Precision determination of b-mass and pseudoscalar B-decay constants

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European Twisted Mass Collaboration

[arXiv: 1603.04306 – to appear in PRD]

New Frontiers in Theoretical Physics
XXXV Convegno di Fisica Teorica
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LQCD: Theoretical framework for 1st principles QCD computations

among many observables LQCD is indispensable for the precise evaluation of ...

- quark masses
- hadronic effects to Weak Matrix Elements

[Experiment] = [*known factors*] × **[CKM]** × **[hadronic matrix elements]**

LQCD

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- *hadronic effects to Weak Matrix Elements*

[Experiment] = [*known factors*] × **[CKM]** × **[hadronic matrix elements]**

LQCD

↓ ↓ ↓

Precision **Flavour Physics** requires broad cooperation of **Theory** and **Experiment**

Indirect probe of higher scales than directly accessible

(New Physics)

m_b f_B f_{Bs}

- fundamental SM parameter

- enters as input

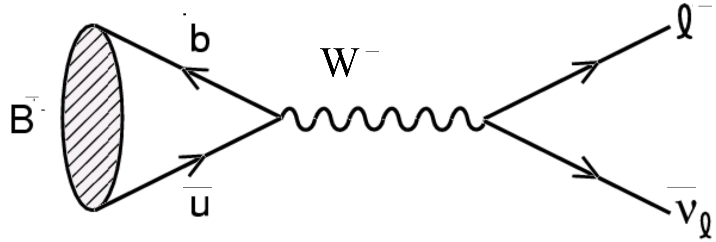
  in Higgs decays: $\mathcal{B}(H \to b\bar{b}) \propto m_b^2$

  in B-decays inclusive: $\mathcal{BR} \propto m_b^5$

m_b **f_B** f_Bs

$\mathcal{B}(B \to \tau\nu) \sim 10^{-4}$

[Belle, BaBar]

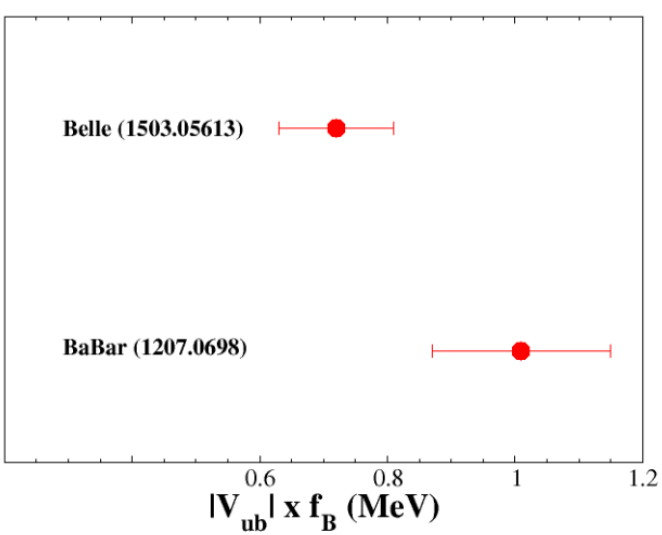


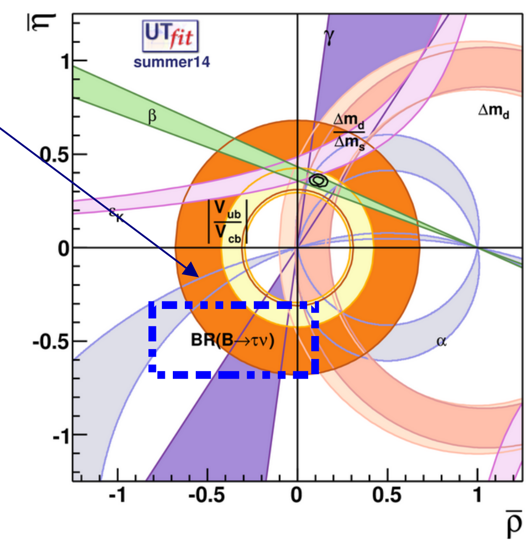
$$\Gamma(B \to \ell\nu_\ell) = \frac{m_B}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$$

(to lowest order)

$\langle 0|A_0|B\rangle = f_B M_B$ lattice

CKM prediction

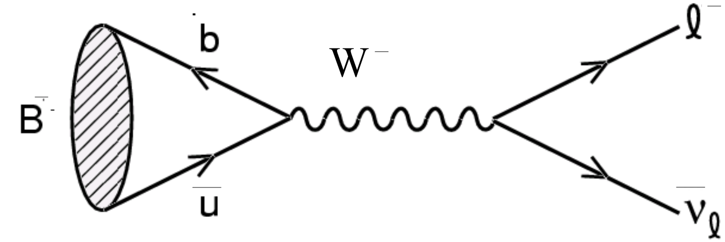




mb | **fB** | fBs

$\mathcal{B}(B \to \tau\nu) \sim 10^{-4}$

[Belle, BaBar]



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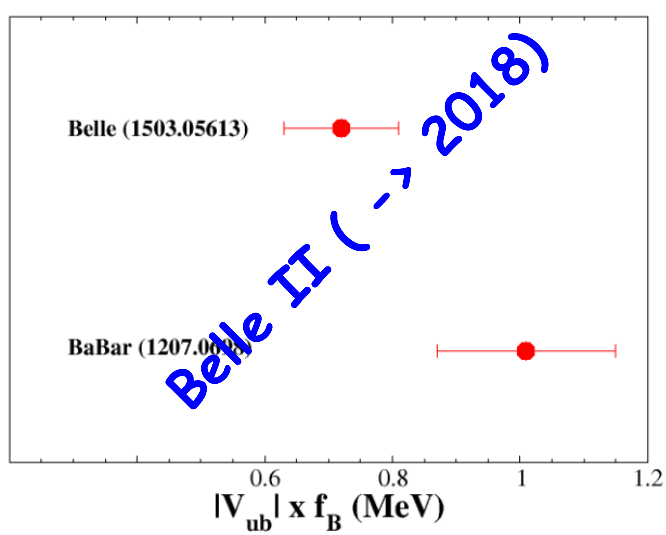
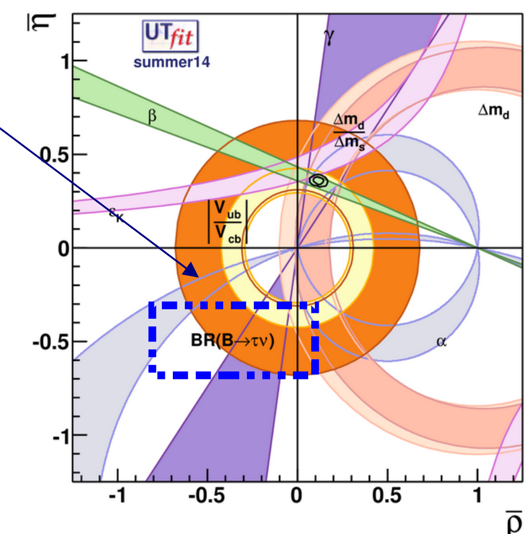




m_b f_B **f_{Bs}**

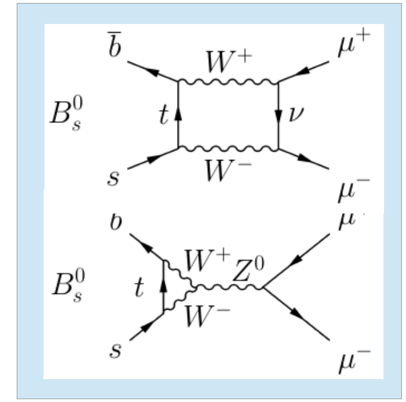
➡ **rare decays**

$$\Gamma(B_q^0 \to \ell^+\ell^-) \propto f_{B_q^0}{}^2 |V_{tb}^* V_{tq}|^2$$

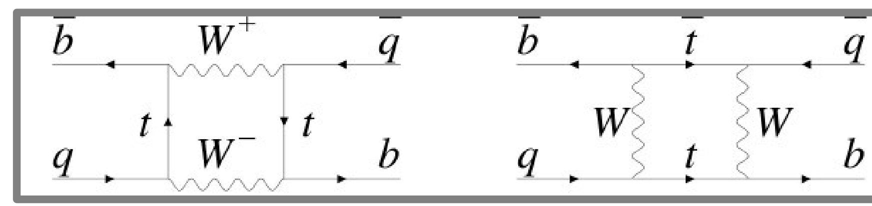
$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) \sim 10^{-9}$$

