Gravity at large scales

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- A cornerstone of modern QFT are the concepts of low-energy effective theory and UV completion

  - e.g. Fermi theory as a low-energy limit of the SM
  - UV completion of the SM, to the latest at the Planck scale

in particle physics we usually assume that the frontier of knowledge is a high-energy frontier.

We do not expect fundamental discoveries in the IR limit

In gravity, several different regimes can still lead to fundamental discoveries

- the UV regime (quantum gravity, string theory)

  the most difficult to probe observationally

- the strong-field regime

  a window that is being opened by GW astronomy (GW150914 !!)

- the far IR limit

  probed by cosmology

Why the IR limit can be interesting?

- wealth of high-quality observational data in the last two decades

  for the first time, we can test this regime in detail

- surprises: dark energy

  maybe a cosmological constant (maybe not?)

- constructing consistent IR modifications of GR is theoretically challenging and highly non-trivial

Intense activity on IR modifications of GR in the last two decades

- DGP model (Dvali-Gabadadze-Porrati 2000)

  gravity becomes effectively 5-dimensional in the IR

  a branch with a self-accelerating solution!

  but with a fatal ghost-like instability

Massive gravity: a long history

- In the linearized theory a specific mass-term is required to avoid a ghost Fierz-Pauli (1939)
- The ghost reappears at the non-linear level Bouleware-Deser (1972)
- construction of a nonlinear ghost-free theory
  de Rham-Gabadadze-Tolley (2011), Hassan-Rosen (2012)
- but no viable cosmological evolution yet found
  flat homogeneous FRW solutions do not exist
  open solutions are plagued by instabilities

Ghost-free bigravity gravity Hassan-Rosen (2012)

- promote a reference metric to a dynamical field
- at the background level, viable cosmological solutions with self-acceleration !
- at the level of cosmological perturbations, fatal instabilities in the scalar and tensor sectors

To build a consistent and cosmologically viable IR modification of GR is highly non-trivial!

Non-locality opens new possibilities

non-locality emerges from fundamental local theories in many situations

- classically, when separating long and short wavelength and integrating out the short wave-length

  (e.g cosmological perturbation theory)

- in QFT, when computing the effective action that includes the effect of radiative corrections of massless or light particles

- the operator $\Box^{-1}$ is relevant in the IR

- a first attempt in this direction Deser-Woodard 2007-2013

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R\left[1 + f(\Box^{-1}R)\right]$$

- the function f(X) is chosen so to give a viable evolution at the background level

$$f(X) = a_1[\tanh(a_2 Y + a_3 Y^2 + a_4 Y^3) - 1]$$

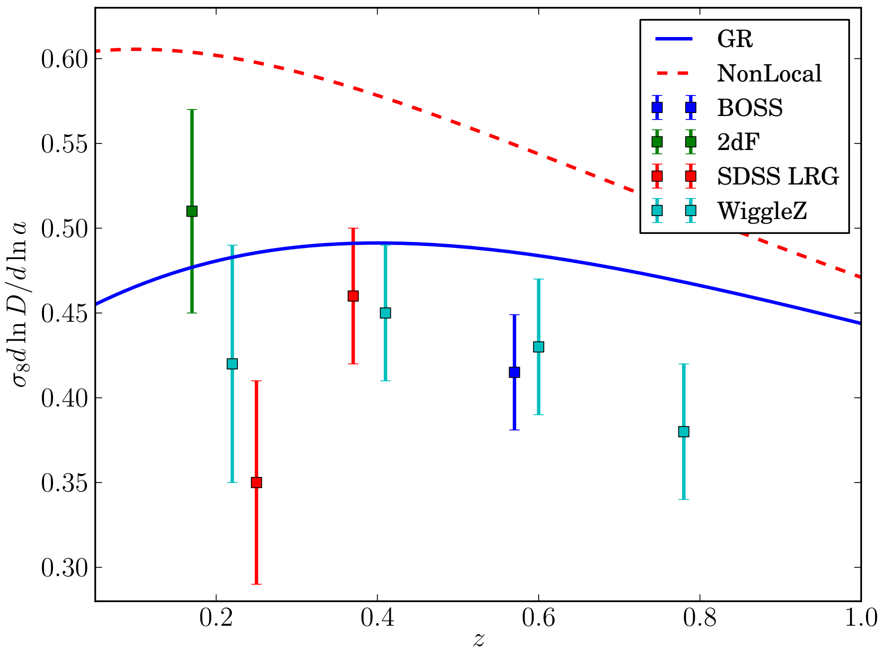
$$Y = X + a_5$$

(not terribly natural!)

a_1,...a_5 fitted to mimick the background evolution of ΛCDM after fixing the background evolution in this way, one can compute cosmological perturbations in the Deser-Woodard model, and compare with data

Deser-Woodard model ruled out at the 8σ level by structure formation

Dodelson and Park 2013



Once again, constructing a viable IR modification of GR is very challenging !

Our approach: we will introduce a mass scale as the coefficient of a non-local term

- phenomenological approach. Identify a non-local modification of GR that works well

- attempt at a more fundamental understanding

Some source of inspiration: a locality / gauge-invariance duality for massive gauge fields

- Proca theory for massive photons

$$S = \int d^4x \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} m_\gamma^2 A_\mu A^\mu - j_\mu A^\mu \right]$$

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = j^\nu \quad \rightarrow \begin{cases} m_\gamma^2 \, \partial_\nu A^\nu = 0 \\ (\Box - m_\gamma^2) A^\mu = 0 \end{cases}$$

- non-local formulation (Dvali 2006)

Stueckelberg trick: $A_\mu \rightarrow A_\mu + \frac{1}{m_\gamma} \partial_\mu \varphi$

we add one field and we gain a gauge symmetry

$$A_\mu \rightarrow A_\mu - \partial_\mu \theta \,, \quad \varphi \rightarrow \varphi + m_\gamma \theta$$

$$S = \int d^4x \, \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_\gamma^2 A_\mu A^\mu - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - m_\gamma A^\mu \partial_\mu \varphi - j_\mu A^\mu \right]$$

$$\partial_\mu F^{\mu\nu} = m_\gamma^2 A^\nu + m_\gamma \partial^\nu \varphi + j^\nu \, ,$$
$$\Box \varphi + m_\gamma \partial_\mu A^\mu = 0 \, .$$

If we choose the unitary gauge $\phi=0$ we get back to the original formulation of Proca theory (and loose the gauge sym because of gauge fixing).

