A Curious Story of Gravity in the Ultraviolet

Galileo Galilei Institute, Firenze

Zvi Bern,

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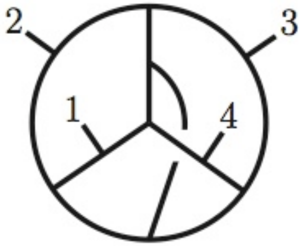
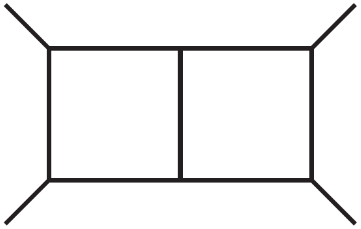
UCLA & KITP

ZB, Clifford Cheung, Huan-Hang Chi, Scott Davies, Lance Dixon, Josh Nohle: arXiv:1507.06118
& to appear

ZB, Huan-Hang Chi, Lance Dixon and Alex Edison: to appear.





Quantum Gravity

Often repeated statement:

"Einstein's theory of General Relativity is incompatible with quantum mechanics."

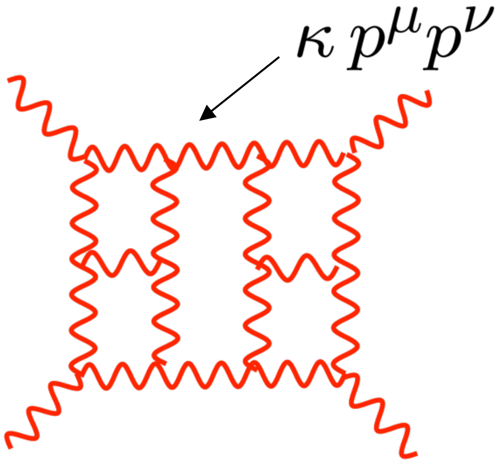
To a large extent this is based on another often repeated statement:

"All point-like quantum theories of gravity are ultraviolet divergent and non-renormalizable."

Where do these statements come from and are they true?

Non-Renormalizability of Gravity?

$$\kappa = \sqrt{32\pi G_N} \longleftarrow$$ Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(\kappa p_j^\mu p_j^\nu)}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g p_j^\nu)}{\text{propagators}}$$

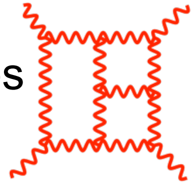
UV: Large momenta in loop integrals

- **Extra powers of loop momenta in numerator means integrals are badly behaved in the UV and must divergence at some loop order.**
- **Much more sophisticated power counting in supersymmetric theories but this is basic idea.**

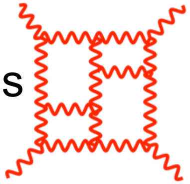
- ***N*** **= 8 supergravity is best theory to look at.**
- **With more supersymmetry expect better UV properties.**
- **High symmetry implies simplicity.**

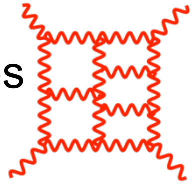
Feynman Diagrams for Gravity

Suppose we want to check UV properties of gravity theories:

3 loops  ~10^{20} TERMS

No surprise it has never been calculated via Feynman diagrams.

4 loops  ~10^{26} TERMS

5 loops  ~10^{31} TERMS

More terms than atoms in your brain!

− Calculations to settle this seemed utterly hopeless!

− Seemed destined for dustbin of undecidable questions.

Supersymmetry helps, but not enough to make a difference.

Where is First Potential $D = 4$ UV Divergence?

3 loops $N = 8$	Green, Schwarz, Brink (1982); Howe and Stelle (1989); Marcus and Sagnotti (1985)	✗
5 loops $N = 8$	Bern, Dixon, Dunbar, Perelstein, Rozowsky (1998); Howe and Stelle (2003,2009)	✗
6 loops $N = 8$	Howe and Stelle (2003)	✗
7 loops $N = 8$	Grisaru and Siegel (1982); Bossard, Howe, Stelle (2009);Vanhove; Björnsson, Green (2010); Kiermaier, Elvang, Freedman(2010); Ramond, Kallosh (2010); Biesert et al (2010); Bossard, Howe, Stelle, Vanhove (2011)	?
3 loops $N = 4$	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops $N = 5$	Bossard, Howe, Stelle, Vanhove (2011)	✗
4 loops $N = 4$	Vanhove and Tourkine (2012)	✓
9 loops $N = 8$	Berkovits, Green, Russo, Vanhove (2009)	✗

ZB, Kosower, Carrasco, Dixon, Johansson, Roiban; ZB, Davies, Dennen, A. Smirnov, V. Smirnov; series of calculations.

Don't bet on divergence

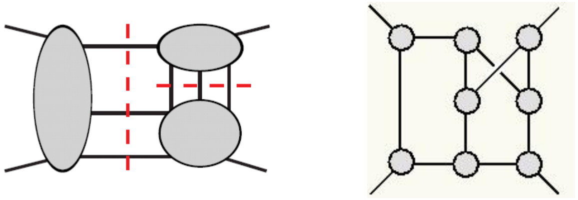
Weird structure. Anomaly-like behavior of divergence.

retracted

- **Conventional wisdom holds that it will diverge soon or later.**
- **But every detailed prediction either wrong of misleading.**

Our Basic Tools

We have powerful tools for computing scattering amplitudes and studying their UV properties:



- **Generalized unitarity method.**

  ZB, Dixon, Dunbar, Kosower
  ZB, Carrasco, Johansson, Kosower

- **Duality between color and kinematics. Gravity scattering amplitudes directly from gauge-theory ones. Double copy.**

  ZB, Carrasco and Johansson (BCJ)

- **Advanced loop-integration technology.**

  Chetyrkin, Kataev and Tkachov; Laporta; A.V. Smirnov; V. A. Smirnov; Vladimirov; Marcus, Sagnotti; Czakon; Laporta; etc.