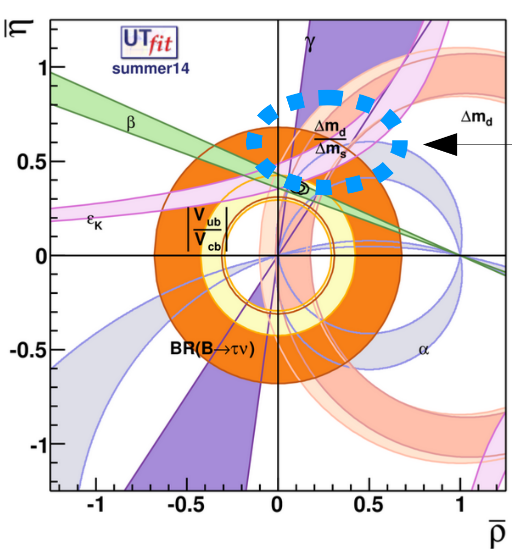
$$\mathcal{B}(B_d^0 \to \mu^+\mu^-) \sim 10^{-10}$$

[see **CMS + LHCb (joint analysis)**]
(1411.4413)



➡ **$B^0_{s(d)} - \bar{B}^0_{s(d)}$ oscillations**

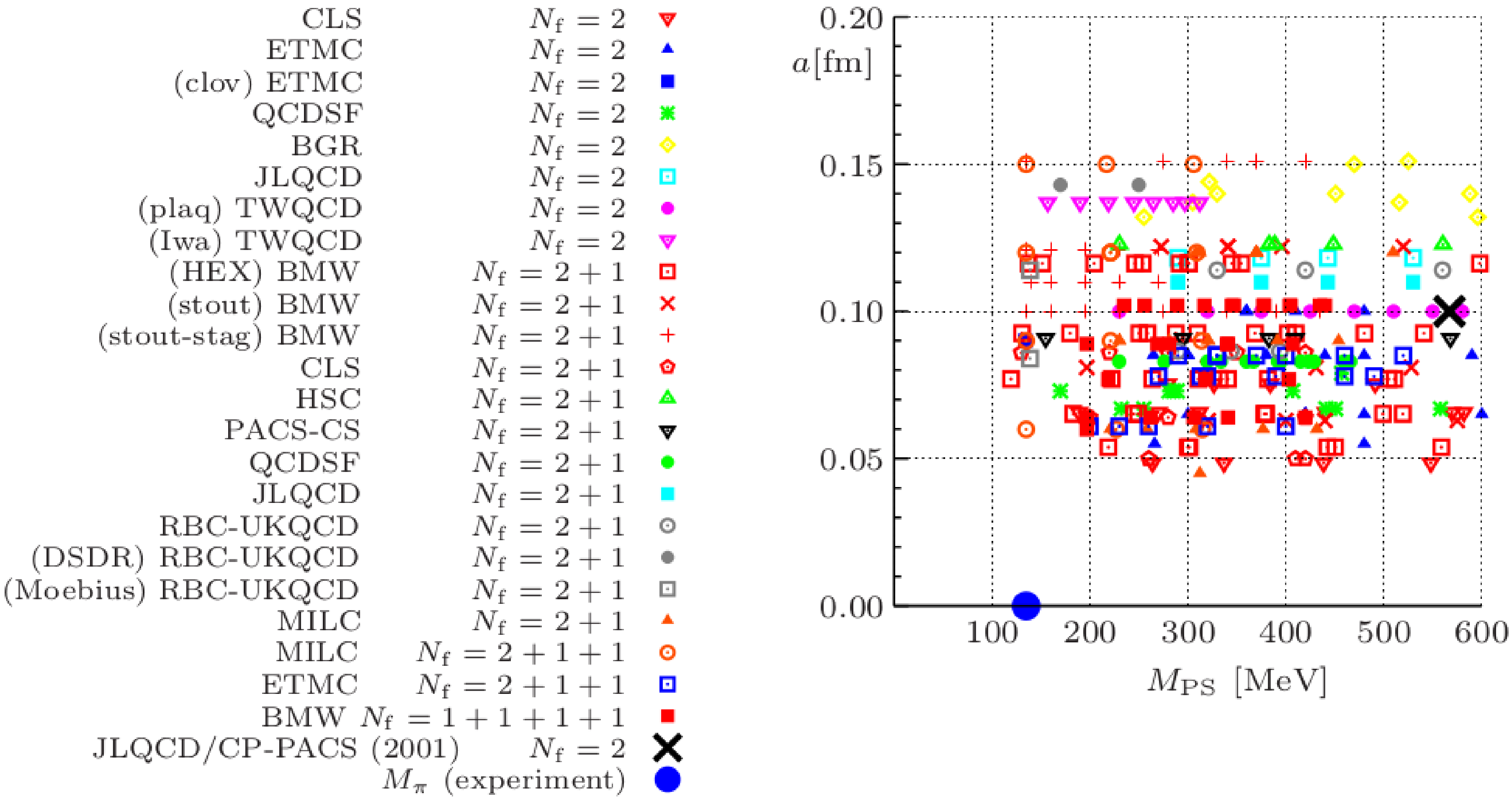




$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2$$

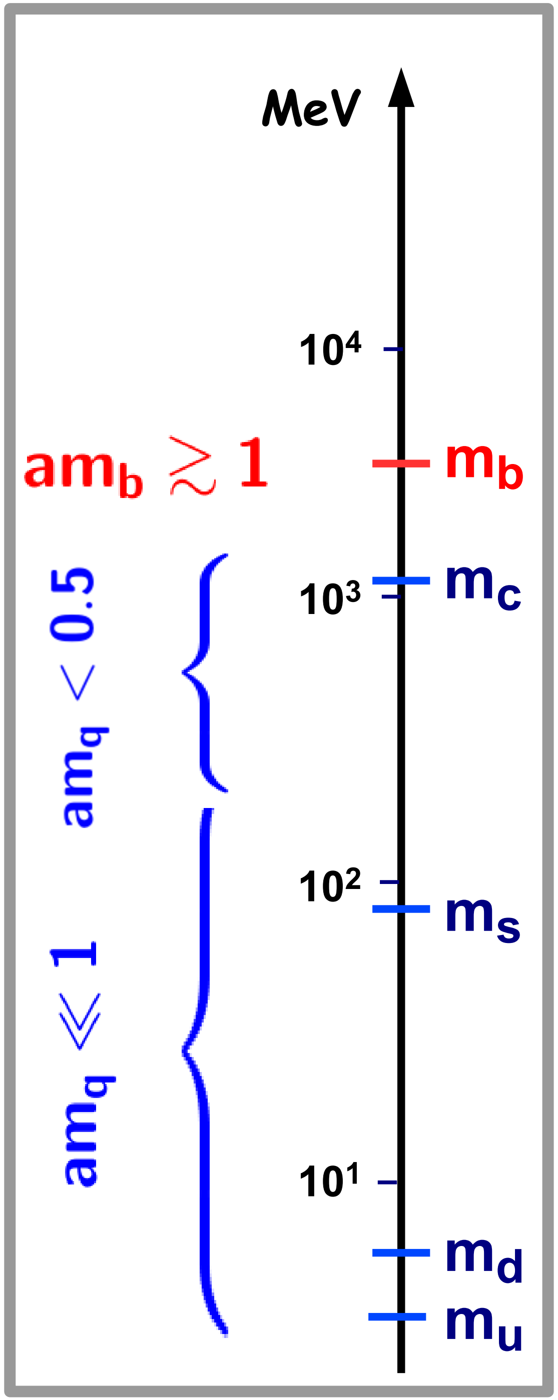
$$\xi = (f_{B_s}\sqrt{B_{B_s}})/(f_{B_d}\sqrt{B_{B_d}})$$

LQCD simulations: landscape



- realistic dynamical quark setup (u/d, s, c: 2+1, 2+1+1, 1+1+1+1)
- C.L. ($a \to 0$) with scaling a^2
- simulations at $m_{ps} \sim m_\pi$ (or very close)
- non perturbative schemes for operators renormalisation

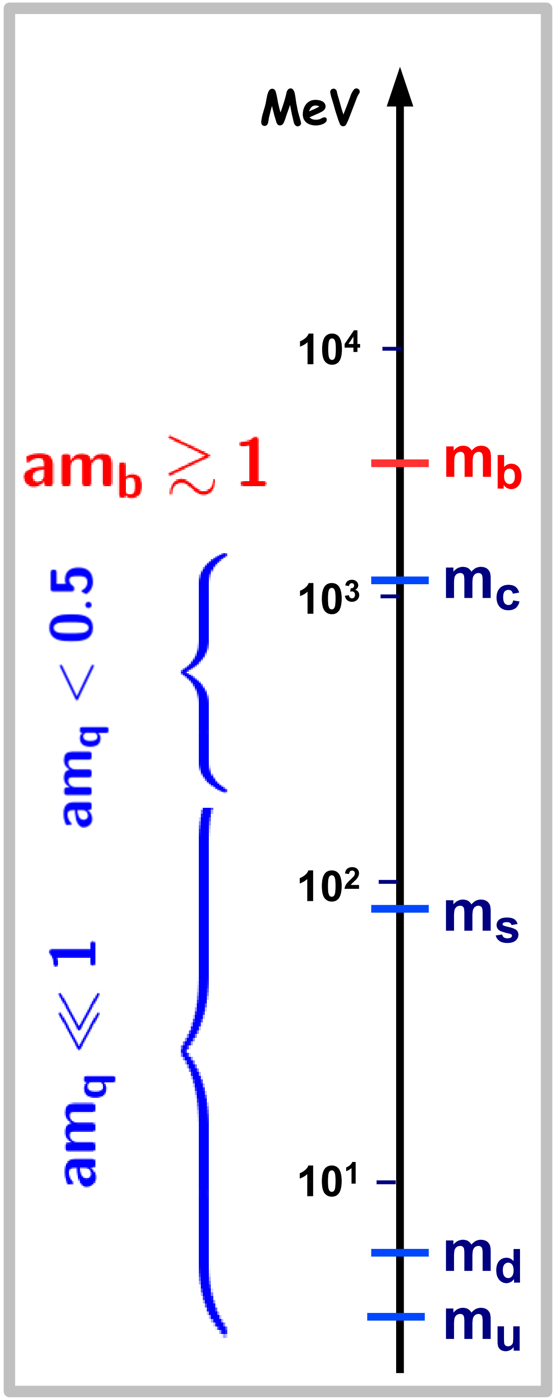
LQCD simulations: scales $(L^{-1} \ll \mu \ll a^{-1})$



Typical current lat. spacings $\mathbf{a^{-1} \in [2, 4]}$ **GeV**

$am_c \sim 0.5$ & $M_\pi L \gtrsim 3 \rightsquigarrow (a \sim 0.085$ fm & $L/a \sim 50)$

LQCD simulations: scales $(L^{-1} \ll \mu \ll a^{-1})$

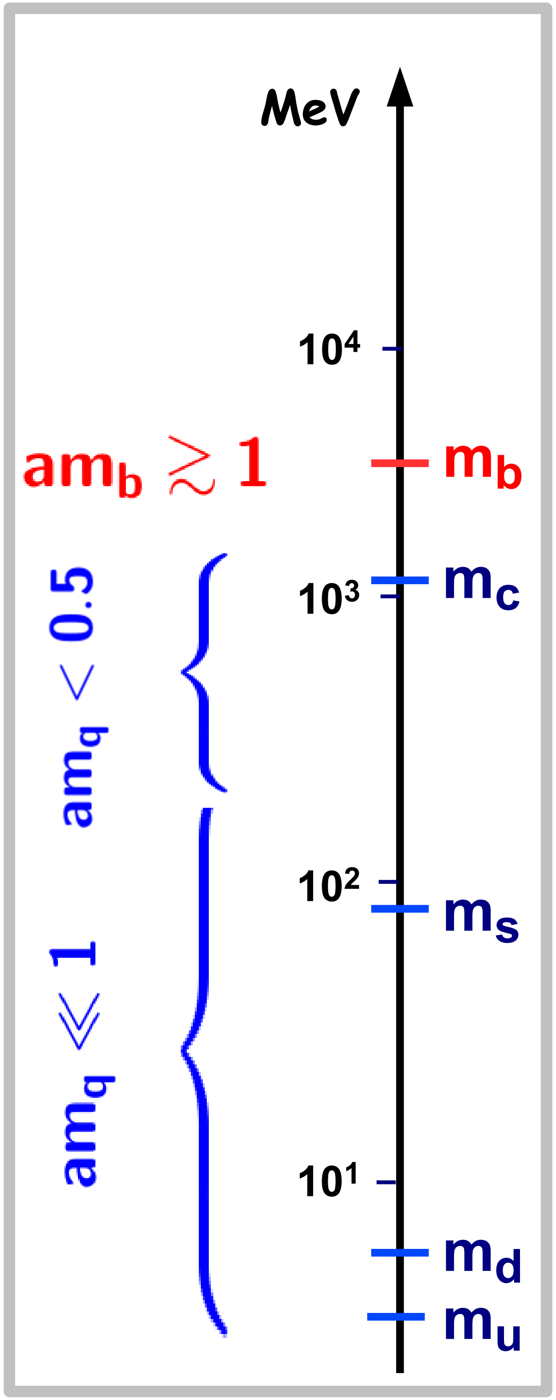


