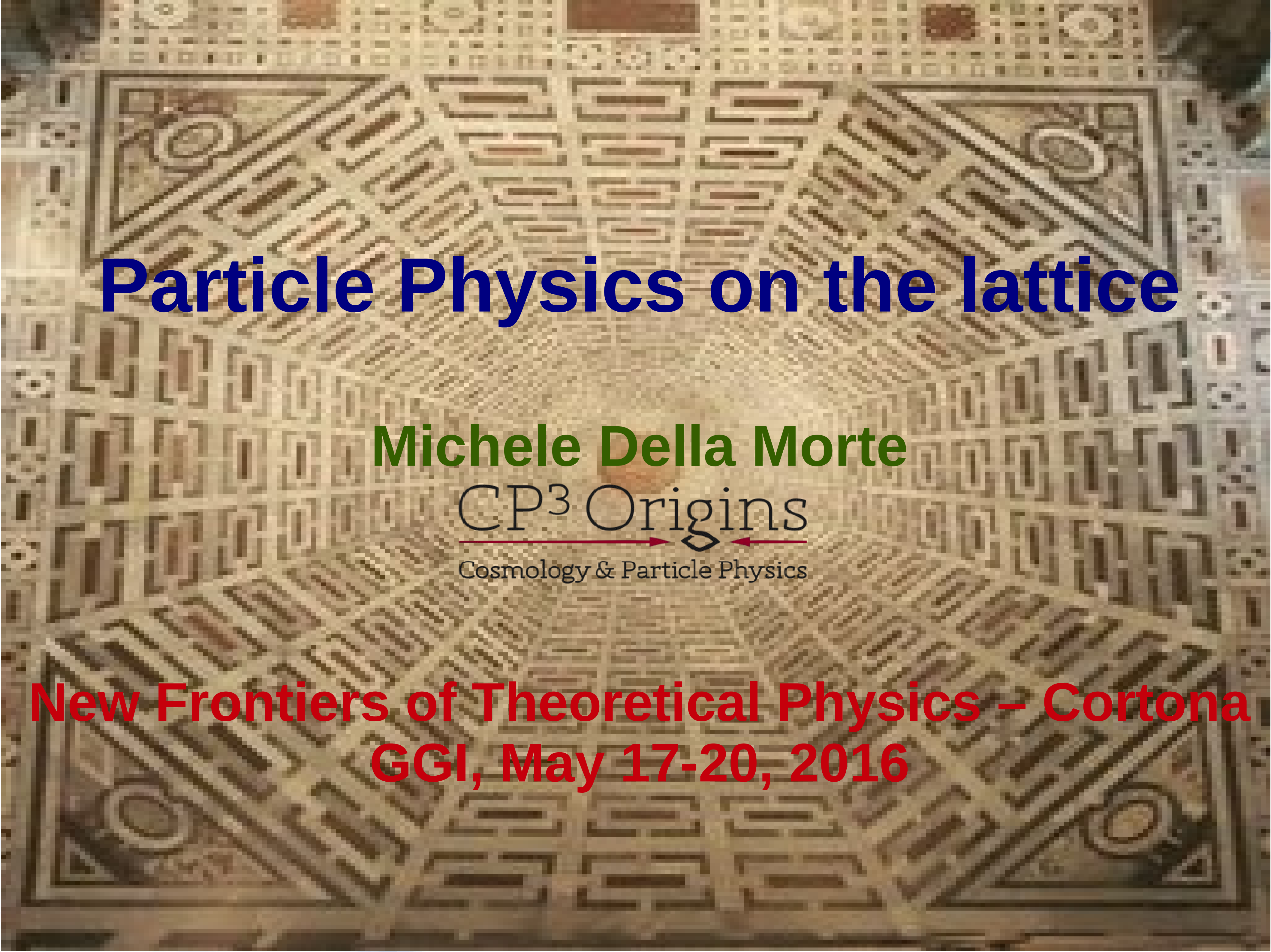


Particle Physics on the lattice

Michele Della Morte

CP³ Origins
Cosmology & Particle Physics

New Frontiers of Theoretical Physics – Cortona
GGI, May 17-20, 2016

Plan of the talk

- Need for lattice phenomenology and introduction to simulations
- FLAG initiative

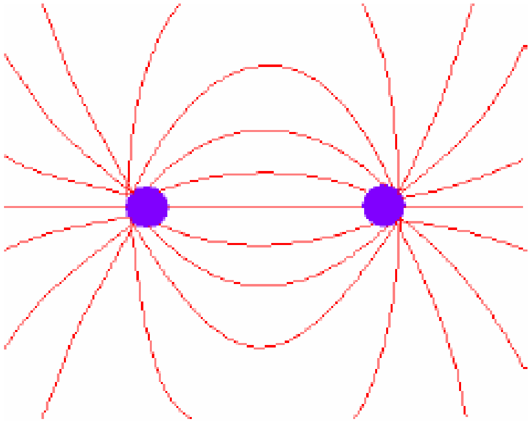
  (b-quark mass for Higgs physics and heavy quarks on the lattice)
- Recent developments:

  - QCD + QED why, what, how ?
  - Hadronic decays

Motivation: non-perturbative phenomena

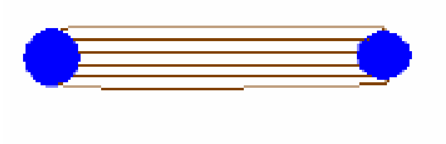
QED → QCD (as prototype of strongly interacting theories)

QED



- Photons and e+, e- act much like free particles.
- Perturabtion theory makes sense.
- Solved by expansion in α_{em}=1/137.

QCD

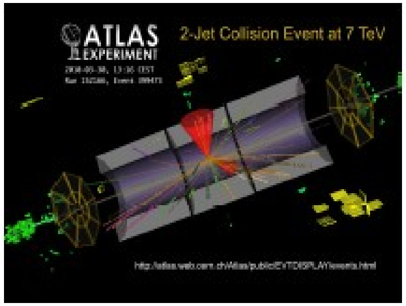


- Quarks and gluons are never observed (confinement).
- Perturbative treatment is absurd at low energies.
- Conventional methods fail.

What theory is QCD ?

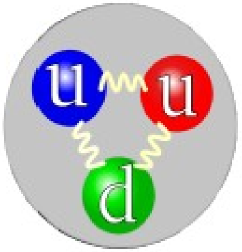
Quite surprisingly it describes

- jets at large energies



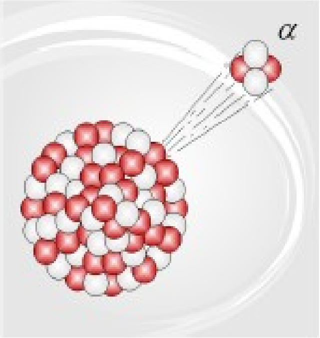
Q C D

- hadrons at small energies



Q C D

- nuclei at even smaller energies

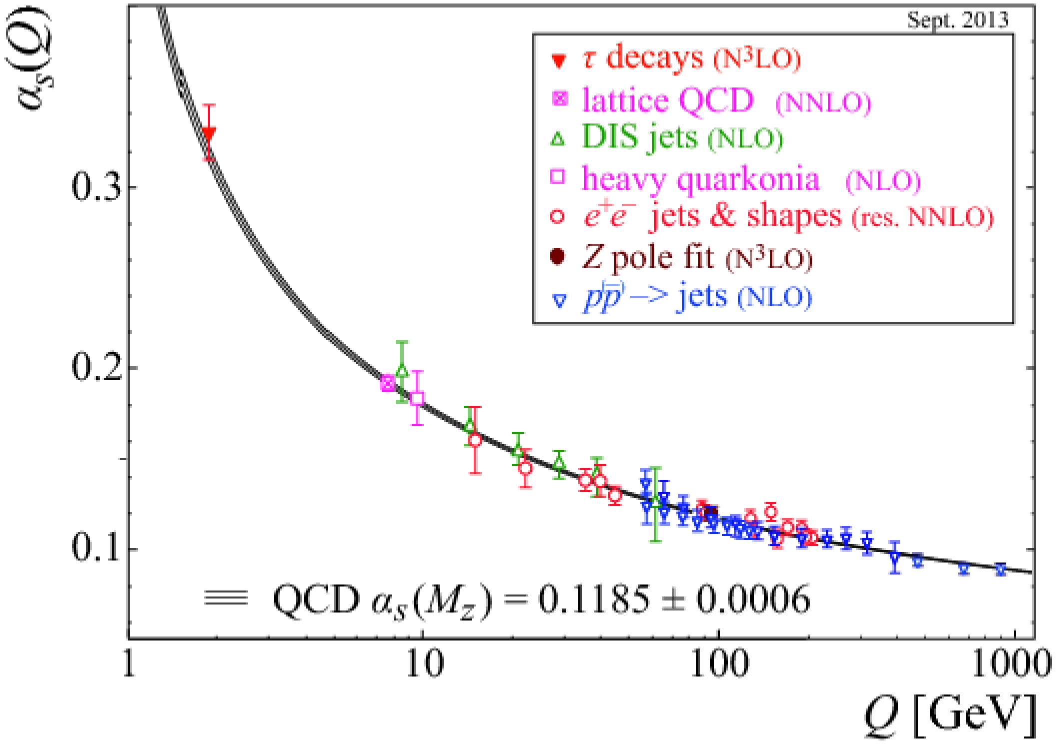


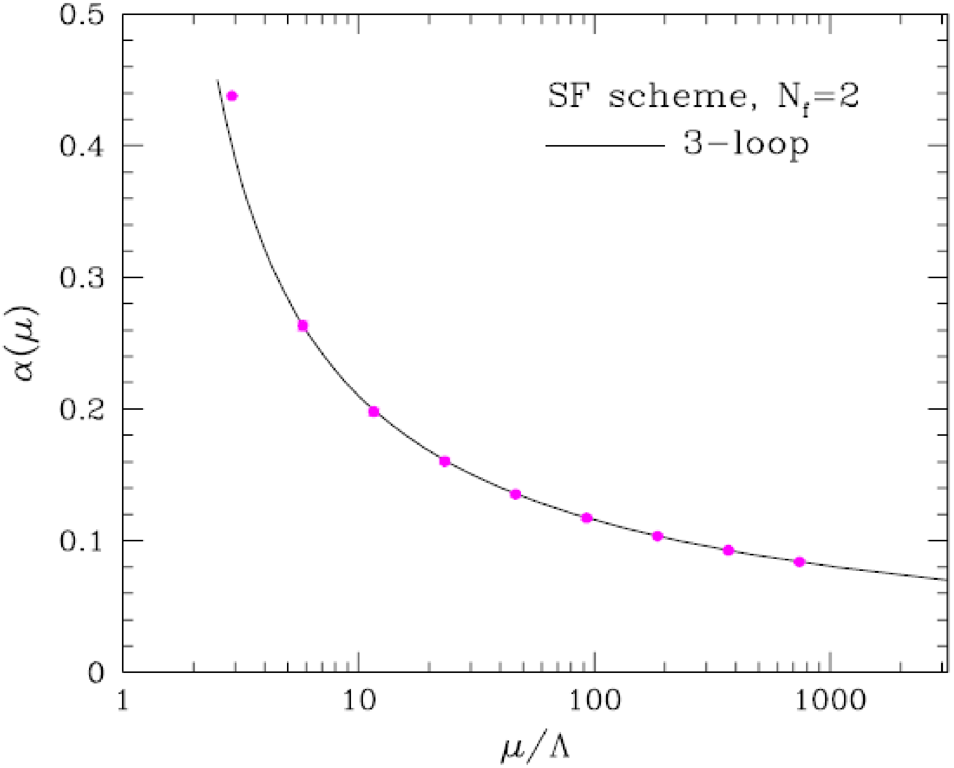
Q C D

$$\boxed{\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2{g_0}^2}\mathrm{tr}\{F_{\mu\nu}F_{\mu\nu}\} + \sum_{f=1}^{N_\mathrm{f}}\overline{\psi}_f\{D + m_{0f}\}\psi_f}$$

