Topological superconductivity and Majorana edge modes in trionic phases

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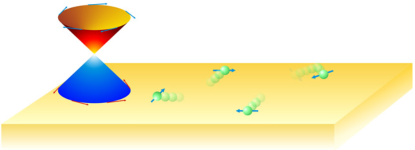
What is a topological phase of matter?

- State of matter beyond Landau's theory
- Characterization: topological quantum numbers (Z, Z_2, etc.)
- Bulk-edge correspondence (gapless edge modes)
- Effective Dirac Hamiltonians in free-fermion models
- Field theories at ground state: Chern-Simons and BF theories
- Topological entanglement entropy with topological order
- 2+1-D non-Abelian phases: non-Abelian anyons
- 3+1-D topological phases: fractional statistics of loops

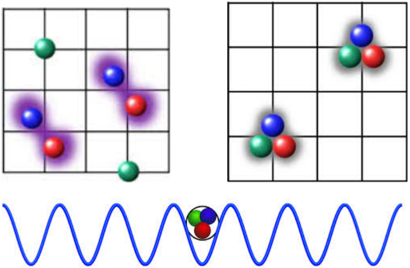
Topological Insulators and Superconductors

Symmetry				Spatial Dimension d								
Class	T	C	S	1	2	3	4	5	6	7	8	$\cdots$
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\cdots$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$\cdots$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\cdots$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\cdots$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$

Protected gapless edge modes: Dirac in TIs and Majorana in TSCs



What is a trionic phase?



(A. Rapp, et al., Phys. Rev. B 77, 144520 (2008))

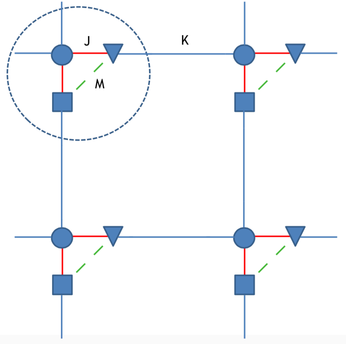
Color superfluidity/superconductivity

So far, there have been no evidences of topological superconductivity in 2D trionic phases.

2D model on the Lieb lattice: Normal state

$$H = \sum_{\boldsymbol{i}} \left[J(a_{\boldsymbol{i}}^\dagger b_{\boldsymbol{i}} + b_{\boldsymbol{i}}^\dagger c_{\boldsymbol{i}}) + K(a_{\boldsymbol{i}}^\dagger b_{\boldsymbol{i}+\hat{x}} + b_{\boldsymbol{i}}^\dagger c_{\boldsymbol{i}+\hat{y}}) + M\, c_{\boldsymbol{i}}^\dagger a_{\boldsymbol{i}} \right] + \mathrm{h.c.},$$

Here, J and K are taken real, while $M = m\, e^{i\theta}$ is complex.

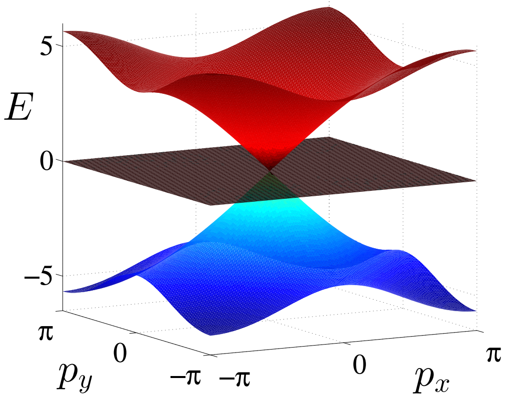


$$H = \sum_{\boldsymbol{p}} \psi_{\boldsymbol{p}}^\dagger h(\boldsymbol{p}) \psi_{\boldsymbol{p}},$$

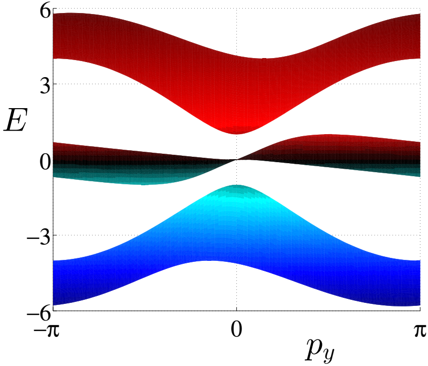
$$h(\boldsymbol{p}) \neq h^*(-\boldsymbol{p})$$

Dirac-like cone and flat band

When $m = 0$, no doubling-fermion problem because of the flat band (Fradkin et al., 1986).



