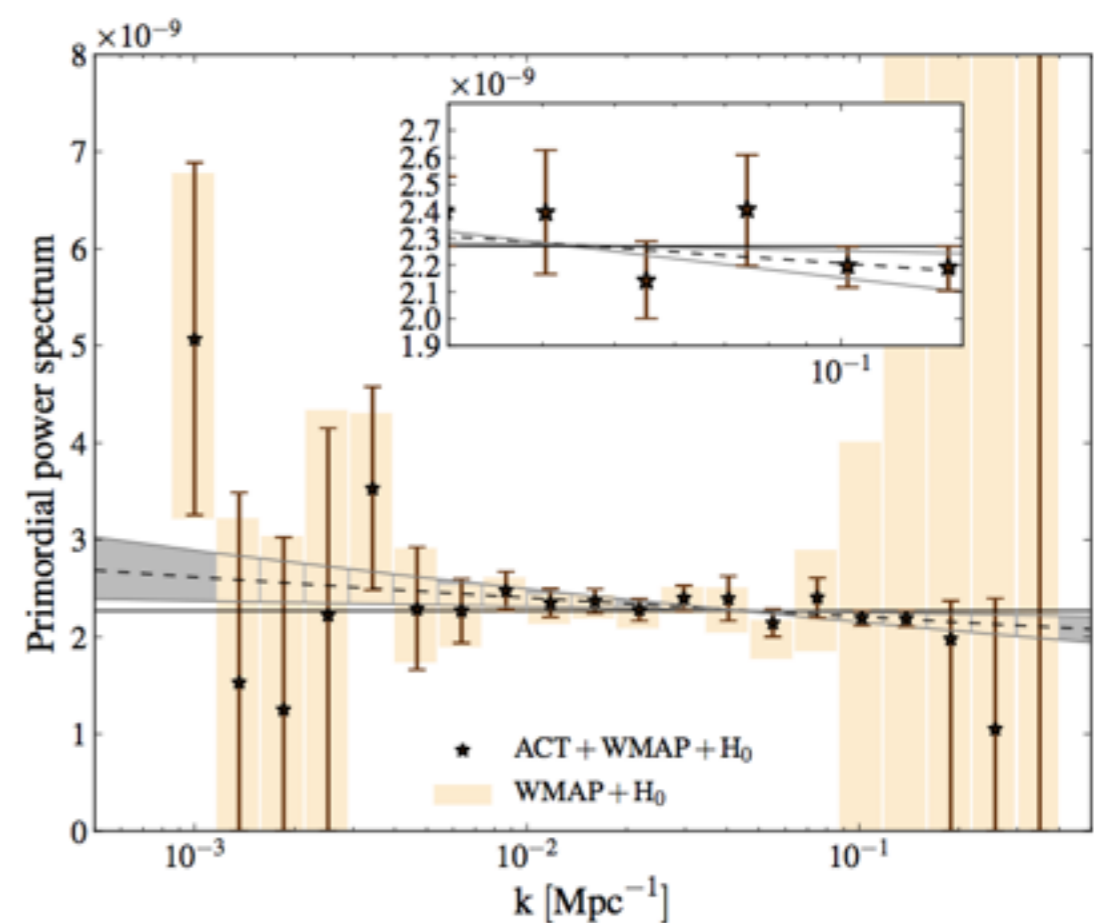
# Observations

Observed:

- Scalar amplitude
- scalar tilt:  $1 - n_s = 2\epsilon + \eta = 0.035$

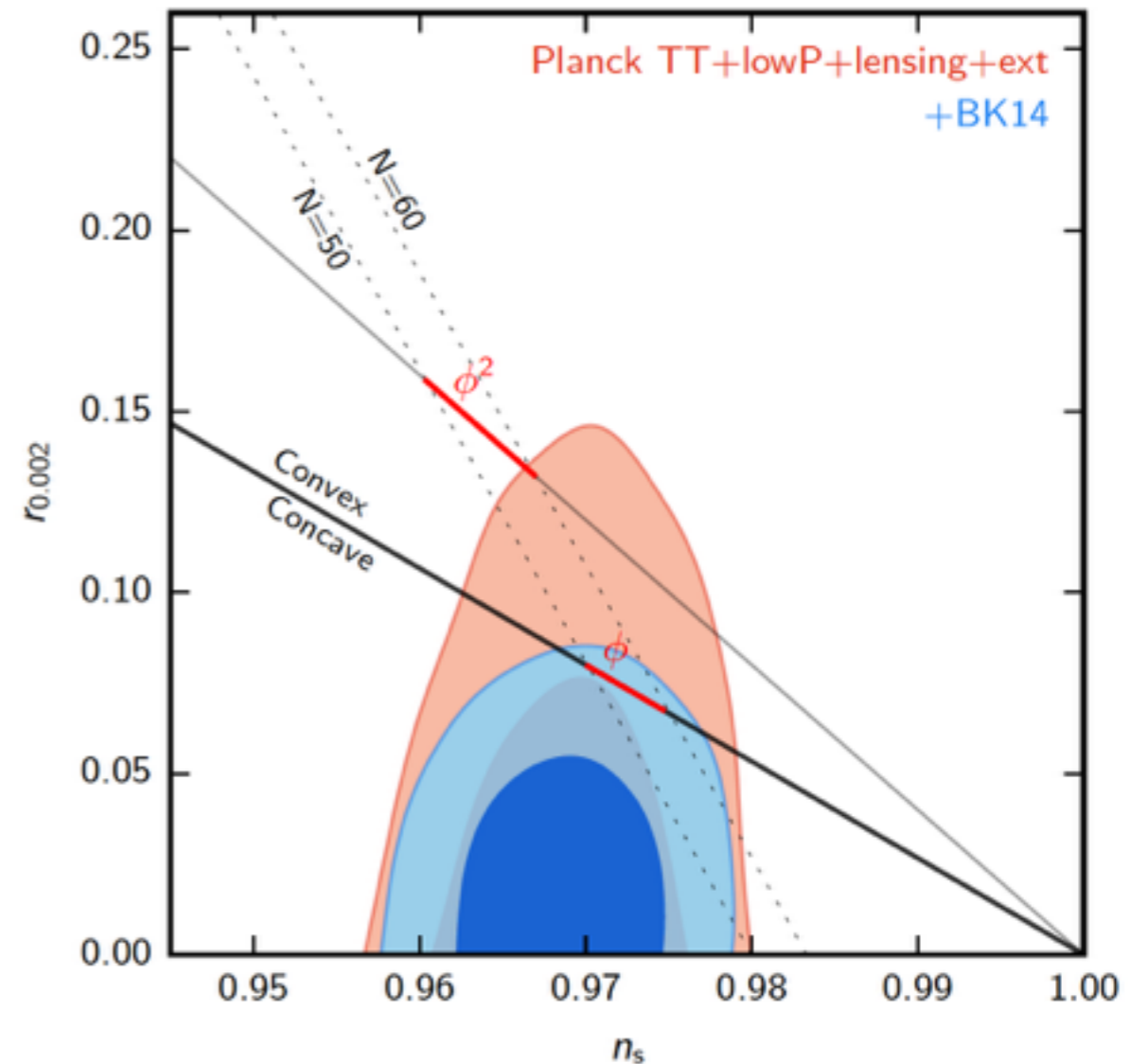
Might be observed in the future

- running of the scalar tilt:  $\alpha_s \sim (1 - n_s)^2$
- tensor amplitude:  $r \equiv \frac{\gamma^2}{g_{ii}^2} = 16\epsilon$       tensor tilt:  $n_T = 8\epsilon$



# No news is interesting news

- Improved constraints from CMB B-mode polarization lead to  $r < 0.07$  [Planck, Bicep2/Keck 15]
- The vanilla quadratic potential is out. Linear is next.
- *A new hierarchy (of scales)*



$$\left. \begin{aligned} 1 - n_s &= 2\epsilon + \eta = 0.035 \\ r &= 16\epsilon < 0.07 \end{aligned} \right\} \Rightarrow \epsilon \ll \eta$$



# The conformal limit of inflation

[with Mirbabayi, Pimentel, Simonovic, van Wijck to appear]

- New hierarchy of scales and slow-roll parameters
- de Sitter isometries and conformal symmetry
- All correlators are fixed by conformal sym.
- Consistency relations + Conformal symmetry
- Wave functional of the universe





# New slow-roll hierarchy

- To describe *our universe* the traditional slow-roll hierarchy needs to be updated for  $\eta \gg \epsilon$
- *Decoupling limit* with fixed power spectrum

$$\epsilon \rightarrow 0 \quad \& \quad M_{\text{Pl}} \rightarrow \infty \quad \text{with} \quad \frac{H^2}{\epsilon M_{\text{Pl}}^2} = 10^{-7}$$

- *Gravitational interaction are turned off.* Gravity is only background
- *Conformal limit* = Decoupling + no breaking of dS isometries (e.g. no speed of sound  $c_s$ )
- Simple class of models

$$\epsilon \sim \frac{1}{N^\beta} \quad \Rightarrow \quad \eta \sim \xi_n \sim \frac{1}{N} \gg \epsilon \quad \left( \xi_{n+1} \equiv \frac{\dot{\xi}_n}{\xi_n H} \right)$$



# dS & CFT

- Conformal limit: inflation is just a *scalar field in dS* (not true at  $O(\epsilon)$ )

$$ds^2 = \frac{-d\tau^2 + dx^2}{\tau^2 H^2}$$

- accelerated expansion: perturbations leave the horizon
- mass determines time dependence outside the horizon

$$\phi(x, \tau) \sim \sum_{\Delta} \tau^{\Delta} O_{\Delta}(x) \quad \Delta = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{V''}{H^2}}$$

- $3+1$  dS isometries,  $SO(4,1)$  Euclidean 3d conformal group
- dS time translation = dilation, dS boost = special conformal transf

$$D : -\tau \partial_{\tau} - x \partial_x \quad \Rightarrow \quad -\Delta - x \partial_x$$



# Two point function

- Dilations fix immediately the two point function

$$\langle \phi(k) \phi(k) \rangle' \stackrel{!}{=} \frac{C(\tau_*)}{k^{3-2\Delta}}$$

- Convert to zeta all modes at the same time

$$\zeta = \phi / \sqrt{2\epsilon(\tau_*)}$$

- this reproduces the right tilt for  $\epsilon=0$

$$1 - n_s = 2\epsilon + \eta \simeq \eta = -2\Delta$$



# Interactions

- Higher n-point functions probe interactions (non-Gaussianities in the CMB and galaxies)
- Interactions in the scalar sector and from gravity
- Maldacena computed

$$\langle \zeta^3 \rangle' = (n_s - 1) \left[ \frac{1}{k_1^3 k_2^3} + 2 \text{ perms} \right] + \epsilon \times \text{equi.} + (\eta^2, \epsilon^2)$$
$$\simeq -\eta \left[ \frac{1}{k_1^3 k_2^3} + 2 \text{ perms} \right]$$

- *The leading bispectrum follows from symmetry!*



# Consistency relations

- In standard single field inflation, the squeezed limit of any  $n$ -point function is related to  $(n-1)$ -point functions
- This can be seen as a Ward identity for diffeomorphism invariance and is a manifestation of the equivalence principle
- A similar (simpler) argument holds without gravity

$$\begin{aligned}\langle \phi_L \phi_S \phi_S \rangle &= \langle \phi_L \langle \phi_S \phi_S \rangle_{\phi_L} \rangle \\ &= \langle \phi_L \phi_L \rangle \partial_{\phi_L} \langle \phi_S \phi_S \rangle\end{aligned}$$





# Bispectrum

- Conformal invariance + the squeezed limit fully fixes the full bispectrum to leading order.
- We recover the bispectrum and derive new terms that are higher order in slow-roll

$$\langle \zeta^3 \rangle' = -(\eta + \eta\xi) \left[ \frac{1}{k_1^3 k_2^3} + 2 \text{ perms} \right] + \eta\xi \times \text{equi.} + (\eta^3)$$

- non-Gaussianity is fixed by the *running of the spectral tilt* (observable)  $\eta\xi$
- This an (observable) *consistency condition for single field* inflation in the scalar sector
- Gauge fixing the graviton breaks conformal invariance. Correlators with a graviton propagator are invariant only up to a gauge transformation (e.g. 4-point function). But gravity decouples...