- $N_\mathrm{f} + 1 = 7$ (bare) parameters

An intuitive picture can be gathered from the running of the coupling





ALPHA Collaboration, 2005

Understanding strong interactions. Stability of light hadrons

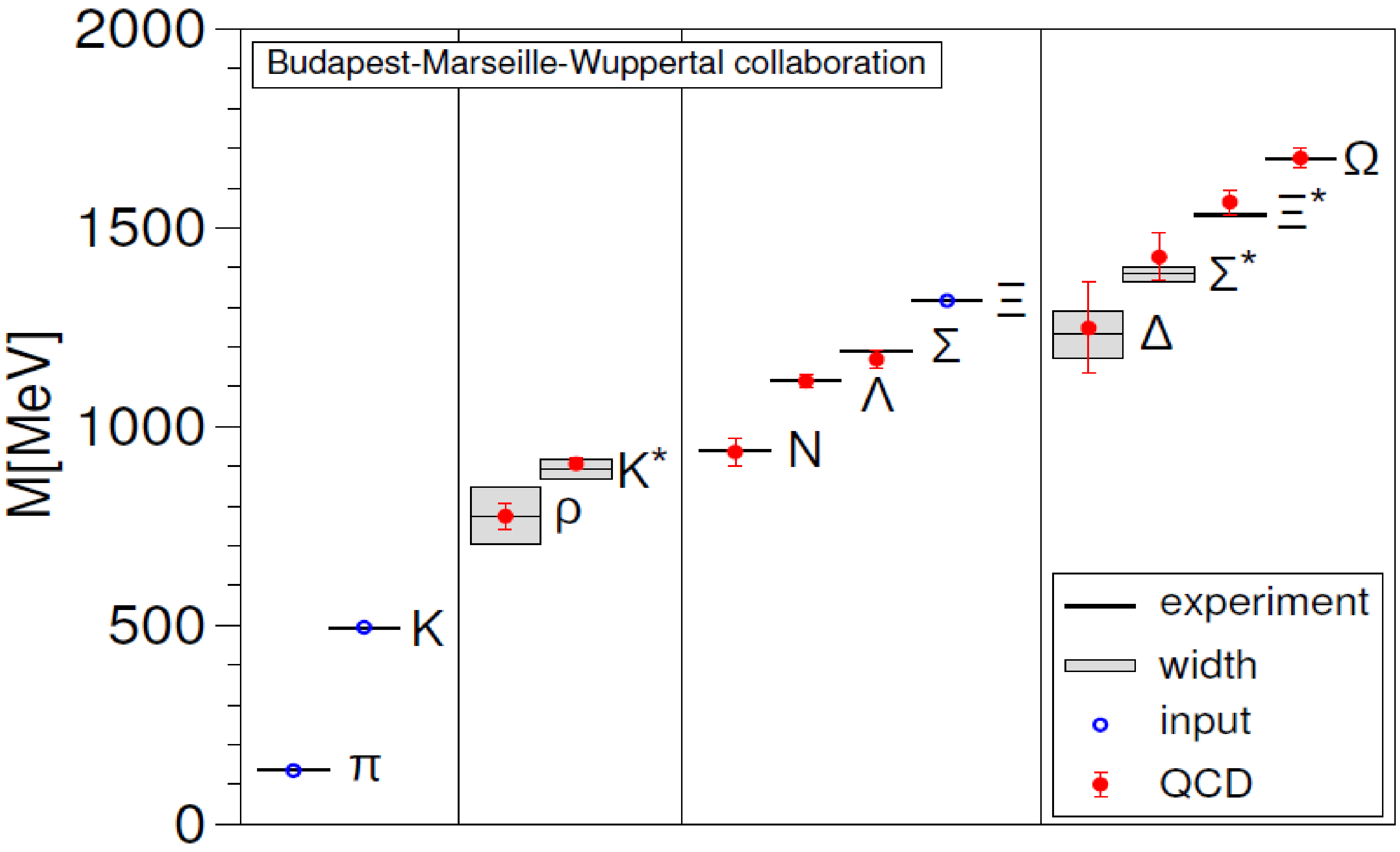
quarks

3 x 5 grams

proton

1 kilogram

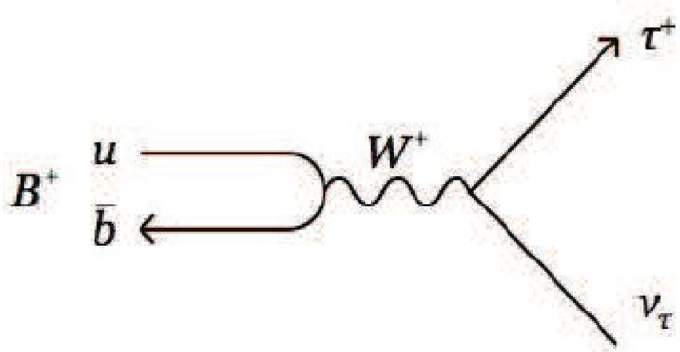
Hadrons are all binding energy !!



Several hadronic processes depend on hadronic contributions. E.g.

(Charged) Decay constants

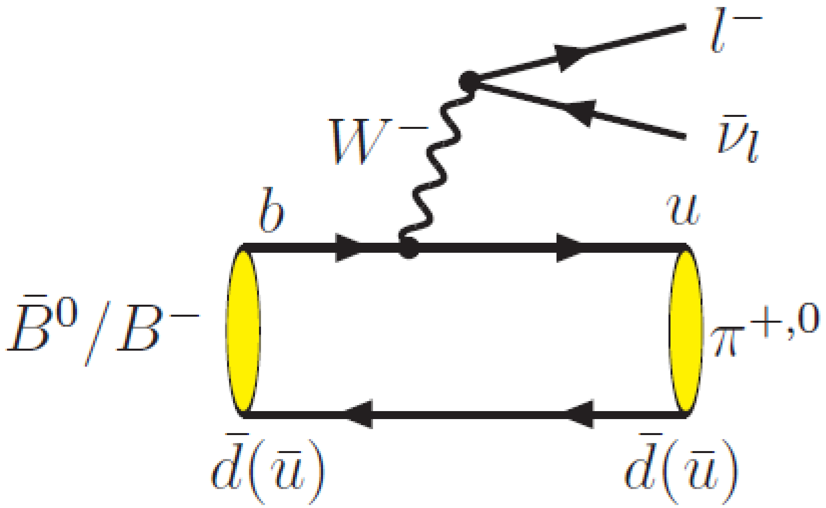
$\langle 0|\overline{u}_f \gamma_\mu \gamma_5 d_{f'}|P(p)\rangle = F_P p_\mu$ are the hadronic parameters entering leptonic decays of pseudoscalar mesons



$$\Gamma(B \to \ell \overline{\nu}_\ell) = \frac{G_F^2}{8\pi} |V_{ub}|^2 F_B^2 \left(\frac{m_\ell}{m_B}\right)^2 m_B^3 \left(1 - \frac{m_\ell^2}{m_B^2}\right)$$

Form factors

Parameterizing semileptonic decay. Simplest: $B \to \pi \ell \nu$



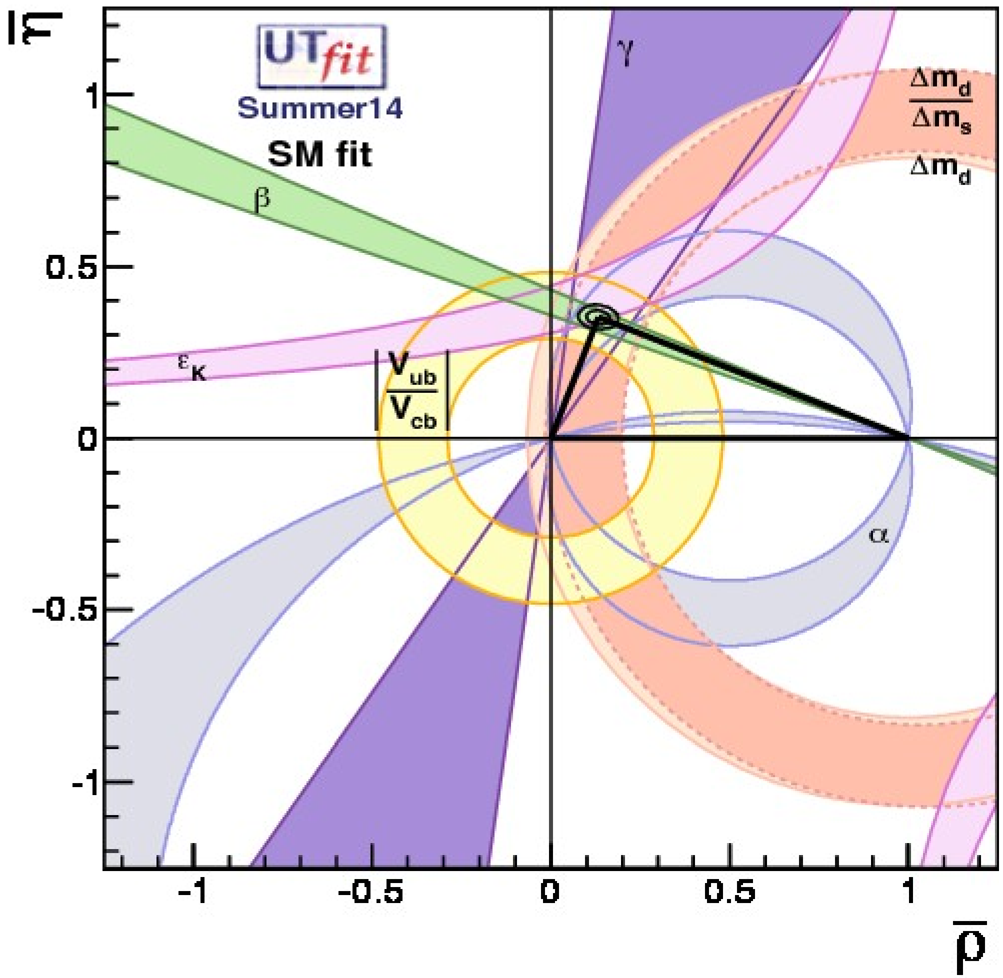
Ignoring the lepton mass:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} p_\pi^3 |V_{ub}|^2 |f_+(q^2)|^2$$

The hadronic matrix element is from a quark bilinear

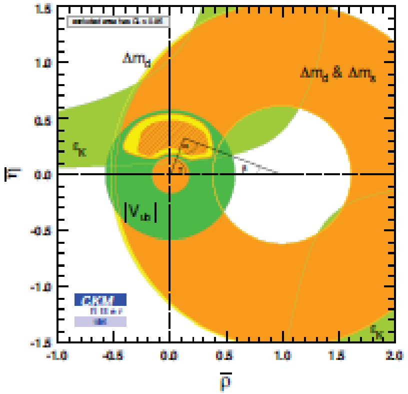
$$\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)(p_\pi + p_B - q\Delta_{m^2})^\mu + f_0(q^2)q^\mu$$

with $\Delta_{m^2} = (m_B^2 - m_\pi^2)/q^2$

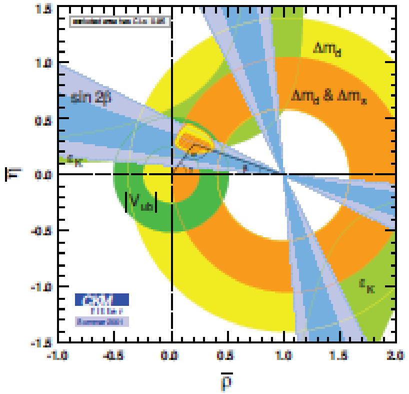


- The Higgs particle gives mass to leptons, quarks and vector bosons (EWSB).
- The matrix of Higgs Yukawa couplings to fermions is non-diagonal. Interactions eigenstates $\neq$ mass eigenstates. They are related through the unitary CKM matrix
- 4 parameters A, λ, ρ, η. The product of two columns closes up in a triangle (in the SM).

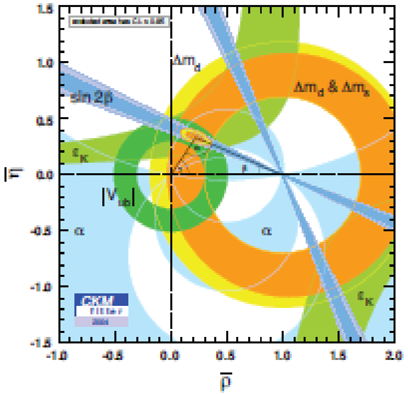
Goals of Flavor Physics: determine V_{CKM}, look for signals of NP, constrain possible BSM models.