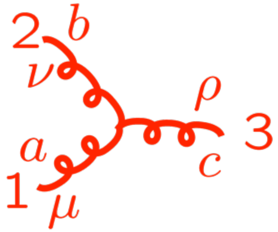
- **I won't explain these tools these but they underlie everything.**
- **Many other tools and advances that I won't discuss here.**

Duality Between Color and Kinematics

ZB, Carrasco, Johansson (BCJ)

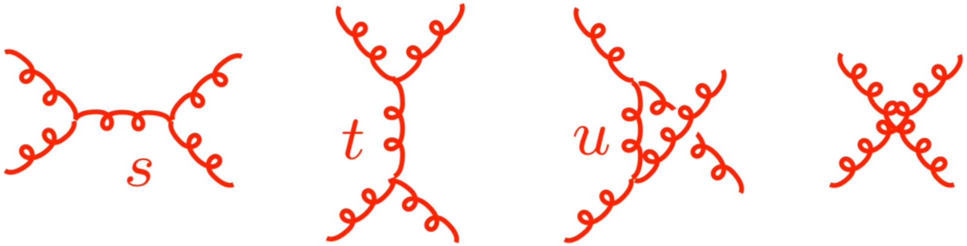
coupling constant **color factor** **momentum dependent kinematic factor**

$$-gf^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho + \text{cyclic})$$



Color factors based on a Lie algebra: $[T^a, T^b] = if^{abc}T^c$

Jacobi Identity $f^{a_1a_2b}f^{ba_4a_3} + f^{a_4a_2b}f^{ba_3a_1} + f^{a_4a_1b}f^{ba_2a_3} = 0$



Use 1 = *s/s* = *t/t* = *u/u* to assign 4-point diagram to others.

$s = (k_1+k_2)^2 \qquad t = (k_1+k_4)^2$

$u = (k_1+k_3)^2$

$$\mathcal{A}_4^{\text{tree}} = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

Color factors satisfy Jacobi identity:

Numerator factors satisfy similar identity:

$$c_u = c_s - c_t$$
$$n_u = n_s - n_t$$

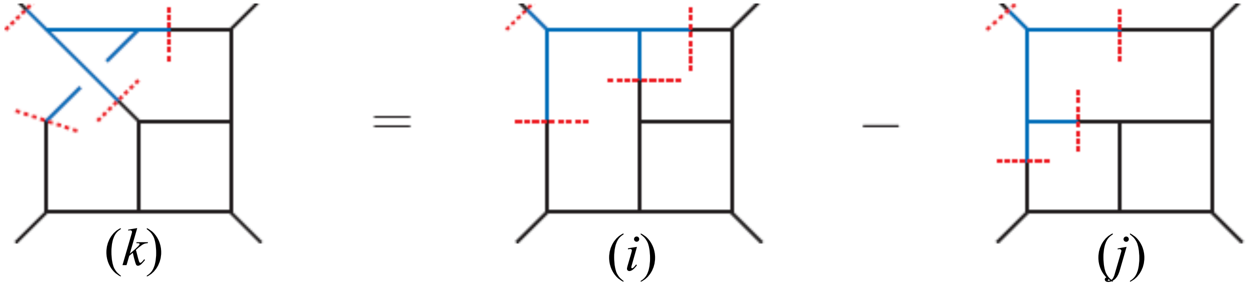
Proven at tree level and conjectured at loop level.

ZB, Carrasco, Johansson; Kiermaier; Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove; Cachazo, etc

Duality Between Color and Kinematics

ZB, Carrasco, Johansson (BCJ)

Conjecture: kinematic numerators exist with same algebraic properties as color factors, even at loop level.



color factor

$$c_k = c_i - c_j$$

$$n_k = n_i - n_j$$

kinematic numerator

$$n_i \sim k_1 \cdot l_1 \, k_3 \cdot l_2 \, \varepsilon_1 \cdot l_3 \, \varepsilon_2 \cdot k_3 \, \varepsilon_3 \cdot l_2 \, \varepsilon_4 \cdot k_3 + \ldots$$

If you have a set of duality satisfying kinematic numerators.

gauge theory ⟶ gravity theory

simply take

color factor ⟶ kinematic numerator

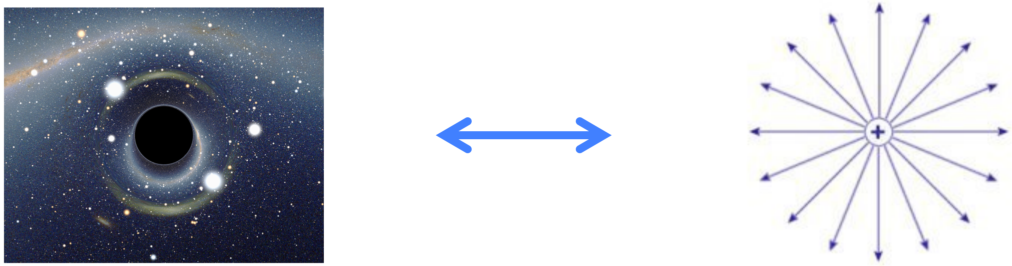
$$c_i \rightarrow n_i$$

You would never know this from studying the respective Lagrangians. Nor is this understood from string theory.