Instead, keep the gauge sym explicit and integrate out ϕ using its own equation of motion:

$$\varphi(x) = -m_\gamma \Box^{-1} (\partial_\mu A^\mu)$$

Substituting in the eq of motion for A^ν :

$$\left(1-\frac{m_\gamma^2}{\Box}\right)\partial_\mu F^{\mu\nu}=j^\nu$$

or

$$(\Box-m_\gamma^2)A^\nu=\left(1-\frac{m_\gamma^2}{\Box}\right)\partial^\nu\partial_\mu A^\mu+j^\nu$$

we have explicit gauge invariance for the massive theory,
at the price non-locality

- a sort of duality between explicit gauge-invariance and explicit locality
- we can fix the gauge $\partial_\mu A^\mu=0$ and the non-local term disappears (and we are back to Proca eqs.)
- with hindsight, the Stueckelberg trick was not needed

- massive photon: can be described replacing

$$\partial_\mu F^{\mu\nu} = j^\nu \quad \to \quad \left(1 - \frac{m^2}{\Box}\right) \partial_\mu F^{\mu\nu} = j^\nu$$

(Dvali 2006)

- for gravity, a first guess for a massive deformation of GR could be

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \to \quad \left(1 - \frac{m^2}{\Box_g}\right) G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

(Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

however this is not correct since $\nabla^\mu(\Box_g^{-1} G_{\mu\nu}) \neq 0$

we lose energy-momentum conservation

- to preserve energy-momentum conservation:

$$G_{\mu\nu} - m^2(\Box^{-1}G_{\mu\nu})^T = 8\pi G T_{\mu\nu}$$

(Jaccard,MM, Mitsou, 2013)

however, instabilities in the cosmological evolution

(Foffa,MM, Mitsou, 2013)

- $$G_{\mu\nu} - m^2(g_{\mu\nu}\Box^{-1}R)^T = 8\pi G T_{\mu\nu}$$

(MM 2013)

stable cosmological evolution! ``RT model"

- a related model:

(MM and M.Mancarella, 2014)

$$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

``RR model"

Absence of vDVZ discontinuity

A. Kehagias and MM 2014

- write the eqs of motion of the non-local theory in spherical symmetry:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

- for mr <<1: low-mass expansion

- for $r \gg r_S$: Newtonian limit (perturbation over Minowski)

- match the solutions for $r_S \ll r \ll m^{-1}$ (this fixes all coefficients)

- result: for $r \gg r_s$
$$A(r) = 1 - \frac{r_S}{r}\left[1 + \frac{1}{3}(1 - \cos mr)\right]$$
$$B(r) = 1 + \frac{r_S}{r}\left[1 - \frac{1}{3}(1 - \cos mr - mr \sin mr)\right]$$

for $r_s \ll r \ll m^{-1}$: $A(r) \simeq 1 - \frac{r_S}{r}\left(1 + \frac{m^2 r^2}{6}\right)$

the limit $m \to 0$ is smooth !

By comparison, in massive gravity the same computation gives

$$A(r) = 1 - \frac{4}{3}\frac{r_S}{r}\left(1 - \frac{r_S}{12 m^4 r^5}\right)$$

vDVZ discontinuity

breakdown of linearity below $r_V = (r_s/m\hat{\ }4)^{1/5}$

Cosmological consequences.

- consider $$S_{\rm NL}=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R-m^2R\frac{1}{\Box^2}R\right]$$

  define $$U=-\Box^{-1}R\,,\qquad S=-\Box^{-1}U$$

  NB: auxiliary non-dynamical fields! U=0 if R=0. It is not the same as a scalar-tensor theory

- in FRW we have 3 variables: H(t), U(t), W(t)=H^2(t)S(t).

  define $x=\ln a(t)$, $h(x)=H(x)/H_0$,
  $\gamma=(m/3H_0)^2$ $\zeta(x)=h'(x)/h(x)$

$$h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(U, U', W, W')$$
$$U'' + (3+\zeta)U' = 6(2+\zeta)$$
$$W'' + 3(1-\zeta)W' - 2(\zeta' + 3\zeta - \zeta^2)W = U$$

- there is an effective DE term, with

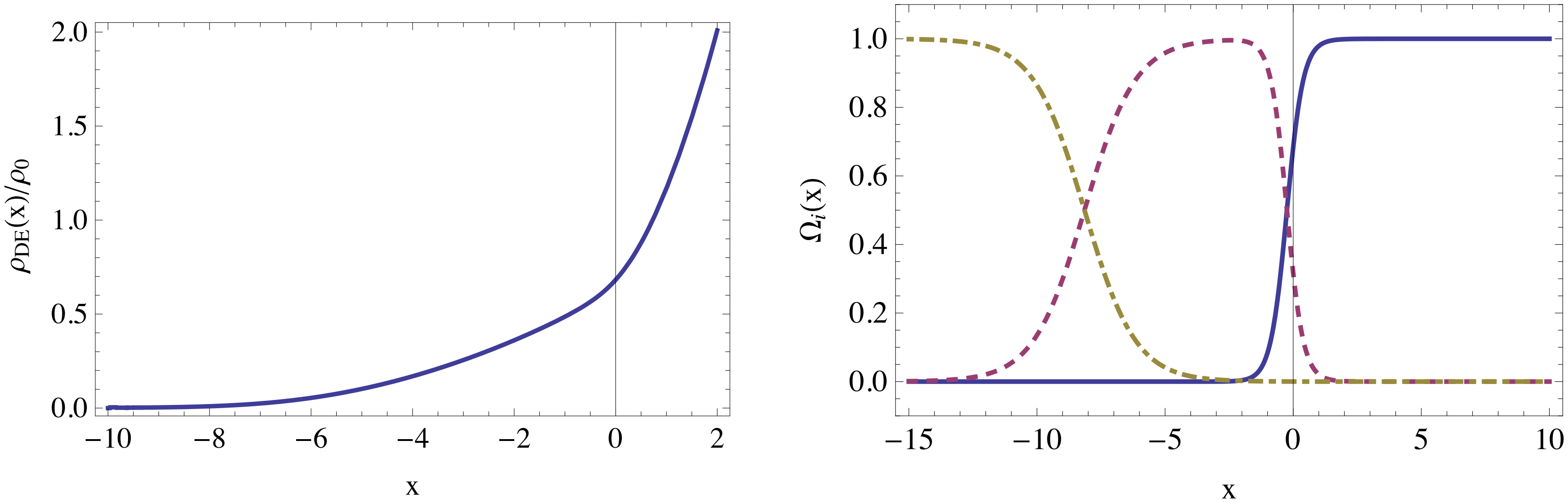
$$\rho_{\rm DE}(x) = \rho_0 \gamma Y(x) \qquad\qquad \rho_0 = 3H_0^2/(8\pi G)$$