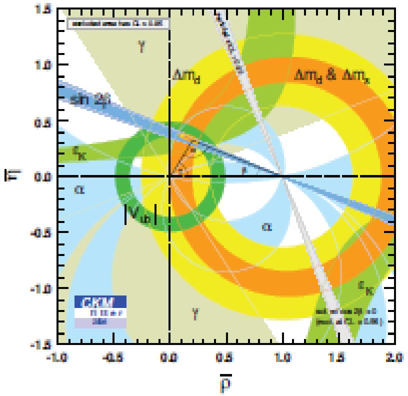
1995



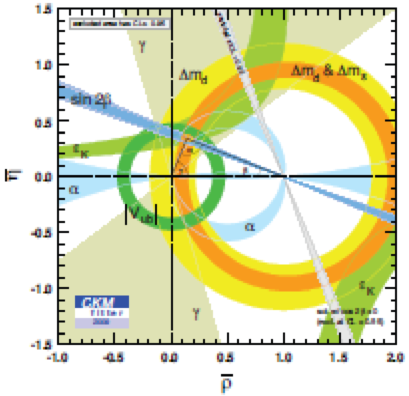
2001



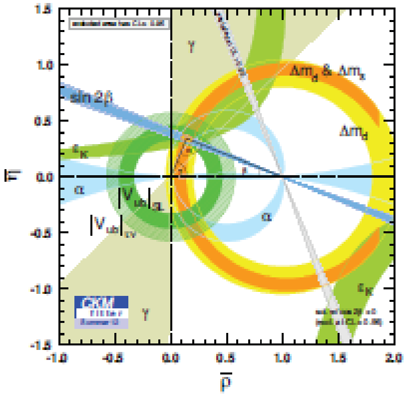
2004



2006



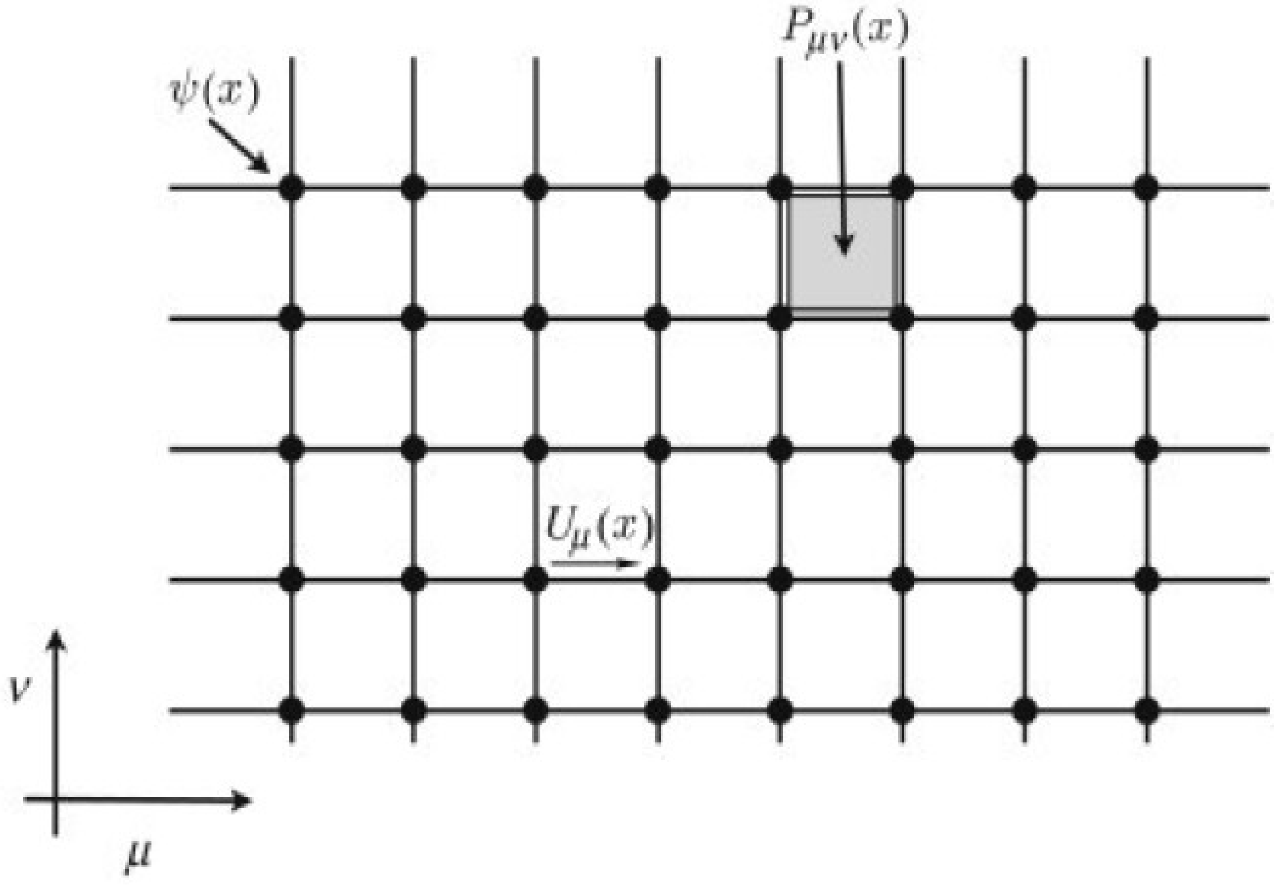
2009



2013

Impressive success of theory, where lattice has an extremely important role, and experiments. The analysis of experimental data, always requires non-perturbative hadronic parameters, as the processes always involve hadrons in the initial/final states.

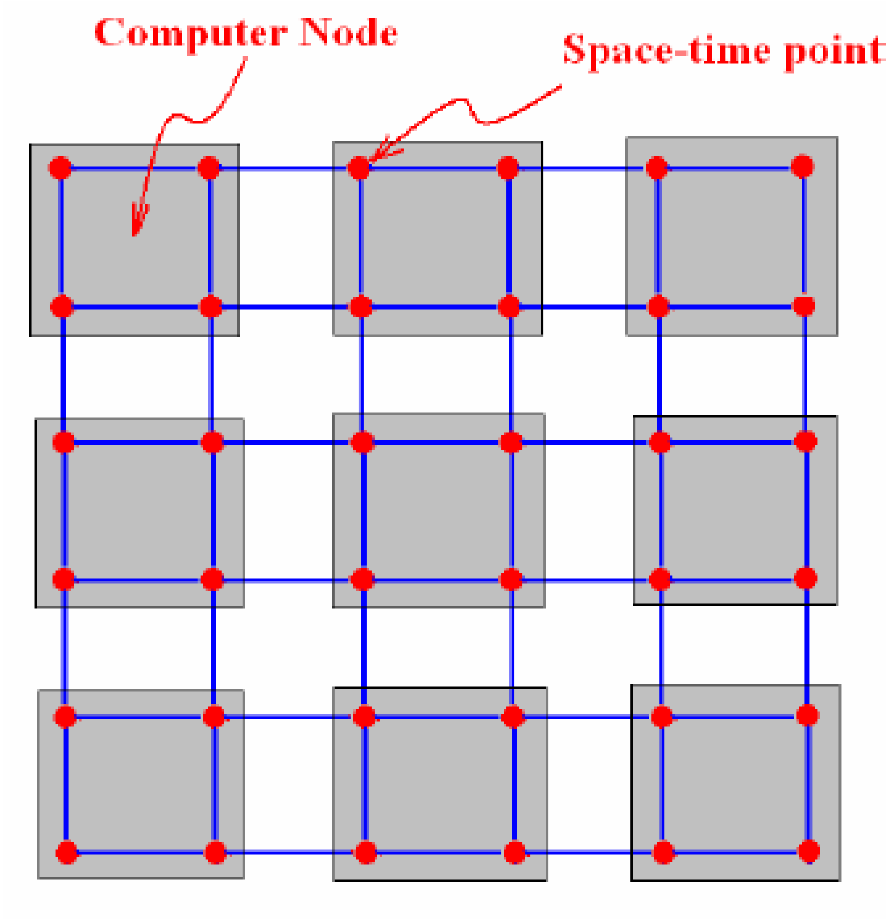
QCD on the lattice



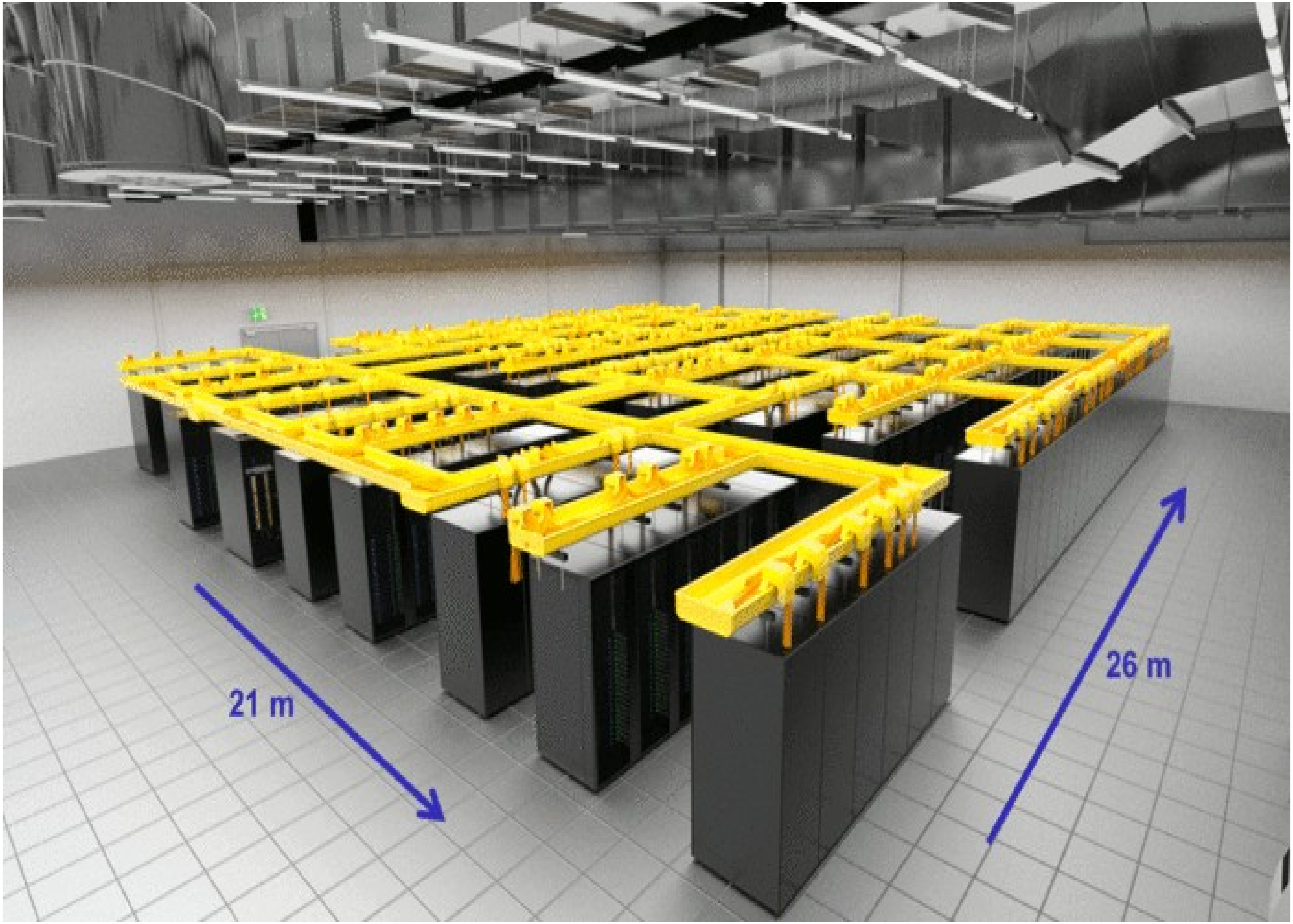
$$S_{\mathrm{W}}^{\mathrm{QCD}} = \overline{\psi}_x D_{xy}^{\mathrm{W}}(U)\psi_y + \beta \sum_{\Box} \left(1 - \frac{1}{N}\mathrm{Re\ Tr}\ U_{\mu\nu}(x)\right)$$

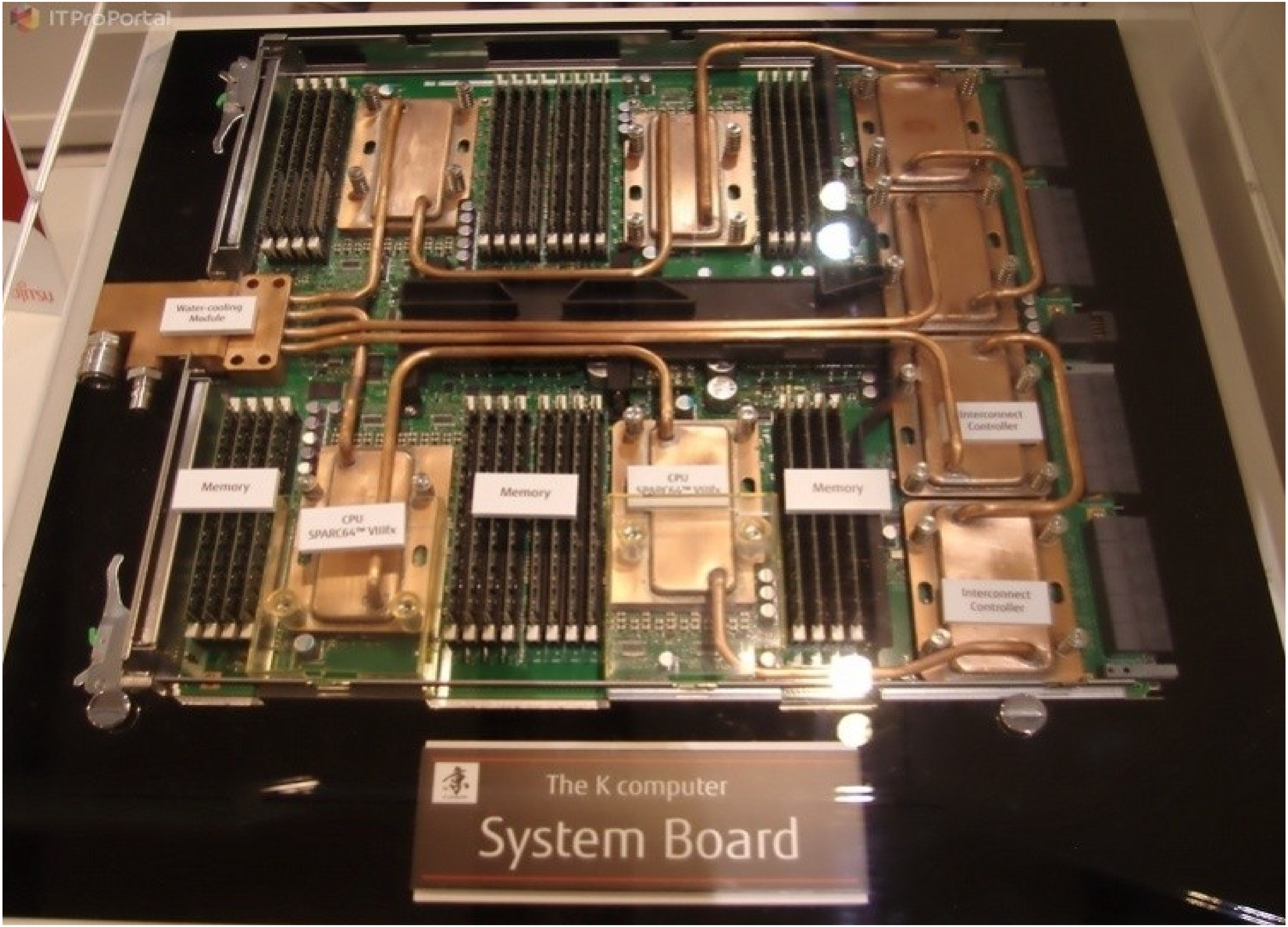
Lattice QCD and computers

- Huge computational demand.
- Uniform space-time structure → parallel computing.
- Assign a cluster of space-time points to each processor (core).
- Evaluate the path integral using importance sampling (Monte Carlo).
- Simple, repetitive arithmetic (10^8x10^8 matrices to vectors).
- Parallel computing was born for these problems (APE) and the progress in the beginning was driven by lattice applications (QCDOC).