# Wave functional of the universe

- Quantum field theory in the Schroedinger picture (common in gauge-gravity duality)
- “The wave function of the Universe is like a diamond. From every angle it looks a different color. Yet it has no color.” [M. Mirbabayi]

$$\langle O(\phi) \rangle = \int [d\phi(x)] |\psi|^2 O(\phi), \quad \psi = \int [d\phi(x, \tau)] e^{iS[\phi]}$$



# Weyl invariance

- The wave function of the universe in any FLRW is invariant under (3d) spatial diff's (Ward identities for squeezed limit consistency relations)
- In dS the wave function is also Weyl invariant

$$g_{ij} \rightarrow e^{2\lambda} g_{ij}, \quad \phi \rightarrow e^{\Delta\lambda} \phi, \quad \zeta \rightarrow \zeta + \lambda$$

- interactions are then constrained

$$\psi \sim \exp \left\{ \int_{x,y} P_\phi(x-y)^{-1} \phi(x)\phi(y) [1 - C\zeta(x)] \right\}$$

- reproduces the *local part of the bispectrum*



# Weyl invariance

- The wave function of the universe in any FLRW is invariant under (3d) spatial diff's (Ward identities for squeezed limit consistency relations)
- In dS the wave function is also Weyl invariant

$$g_{ij} \rightarrow e^{2\lambda} g_{ij}, \quad \phi \rightarrow e^{\Delta\lambda} \phi, \quad \zeta \rightarrow \zeta + \lambda$$

- interactions are then constrained

$$\psi \sim \exp \left\{ \int_{x,y} P_\phi(x-y)^{-1} \phi(x)\phi(y) [1 - \Delta \zeta(x)] \right\}$$

- reproduces the *local part of the bispectrum*



# Conformal limit

Hard to test experimentally, but interesting theoretically:

- All n-point correlators are *dS invariant* (leading order)
- new “scalar” consistency relation *away from the squeezed limit*
- new (physical!) relation:  $f_{\text{NL}}^{\text{eq}} \sim a_s$
- Bispectrum gives the 3-point function of a CFT in Fourier space (triple K-integral)



# The origin of the adiabatic mode

- Adiabatic perturbations
- Two paradigms
- Slow-roll conditions in multifield
- Slow-descent inflation



# Adiabatic perturbations

- primordial pert's are adiabatic to few % [Planck]

$$\left(\frac{\delta\rho}{\bar{\rho} + \bar{p}}\right)_{DM} = \left(\frac{\delta\rho}{\bar{\rho} + \bar{p}}\right)_b = \left(\frac{\delta\rho}{\bar{\rho} + \bar{p}}\right)_\gamma = \left(\frac{\delta\rho}{\bar{\rho} + \bar{p}}\right)_\nu = \dots$$

- Explaining it might be a legacy of our generation





# Adiabatic perturbations



- primordial pert's are adiabatic to few % [Planck]

$$\left( \frac{\delta\rho}{\bar{\rho} + \bar{p}} \right)_{DM} = \left( \frac{\delta\rho}{\bar{\rho} + \bar{p}} \right)_b = \left( \frac{\delta\rho}{\bar{\rho} + \bar{p}} \right)_\gamma = \left( \frac{\delta\rho}{\bar{\rho} + \bar{p}} \right)_\nu = \dots$$

- Explaining it might be a legacy of our generation



# Adiabatic perturbations

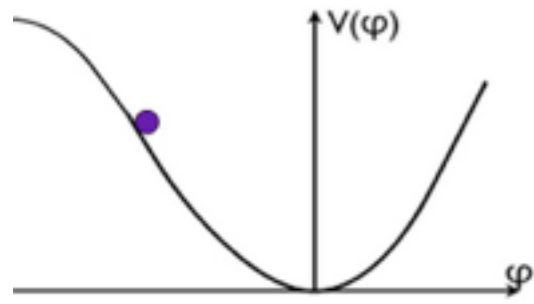


- primordial pert's are adiabatic to few % [Planck]

$$\left( \frac{\delta\rho}{\bar{\rho} + \bar{p}} \right)_{DM} = \left( \frac{\delta\rho}{\bar{\rho} + \bar{p}} \right)_b = \left( \frac{\delta\rho}{\bar{\rho} + \bar{p}} \right)_\gamma = \left( \frac{\delta\rho}{\bar{\rho} + \bar{p}} \right)_\nu = \dots$$

- Explaining it might be a legacy of our generation





# Two paradigms



## Single field inflation

- Only one (light) field has superhorizon-scale pert's
- These freeze and re-enter at late times
- Pert's *are born* adiabatic
- Weinberg's adiabatic mode

## Multi-field inflation

- Many fields are perturbed on super horizon scales
- Subhorizon *thermalization* (no conserved charges) erases isocurvature pert's  

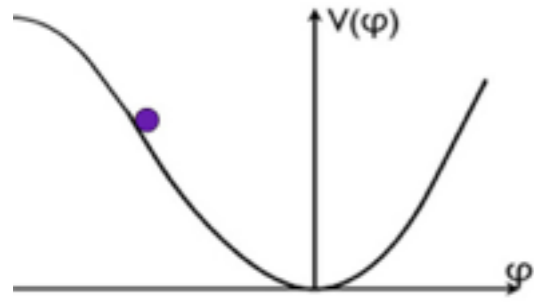
$$n_i \sim e^{-E_i/T} \rightarrow \delta n_i \sim \delta T$$
- Pert's *become* adiabatic



# Adiabatic modes à la Weinberg

- Weinberg's argument mimics the proof of massless Goldstone bosons
- Consider classical GR perturbations around FLRW
- Fix the gauge *at finite momentum* (pert's vanish at infinity), e.g. Newton gauge (cosmological pert. theory)
- Apply a large diff. By diff-invariance it must generate a new solution that does not vanish at infinity (zero momentum)
- Two (non-decaying) solutions survive to finite momentum: the adiabatic mode and gravitational waves
- *If there is only one scalar mode, it is the adiabatic mode*





# Two paradigms for (string) inflation



UV considerations suggest many fields:

- RG flow, explicit String Theory constructions,
- Hickam's dictum (cfr Occam's razor): "Patients can have as many diseases as they damn well please"

How did a simple (adiabatic) universe emerged from the (complicated) UV theories of inflation?

- 99% of searched in the (ST) landscape are for single field
- Why not embrace the multi-field paradigms and its complexity?