- define w_{DE} from

$$\dot{\rho}_{\rm DE} + 3(1+w_{\rm DE})H\rho_{\rm DE} = 0$$

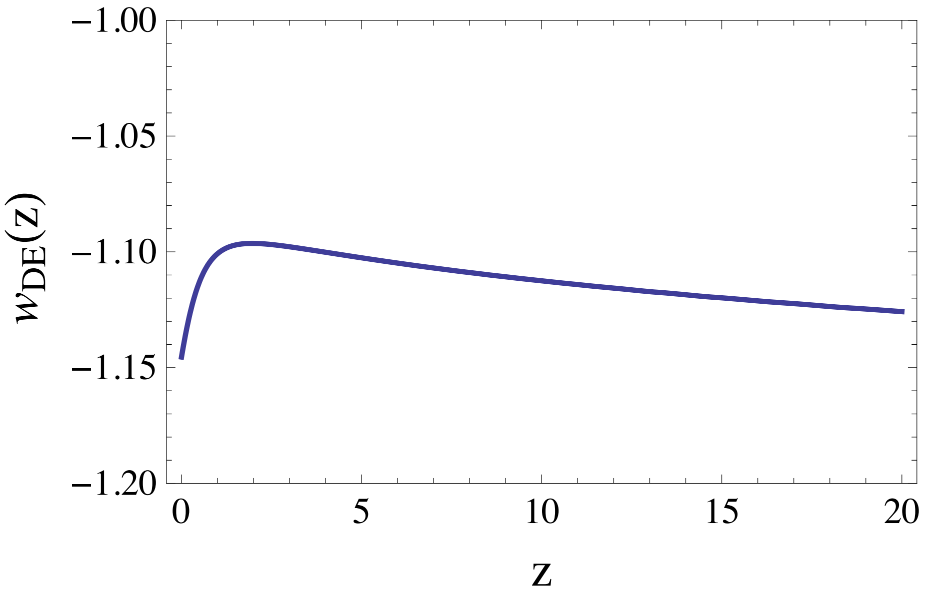
- the model has the same number of parameters as ΛCDM, with $\Omega_\Lambda \leftrightarrow \gamma$.

- results:



- Fixing $\gamma = 0.0089..$ ($m=0.28\ H_0$) we reproduce $\Omega_{DE}=0.68$

- having fixed γ we get a pure prediction for the EOS:



fit $w(a)=w_0+(1-a)\,w_a$

(in the region $0< z <1.6$)

$w_0= -1.14,\quad w_a=0.08$

on the phantom side ! general consequence of

$$\dot{\rho}_{\rm DE} + 3(1+w_{\rm DE})H\rho_{\rm DE} = 0$$

together with $\rho>0$ and $d\rho/dt>0$

The RT model $G_{\mu\nu} - m^2(g_{\mu\nu}\Box^{-1}R)^T = 8\pi G T_{\mu\nu}$

gives $w_0= -1.04,\quad w_a=-0.02$

warning. This is not wCDM !!!

Cosmological perturbations

- well-behaved? YES

Dirian, Foffa, Khosravi, Kunz, MM
JCAP 2014

this step is already non-trivial, as we already saw with DGP or bigravity

- consistent with data? YES

this step ruled out the Deser-Woodard non-local model

- comparison with ΛCDM

Dirian, Foffa, Kunz, MM, Pettorino,
JCAP 2015 and 1602.03558

implement the perturbations in a Boltzmann code

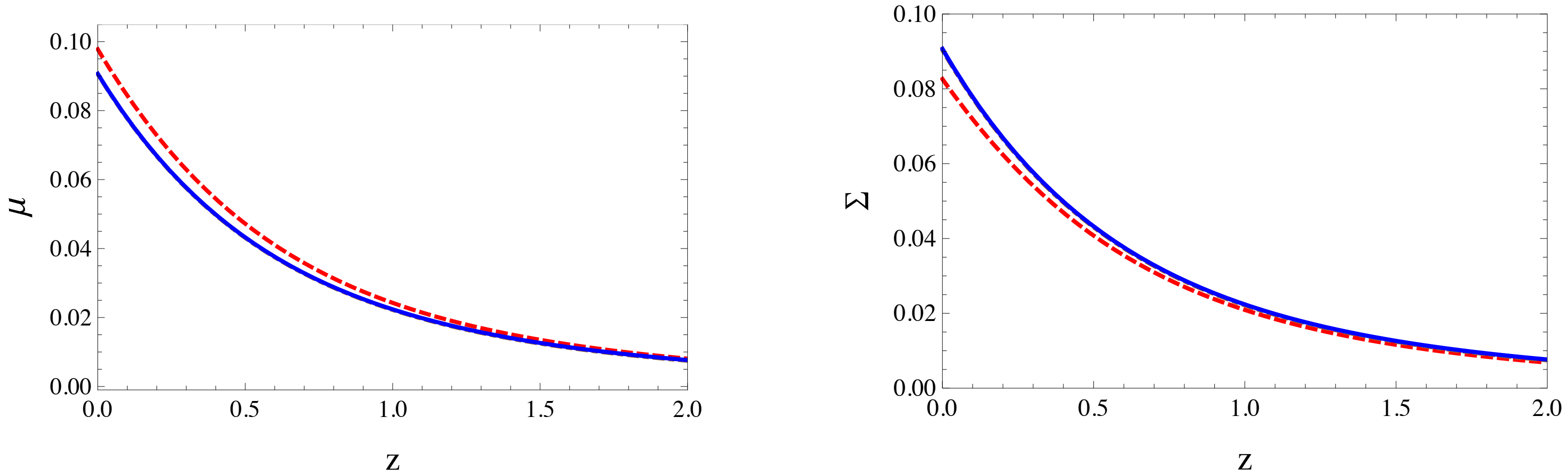
compute likelihood, χ^2, perform parameter estimation

no other IR modifications of GR has ever reached this stage!