Applications to Black Hole Physics

Wouldn't it be really cool if every classical solution in gravity could be mapped to to a double of gauge theory classical solutions?

Where to start? **Obviously the coolest place possible: black holes.**



black hole **point charge**

Monteiro, O'Connell and White

Special coordinates: Kerr-Schild coordinates:

Schwarzschild black hole

$$g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu \qquad \phi(r) = \frac{2m}{r}$$

k is null

Coulomb point charge

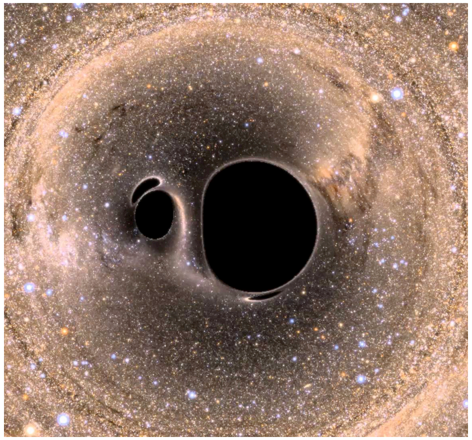
$$A_\mu = \phi k_\mu \qquad \phi(r) = \frac{Q}{r}$$

Schwarzschild ~ (Coulomb)^2

Applications to Black Hole Physics

Luna, Monteiro, O'Connell and White;
Luna, Monteiro, Nicholsen, O'Connell and White;
Ridgway and Wise.

A variety of other cases:



- **Kerr (rotating) black hole.**
- **Taub-NUT space.**
- **Solutions with cosmological constant.**
- **Radiation from accelerating black hole.**

It may be possible to extend this to more general cases.
Need good coordinates!

UV in Gravity

Most theorists believe that UV properties of quantum field theories of gravity are "well understood", up to "minor" details, e.g. the precise loop order where divergences occur.

The main purpose of my talk is to try to convince you that the UV structure of gravity is strange and surprising and most certainly *not* "well understood".

1. Examples of no divergence even when no known symmetry arguments prevent them. "Enhanced cancellations". Unlike gauge theory.
2. When UV divergences are present in pure (super) gravity, properties are strange and unexpected.

Predictions of Ultraviolet Divergences

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Björnsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; Bossard, Howe, Stelle, Vanhove, etc

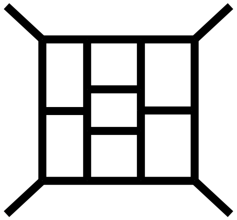
- **First quantized formulation of Berkovits' pure-spinor formalism.**
- **Unitarity method.**

Bjornsson and Green
ZB, Davies, Dennen

Key point: *all* supersymmetry cancellations are exposed.

Poor UV behavior, unless new types of cancellations between diagrams exist that are "not consequences of supersymmetry in any conventional sense" Bjornsson and Green

- **$N = 8$ sugra should diverge at 5 loops in $D = 24/5$.** ?
- **$N = 8$ sugra should diverge at 7 loops in $D = 4$.** ?
- **$N = 4$ sugra should diverge at 3 loops in $D = 4$.** ✗
- **$N = 5$ sugra should diverge at 4 loops in $D = 4$.** ✗

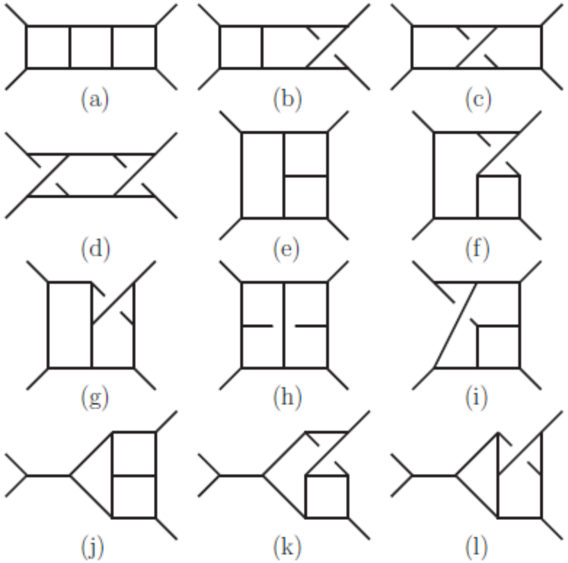


Consensus agreement from all methods

These new types of cancellations do exist: "enhanced cancellations".