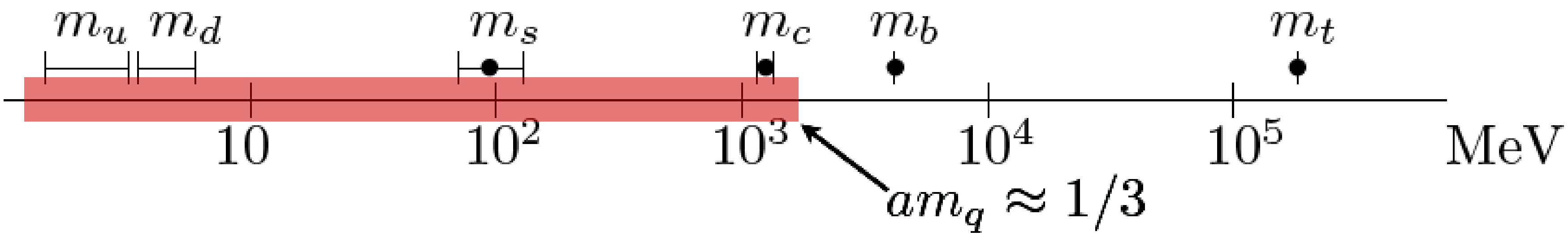


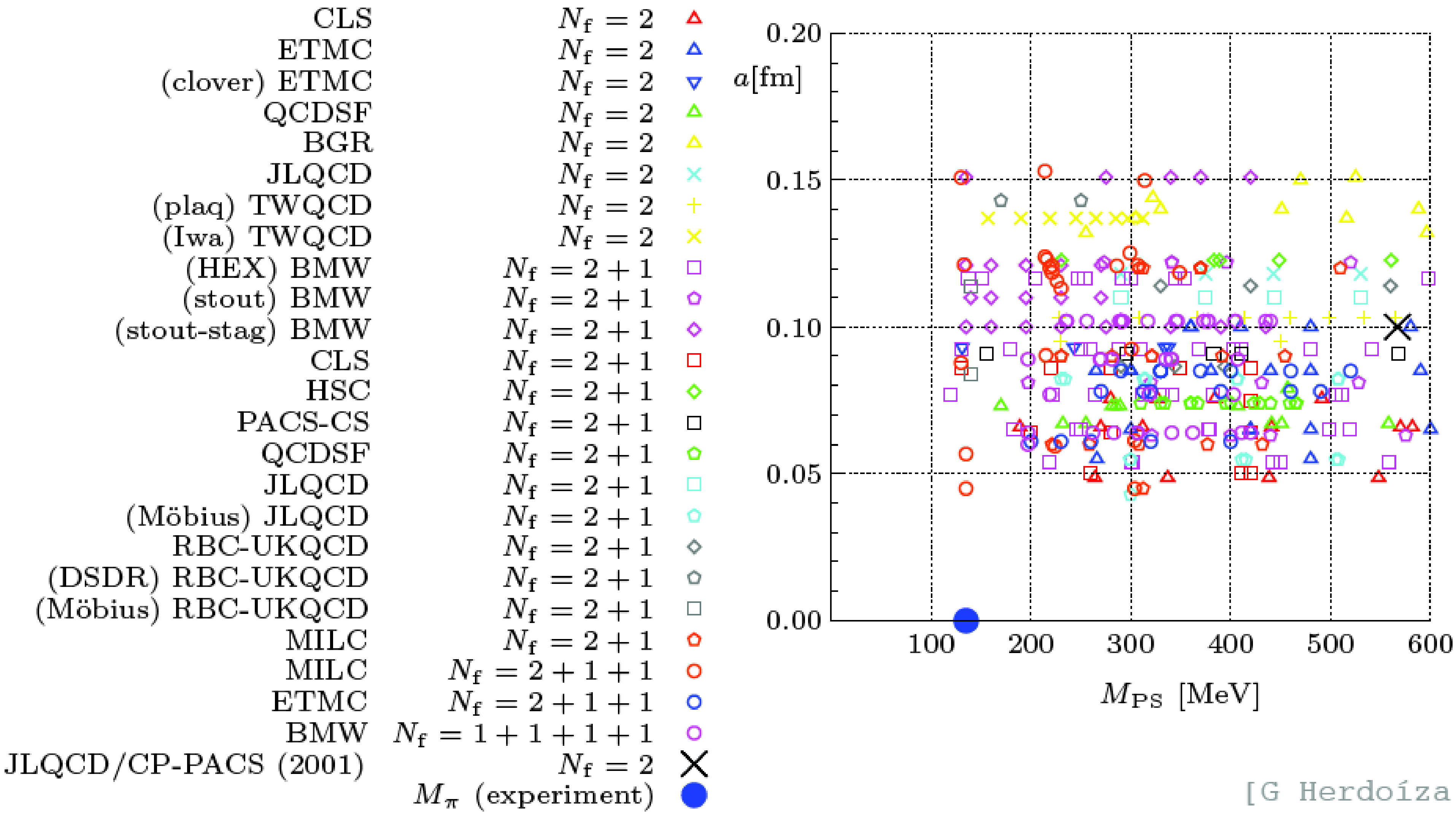
APE = Array Processor Experiment, 1 Gflop/s in 1987


21 m
26 m


ITProPortal
Water-cooling Module
Memory
CPU SPARC64™ VIIIfx
Memory
CPU SPARC64™ VIIIfx
Memory
Interconnect Controller
Interconnect Controller
The K computer
System Board

Ensembles





[G Herdoíza]

Lattice can provide first principle – systematically improvable determinations of such parameters. However they are not free from approximations / systematics

- Number of dynamical flavours
- Unphysical quark masses (and no isospin breaking)
- Finite lattice spacing
- Finite volume
- Renormalization

FLAG's goal is to walk users of lattice results through systematics and the way they have been addressed

Eur. Phys. J. C (2014) 74:2890
DOI 10.1140/epjc/s10052-014-2890-7

THE EUROPEAN
PHYSICAL JOURNAL C

Review

Review of lattice results concerning low-energy particle physics

FLAG Working Group

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Criteria, as of now

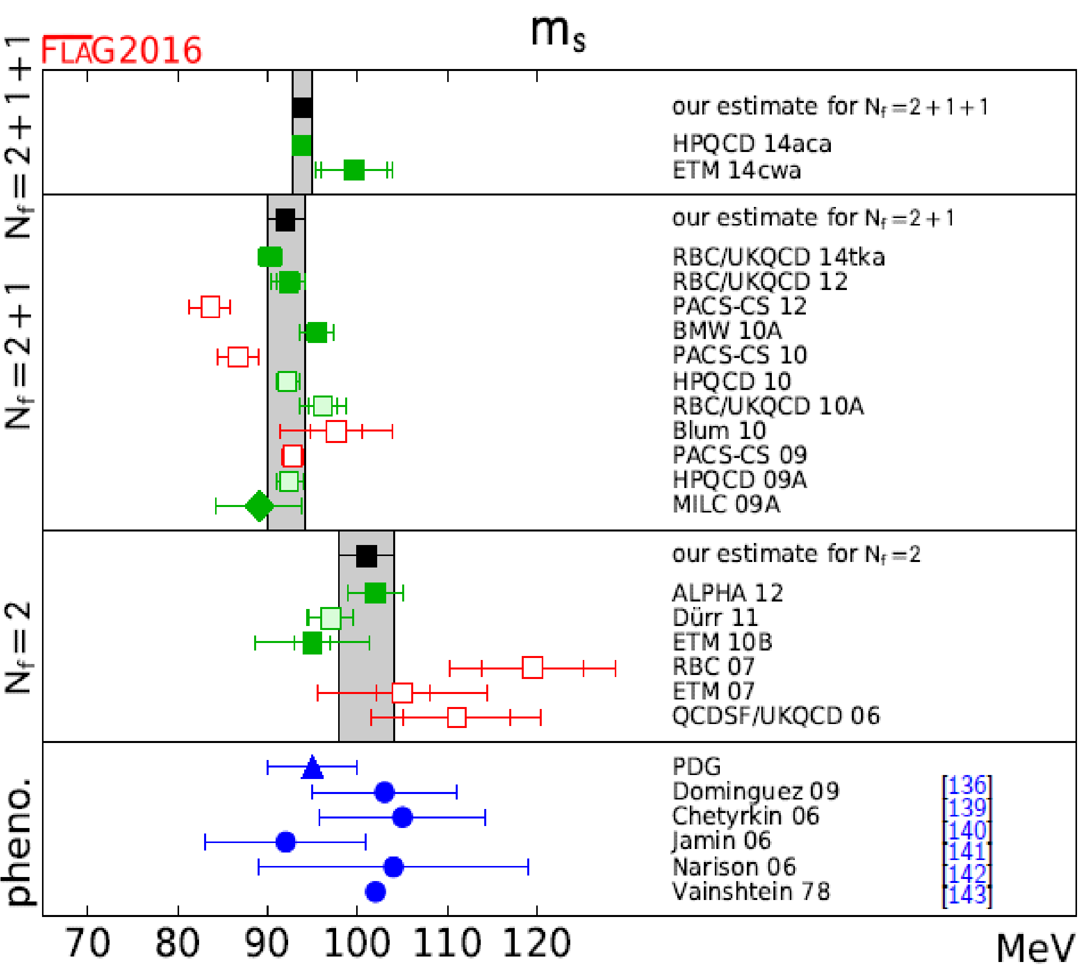
- Chiral extrapolation:
  - ★ $M_{\pi,\mathrm{min}} < 200$ MeV
  - ○ $200 \text{ MeV} \leq M_{\pi,\mathrm{min}} \leq 400$ MeV
  - ■ $400 \text{ MeV} < M_{\pi,\mathrm{min}}$

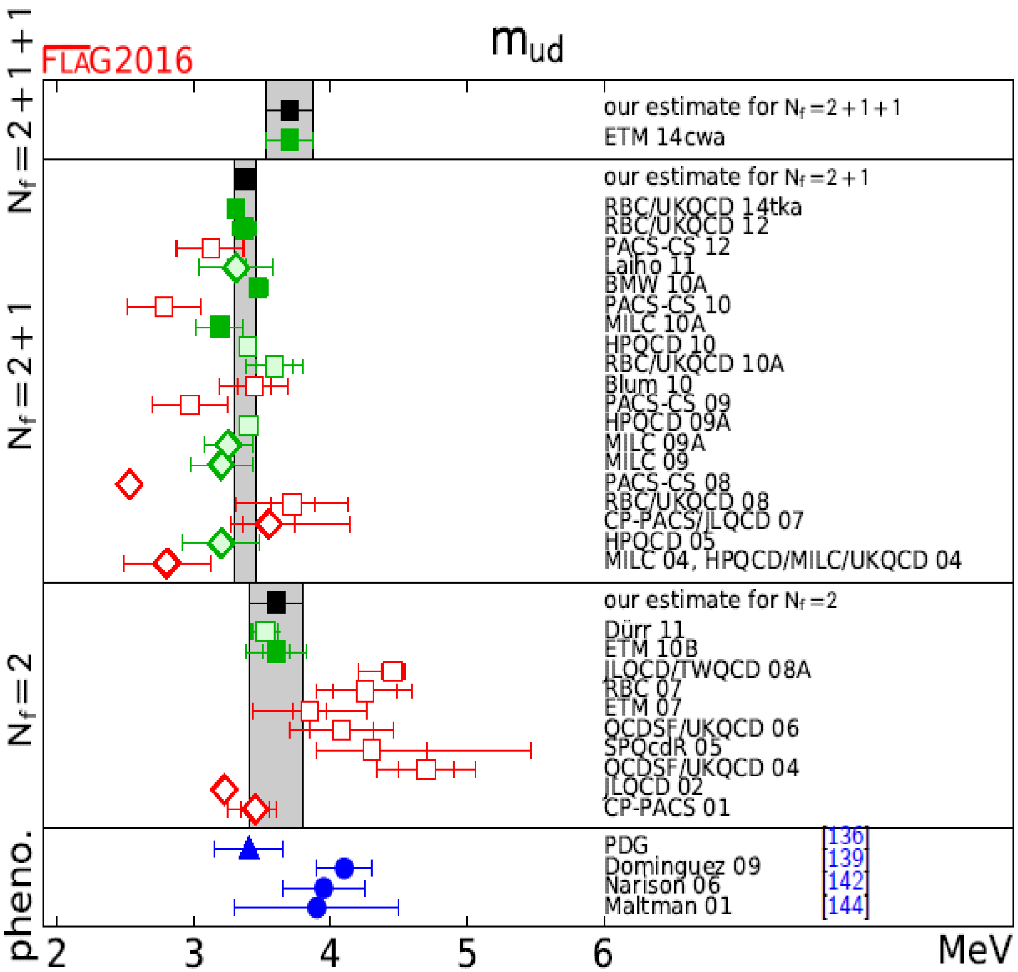
- Continuum extrapolation:
  - ★ at least 3 lattice spacings and at least 2 points below 0.1 fm and a range of lattice spacings satisfying $[a_{\mathrm{max}}/a_{\mathrm{min}}] \geq 2$
  - ○ at least 2 lattice spacings and at least 1 point below 0.1 fm and a range of lattice spacings satisfying $[a_{\mathrm{max}}/a_{\mathrm{min}}] \geq 1.4$
  - ■ otherwise

- Finite-volume effects:
  - ★ $[M_{\pi,\mathrm{min}}/M_{\pi,\mathrm{fid}}]^2 \exp\{4 - M_{\pi,\mathrm{min}}[L(M_{\pi,\mathrm{min}})]_{\mathrm{max}}\} < 1$, or at least 3 volumes
  - ○ $[M_{\pi,\mathrm{min}}/M_{\pi,\mathrm{fid}}]^2 \exp\{3 - M_{\pi,\mathrm{min}}[L(M_{\pi,\mathrm{min}})]_{\mathrm{max}}\} < 1$, or at least 2 volumes
  - ■ otherwise