(furthermore, bigravity tunes 5 parameters more than LCDM, and in Deser-Woodard model one tunes whole function)

- the perturbations are well-behaved and differ from ΛCDM at a few percent level

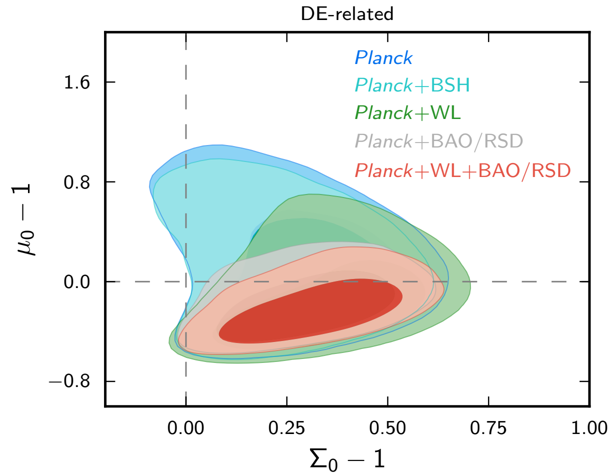
$$\Psi = [1+\mu(a;k)]\Psi_{\rm GR}$$
$$\Psi - \Phi = [1+\Sigma(a;k)](\Psi-\Phi)_{\rm GR}$$



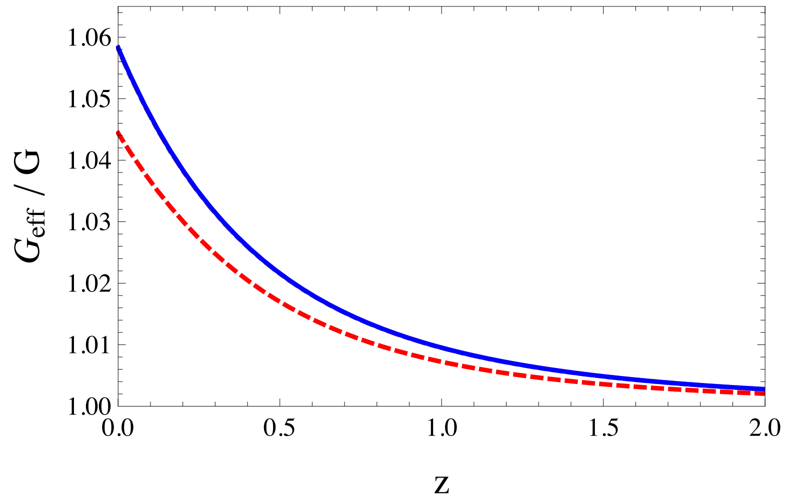
- deviations at z=0.5 of order 4%

- consistent with data:

(Ade et al., Planck XV, 2015)

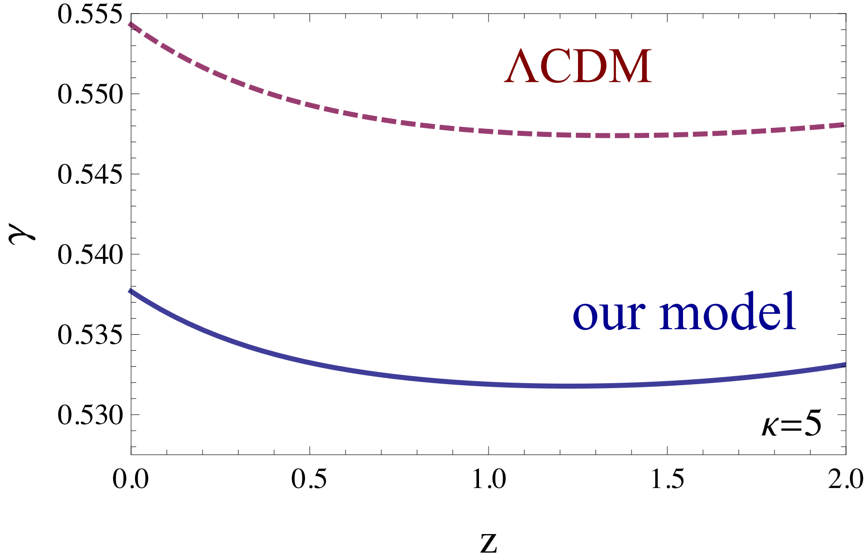


effective Newton constant



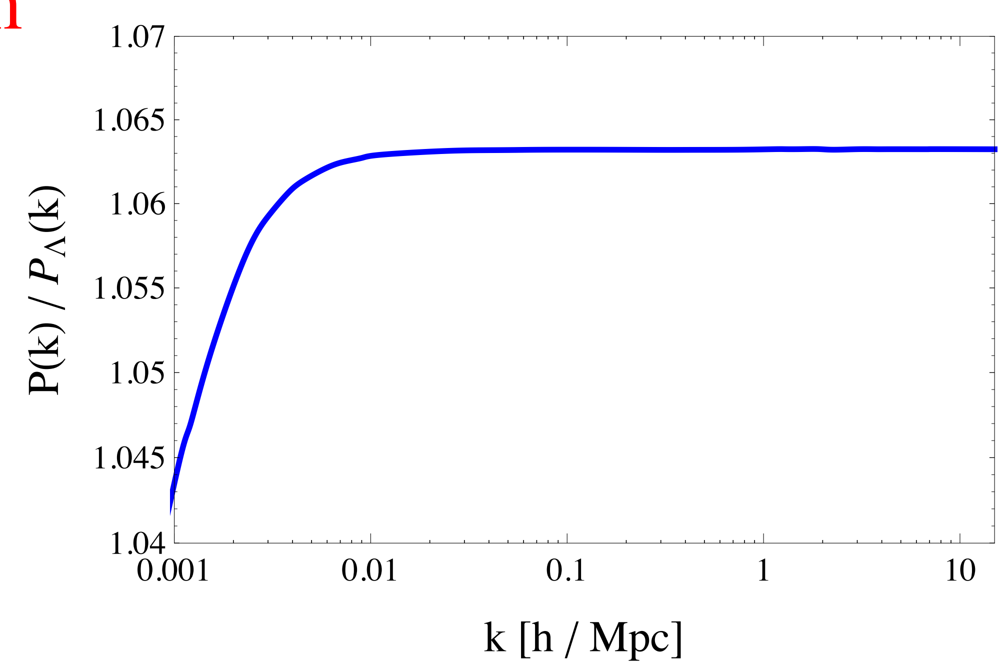
growth index:

$$\frac{d\log\delta_M(a;k)}{d\ln a} = [\Omega_M]^{\gamma(z;k)}$$

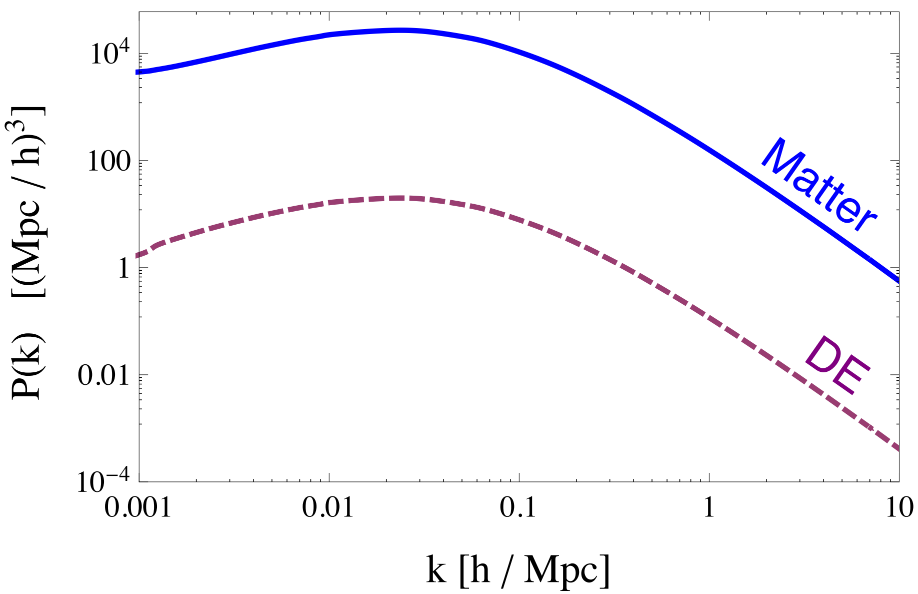


- linear power spectrum

matter power spectrum
compared to ΛCDM



DE clusters but its linear
power spectrum is small
compared to that of matter



- sufficiently close to ΛCDM to be consistent with existing data, but distinctive prediction that can be clearly tested in the near future

  - phantom DE eq of state: $w(0) = -1.14$ (RR) (or -1.04 RT) + a full prediction for $w(z)$
    - DES $\Delta w = 0.03$
    - EUCLID $\Delta w = 0.01$
  - linear structure formation

    $$\mu(a) = \mu_s a^s \to \mu_{\mathbf{s}} = \mathbf{0.09}, \mathbf{s} = \mathbf{2}$$

    - Forecast for EUCLID, $\Delta\mu = 0.01$

  - non-linear structure formation: 10% more massive halos

Barreira, Li, Hellwing, Baugh, Pascoli 2014

  - lensing: deviations at a few %

Boltzmann code analysis and comparison with data

Dirian, Foffa, Kunz, MM, Pettorino, JCAP 2015 and 1602.03558

- We test the non-local models against
  - Planck 2015 TT, TE, EE and lensing data,
  - isotropic and anisotropic BAO data,
  - JLA supernovae,
  - local H_0 measurements,
  - growth rate data

  and we perform Bayesian parameter estimation.

- we modified the CLASS code and use Montepython MCMC
- we vary $\omega_b = \Omega_b h_0^2,\ \omega_c = \Omega_c h_0^2,\ H_0,\ A_s,\ n_s,\ z_{\rm re}$

In ΛCDM, Ω_Λ is a derived parameter, fixed by the flatness condition. Similarly, in our model the mass parameter m^2 is a derived parameter, fixed again from $\Omega_{tot}=1$

we have the same free parameters as in ΛCDM

- # Results

χ^2 statistically equivalent between LCDM and RT

RR disfavored (with standard neutrino masses and no large prior on H_0)

confirmed from computation of the Bayes factors

Table 1: BAO+Planck+JLA

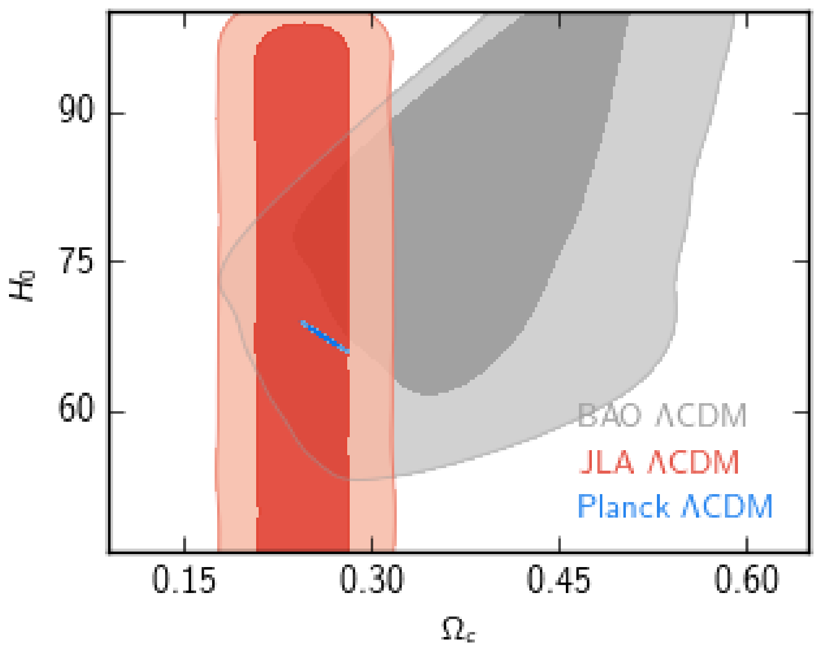
Param	ΛCDM	RT	RR
$100\ \omega_b$	$2.228^{+0.014}_{-0.015}$	$2.223^{+0.014}_{-0.014}$	$2.213^{+0.014}_{-0.014}$
ω_c	$0.119^{+0.0011}_{-0.0011}$	$0.1197^{+0.0011}_{-0.00096}$	$0.121^{+0.001}_{-0.001}$
H_0	$67.67^{+0.47}_{-0.5}$	$68.76^{+0.46}_{-0.51}$	$70.44^{+0.56}_{-0.56}$
$\ln(10^{10}A_s)$	$3.066^{+0.019}_{-0.026}$	$3.056^{+0.021}_{-0.023}$	$3.027^{+0.027}_{-0.023}$
n_s	$0.9656^{+0.0041}_{-0.0043}$	$0.9637^{+0.0039}_{-0.0041}$	$0.9601^{+0.004}_{-0.0039}$
$\tau_{\rm re}$	$0.06678^{+0.011}_{-0.013}$	$0.0611^{+0.011}_{-0.013}$	$0.04516^{+0.014}_{-0.012}$
$z_{\rm re}$	$8.893^{+1.1}_{-1.2}$	$8.359^{+1.2}_{-1.2}$	$6.707^{+1.7}_{-1.2}$
σ_8	$0.817^{+0.0076}_{-0.0095}$	$0.8283^{+0.0085}_{-0.0093}$	$0.8443^{+0.01}_{-0.0099}$
$\chi^2_{\rm min}$	13631.0	13631.6	13637.0
$\Delta\chi^2_{\rm min}$	0	0.6	6.0