Typical current lat. spacings $\mathbf{a^{-1} \in [2,4]}$ **GeV**

$am_b \sim 0.5$ & $M_\pi L \gtrsim 3 \rightsquigarrow (a \sim 0.025$ fm & $L/a \sim 175)$

$am_c \sim 0.5$ & $M_\pi L \gtrsim 3 \rightsquigarrow (a \sim 0.085$ fm & $L/a \sim 50)$

LQCD simulations: scales $(L^{-1} \ll \mu \ll a^{-1})$



Typical current lat. spacings $a^{-1} \in [2, 4]$ GeV

$\text{CPU} \propto L^5 \quad \text{CPU} \propto a^{-4-6} \quad \text{CPU} \propto 1/m_\ell$

$am_b \sim 0.5$ & $M_\pi L \gtrsim 3 \rightsquigarrow (a \sim 0.025$ fm & $L/a \sim 175)$

$am_c \sim 0.5$ & $M_\pi L \gtrsim 3 \rightsquigarrow (a \sim 0.085$ fm & $L/a \sim 50)$

➡ But even for simulations at higher pion masses M_{ps} ~ 200 MeV, the CPU cost is O(50-100) larger than the most expensive todays LQCD simulations

➡ **Hence, in practice it is <u>still not</u> possible to simulate reliably the b-quark mass**

B-physics on the Lattice

- Is it possible to set under control discretisation errors when $am_b \gtrsim 1$?