- Renormalization (where applicable):
  - ★ non-perturbative
  - ○ 1-loop perturbation theory or higher with a reasonable estimate of truncation errors
  - ■ otherwise

Light quark masses

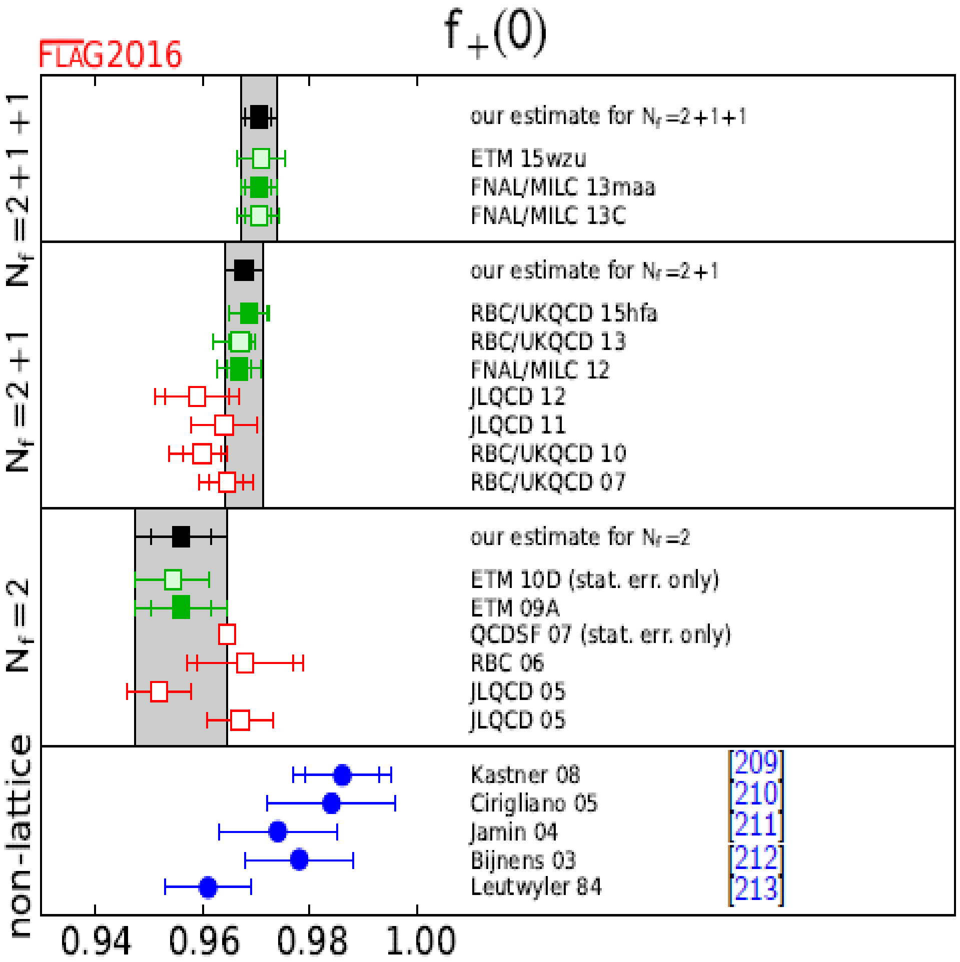


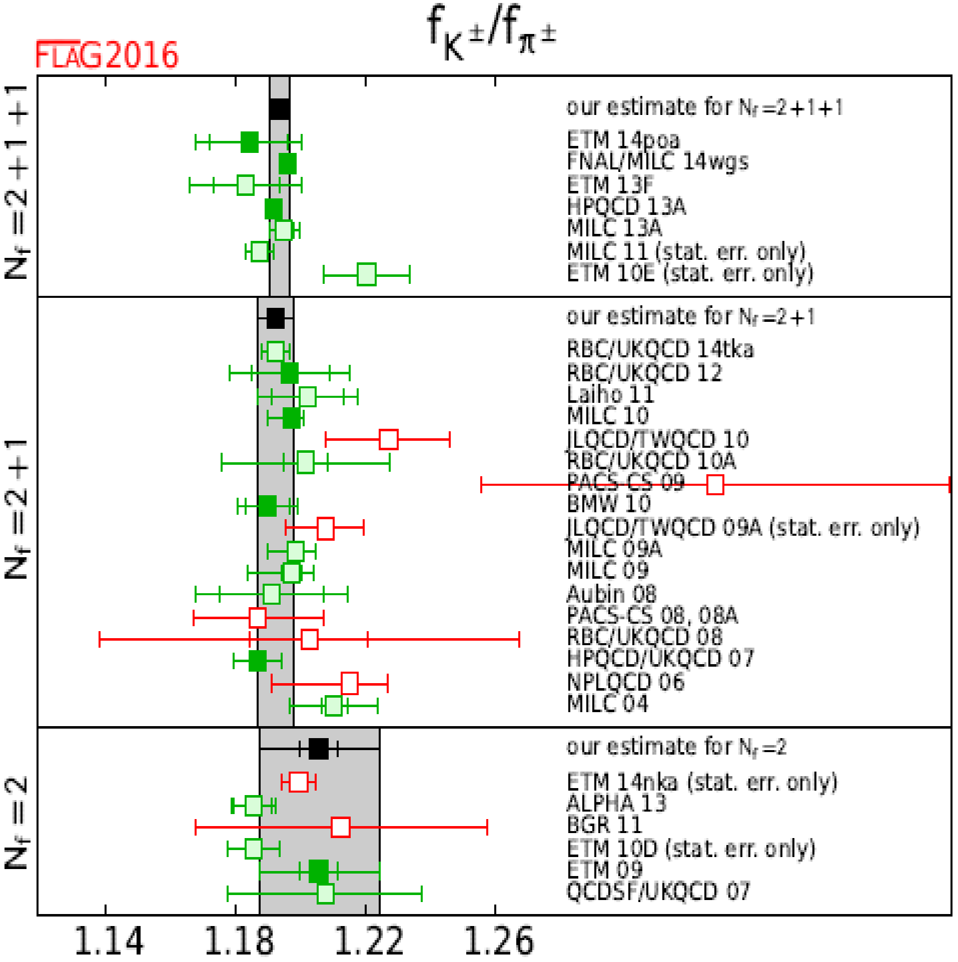


- '*Estimates*' differ from '*averages*'. For N_f=2+1 an error coming from quenching of the charm has been included

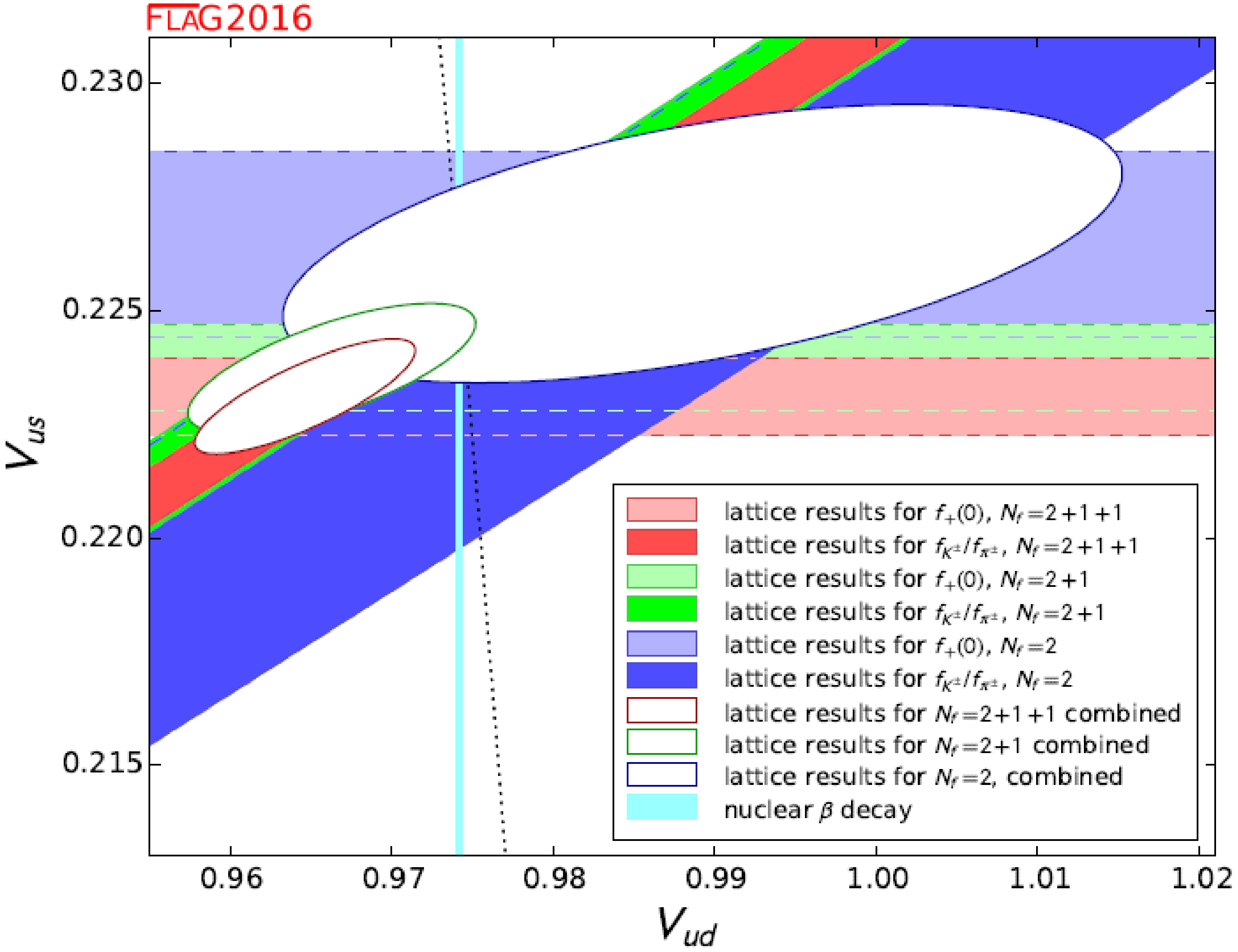
PRELIMINARY

Leptonic and semileptonic Kaon and pion decays



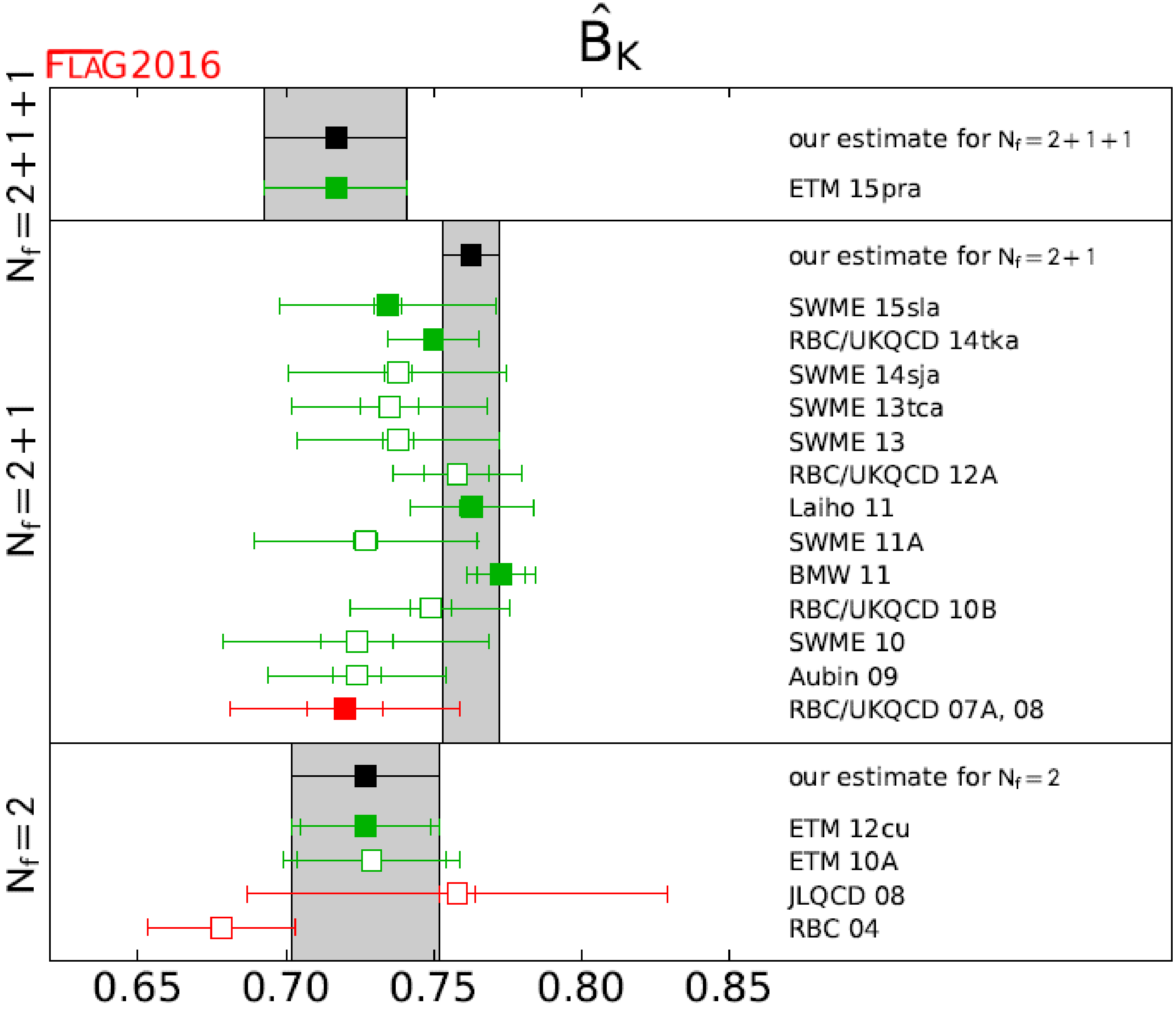


PRELIMINARY



$|V_u|^2=0.980(9)$ for $N_f=2+1+1$. The consistency among leptonic and semi-leptonic determinations of $|V_{us}|$ is a check of the equality of the Fermi constant describing interactions among leptons and the one describing interactions among leptons and quarks (may not be in BSM).