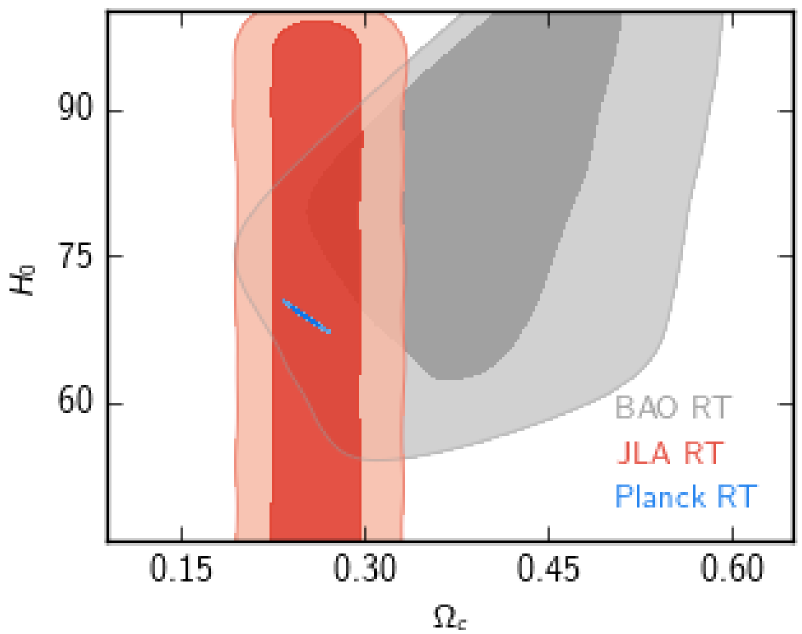
Table 2: BAO+Planck+JLA+($H_0 = 73.8$)

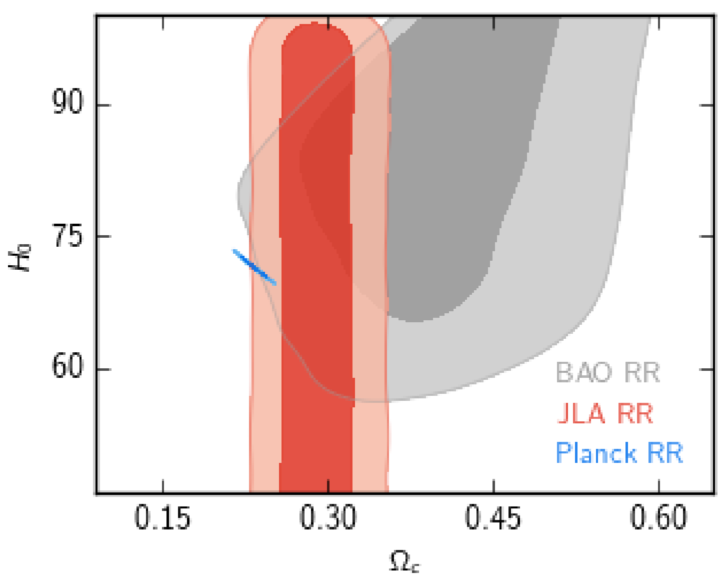
Param	ΛCDM	RT	RR
$100\ \omega_b$	$2.233^{+0.014}_{-0.014}$	$2.226^{+0.014}_{-0.014}$	$2.217^{+0.014}_{-0.014}$
ω_c	$0.1185^{+0.00097}_{-0.0011}$	$0.1194^{+0.001}_{-0.001}$	$0.1207^{+0.00096}_{-0.00097}$
H_0	$67.93^{+0.48}_{-0.43}$	$68.91^{+0.49}_{-0.5}$	$70.65^{+0.52}_{-0.54}$
$\log(10^{10}A_s)$	$3.077^{+0.026}_{-0.019}$	$3.061^{+0.026}_{-0.022}$	$3.031^{+0.018}_{-0.022}$
n_s	$0.9671^{+0.0041}_{-0.0041}$	$0.9645^{+0.004}_{-0.0041}$	$0.9611^{+0.0038}_{-0.004}$
$\tau_{\rm re}$	$0.07275^{+0.014}_{-0.01}$	$0.0641^{+0.013}_{-0.012}$	$0.04791^{+0.01}_{-0.011}$
$z_{\rm re}$	$9.435^{+1.3}_{-0.85}$	$8.636^{+1.3}_{-1.1}$	$7.02^{+1.1}_{-1.2}$
σ_8	$0.8197^{+0.0096}_{-0.0075}$	$0.8298^{+0.0095}_{-0.0086}$	$0.8456^{+0.0081}_{-0.0088}$
$\chi^2_{\rm min}$	13637.5	13636.1	13638.9
$\Delta\chi^2_{\rm min}$	1.4	0	2.8

large value for H_0 suggested by local measurements.