➡ A variety of proposals based on the use of effective theories for dealing with valence b-quark effects:

- Relativistic heavy quark actions designed to reduce lattice artifacts [FNAL, RBC]
- NRQCD action [HPQCD]
- HQET and non perturbative matching to QCD [ALPHA]
- Chain of ratios for (relativistic) $m_h > m_c$ & HQET scaling laws **(ratio method)** [ETMC]

The more (and of different systematics) setups are in play the better control (and confidence) on h-quark systematic uncertainties is gained for the lattice evaluation of B-observables.

ETMC simulations

$N_f = 2 + 1 + 1$
$(u/d, s, c)$

a (fm)	M_{ps}^{min}(MeV)	$L^3 \times T/a^4$	L (fm)	$M_{ps}^{min} L$
0.089	245	$32^3 \times 64$	2.84	3.53
0.082	239	$32^3 \times 64$	2.61	3.16
0.062	211	$48^3 \times 96$	2.97	3.19

Action: MtmWilson + Iwasaki glue (+ valence tm/OS)

$(m_{u/d}, m_s, m_c, a) \longleftarrow (M_\pi, M_K, M_D \& M_{Ds}, f_\pi)$ phys. input

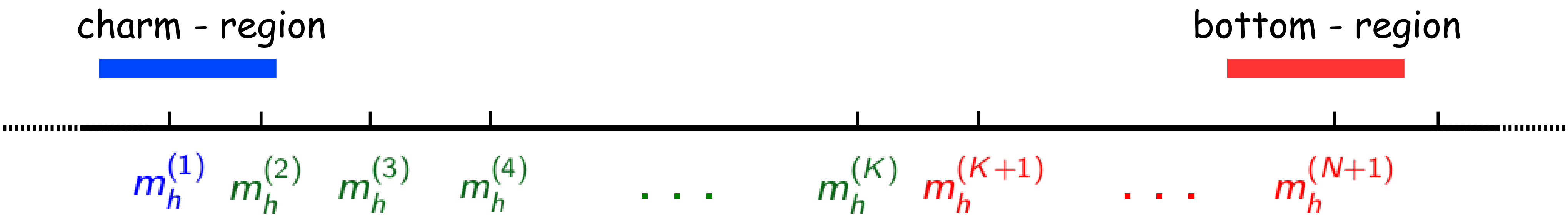
$m_h \sim [m_c, 3.5 m_c]$ (Smearing to isolate ground state)

Frezzotti & Rossi, hep-lat/0306014, hep-lat/0311008, hep-lat/0407002
ETMC 1004.5284, 1005.2042, 1403.4504

Ratio Method - 1

[ETMC, 0909.3187, 1107.1441, 1308.1851]

- Sequence of h-quark masses: $m_h^{(n)} = \lambda\, m_h^{(n-1)} \quad n = 2, \ldots, K \qquad (\text{e.g } \lambda \approx 1.1 - 1.2)$



- Compute on the lattice a quantity that depends on h-quark:

$$\left[Q(m_h^{(n)})\right]_{lat} = \left[Q(m_h^{(n)})\right]_{C.L} + O((am_h^{(n)})^2)$$

→ smooth scaling behaviour (cutoff errors under good control) is expected only for m_h in charm-region (e.g n=1 in our h-mass sequence).

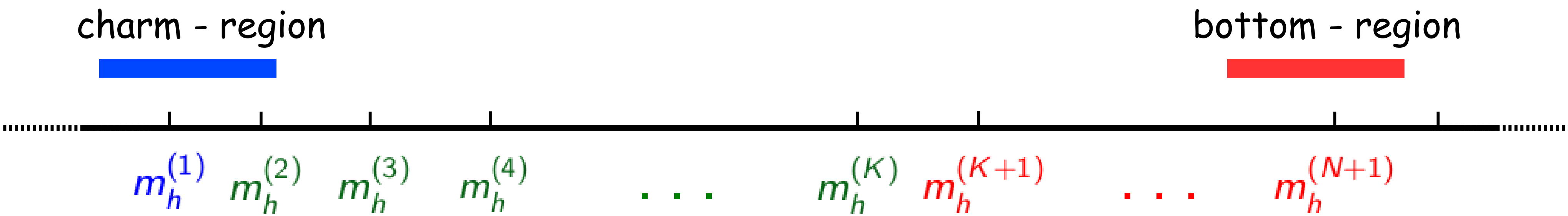
- Ratios are also expected to show good scaling behaviour even for m_h away from charm-region:

$$\left[\frac{Q(m_h^{(n)})}{Q(m_h^{(n-1)})}\right]_{lat} = \left[\frac{Q(m_h^{(n)})}{Q(m_h^{(n-1)})}\right]_{C.L} + (\lambda^2 - 1)O((am_h^{(n)})^2)$$

Ratio Method - 2

[ETMC, 0909.3187, 1107.1441, 1308.1851]

- Sequence of h-quark masses: $m_h^{(n)} = \lambda\, m_h^{(n-1)} \quad n = 2, \ldots, K \qquad (\text{e.g } \lambda \approx 1.1 - 1.2)$



- Choose $Q(m_h)$ described by some HQET scaling law;

$$Q(m_h) = Q_{stat} + \frac{c_1}{m_h} + \ldots$$

→ Ratios:

$$R_Q(m_h) = 1 + \frac{c_1'}{m_h} + \left(\frac{c_2'}{m_h^2}\right) + \ldots \xrightarrow{m_h \to \infty} \mathbf{1}$$

[static limit for ratios is exactly known]

Ratio Method - 3

[ETMC, 0909.3187, 1107.1441, 1308.1851]

- Sequence of h-quark masses: $m_h^{(n)} = \lambda \, m_h^{(n-1)} \quad n = 2, \ldots, K \qquad (\text{e.g } \lambda \approx 1.1 - 1.2)$

charm - region   bottom - region

$m_h^{(1)} \quad m_h^{(2)} \quad m_h^{(3)} \quad m_h^{(4)} \quad \ldots \quad m_h^{(K)} \quad m_h^{(K+1)} \quad \ldots \quad m_h^{(N+1)}$

<u>chain equation</u>

$$Q(m_h^{(1)}) \times \underbrace{\frac{Q(m_h^{(2)})}{Q(m_h^{(1)})}} \times \underbrace{\frac{Q(m_h^{(3)})}{Q(m_h^{(2)})}} \times \ldots \times \underbrace{\frac{Q(m_h^{(K)})}{Q(m_h^{(K-1)})}} \times \underbrace{\frac{Q(m_h^{(K+1)})}{Q(m_h^{(K)})}} \times \ldots \times \underbrace{\frac{Q(m_h^{(N+1)})}{Q(m_h^{(N)})}}$$