# Single-field slow roll

- Hubble slow-roll conditions are nice and clear

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1 \quad \& \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \ll 1 \quad \dots$$

- In single field they imply the potential slow-roll conditions

$$\epsilon_V \equiv \frac{M_{Pl}^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta \equiv M_{Pl}^2 \frac{V''}{V}$$

- Single field Inflation requires flat potentials and proceeds along the gradient (*gradient flow*)

$$3H\dot{\phi} \simeq -V'$$



# Single field lamppost

- Hubble slow-roll conditions are again nice & clear

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1 \quad \& \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \ll 1 \quad \dots$$

- But “potential” slow-roll conditions are poorly understood and often misquoted

$$3H\dot{\phi}_i \simeq -V_{,\phi_i} \quad \text{not necessary}$$

$$\epsilon_V = \frac{M_P^2}{2} \frac{V_i V_i}{V^2} \ll 1 \quad \text{not necessary}$$

$$\eta_V = M_P^2 \frac{\min \text{Evalue}(V_{,ij})}{V^2} \ll 1 \quad \text{neither necessary nor sufficient [see e.g. Yang 12]}$$





# Multi-field slow-roll conditions

[with urzainqui & Cancer]

- Single field space geometry is always flat
- Multi-field space is curved (sigma models in String and Effective theories)

$$X \equiv G_{IJ} \dot{\phi}^I \dot{\phi}^J / 2$$

- Slow-roll condition must be covariant under field space re-parameterization

$$3M_P^2 H^2 = V + X, \quad \dot{X} + 6HX + V_{,I} \dot{\phi}^I = 0$$

$$D_t \dot{\phi}^I + 3H \dot{\phi}^I + G^{IJ} V_{,J} = 0$$



# Beyond gradient flow

$$\epsilon = \frac{3X}{V+X} \quad \Rightarrow \quad \eta = 2\epsilon + \frac{\dot{X}}{HX}$$

- Prolonged inflation requires

$$\dot{X} \ll 6HX \simeq -V_{,K} \dot{\phi}^K$$

- instead of the common gradient flow assumption

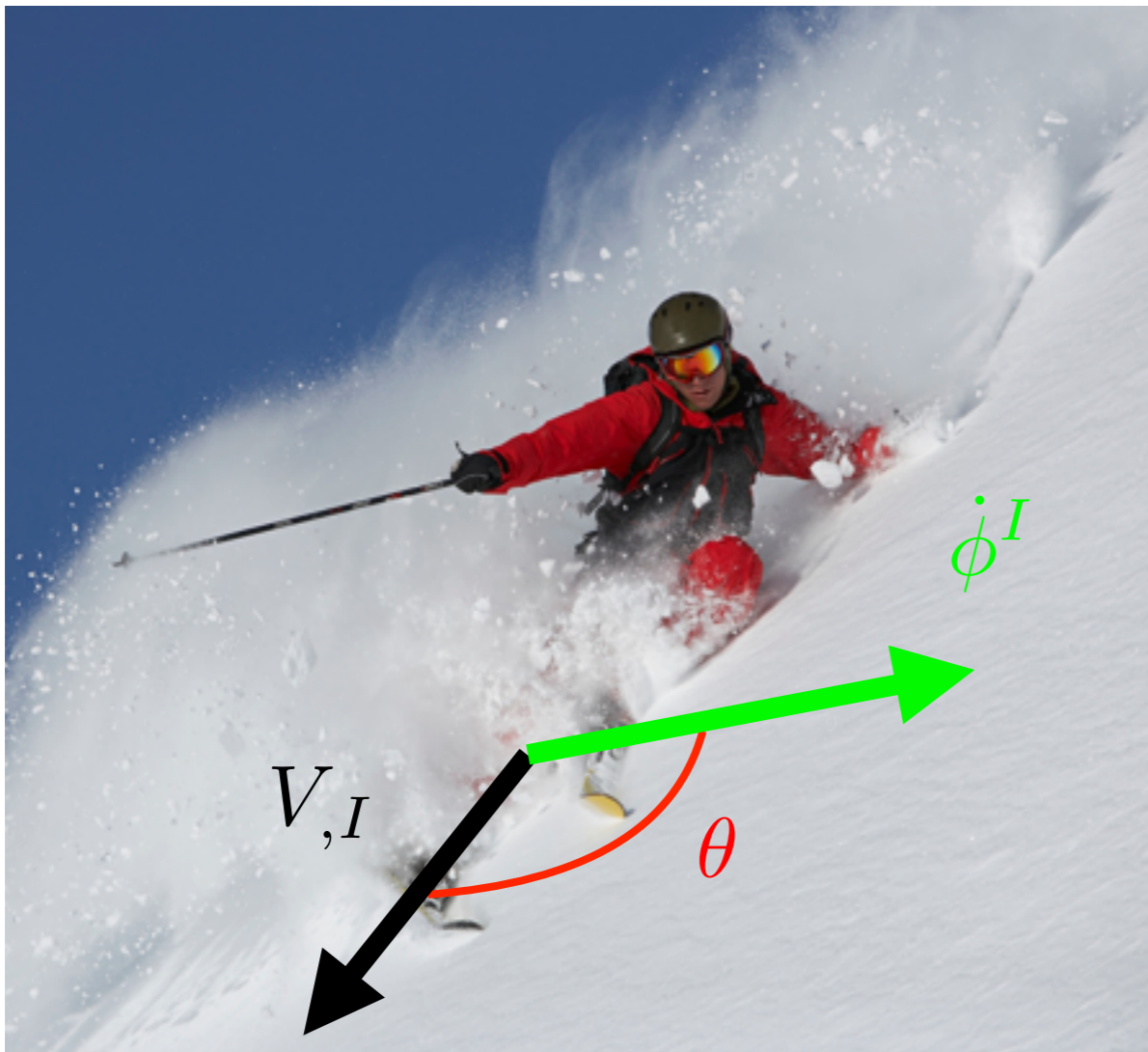
$$D_t \dot{\phi}^I + 3H \dot{\phi}^I + G^{IJ} V_{,J} = 0$$

- for  $\theta$  the angle between gradient and velocity

$$\epsilon_V = \frac{M_P^2}{2} \frac{V_{,K} V^{,K}}{V^2} \cos^2 \theta$$



# Slow-descent



If velocity is uncorrelated with gradient, then inflation is generic for sufficiently large  $N$ :

$$\langle \cos \theta^2 \rangle = \frac{1}{N}$$

This is a genuine multi-field behavior.

Does this happen in the landscape?