Latest value $H_0 = 73.02 \pm 1.79$ (Riess et al 1604.01424)



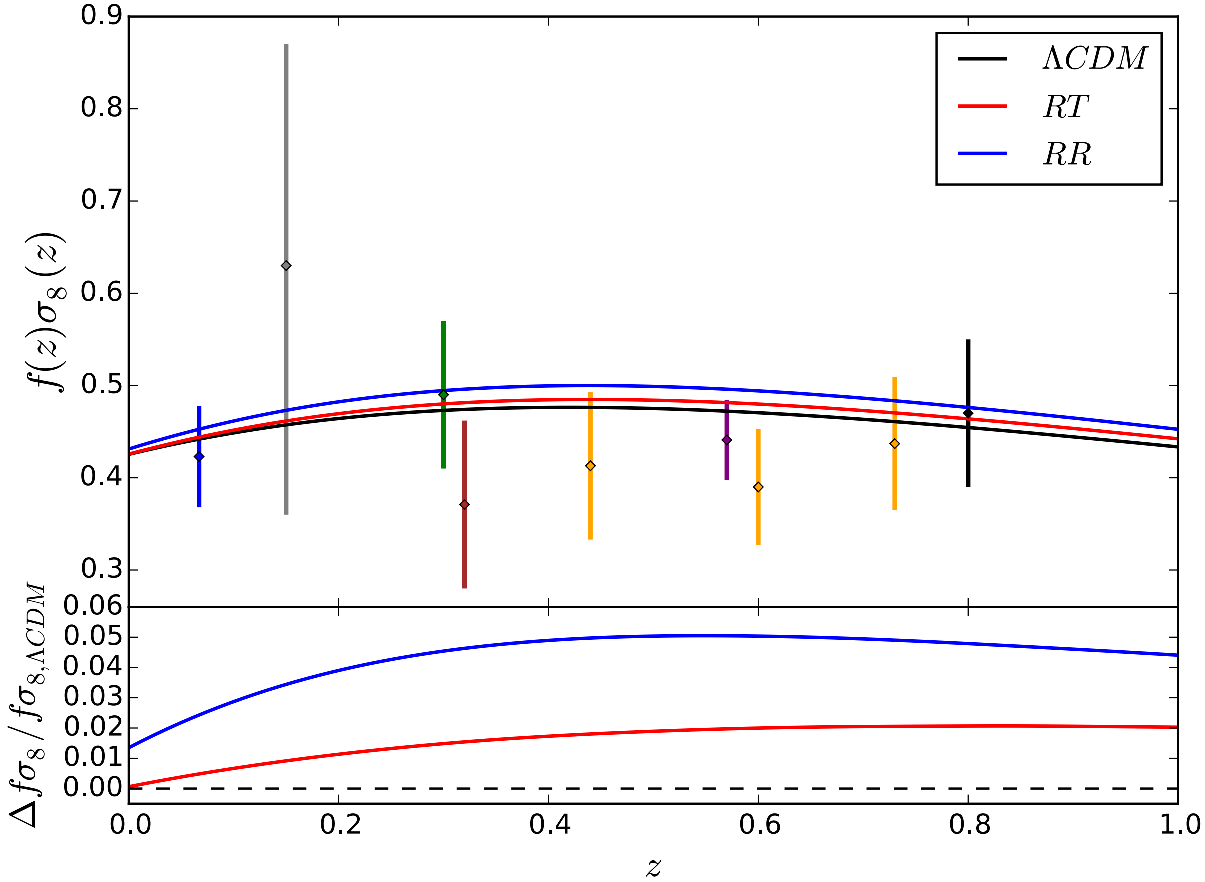




The RT model works perfectly well

The RR model has a
2σ tension between CMB and SN

growth rate and structure formation



Conclusion: at the phenomenological level, these non-local models work very well

- solar system tests OK
- generates dynamically a dark energy
- cosmological perturbations work well
- passes tests of structure formation
- comparison with CMB,SNe,BAO with modified Boltzmann code ok
- higher value of H0

They are the only existing models, with the same number of parameters as ΛCDM, which are competitive with ΛCDM from the point of view of fitting the data

Where such non-local term comes from?

- loop corrections involving massless or light particles give non-local terms

  e.g. in QED $$S_{\text{eff}} = -\frac{1}{4}\int d^4x\, F_{\mu\nu}\frac{1}{e^2(\Box)}F^{\mu\nu}$$

  $$\frac{1}{e^2(\Box)} = \frac{1}{e^2(\mu)} - \beta_0 \log\left(\frac{-\Box}{\mu^2}\right)$$

- in gravity
  - loops of scalar, spinor and vector field in a fixed curved background

    Barvinsky-Vilkovisky 1985,1987, [.....]
    decoupling: Gorbar-Shapiro 2003

  - graviton loops

    Fradkin-Tseytlin 1982
    Avramidi-Barvinski 1985

However, perturbative loop corrections do not help (MM, PRD 2016)

to one-loop, the quantum effective action reads

$$S=\int d^4x\sqrt{-g}\left[\frac{m_{\rm Pl}^2}{2}R-R\,k_R(\Box)R-C_{\mu\nu\rho\sigma}k_W(\Box)C^{\mu\nu\rho\sigma}\right]$$

the form factors are known. The contribution of a particle with mass M, in the regime where M is light (M<<E, H) is non-local,

$$k_R\left(\frac{-\Box}{M^2}\right)=\alpha\log\left(\frac{-\Box}{M^2}\right)+\beta\left(\frac{M^2}{-\Box}\right)+\gamma\left(\frac{M^2}{-\Box}\right)\log\left(\frac{-\Box}{M^2}\right)+\delta\left(\frac{M^2}{-\Box}\right)^2+\ldots$$

e.g. Gorbar-Shapiro 2003, Codello-Jain 2015

$$m_{\rm Pl}^2R-R\frac{M^4}{\Box^2}R=m_{\rm Pl}^2(R-R\frac{m^2}{\Box^2}R),\qquad m^2=M^4/m_{\rm Pl}^2$$

$$m\sim H_0\to M\sim(m_{\rm Pl}H_0)^{1/2}\gg H_0$$

in this regime $M\gg\Box$ and $k_R(\Box)=\Box/M^2$ is local

We need a deeper, and probably non-perturbative mechanism.

Can gravity have non-perturbative effects in the IR ?

MM, 1506 and 1603 (PRD 2016)

we are used to think to gravity as becoming weaker and weaker at low energy, so the answer would seem `obviously no''.