$m_h^{(1)} \sim \mathbf{m_c}$

well-controlled **CL** computation in the c-region

ratios safely computed to C.L. thanks to partial cancellation of systematics

<u>Interpolation</u>

(HQET inspired)

of ratios in the **<u>b-region</u>** using *safely* computed relativistic data and the **exact static limit**

Ratio Method - 4

[ETMC, 0909.3187, 1107.1441, 1308.1851]

- Sequence of h-quark masses: $m_h^{(n)} = \lambda \, m_h^{(n-1)} \quad n = 2, \ldots, K \qquad (\text{e.g } \lambda \approx 1.1 - 1.2)$

charm - region ... bottom - region

$m_h^{(1)} \quad m_h^{(2)} \quad m_h^{(3)} \quad m_h^{(4)} \quad \ldots \quad m_h^{(K)} \quad m_h^{(K+1)} \quad \ldots \quad m_h^{(N+1)}$

chain equation

$$\mathbf{Q}(\mathbf{m}_h^{(N+1)}) = Q(m_h^{(1)}) \times \frac{Q(m_h^{(2)})}{Q(m_h^{(1)})} \times \frac{Q(m_h^{(3)})}{Q(m_h^{(2)})} \times \ldots \times \frac{Q(m_h^{(K)})}{Q(m_h^{(K-1)})} \times \frac{Q(m_h^{(K+1)})}{Q(m_h^{(K)})} \times \ldots \times \frac{Q(m_h^{(N+1)})}{Q(m_h^{(N)})}$$

$m_h^{(1)} \sim \mathbf{m_c}$

well-controlled **CL** computation in the c-region

ratios safely computed to C.L. thanks to partial cancellation of systematics

Interpolation *(HQET inspired)* of ratios in the **b-region** using *safely* computed relativistic data and the **exact static limit**

➡ Tune λ such that

1) set $Q(m_h^{(N+1)}) = Q(m_b)|_{(\text{expt.})} \longrightarrow m_b = \lambda^N m_h^{(1)}$

2) make predictions for any other h-quark quantity through a similar chain procedure that ends up at $m_b = m_h^{(N+1)}$

b-mass computation

$$Q_m = \frac{M_{hs}}{(M_{h\ell})^{\gamma}(M_{cs})^{(1-\gamma)}}$$

[ratio of pseudoscalar masses with heavy-light/srange and charm-strange quark content]

B-decay constants computation

$$\mathcal{F}_{hq} = f_{hq}/M_{hq}, \quad q = \ell, s$$

[decay constant to pseudoscalar mass ratio with heavy-light/strange quark content]

dimensionless quantities:

- smoother scaling behaviour to the C.L.
- scale setting is provided once extrapolations to C.L. @ m_π are done

* γ : variation of this parameter allows further control on systematics

b-mass computation

$$Q_m = \frac{M_{hs}}{(M_{h\ell})^{\gamma}(M_{cs})^{(1-\gamma)}}$$

B-decay constants computation

$$\mathcal{F}_{hq} = f_{hq}/M_{hq}, \quad q = \ell, s$$

asymptotic HQET conditions

$$\lim_{m_h^{\rm pole}\to\infty} \left(\frac{Q_m}{(m_h^{\rm pole})^{(1-\gamma)}} \right) = {\rm const.}$$

$$\lim_{m_h^{\rm pole}\to\infty} \mathcal{F}_{hq}\,(m_h^{\rm pole})^{3/2} = {\rm const.}$$

static limit of ratios of Q_m and F_{hq} exactly known (=1)

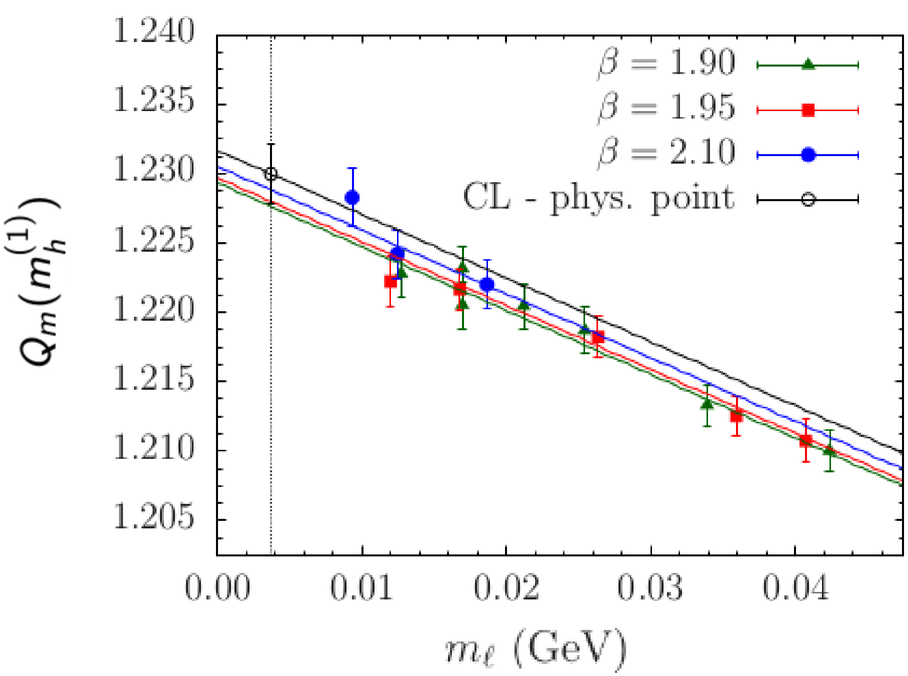
b-mass computation

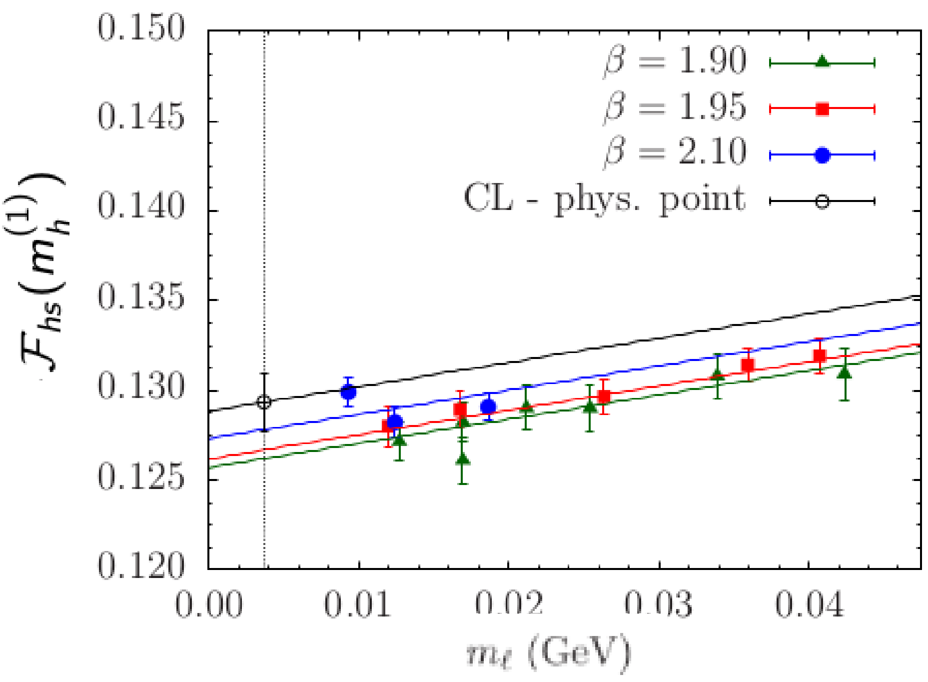
$$Q_m = \frac{M_{hs}}{(M_{h\ell})^{\gamma}(M_{cs})^{(1-\gamma)}}$$