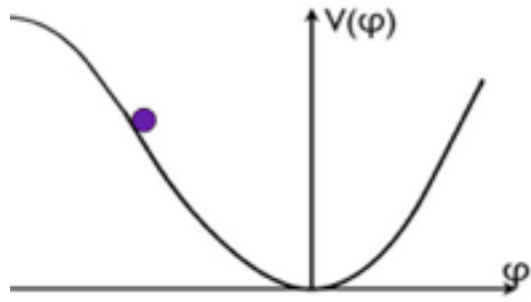
# Existence proof

- Two (or more) fields. Steep gradient along  $y$ , canceled by “field-space gravity”

$$\ddot{y} + \Gamma_{yx}^y \dot{y} \dot{x} + 3H \dot{y} + \Gamma_{xx}^y \dot{x} \dot{x} + V_{,y} = 0$$

- evolution along  $x$ , where  $V$  is much flatter ( $\perp$  to gradient)
- Formally analogous to Newtonian orbits [Yang '12]
- Robustness and generality?





# Single vs multi-field



A few observables can rule out single field:

- isocurvature perturbations
- local non-Gaussianity

*But can we rule out the multi-field paradigm?*

- Computing multi-field predictions requires understanding: inflation, reheating, coupling to SM physics and thermalization



# How to rule out multi-field

- Single field consistency relation:
  - Tensors:  $r = 8n_T$
  - Scalars (new):  $f_{NL}^{equi.} = \alpha_s$
- Data suggest the simple relation [Mukhanov 13; Roest 13; Creminelli 14; Zavala 14; Gobbetti, EP, Roest 15]

$$1 - n_s = 2/N$$

- In single field this can easily happen (e.g. chaotic or Starobinski inflation). In multifield, it requires *tuning of the potential WITH of the initial conditions* [EP in progress]
- observation of running  $-2/N^2$  would *rule out multifield*





# Definition of multi-field

- Assume gradient flow, and at any point in field space compute efolds  $N(\Phi)$  until the adiabatic attractor is reached

- On superhorizon scales, delta-N formalism gives

$$\zeta = \delta N = \frac{\partial N}{\partial \phi^I} \delta \phi^I + \dots$$

- The gradient of  $N(\Phi)$  generated the observed pert's



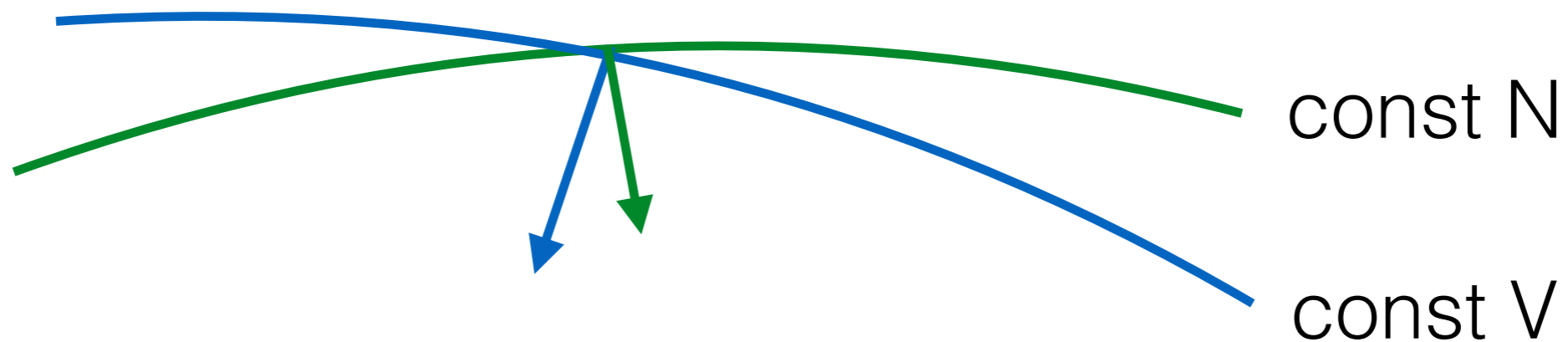
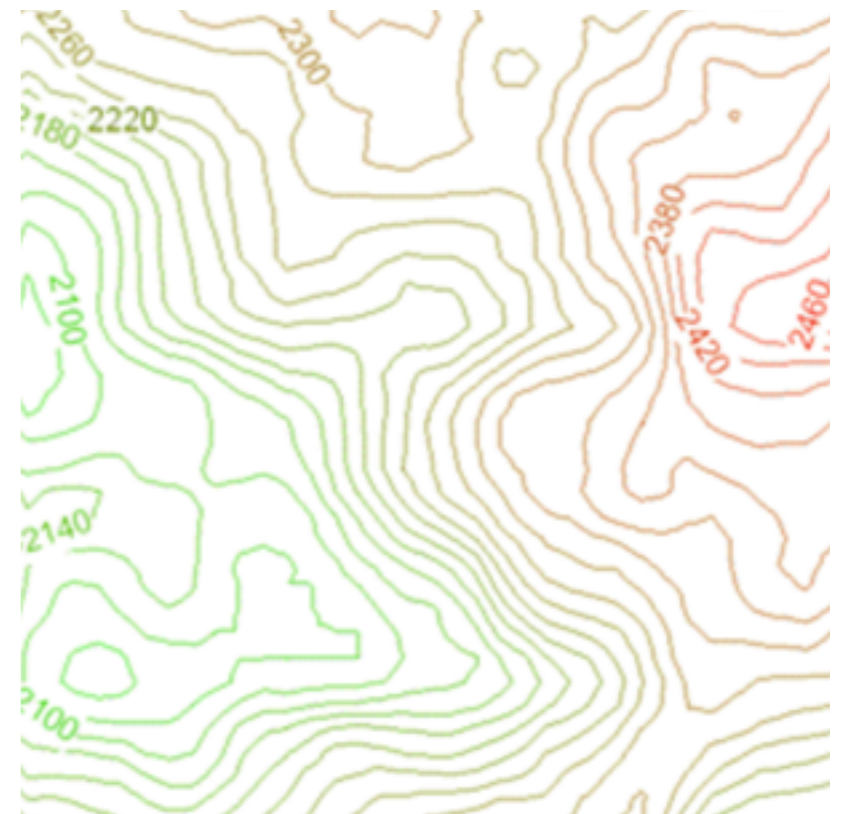
Constant N lines





# Definition of multi-field

- consider constant  $V$  surfaces normal to the gradient  $V_{,\phi}$
- In effective single field, the gradients of  $V$  and  $N$  are parallel



# Simple scalings

- Data suggest the simple relation [Mukhanov 13; Roest 13; Creminelli 14; Zavala 14;