The hadronic parameter in ε_K at LO in the EWH



PRELIMINARY

New quantities in FLAG II and III

- For light flavors, lattice computations are quite precise, mature and advanced, to the point that isospin breaking and QED effects have to be included soon (see later).
- “Heavy quantities” included in FLAG-II an III are less advanced. Fewer computations (sometimes one only) passing the criteria.
- In FLAG III we have included lattice dterminations of heavy quark masses, charm and bottom. The latter is relevant for Higgs physics.

Largest Higgs BR is to b's

- Sensitive to m_b: $\Gamma(H \to b\bar{b}) = \frac{G_F N_c}{4\sqrt{2}\pi} m_H \beta^3 M_b^2$
- QCD included to N^3LO for H→bb predictions

Input values for Higgs BR fits

Parameter	Central Value	Uncertainty
$\alpha_s(M_Z)$	0.119	±0.002 (90%CL)
m_b	4.49 GeV	±0.06 GeV
M_t	172.5 GeV	±2.5 GeV

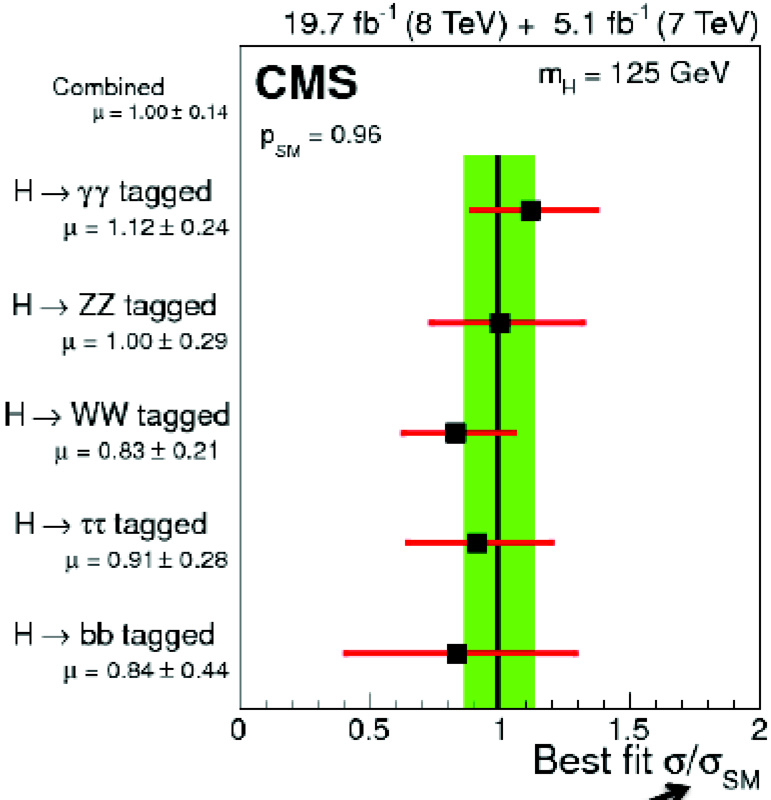
M_b is pole mass calculated with 1 loop running of $m_b(m_b)$=4.16 GeV

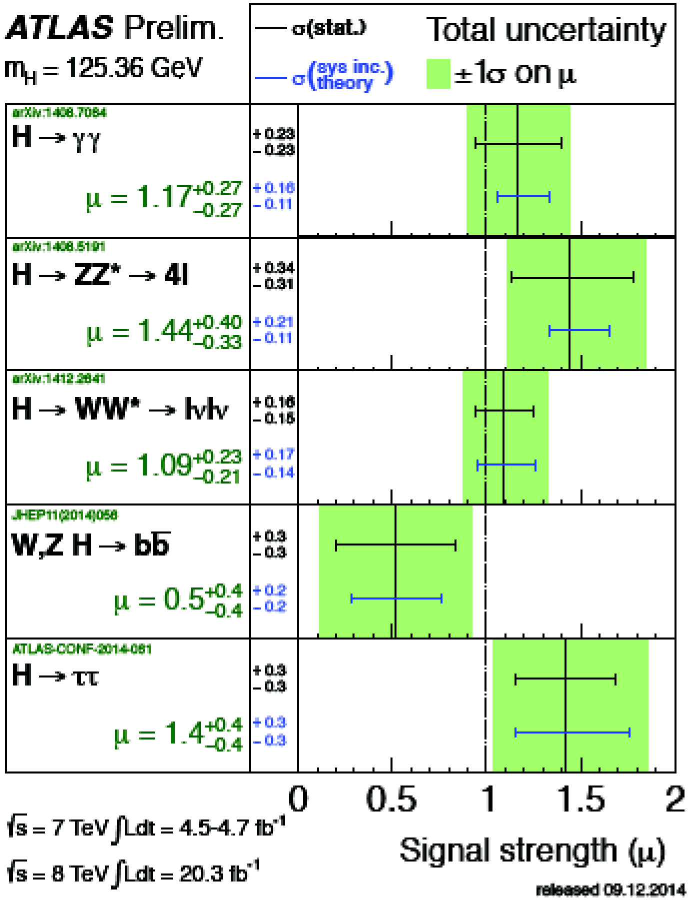
$$\frac{\delta\Gamma_b}{\Gamma_b} \sim \pm 3\%$$

Also, dominating uncertainty in:

$$\Gamma_H(m_H = 125\ GeV) = 4\ MeV \pm 4\%$$

Consistent with SM Hypothesis





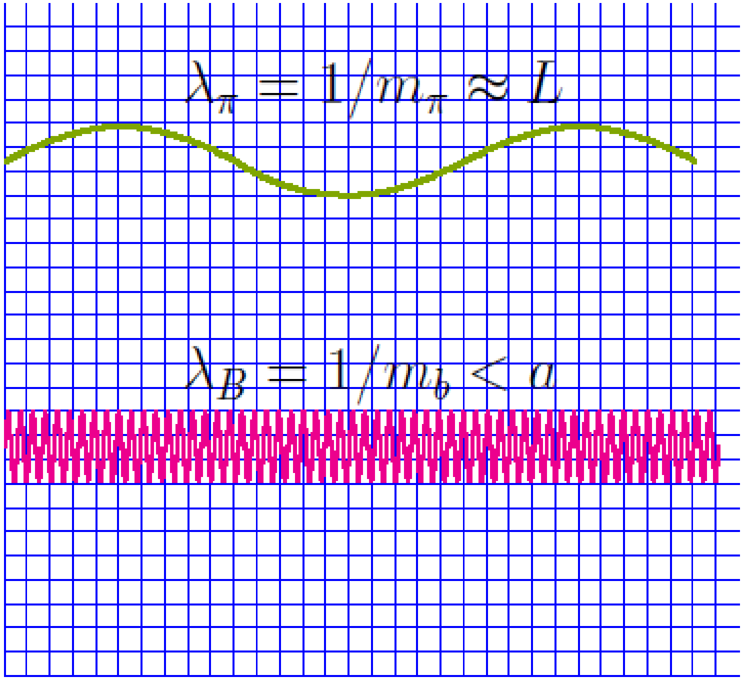
Requires theory input!

Errors soon to be dominated by theory

Heavy quarks on the lattice

- finite volume effects are mainly triggered by the light degrees of freedom. The usual requirement is $m_{PS}L > 4$ and m_{PS} is now getting to the physical point $\Rightarrow L \simeq 7\ \mathrm{fm}$.
- cutoff effects are related to the heavy quark mass. $a << 1/m_b \simeq 0.03\ \mathrm{fm}$.

$\Rightarrow L/a \simeq 200$ is needed to have those systematics under control !!
Integrating out the heavy quark mass in this case is useful !!



In addition the autocorrelation of observables grows as $1/a^n$ with $n \geq 2$ [Schäfer, Sommer and Virotta '10, Lüscher and Schäfer, '11]

Different methods have different systematics. It is crucial to compare results from a variety of them. Briefly:

Most approaches directly apply some EFT (typically valid in a particular kinematic regime)

- NRQCD [Thacker, Lepage 1991]: Expansion in v_h and in $1/m_h$.
  Dim. 5 ops at leading order $\Rightarrow$ non-renormalizable.
  One has to look for a *window* where cutoff effects $(O(a^n))$and power divergences $(O(1/a^m))$ are both small. Typically $am_h \geq 1$.
- Combinations of HQET and Symanzik effective theory:
  - O(a) improved HQET [ALPHA ...]
  - First Symanzik EFT, then HQET (expand in $1/m_h$ the improvement coefficients) → Fermilab action [El-Khadra, Kronfeld, Mackenzie 1996], RHQ [Christ, Li, Lin 2007] and Tsukuba action [Aoki, Kuramashi, Tominaga 2003].

Having introduced operators of higher dimensions all these theories produce power divergences (also in the Fermilab approach when $m_h \to \infty$ at fixed a).

The continuum limit exists only if these divergences are subtracted non-perturbatively. At any order in g_0^2:

$$\frac{g_0^{2n}}{a} \approx \frac{1}{\ln(a)^n a} \to \infty \quad \text{as } a \to 0$$

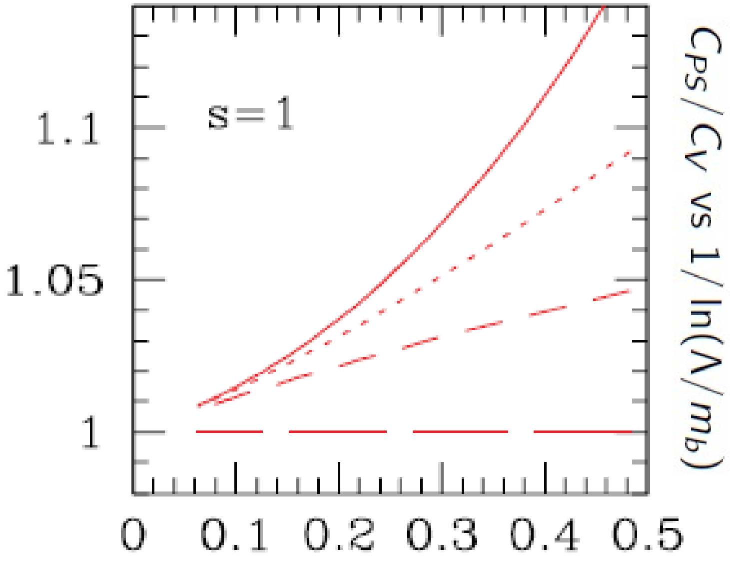
We devised a completely non-perturbative setup for lattice HQET.

Non (directly) EFT based approaches

- HISQ [HPQCD 2011]: at lattice spacings of $a \approx 0.05$ fm and $L/a \approx 100$ as currently produced by MILC, $am_b \approx 1$ so one can simulate directly at $m_b/2$ and then extrapolate to the b (using HQET).
  Getting there but autocorrelations seem a severe problem ...
- Interpolation method [Guazzini, Sommer, Tantalo 2006, ...]: using data around the charm and results in the static limit + fits in powers of $1/m_h$.

  In both methods matching factors from PT are used to define observables with the proper scaling (removing $\ln(m_h)$).

[R. Sommer, 2010]

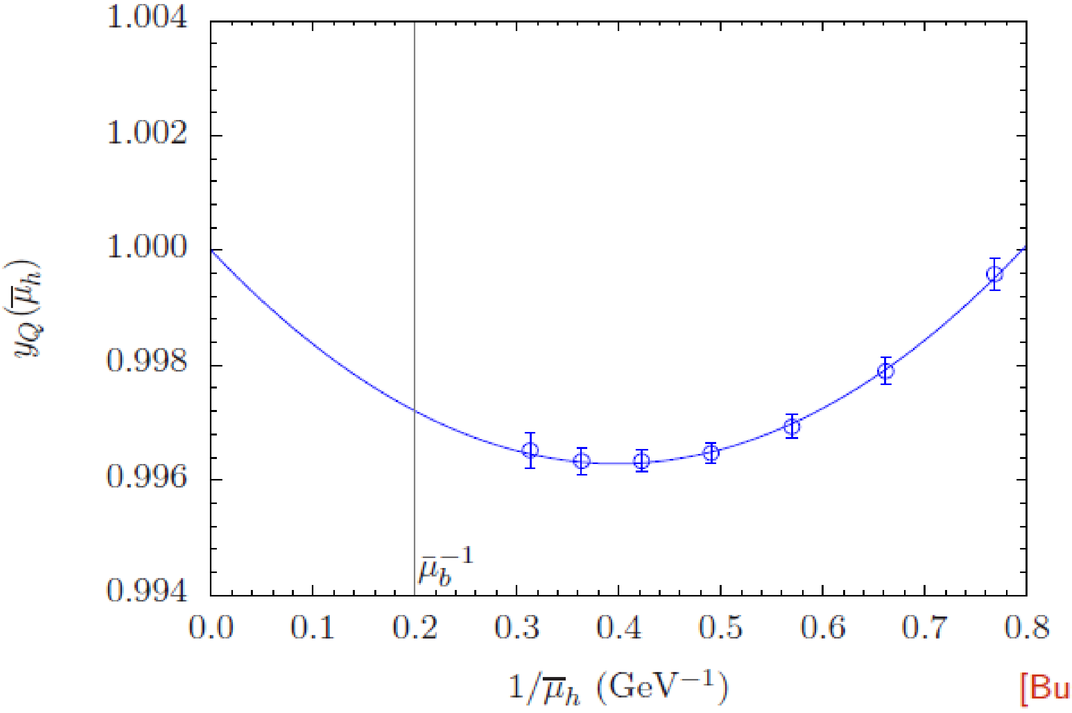


[Chetyrkin and Grozin 2003, Broadhurst and Grozin '91, '95, Bekavac et al. 2010]