ZB, Davies, Dennen

$N = 4$ Supergravity UV Cancellation



$D = 4 - 2\epsilon$ **ZB, Davies, Dennen, Huang**

Graph	$(\text{divergence})/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3} + \frac{205}{27648}\frac{1}{\epsilon^2} + \left(-\frac{5551}{768}\zeta_3 + \frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3} - \frac{1}{4}\frac{1}{\epsilon^2} + \left(\frac{593}{288}\zeta_3 - \frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3} + \frac{2057}{6912}\frac{1}{\epsilon^2} + \left(\frac{10769}{2304}\zeta_3 - \frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3} - \frac{41}{1536}\frac{1}{\epsilon^2} + \left(\frac{3227}{2304}\zeta_3 - \frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3} - \frac{29}{1024}\frac{1}{\epsilon^2} + \left(-\frac{2087}{2304}\zeta_3 - \frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3} + \frac{9}{64}\frac{1}{\epsilon^2} + \left(\frac{101}{12}\zeta_3 - \frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3} + \frac{89}{1152}\frac{1}{\epsilon^2} + \left(-\frac{377}{144}\zeta_3 + \frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3} - \frac{251}{1152}\frac{1}{\epsilon^2} + \left(-\frac{835}{144}\zeta_3 + \frac{7385}{3456}\right)\frac{1}{\epsilon}$

All **three-loop divergences and subdivergences cancel completely!**

Still no standard-symmetry explanation, despite valiant attempt.

Bossard, Howe, Stelle; ZB, Davies, Dennen

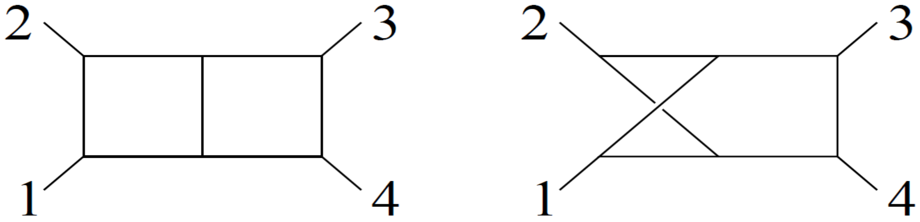
Prediction based on supergravity imply divergences.
A nontrivial example of "enhanced cancellations".

Where does new magic come from?

ZB, Davies, Dennen, Huang; Bossard, Howe, Stelle

To analyze we need a simpler example: Half-maximal supergravity in $D = 5$ at 2 loops.

Similar to $N = 4$, $D = 4$ sugra at 3 loops, except much simpler.



Quick summary:

- **Finiteness in $D = 5$ tied to double-copy structure.**
- **Cancellations in certain forbidden gauge-theory color structures imply hidden UV cancellations in supergravity, even though no standard symmetry explanation.**

- **Double copy structure implies extra cancellations!**
- **Quite a nonstandard explanation for a cancellation.**

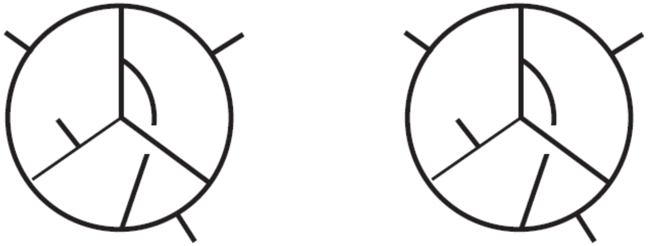
Unfortunately, not easy to extend beyond 2 loops.

$\mathcal{N} = 4$ Supergravity at Four Loops

ZB, Davies, Dennen, Smirnov, Smirnov

We also calculated four-loop divergence in $\mathcal{N} = 4$ supergravity.

$\mathcal{N} = 4$ sugra: ($\mathcal{N} = 4$ sYM) x ($\mathcal{N} = 0$ YM)



Integration uses state-of-the-art software developed for QCD. Industrial strength software needed: FIRE5 and special purpose C++ code.

The 4 loop Divergence of $N = 4$ Supergravity

ZB, Davies, Dennen, A.V. Smirnov, V.A. Smirnov

dim. reg. UV pole

$$\boxed{\mathcal{M}^{\text{4-loop}}\Big|_{\text{div.}} = \frac{1}{(4\pi)^8}\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^{10}\frac{1}{144}(1-264\zeta_3)\,\mathcal{T}}$$

kinematic factor

$D = 4 - 2\epsilon$

It diverges but it has strange properties:

- **Contributions to helicity configurations that vanish were it not for a quantum anomaly in $U(1)$ subgroup of duality symmetry.**
- **These helicity configuration have vanishing integrands in $D = 4$. Divergence is 0/0. Anomaly-like behavior not found in $N \geq 5$ sugra.**

Carrasco, Kallosh, Tseytlin and Roiban

Motivates closer examination of divergences.
Want simpler example: Pure Einstein gravity is simpler.

Pure Einstein Gravity

Standard argument for 1 loop finiteness of pure gravity:

’t Hooft and Veltman (1974)

$\cancel{R^2}$ $\cancel{R_{\mu\nu}^2}$

Divergences vanish by equation of motion and can be eliminated by field redefinition.

$\cancel{R_{\mu\nu\rho\sigma}^2}$

In *D* = 4 topologically trivial space, Gauss-Bonnet theorem eliminates Riemann square term.

$$\int d^4x \sqrt{-g}(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2) = 32\pi^2\chi$$

Euler characteristic

Pure gravity divergence with nontrivial topology:

$$\mathcal{L}^{\rm GB} = -\frac{1}{(4\pi)^2}\frac{53}{90\epsilon}(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2)$$

Capper and Duff (1974)
Tsao (1977); Critchley (1978)
Gibbons, Hawking, Perry (1978)
Goroff and Sagnotti (1986)
Bornsen and van de Ven (2009)

- **Euler characteristic vanishes in flat space.** ’t Hooft and Veltman (1974)
- **Dimensional regularization makes it subtle.** Capper and Kimber (1980)

This is an “enhanced cancellation”, but here it is well understood.