Gobbetti, EP, Roest 15]

$$1 - n_s = \frac{2}{N}$$

- In single field this can easily happen (e.g. chaotic inflation or Starobinski)
- In multifield, even tuning the potential, the region of initial condition that satisfy  $1/N$  has measure zero [EP in progress]
- In multifield,  $1/N$  requires (more) *tuning of the potential AND of the initial conditions*
- observation of running  $-2/N^2$  would *rule out multifield*

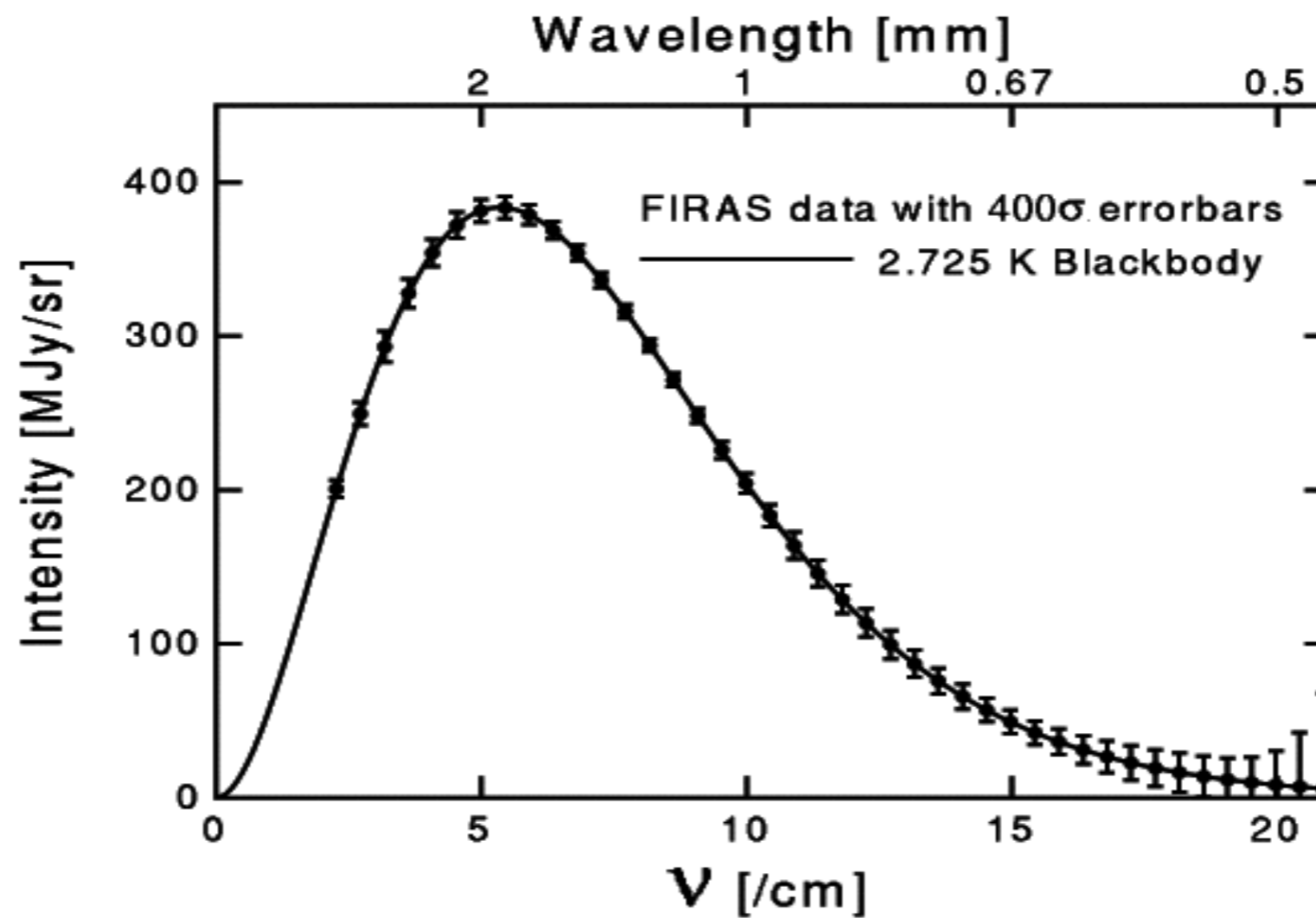


# Conclusions

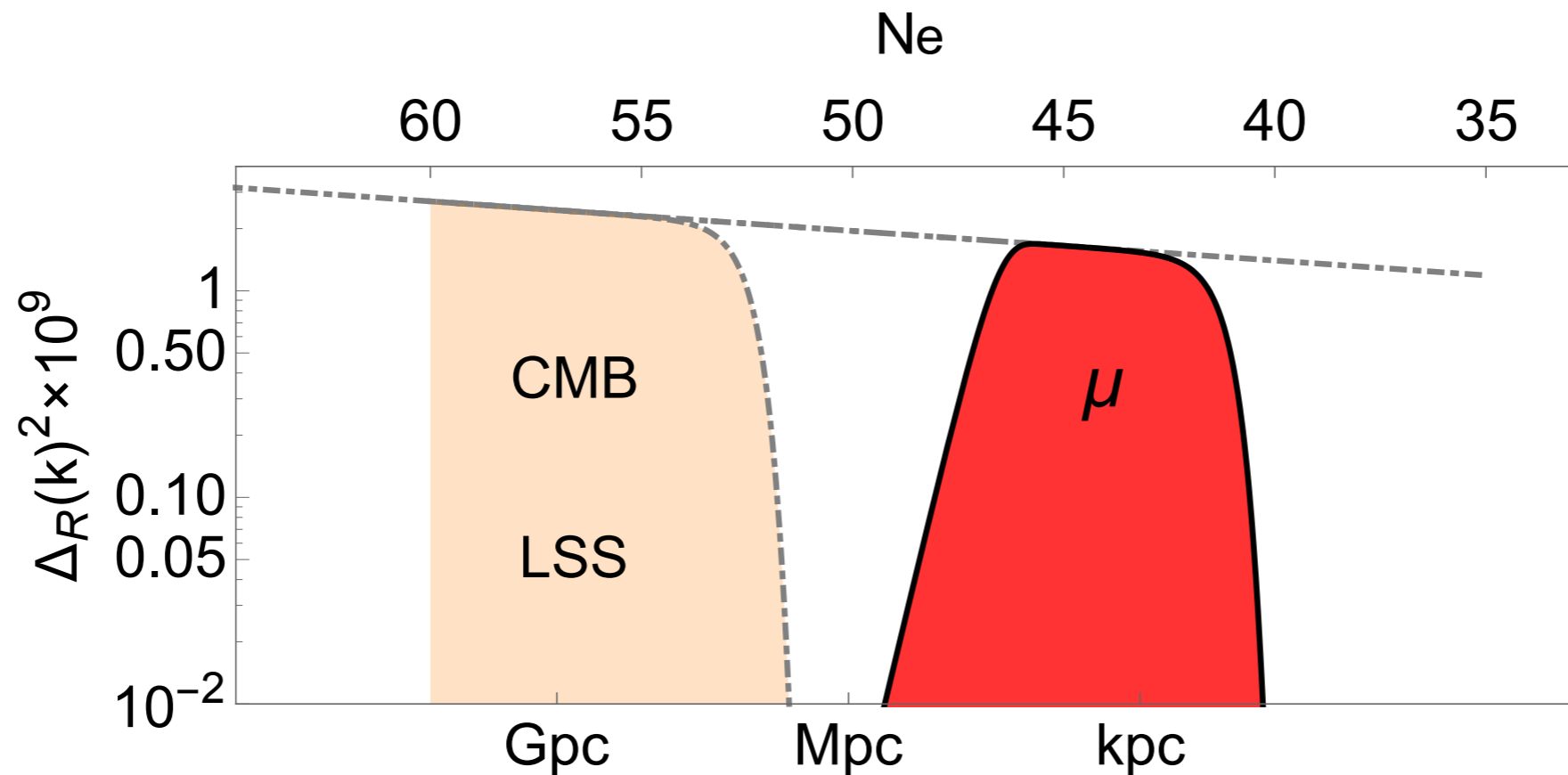
- Observations dictate a new hierarchy:  $\varepsilon \ll \eta$ 
  - Primordial correlators are fixed by conformal symmetry
  - *New consistency condition*:  $f_{\text{NL}}^{\text{eq}} \sim \alpha_s$  (physical)
  - Multi-field conformal limit?
- The origin of the adiabatic mode
  - Adiabaticity from complexity through thermalization
  - Embrace the complexity of multi-field dynamics
  - There's more to life than slow-roll gradient flow inflation
  - Does the landscape support slow-descent inflation?