Those factors *mostly* drop out in the Ratio Method [ETM 2010 ...]:

$$P(m_b) = P(m_c)\,\frac{P(\lambda m_c)}{P(m_c)}\,\frac{P(\lambda^2 m_c)}{P(\lambda m_c)}\cdots$$

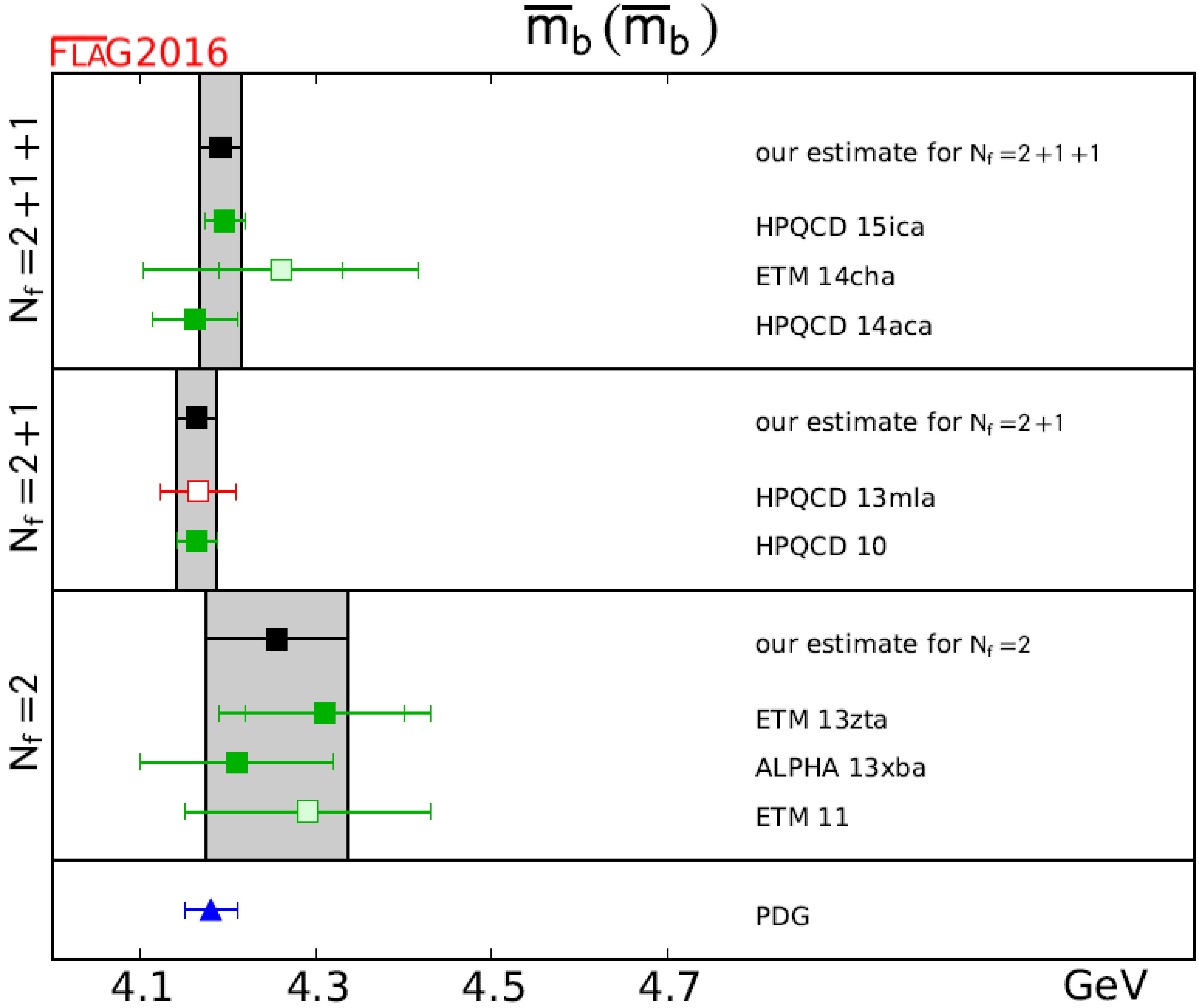
The ratios have static limit=1.



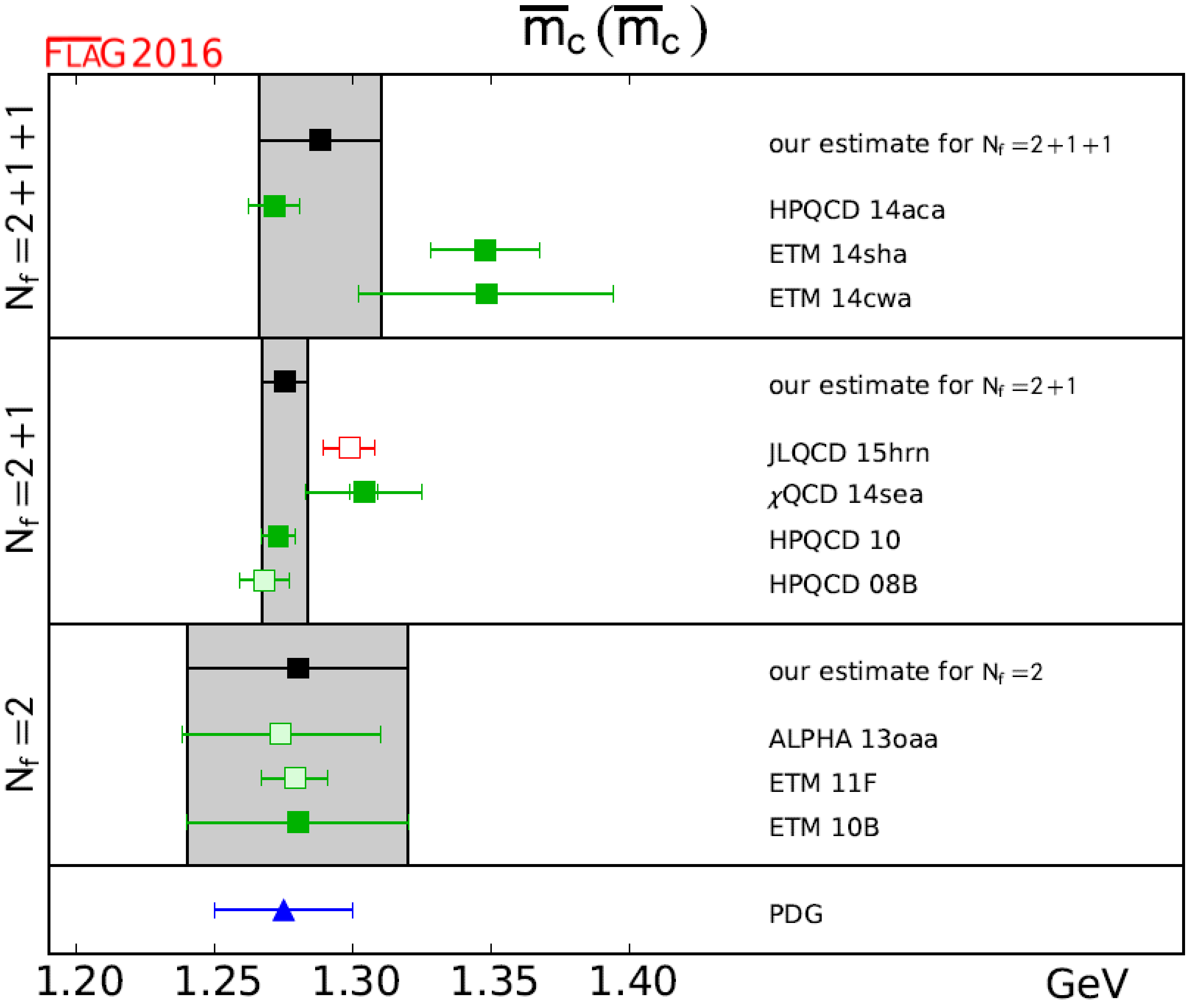
[Bussone et al., 2014]

- At the masses explored the $1/m_h^2$ corrections seem to be as big as $1/m_h$.

Talks by: P. Dimopoulos, A. Melis



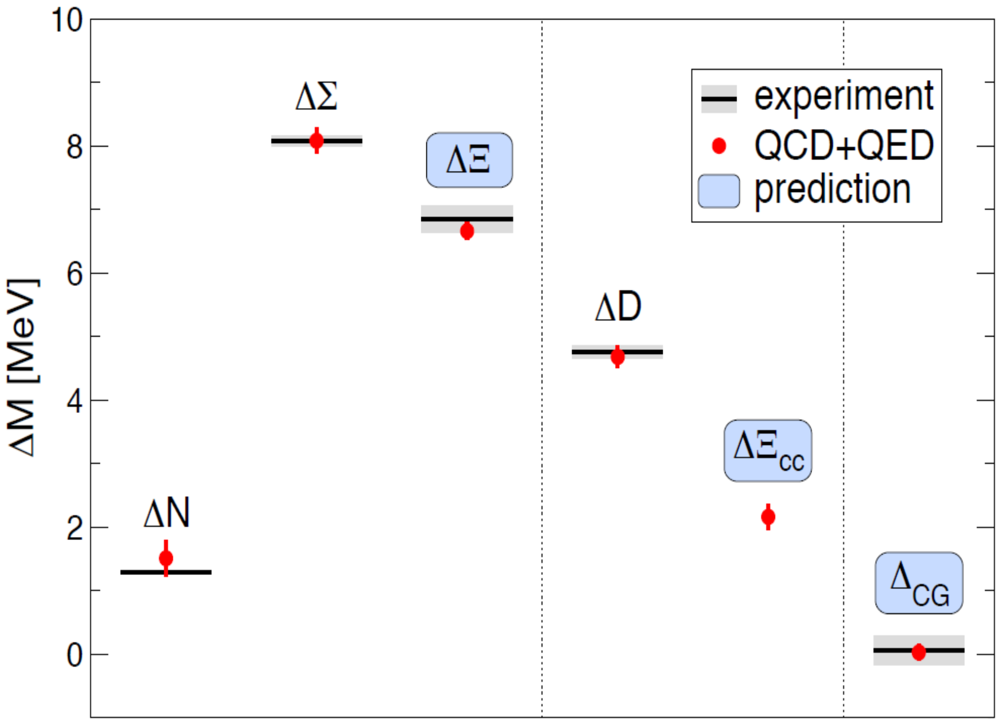
PRELIMINARY

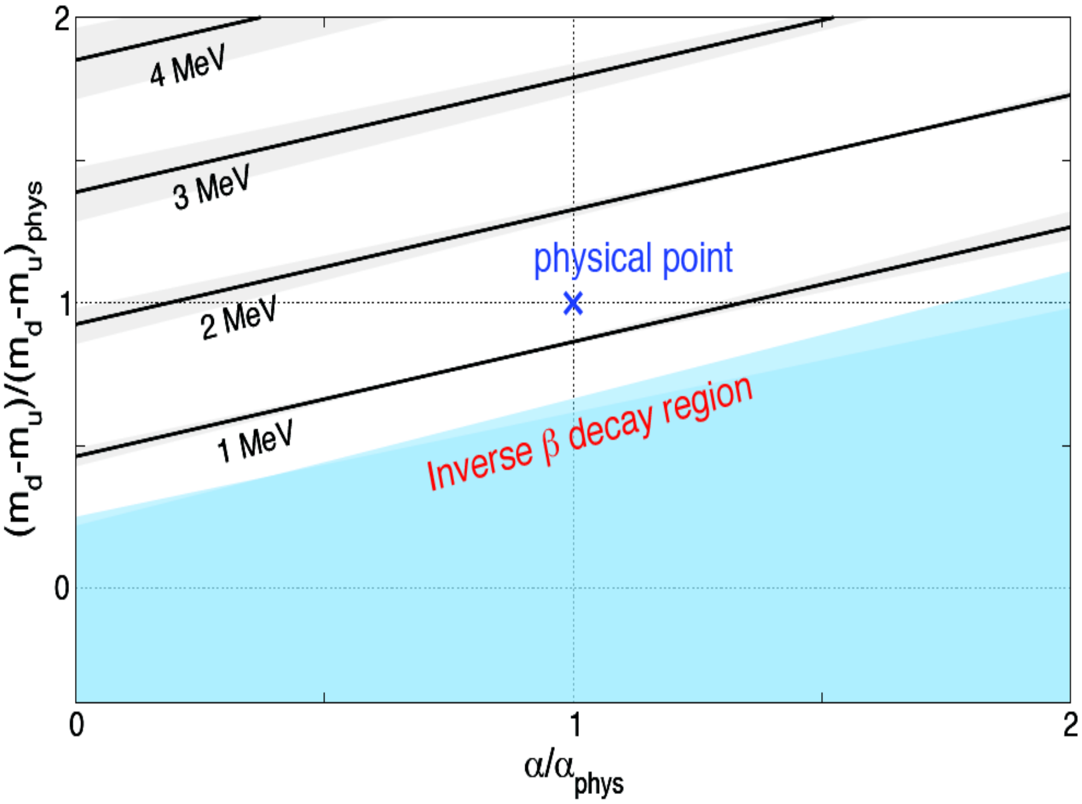


PRELIMINARY

QED effects are becoming relevant for light quantities.