The Trace Anomaly

Capper and Duff (1974); Tsao (1977); Critchley (1978); Gibbons, Hawking, Perry (1978); Duff and van Nieuwenhuizen (1980); Siegel (1980); Grisaru, Nielsen, Siegel, Zanon (1984); Goroff and Sagnotti (1986); Bornsen and van de Ven (2009); Etc.

The Gauss-Bonnet divergence exactly corresponds to trace anomaly.

$$D = 4 - 2\epsilon$$

$$\mathcal{L}^{\rm GB} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\Big(\underset{\text{graviton}}{\underset{\nearrow}{4\cdot 53}} + \underset{\text{scalar}}{\underset{\nearrow}{1}} + \underset{\text{2 form}}{\underset{\uparrow}{91}} - \underset{\text{3 form}}{\underset{\nwarrow}{180}}\Big)\underset{\text{Gauss-Bonnet}}{\underset{\nwarrow}{(R^2 - 4R_{\mu\nu} + R^2_{\mu\nu\rho\sigma})}}$$

$$T^\mu{}_\mu = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(4\cdot 53 + 1 + 91 - 180\Big)(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma})$$

Duff and van Nieuwenhuizen (1980);

Referred to as trace, conformal, trace or Weyl anomaly.

Quantum Inequivalence?

$D = 4 - 2\epsilon$

$D \to 4$

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{2}{360}\Big(4\cdot 53 + 1 + 91 - 180\Big)\left(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}\right)$$

graviton **scalar** **2 form** **3 form** **Gauss-Bonnet**

two form dual to scalar **three form not dynamical**

$$\partial_\mu \phi \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma} \qquad \Lambda^{1/2} \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma} \qquad D = 4$$

Classically equivalent. But is it quantum mechanically equivalent?

- **Quantum *in*equivalence under duality transformations.**
  Duff and van Nieuwenhuizen (1980)
- **Quantum equivalence under duality. Gauge artifact.**
  Siegel (1980)
- **Quantum equivalence of effective action (ignoring trace anomaly).**
  Fradkin and Tseytlin (1984)
- **Quantum equivalence of susy 1 loop effective action (with Siegel's argument for higher loops)**
  Grisaru, Nielsen, Siegel, Zanon (1984)
- **Quantum *in*equivalence and boundary modes.**
  Finn Larsen and Pedro Lisbao (2015)

Quantum Inequivalence?

Need to ask the question in term of physically measureable quantitites

- Scattering amplitudes good to look at. Cross sections physical.
- One loop too trivial. Need to look at two loops.
- We will see this question has a lot to do with UV properties.

Two-Loop Pure gravity

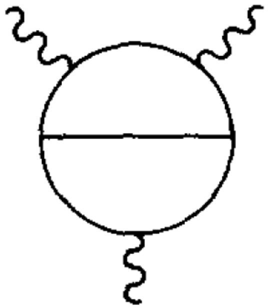
By two loops there is a valid R^3 divergence.

Goroff and Sagnotti (1986); Van de Ven (1992)

Divergence in pure gravity:

UV divergence

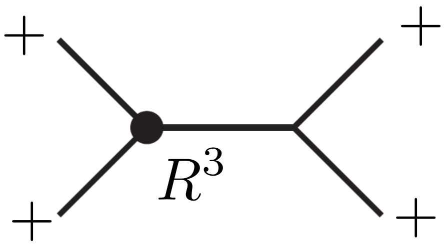
$$\mathcal{L}^{R^3} = \frac{209}{2880}\frac{1}{(4\pi)^4}\frac{1}{2\epsilon}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$



- **Based on dimensional regularization.** $D = 4 - 2\epsilon$
- **On surface nothing weird going on.**
- **The Goroff and Sagnotti result is correct in all details.**

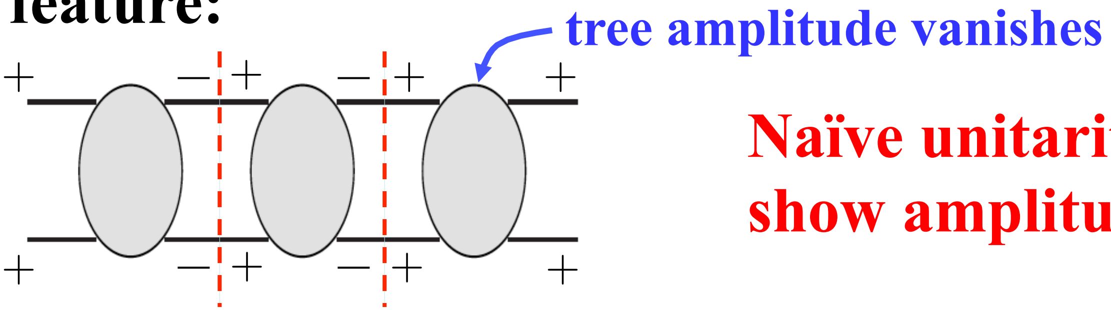
However, a goal of this talk is to show you that UV divergences in pure (super)gravity is subtle and weird, once you probe carefully.