# CMB spectral distortions



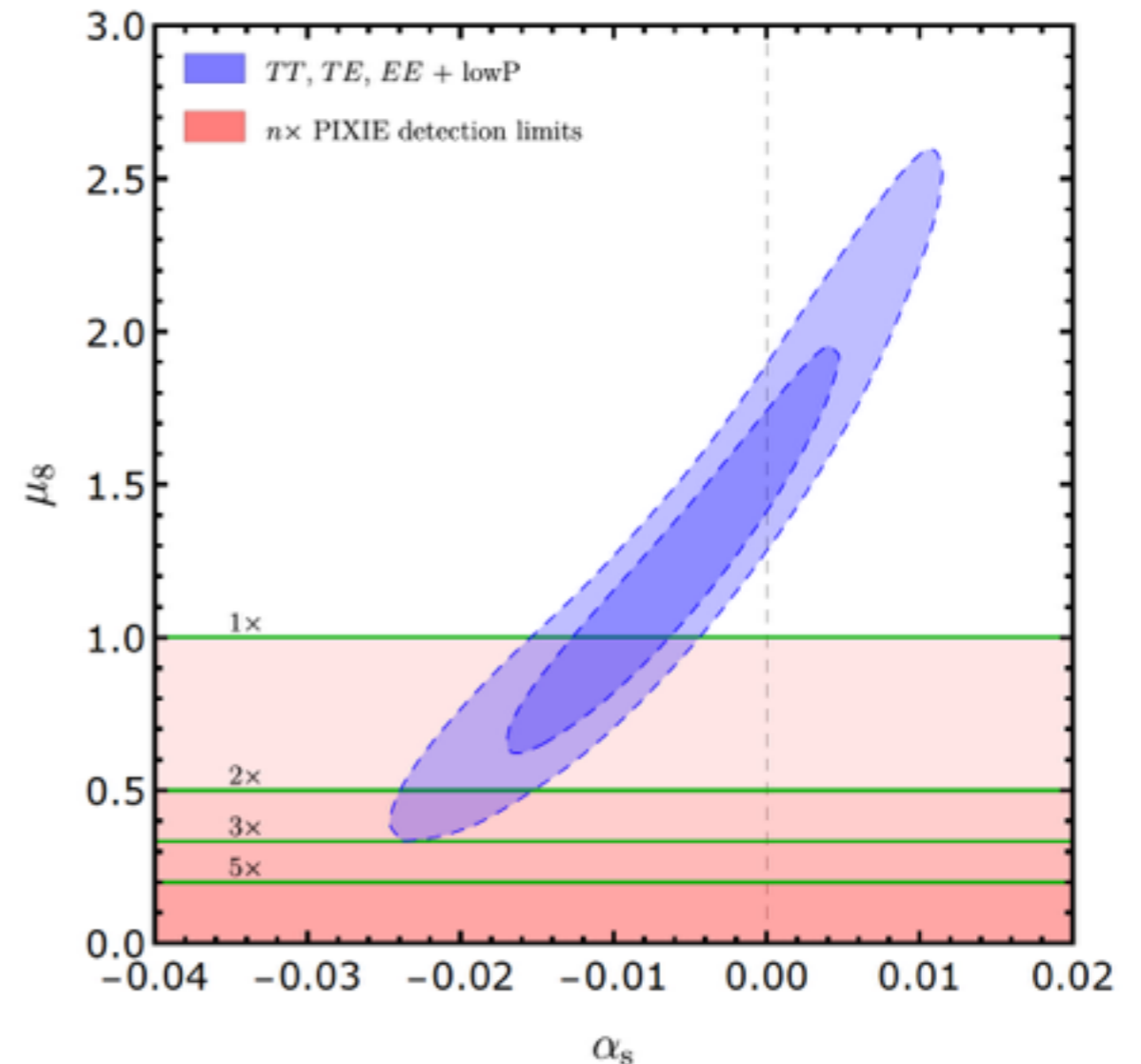
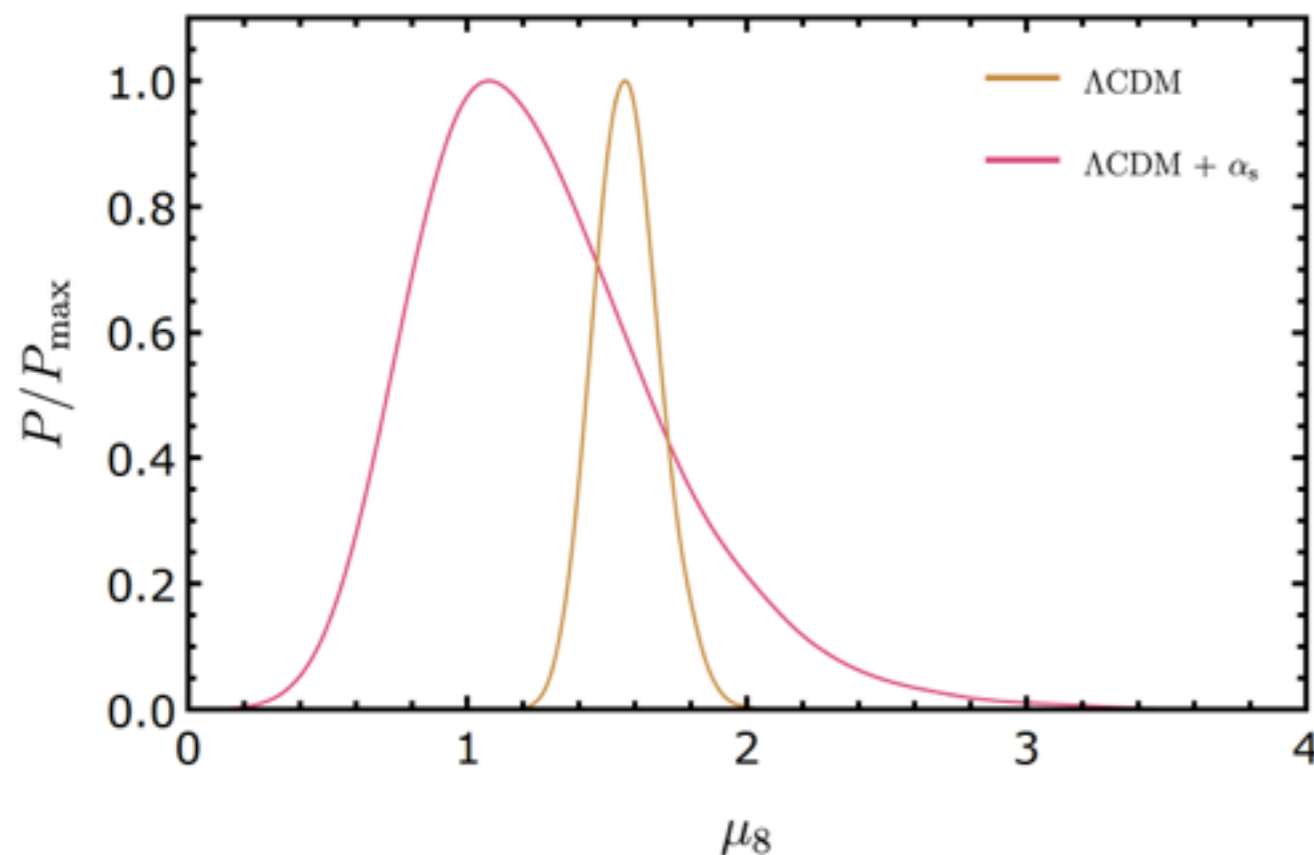
# Another 10 e-folds