QCD + QED direct simulations [Borsanyi et al., BMW group, 2014]





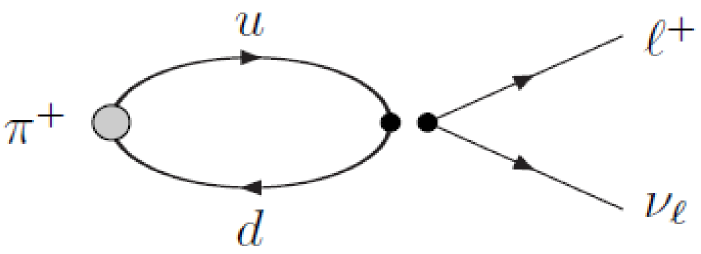
- Large volume 1+1+1+1 simulations of QCD + QED (at unphysical *e* due to noise to signal problem). 300 times more expensive than $N_f=2$ QCD. Pilot and benchmark computation concerning the setup.

- Separation of effects using $\Delta M_\Sigma^{QED}=0$

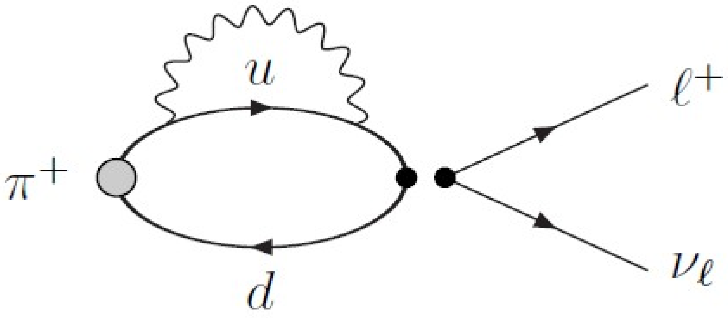
QED corrections to hadronic processes

Let's consider the leptonic decay at O(α) in the WEH [N. Carrasco et al., 1502.00257]

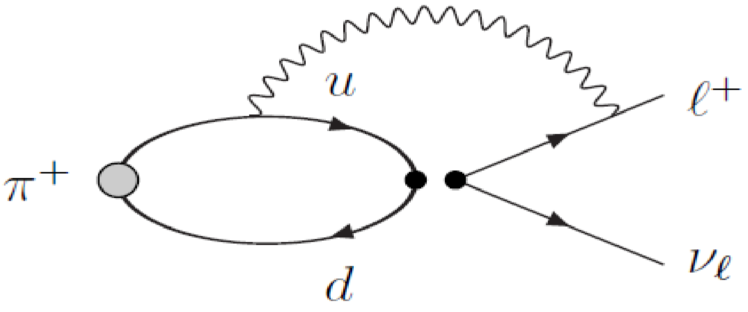
Pure QCD



factorizable. Hadronic part → f_P. Then $\underset{\text{Number external photons}}{\underset{||}{\Gamma_0}}$ at O(α)



Still factorizable.



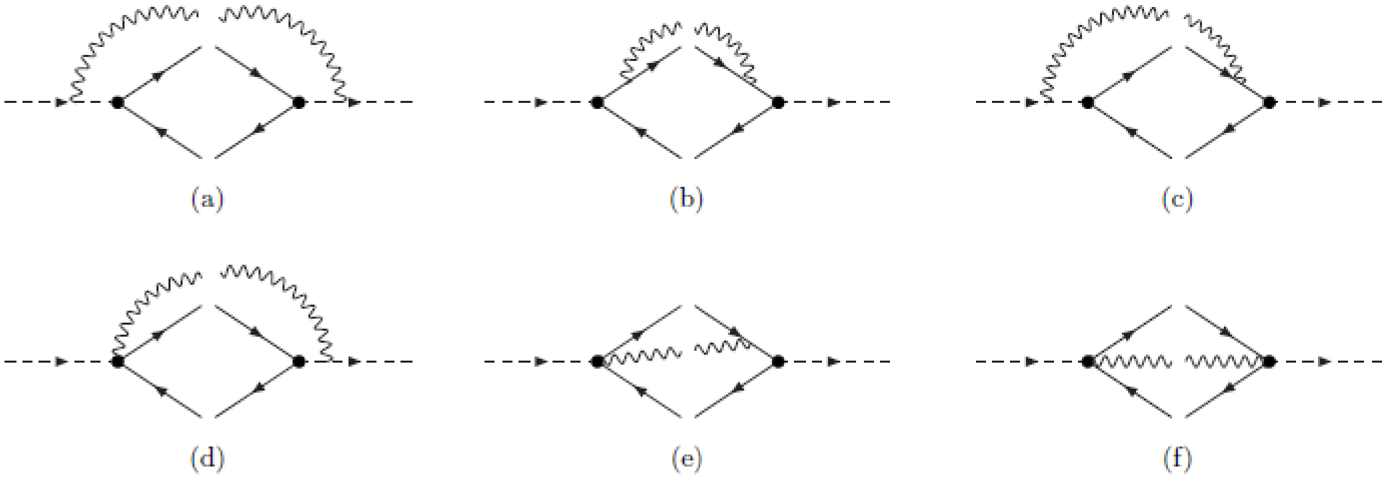
NON factorizable.

Also, Γ_0 is infrared divergent, one needs to consider (one) real photon emission as well. No such problems for spectrum.

F.Bloch, A.Nordsieck Phys.Rev. 52 (1937)
T.D.Lee, M.Nauenberg Phys.Rev. 133 (1964)

⇒ Not much sense of QED corrections to a decay constant ...

$\Gamma_1(\Delta E)$ with $\Delta E = E_\gamma^{\mathrm{max}}$



The combination $\Gamma_0 + \Gamma_1(\Delta E)$ is free from IR divergencies at $O(\alpha)$. One can split it as

$$\Gamma(\Delta) = \left\{\Gamma_0 - \Gamma_0^{pt}\right\} + \left\{\Gamma_0^{pt} + \Gamma_1(\Delta)\right\} \quad = \quad \overbrace{\lim_{L\mapsto\infty} \left\{\Gamma_0(L) - \Gamma_0^{pt}(L)\right\}}^{\text{lattice}} \quad + \quad \overbrace{\left\{\Gamma_0^{pt} + \Gamma_1(\Delta)\right\}}^{\text{continuum pt for } \Delta \ll M_P}$$

- *pt*=pointlike approximation (perturbative). OK for soft photons, they can't resolve the hadron structure. For K and π, $\Delta E \simeq 20$ MeV. Currently main limitation of the approach.
- Both terms are IR-safe and have a $L \to \infty$ limit.
- $\Gamma_0(L)$ is computed on the lattice. It requires rather involved Euclidean correlators, with lepton propagators in the numerical computation of the non-factorizable contributions.

Hadronic decays. A_2 amplitude of $K \to \pi\pi$ [many years of work by RBC/UKQCD, arXiv:1502.00263]

Z. Bai, T. Blum, P. A. Boyle, N. H. Christ, J. Frison, N. Garron, T. Izubuchi and C. Jung *et al.*, arXiv:1505.07863 [hep-lat].
N. Ishizuka, K.-I. Ishikawa, A. Ukawa and T. Yoshi, arXiv:1505.05289 [hep-lat].

$$K \to (\pi\pi)_{I=2}$$
$$K \to (\pi\pi)_{I=0}$$

$$\frac{\mathrm{Re}(A_0)}{\mathrm{Re}(A_2)} \approx 22.5$$

This $\Delta I = 1/2$ rule is unexplained and must be of non-perturbative nature.

$$A_{2/0} = F\langle(\pi\pi)_{I=2/0} \mid H_W \mid K\rangle$$

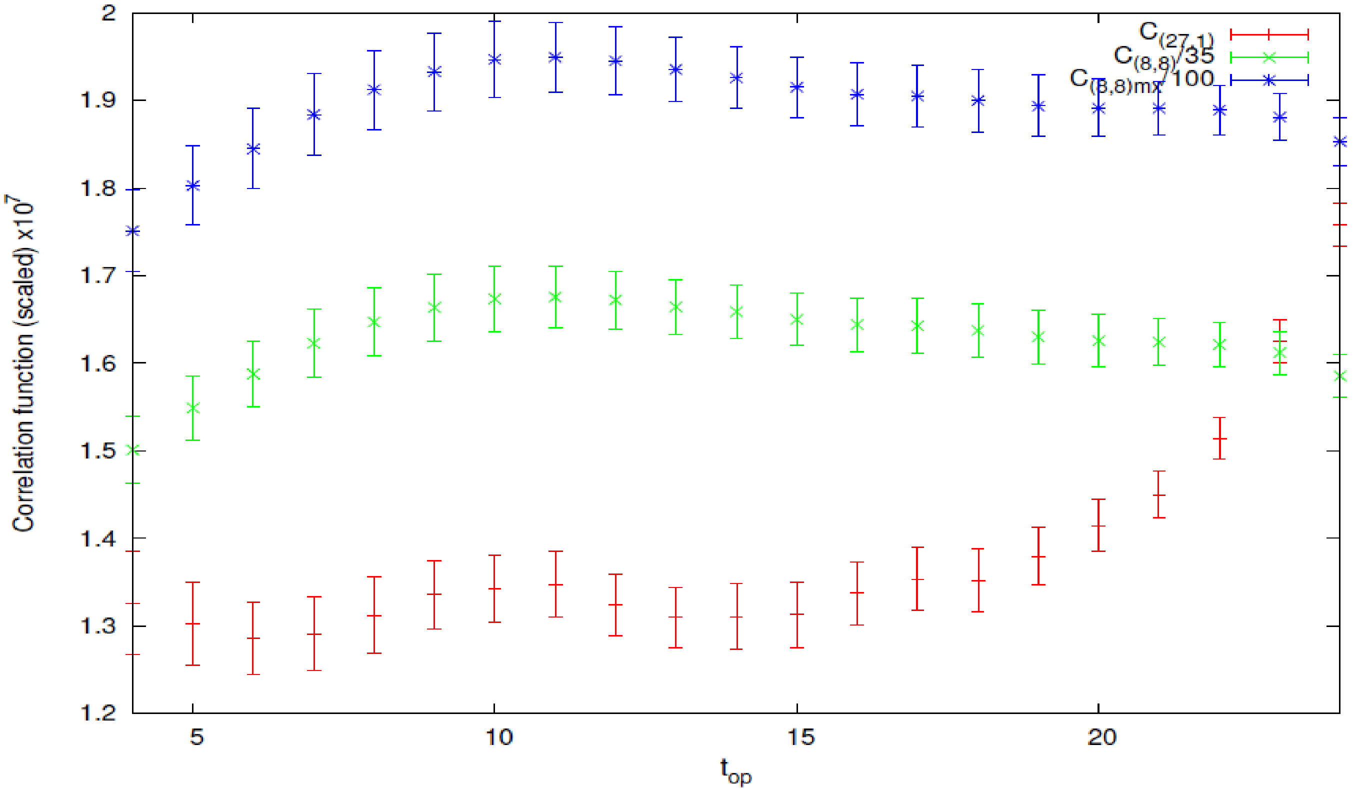
- 3 (four-fermion) operators in the Weak Eff. Hamiltonian contribute.
- F is a factor relating the finite volume matrix elements to the infinite volume ones. It depends on the $\pi\pi$ phase shift [Lellouch and Lüscher, '01]
- Kinematics should be matched, i.e. $E_{\pi\pi} = m_K$. That is achieved using antiperiodic boundary conditions for the d, s.t. $p = \pm\pi/L$....

64^3 K$\rightarrow$ $\pi\pi$ 3-point correlation functions

Kaon - 2 pion separation 26

Tadeusz Janowski

$$C_i^{K\rightarrow\pi\pi}(t) = N_{\pi\pi} N_K M_i e^{-(m_K - E_{\pi\pi}) t_{op}} e^{-E_{\pi\pi} t_{\pi\pi}}$$



- 2 ensembles of 2+1 DW fermions with L~5 fm and physical m_π.
- NP renormalization in RI-SMOM scheme. Matching to $\overline{\text{MS}}$ at 1-loop. Currently dominating error budget. (Wilson coeffs. at NLO)

$$\boxed{\text{Re}(A_2) = 1.50(4)_{\text{stat}}(14)_{\text{syst}} \times 10^{-8}\ \text{GeV}; \quad \text{Im}(A_2) = -6.99(20)_{\text{stat}}(84)_{\text{syst}} \times 10^{-13}\ \text{GeV}\,.}$$

experimental value $1.570(53) \times 10^{-8}$ GeV from neutral kaon decays

Hadronic decays. Multiple-channel generalization of the LL approach [Sharpe and Hansen, 2012, Briceno and Davoudi, 2012]

- The LL method, derived in Minkowski pace, first relates the finite volume dependence of the energy levels of two-particle states (accessible in Euclidean) to the (∞-L) S-matrix and phase shifts (not accessible, due to Maiani-Testa no-go theorem, '90).
- In a second step a new state (e.g. K) is introduced with a perturbative interaction term H_W with $\pi\pi$. Matching the kinematic and considering degenerate PT, the finite L correction to the energy levels is related to the ∞-L scattering amplitude (i.e. the finite and ∞ L, matrix elements of $\langle K|H_W|\pi\pi\rangle$ are related).
- The explicit generalization includes several two-particle states ($\pi\pi \text{ and } \bar{K}K$).
- Now the S-matrix does not only include phase shifts and different kinematics are needed to determine the parameters. Also, one gets a system of equations relating finite and infinite volumes matrix elements.
- This is a first step towards hadronic decays of e.g. D-mesons.

Conclusions

- I have given an incomplete review of (mostly) flavor physics on the lattice. Review of FLAG review ...

- Higgs less of a portal to New Physics than hoped. To establish that precise results in the b-sector are needed.

- If the keywords are precise and rare, we are getting there. Approaches to include sub-leading systematics being developed (QED, multi-hadron channels).