B-decay constants computation

$$\mathcal{F}_{hs} = f_{hs}/M_{hs}$$

safe scaling behaviour in the charm region





combined continuum & chiral fits

"$A + Bm_\ell + Da^2$"

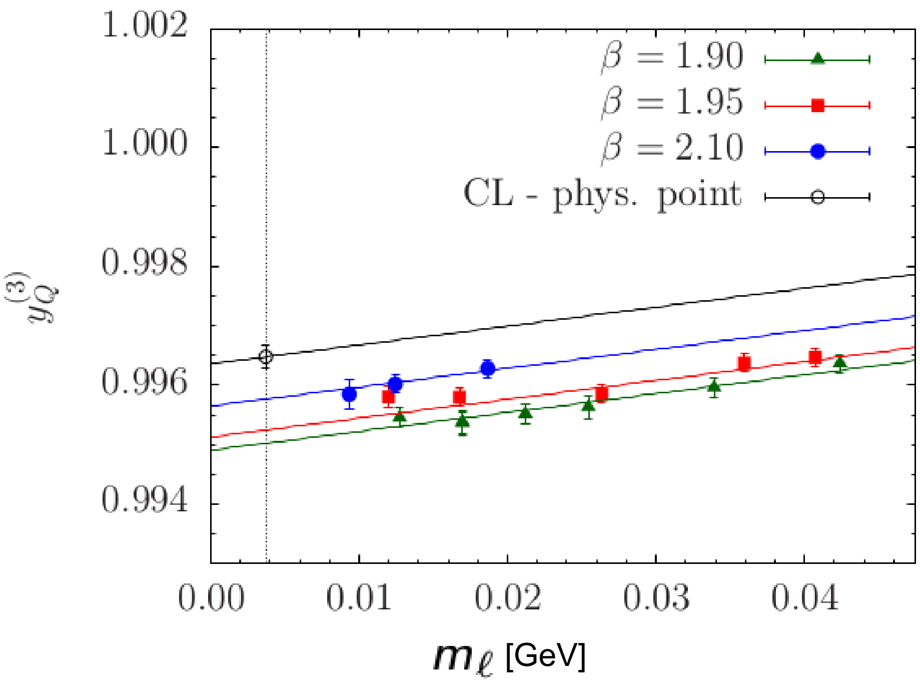
b-mass computation

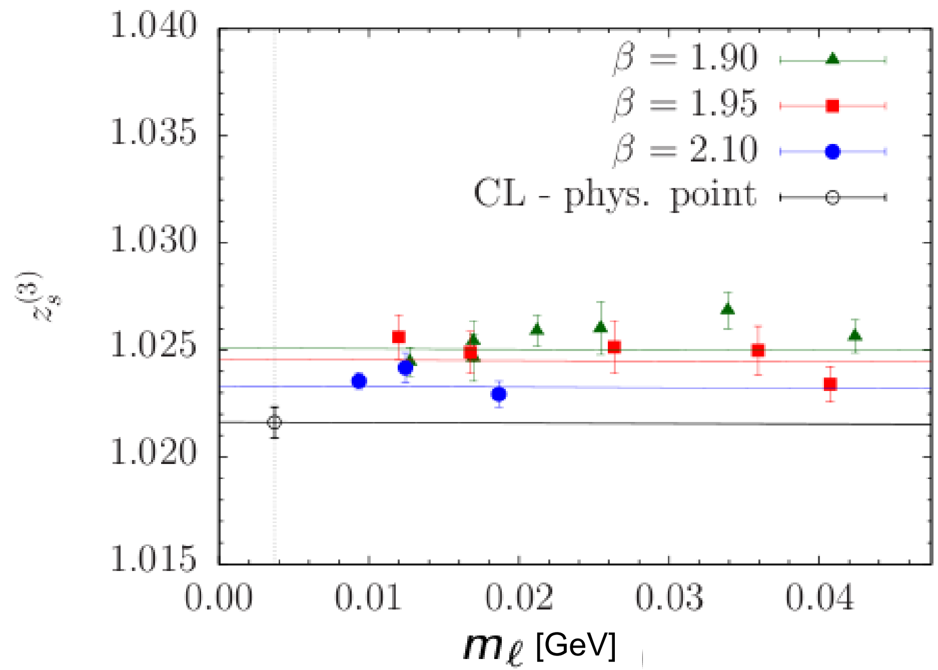
$$Q_m = \frac{M_{hs}}{(M_{h\ell})^{\gamma}(M_{cs})^{(1-\gamma)}}$$

B-decay constants computation

$$\mathcal{F}_{hs} = f_{hs}/M_{hs}$$

well controlled discretisation effects of ratios (for $m_h > m_c$ - region)





combined continuum & chiral fits

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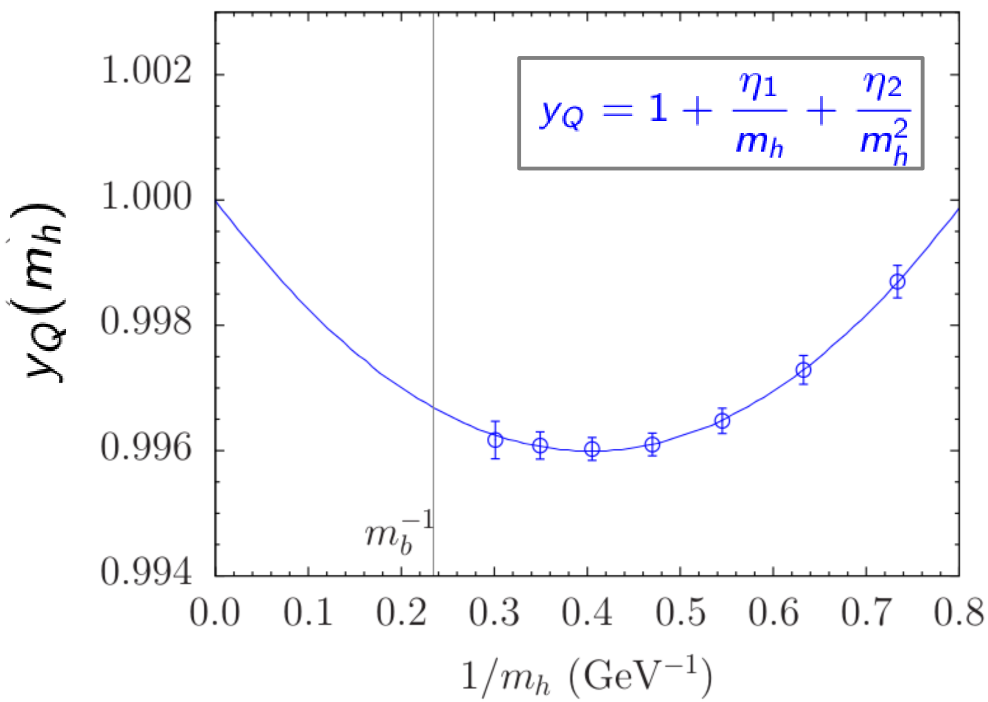
b-mass computation