- CMB/LSS probe only a narrow window of scales ( $\sim 7$  e-folds)
- Dissipation at  $2 \times 10^6 > z > 4 \times 10^4$  (frozen photon number) creates CMB spectral distortions
- Spectral distortion probes **another 5 e-folds**  $50 < k \text{ Mpc} < 10^4$  [Sunyaev, Zel'dovich, Silk, Peebles, Hu, Danese, de Zotti, Chluba, ...]
- independent modes: CMB  $\sim 10^6$ , LSS  $\sim 10^9$ ,  $\mu \sim 10^{12}$ . Tiny cosmic variance!



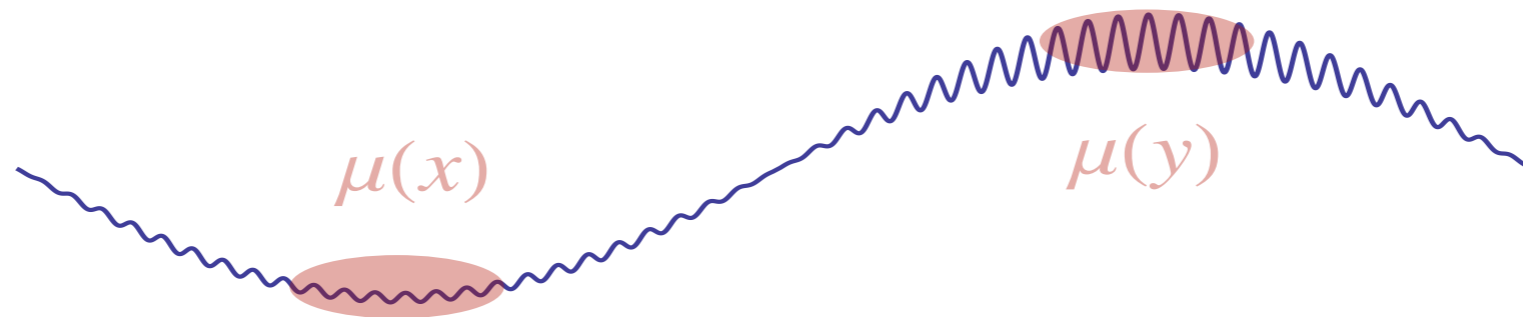
# “If you build it, he will come” [Field of Dreams '89]



- Planck puts lower bound on  $\mu$ , measurable by next CMB satellite (PIXIE, LightBIRD, PRISM)
- **Guaranteed discovery:** detection of  $\mu$ , or of negative running [Cabass, Melchiorri, EP 16]



# primordial non-Gaussianity



- *Anisotropies* in  $\mu$  probe primordial non-Gaussianity  
[EP & Zaldarriaga '12]
- since  $\mu \propto \delta^2$ , correlation  $C_l^{\mu T}$  gives local bispectrum

$$\langle a_{lm}^{\mu} a_{lm}^{\Delta T} \rangle \simeq 50 f_{NL} \frac{\Delta_R^4}{l(l+1)}$$

- Cosmic Variance limit (futuristic) is  $f_{NL} \sim 10^{-3}$