Two Loop Identical Helicity Amplitude



Pure gravity identical helicity amplitude sensitive to Goroff and Sagnotti divergence.

Curious feature:



Naïve unitarity arguments show amplitude vanishes!

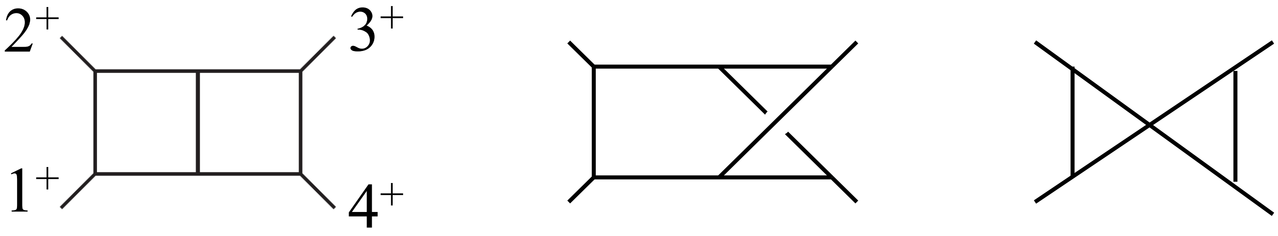
- Gravity amplitude proportional to 0/0, resolved in dim reg.
- Characteristic of quantum anomalies.
- Bardeen and Cangemi pointed out nonvanishing of identical helicity is connected to an anomaly in self-dual sector.

A surprise:

Divergence is *not* generic but tied to anomaly-like behavior.

Two-Loop Divergence

ZB, Cheung, Chi, Davies, Dixon and Nohle (to appear)



$$209 = 11 \cdot 19$$

Integrating we obtain:

$$3431 = 47 \cdot 73$$

$$\mathcal{M}_4^{\text{2-loop}}\Big|_{\text{bare div.}} = -\frac{1}{\epsilon}\,\frac{3431}{5400}\,\mathcal{K} \qquad \mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

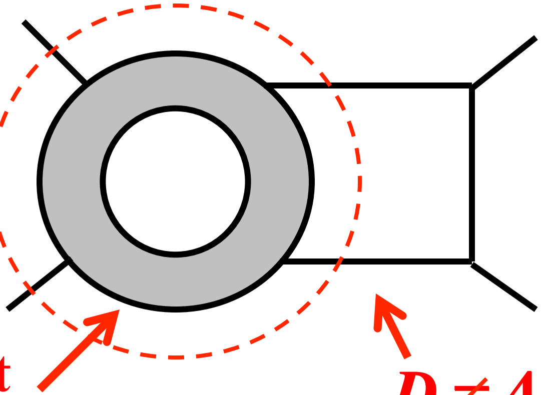
Not the same as the Goroff and Sagnotti result

However, Goroff and Sagnotti subtracted subdivergences integral by integral, following standard procedures.

Subdivergences? What subdivergences?
There are no one-loop divergences. Right?

Subdivergences?

The integrand has subdivergences



Representative diagram.

Gauss-Bonnet subdivergence

D **≠ 4, no subdivergence!**

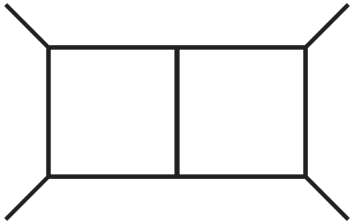
subdivergences

A strange phenomenon: no one loop divergences, yet there are one-loop subdivergences to subtract!

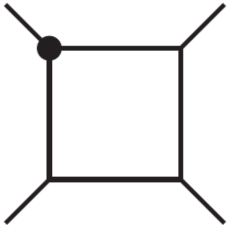
- **To match the G&S result we need to subtract subdivergences.**
- **Using modern methods we can track the pieces.**

Two Loop Identical Helicity Divergence

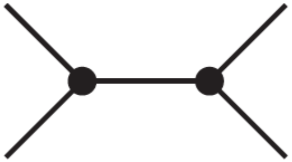
ZB, Cheung, Chi, Davies, Dixon and Nohle



2 loop bare



single GB subtraction



double GB subtraction

$$\left.\begin{aligned} \mathcal{M}_4^{\text{2-loop}}\Big|_{\text{div.}} &= -\frac{1}{\epsilon}\frac{3431}{5400}\mathcal{K} \\ \mathcal{M}_4^{\text{1-loop GB}}\Big|_{\text{div.}} &= \frac{1}{\epsilon}\frac{689}{675}\mathcal{K} \\ \mathcal{M}_4^{\text{tree GB}^2}\Big|_{\text{div.}} &= \frac{1}{\epsilon}\frac{5618}{675}\mathcal{K} \end{aligned}\right\} \quad \mathcal{M}_4^{\text{total}}\Big|_{\text{div.}} = \frac{1}{\epsilon}\boxed{\frac{209}{24}}\mathcal{K}$$

Goroff and Sagnotti divergence reproduced

Trace anomaly plays central role in divergence!

Meaning of Divergence?