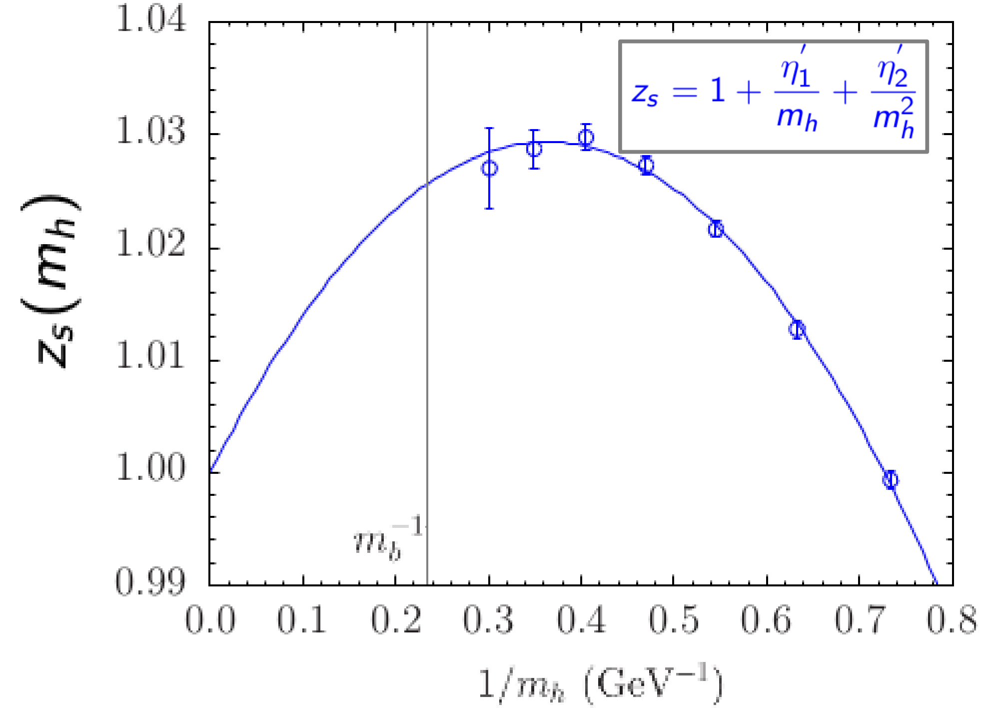
$$Q_m = \frac{M_{hs}}{(M_{h\ell})^{\gamma}(M_{cs})^{(1-\gamma)}}$$

B-decay constants computation

$$\mathcal{F}_{hs} = f_{hs}/M_{hs}$$

ratio interpolation in the b-region





b-mass computation

$$Q_m = \frac{M_{hs}}{(M_{h\ell})^{\gamma}(M_{cs})^{(1-\gamma)}}$$

<u>chain equation</u>

$$y_Q(m^{(2)})\, y_Q(m^{(3)})\ \ldots\ y_Q(m^{(N+1)}) =$$

$$\lambda^{K(\gamma-1)}\ \frac{Q_m(m^{(N+1)})}{Q_m(m^{(1)})} \cdot \left(\frac{\rho(m^{(N+1)},\mu)}{\rho(m^{(1)},\mu)}\right)^{\gamma-1}$$

Satisfy chain equation by tuning *step* λ, such that

$Q(m_h^{(N+1)})$ matches $(M_{Bs}/(M_B)^{\gamma})(M_{Ds})^{(\gamma-1)}$

$$m_b = \lambda^N m_h^{(1)}$$

b-mass computation

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$Q(m_h^{(N+1)})$ matches $(M_{Bs}/(M_B)^{\gamma})(M_{Ds})^{(\gamma-1)}$

$$m_b = \lambda^N m_h^{(1)}$$

B-decay constants computation

$$\mathcal{F}_{hs} = f_{hs}/M_{hs}$$

$$z_s(m_h^{(2)})\, z_s(m_h^{(3)}) \;\ldots\; z_s(m_h^{(N+1)}) \quad =$$

$$\lambda^{3N/2} \frac{\mathcal{F}_{hs}(m_h^{(N+1)})}{\mathcal{F}_{hs}(m_h^{(1)})} \cdot \frac{C_A^{stat}(\mu^*, m_h^{(1)})}{C_A^{stat}(\mu^*, m_h^{(N+1)})} \left(\frac{\rho(m_h^{(N+1)},\mu)}{\rho(m_h^{(1)},\mu)}\right)^{3/2}$$

Apply the chain equation up to $m_b = \lambda^N m_h^{(1)}$

to obtain f_{Bs}/M_{Bs}

and multiplication with $M_{Bs}^{(expt)}$ provides

$$f_{Bs}$$

Similar strategy for the evaluation of

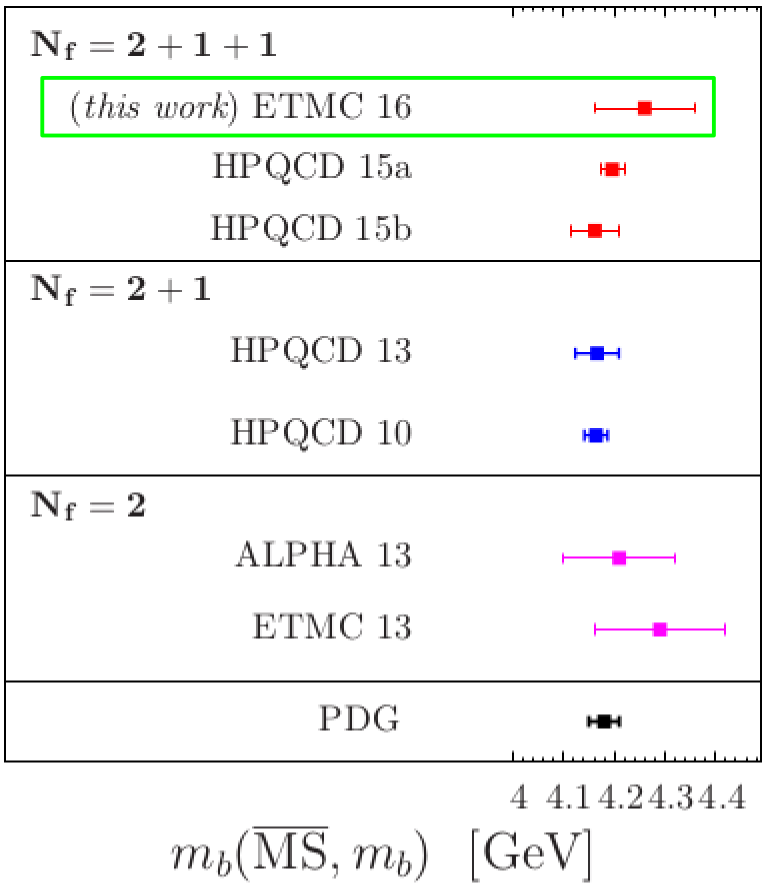
- m_b / m_c
- f_{Bs} / f_B

➡ $f_B = f_{Bs} / (f_{Bs} / f_B)$

Results

b-mass

$$m_b(\overline{\text{MS}}, m_b) = 4.26(3)_{stat+fit}(10)_{syst}[10] \text{ GeV}$$