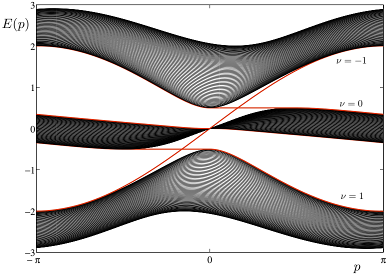
Chern number in semimetals

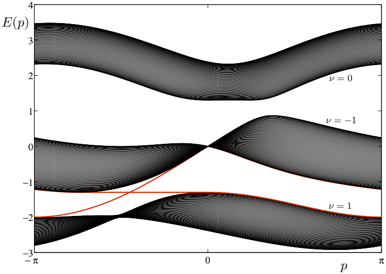


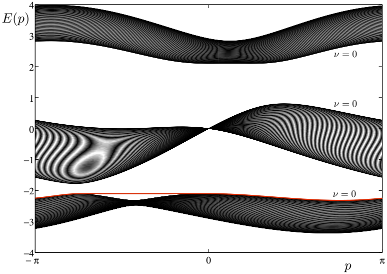
$$\nu_n = \frac{1}{2\pi i} \int_{BZ} d^2p\, F_{xy},$$

where $F_{xy} = \partial_x A_y - \partial_y A_x$, with $A_\alpha = \langle n(\boldsymbol{p}) | \frac{\partial}{\partial p_\alpha} | n(\boldsymbol{p}) \rangle$.

(in the picture above, $\nu = 1$ for the lower band with $M = 0.5\, i$)


$E(p)$
$\nu=-1$
$\nu=0$
$\nu=1$
$-\pi$
0
π
p


$E(p)$
$\nu=0$
$\nu=-1$
$\nu=1$
$-\pi$
0
π
p


$E(p)$
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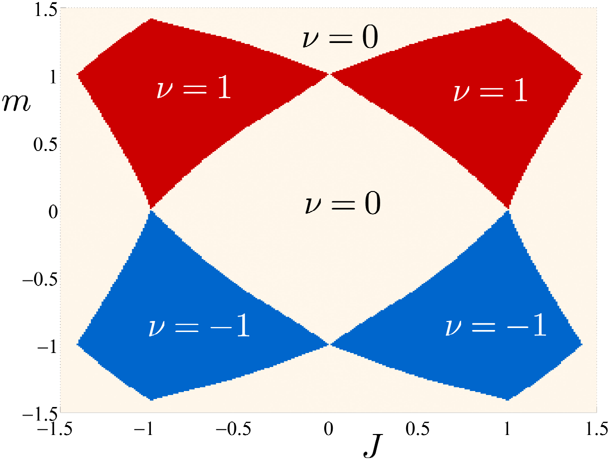
Phase diagram



Chern insulator vs Chern semimetal

- Free-fermion Hamiltonian
- Gapped bulk
- Time-reversal symmetry is broken
- Non-zero Chern number
- Topologically protected gapless edge states
- Topological phase transitions: the gap closes

- Free-fermion Hamiltonian
- Gapless bulk
- Time-reversal symmetry is broken
- Non-zero Chern number
- Topologically protected gapless edge states
- Topological phase transitions: the bands touch each other