What does the divergence mean? $\Lambda^{1/2} \leftrightarrow \varepsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma}$

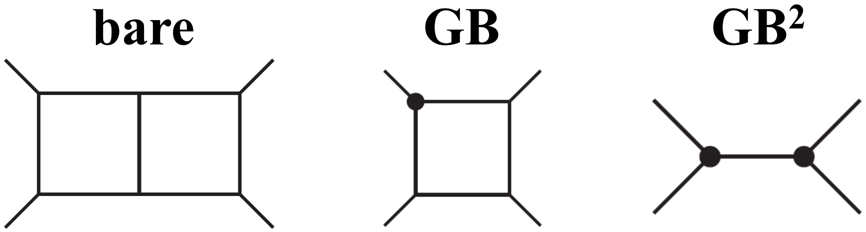
Adding n_3 3-form field offers good way to understand this:

- **On the one hand, no degrees of freedom in $D = 4$, so no change in divergence expected.**
- **On the other hand, the trace anomaly is affected, so expect change in divergence.**

- **Note that 3 form proposed as way to dynamically neutralize cosmological constant.**

Brown and Teitelboim; Bousso and Polchinski

	$1/\epsilon$
bare	$-\frac{3431}{5400} - \frac{199 n_3}{30} + 6 n_3^2$
GB	$\frac{4\cdot 53 - 180 n_3}{360} \cdot \frac{2\cdot(13+180 n_3)}{15}$
GB^2	$24\left(\frac{4\cdot 53 - 180 n_3}{360}\right)^2$
total	$\frac{209}{24} - \frac{15}{2} n_3$



Divergence depends on nondynamical 3-form fields. Quantum inequivalence?

But wait: what about finite parts? Need physical quantity!

Scattering Amplitudes

Pure Gravity:

IR singularities subtracted and independent of 3 form

$$\mathcal{M}_G^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big[\boxed{\frac{1}{\epsilon}\frac{209}{24}stu} + \frac{117617}{21600}stu$$
$$+\left(\frac{1}{10}stu - \frac{1}{60}s^3\right)\log\left(\frac{-s}{\mu^2}\right) + \frac{1}{120}\left(s^2+t^2+u^2\right)s\log^2\left(\frac{-s}{\mu^2}\right) + \text{perms}\Big]$$

Gravity + 3 Form:

divergences differ.
logarithms identical!

$$\mathcal{M}_{G3}^{(2)}(1^+,2^+,3^+,4^+) = \mathcal{N}\Big[\boxed{\frac{1}{\epsilon}\frac{29}{24}stu} + \frac{411617}{21600}stu$$
$$+\left(\frac{1}{10}stu - \frac{1}{60}s^3\right)\log\left(\frac{-s}{\mu^2}\right) + \frac{1}{120}\left(s^2+t^2+u^2\right)s\log^2\left(\frac{-s}{\mu^2}\right) + \text{perms}\Big]$$

- **Value of divergence not physical. Renormalize away.**
- **3 form is a Cheshire Cat field: physical scattering unaffected.**

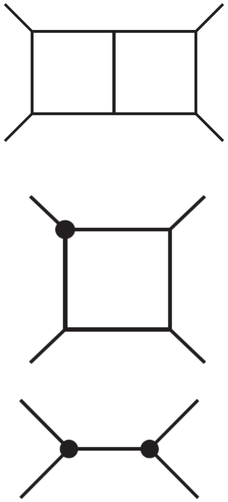
- **Results consistent with quantum equivalence under duality.**
- **For carefully defined physically measurable quantities it seems that duality transformations should not alter the physics.**

$N = 1$ Supergravity

ZB, Chi, Dixon, Edison (to appear)

Divergence violates susy ward identity even though regulator should be supersymmetric! Due to trace anomaly.

Result for $N = 1$ supergravity with 1 matter multiplet



$$\mathcal{M}_4\Big|_{\text{div}} = \frac{1}{\epsilon}\,\frac{81871}{21600}\,\mathcal{K} + 0\ln(\mu^2)\mathcal{K}$$

Very strange, but no stranger than earlier results.

Have no fear: no physical effect! Local counterterm eats the divergence restoring susy.

Still working on case with no matter multiple, but no reason to expect different outcome.

Simple Two-Loop Formula

ZB, Cheung, Chi, Davies, Dixon and Nohle; ZB, Chi, Dixon, Edison (to appear)

Focus on renormalization scale dependence *not* divergences!
In QCD these are effectively the same. In gravity *not* related!

Looking at various theories, we wind up with a simple 2 loop formula:

$$\boxed{\mathcal{M}_4^{(2)}\Big|_{\ln\mu^2} = -\mathcal{K}\,\frac{N_b - N_f}{8}\,\ln\mu^2}$$

N_b is number of bosonic states.
N_f is number of fermionic states.

- **Confident this is robust and does not depend on dimensional regularization or details of theory.**
- **Vanishes at two loops in susy theory, as expected.**
- **Unless lnμ^2 dependence vanishes, theory should still be considered nonrenormalizable.**

Anomaly-like structure leads to a remarkably simple formula for UV properties in any minimally coupled gravity theory! Who ordered this??

Summary

1. Gravity integrands from gauge theory. Very powerful tool.
2. Standard view of gravity UV too naive:
   - — New phenomenon: "Enhanced" UV cancellations in gravity.
   - — So far divergences of pure (super)gravity theories appear to be due to anomalous behavior! No anomaly no divergence.
   - — Renormalized scattering amplitudes independent of duality transformations.
3. Focus on renormalization scale dependence rather than divergences. Equivalent in gauge theory, but not in gravity.
4. Simple two-loop formula for renormalization scale behavior in any gravity theory. Who ordered that?