However, some interesting hint comes from the dynamics of the conformal mode

$$g_{\mu\nu}(x) = e^{2\sigma(x)}\bar{g}_{\mu\nu}(x)$$

Antoniadis and Mottola 1992
Antoniadis, Mazur and Mottola 2007

in classical GR σ is a constrained variable. At the quantum level it acquires dynamics because of the conformal anomaly

– in D=2: Polyakov action (which becomes local in terms of σ)

$$S_{\rm anom} = -\frac{N}{96\pi}\int d^2x\sqrt{-g}\,R\frac{1}{\Box}R$$
$$= \frac{N}{24\pi}\int d^2x\sqrt{-\bar{g}}\,(-\sigma\bar{\Box}\sigma + \bar{R}\sigma)$$

– in D=4: covariant non-local anomaly-induced action (again local in terms of σ)

$$S_{\rm anom} = -\frac{Q^2}{16\pi^2}\int d^4x\,(\Box\sigma)^2$$

– the propagator of σ is $1/k^4$

fluctuations in σ become large in the IR

$$(\Box\sigma)^2 \to G(x,x') = -\frac{1}{2Q^2}\log\left[\mu^2(x-x')^2\right]$$

we expect strong IR effects due to σ

The situation is quite similar to D=2, where a $1/k^2$ propagator again gives a G(x)=log x in coordinate space, leading often to a rich IR physics

- BKT transition in d=2 (which also triggered by a logarithmic growth of fluctuations) with generation of a mass gap

- confinement in the Schwinger model, mass gap generation in O(N) sigma-models, etc.

Natural expectation: dynamical generation of a mass term for σ ?

however, we do not want to spoil diff invariance.

no local term starts with $m^2\sigma^2$

However, writing $g_{\mu\nu} = e^{2\sigma(x)}\eta_{\mu\nu}$

$$\begin{aligned} R &= -6e^{-2\sigma}\left(\Box\sigma + \partial_\mu\sigma\partial^\mu\sigma\right) \\ &= -6\Box\sigma + \mathcal{O}(\sigma^2) \end{aligned}$$

$$m^2 R \frac{1}{\Box^2} R = 36 m^2 \sigma^2 + \mathcal{O}(\sigma^3)$$

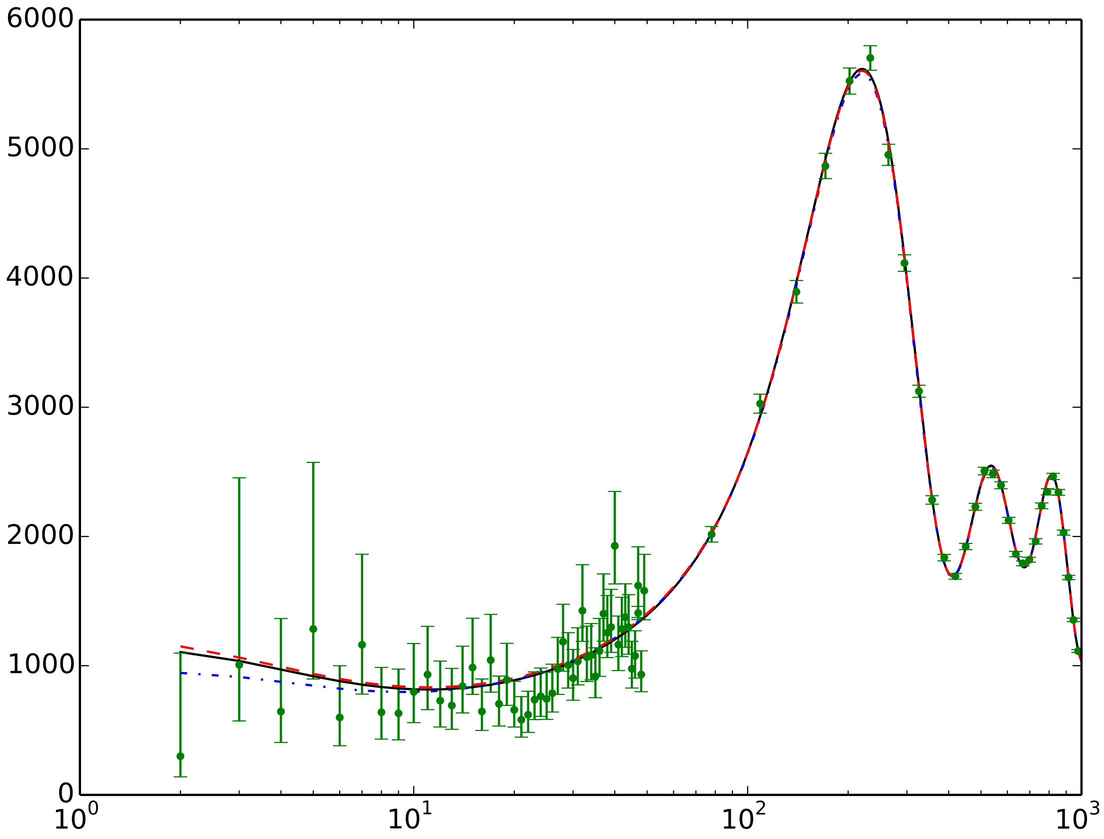
our non-local term is just a mass-term for σ, plus a non-linear completion that makes it diff-invariant !

An interesting direction for future investigations....

Thank you!

based on

Jaccard, MM, Mitsou,	PRD 2013, 1305.3034
MM,	PRD 2014, 1307.3898
Foffa, MM, Mitsou,	PLB 2014, 1311.3421
Foffa, MM, Mitsou,	IJMPA 2014, 1311.3435
Kehagias and MM,	JHEP 2014, 1401.8289
MM and Mancarella,	PRD 2014, 1402.0448
Dirian, Foffa, Khosravi, Kunz, MM,	JCAP 2014, 1403.6068
Dirian, Foffa, Kunz, MM, Pettorino,	JCAP 2015, 1411.7692
MM	PRD 2016, 1603.01515 (and 1506.06217)
Cusin, Foffa, MM,	PRD 2016, 1512.06373
Cusin, Foffa, MM, Mancarella,	PRD 2016, 1602.01078
Dirian, Foffa, Kunz, MM, Pettorino,	1602.03558



LCDM and RT model almost indistinguishable

RR (blue dot-dashed) lower at low multipoles