Duffin-Kemmer-Petiau theory

Dirac Hamiltonian and Clifford algebra

$$H_{eff}^{Dirac} = \sigma_x p_x + \sigma_y p_y + \sigma_0 m,$$

$$\{\sigma_\mu, \sigma_\nu\} = 2\eta_{\mu\nu} I$$

DKP Hamiltonian ($J = -K$)

$$H_{eff}^{DKP} = K\left[\beta^x, \beta^0\right] p_x + K\left[\beta^y, \beta^0\right] p_y + M\beta^0,$$

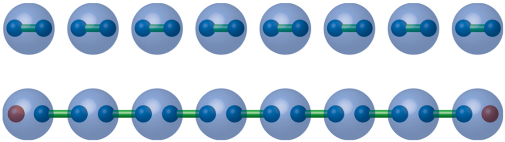
$$\beta^\mu \beta^\nu \beta^\sigma + \beta^\sigma \beta^\nu \beta^\mu = \beta^\mu \eta^{\nu\sigma} + \beta^\sigma \eta^{\nu\mu}$$

the 3×3 β^μ matrices satisfy the Duffin-Kemmer-Petiau algebra (Kemmer, 1939).

Topological superconductors

- TSCs = Insulator/semimetal + Cooper pairs
- Class D = PH symmetry + TR symmetry broken

Simplest example: 1D Kitaev chain



Nearest-neighbor Copper pairings (Δ_{NN})

In 2D: p-wave superconductors (very hard to simulate in cold atoms)

TSCs on the Lieb lattice

We now consider the spinful (or double layer) Chern semimetal

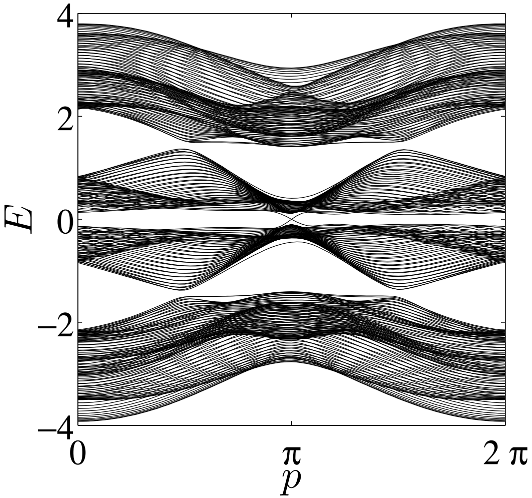
$$H_{bdG} = \begin{pmatrix} H(\boldsymbol{p}) & \Delta_C \\ \Delta_C^\dagger & -H(-\boldsymbol{p})^* \end{pmatrix}$$

In the real space, the Cooper pairings are given by

$$\Delta_C = \sum_{\boldsymbol{i}} \left(\Delta_1 b_{\uparrow\boldsymbol{i}} b_{\downarrow\boldsymbol{i}} + \Delta_2 a_{\uparrow\boldsymbol{i}} c_{\downarrow\boldsymbol{i}} + \Delta_3 a_{\uparrow\boldsymbol{i}} b_{\downarrow\boldsymbol{i}} + \Delta_{NN} b_{\uparrow\boldsymbol{i}+1} b_{\downarrow\boldsymbol{i}}\right) + h.c.$$

Majorana edge modes

For $J = K = 1$, $M = 1.5i$, $\Delta_1 = 0.6$, $\Delta_2 = 0.2$, $\Delta_3 = 0.2$, $\Delta_{NN} = 0.3$, we get Majorana edge states crossing zero



Conclusions and outlook

We have proposed a new lattice model that supports topological superconducting phases.
These phases are characterized by a non-zero Chern number in the bulk and topologically protected gapless Majorana edge modes.

There are several possible extensions of our model:

- 3D generalization, $\nu_{2D} \rightarrow \nu_{3D}$
- Fractional Topological Superconductors (Fibonacci anyons)

References:

G. P. and K. Meichanetzidis, Phys. Rev. B 92, 235106 (2015).
G. P. and K. Meichanetzidis, "Two-dimensional topological superconductors on the Lieb lattice", in preparation.