Expect many more surprises as we probe gravity theories using modern perturbative tools.

Extra slides

Divergences and Duality

ZB, Cheung, Chi, Davies, Dixon and Nohle

$$\mathcal{M}_4 = \left[\frac{1}{\epsilon}\left(\frac{209}{24} - \frac{15}{2} n_3\right) - \frac{1}{4}\ln\mu^2\right]\mathcal{K} + \text{finite}$$

divergence

number of 3 forms

independent of 3 forms

Renormalization scale

$$\mathcal{K} = \left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\, stu \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2$$

- **Weird that renorm. scale and UV divergence not linked! Happens because of Gauss-Bonnet subdivergence.**
- **The renormalization scale dependence is robust and almost certainly not an artifact of dimensional regularization.**
- **For carefully defined physically measurable quantities it appears that duality transformations should not alter the physics.**

Gravity From Gauge Theory

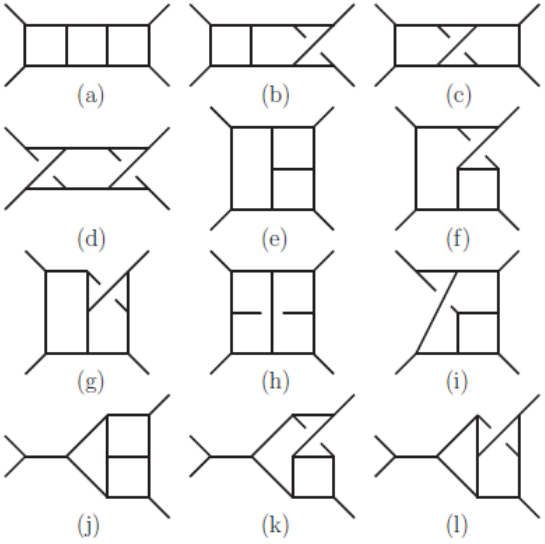
$$\begin{array}{lll} & n & \tilde{n} \\ \mathcal{N}=8 \text{ sugra:} & (\mathcal{N}=4 \text{ sYM}) \times & (\mathcal{N}=4 \text{ sYM}) \\ \mathcal{N}=5 \text{ sugra:} & (\mathcal{N}=4 \text{ sYM}) \times & (\mathcal{N}=1 \text{ sYM}) \\ \mathcal{N}=4 \text{ sugra:} & (\mathcal{N}=4 \text{ sYM}) \times & (\mathcal{N}=0 \text{ sYM}) \end{array}$$

Spectrum controlled by simple tensor product of YM theories.
Recent papers show more sophisticated lower-susy cases.

Anastasiou, Bornsten, Duff; Duff, Hughs, Nagy; Johansson and Ochirov; Carrasco, Chiodaroli, Günaydin and Roiban; ZB, Davies, Dennen, Huang and Nohle; Nohle; Chiodaroli, Günaydin, Johansson, Roiban.

Enhanced UV Cancellations

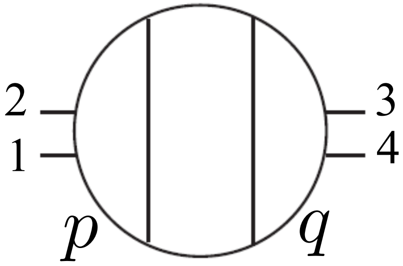
ZB, Davies, Dennen (2014)



Suppose diagrams in *all* possible Lorentz covariant representations are UV divergent, but the amplitude is well behaved.

- **By definition this is an enhanced cancellation.**
- **Not the way gauge theory works.**

***N* = 4 sugra**



already log divergent

***N* = 4 sugra: pure YM x *N* = 4 sYM**

$$n_i \sim s^3 t A_4^{\text{tree}} (p\cdot q)^2\, \varepsilon_1\cdot p\, \varepsilon_2\cdot p\, \varepsilon_3\cdot q\, \varepsilon_4\cdot q + \ldots$$

This diagram is log divergent

3 loop UV finiteness of *N* = 4 supergravity proves existence of "enhanced cancellation" in supergravity theories.

Some New Directions in Gravity Loops

If you want to solve a difficult problem get an army of energetic young people to help with new ideas:

- **Better understanding and applications of BCJ duality.**
  Chiodaroli, Gunaydin, Johansson and Roiban,; Johannsson, Ochirov; O'Connell, Montiero, White; ZB, Davies, Nohle; Boels, Isermann, Monteiro, and O'Connel; Mogull and O'Connell, He, Monteiro, and Schlotterer
- **Scattering equations and double-copy relations.**
  Cachazo, He, Yuan
- **Twistor strings now at loop level for *N* = 8 supergravity.**
  Adamo, Casali and Skinner; Geyer, Mason, Monteiro and Tourkine
- **New ideas on unitarity cuts based on Feynman Tree Theorem**
  Baadsgaard, Bjerrum-Bohr, Bourjaily, Caron-Huot, Damgaard and Feng
- **Important advances in related string theory amplitudes.**
  Carlos Mafra and Oliver Schlotterer
- **Nonplanar analytic hints from Amplituhedron.**
  ZB, Hermann, Litsey, Stankowicz, Trnka
- **Awesome equation solver. Millions of equations encountered at 5 loops can be dealt with! Very cool algorithm!**
  Schabinger and von Manteuffel