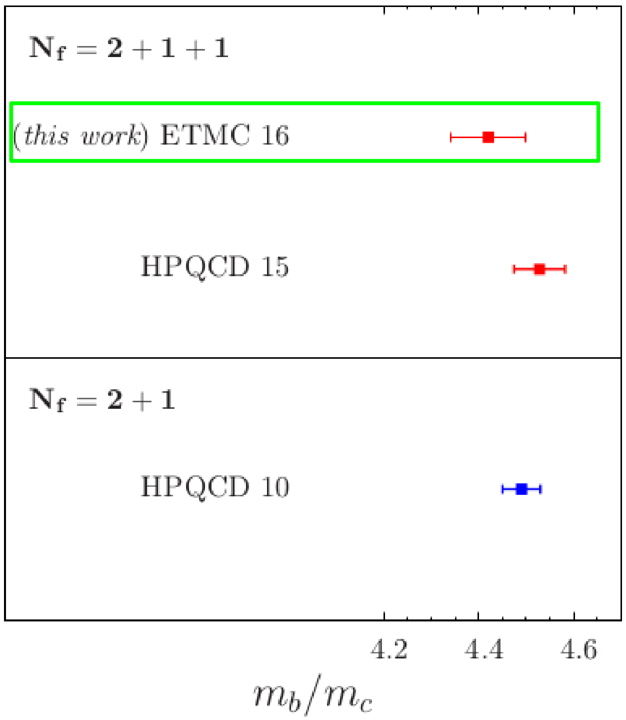
error budget

uncertainty (in %)	m_b
stat+fit	0.9
syst. discr.	1.6
syst. ratios	0.8
syst. chiral	0.4

RI′ - $\overline{\text{MS}}$ matcing	1.3
Total	2.4

m_b / m_c

m_b/m_c = 4.42 $(3)_{stat+fit}$ $(8)_{syst}$ [8]



error budget

uncertainty (in %)	m_b/m_c
stat+fit	0.7
syst. discr.	0.9
syst. ratios	0.8
syst. chiral	0.3
syst. trig. point	1.2
Total	1.9

➡ From m_c/m_s = 11.62(16)(1) (ETMC 1403.4504) we can easily determine

m_b/m_s =51.4 $(1.1)_{stat+fit}$ $(0.9)_{syst}$ [1.4]

(HPQCD '14: m_b/m_s =52.55 (55), Chakraborty et al. 1408.4169)

B-decay constants

$$f_{Bs} = 229(4)_{stat+fit}(3)_{syst}[5] \text{ MeV}$$
$$f_{Bs}/f_B = 1.184(18)_{stat+fit}(18)_{syst}[25]$$
$$f_B = 193(5)_{stat+fit}(3)_{syst}[6] \text{ MeV}$$

error budget

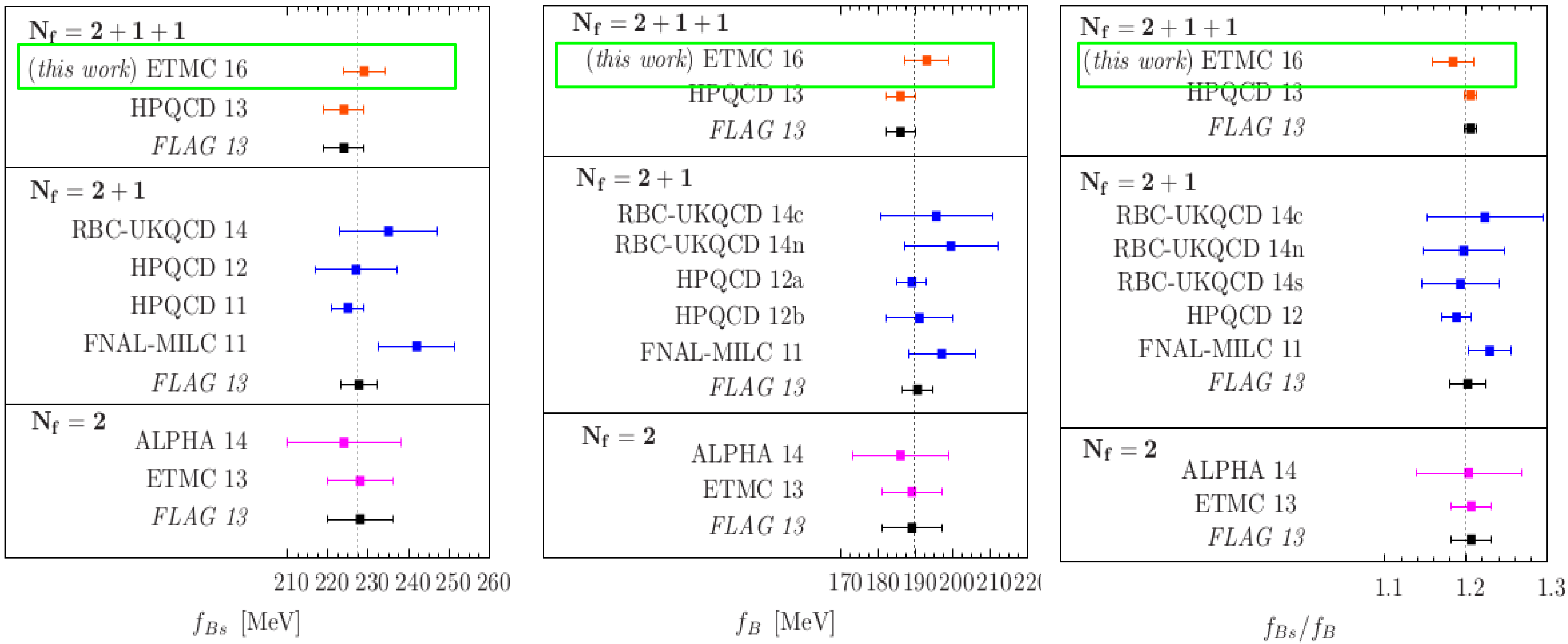
uncertainty (in %)	f_{Bs}	f_{Bs}/f_B	f_B
stat+fit	1.7	1.5	2.5
syst. discr.	1.3	0.6	0.7
syst. ratios	0.5	0.3	0.6
syst. chiral	0.3	0.2	0.4
syst. from f_K/f_π	-	1.3	1.3
Total	2.2	2.1	3.0

$$\frac{f_{Bs}/f_B}{f_K/f_\pi} = 0.997(15)_{stat+fit}(7)_{syst}[17]$$

$$\frac{f_{Ds}/f_D}{f_K/f_\pi} = 1.003(13)_{stat+fit}(6)_{syst}[14]$$

[ETMC 1411.7908]

B-decay constants



[weak dependence on number of dynamical flavours]

[all results compatible within errors]

Conclusions

- High precision computation of b-quark mass and pseudoscalar B-decay constants with ETMC 2+1+1 gauge ensembles employing ratio method.

- Ratio Method
  - computationally cheap
  - no need for explicit computation in effective theory

- Accurate study of the systematics thanks to new variants of ratios used in the present analysis.
  (→ Much reduced total uncertainty wrt ETMC Nf=2 (2013) results)

Conclusions

- High precision computation of b-quark mass and pseudoscalar B-decay constants with ETMC 2+1+1 gauge ensembles employing ratio method.

- Ratio Method
  - computationally cheap
  - no need for explicit computation in effective theory

- Accurate study of the systematics thanks to new variants of ratios used in the present analysis.
  (→ Much reduced total uncertainty wrt ETMC Nf=2 (2013) results)

Thank you for your attention!

Extra slides

N_f=2+1+1 MtmWilson + Iwasaki glue (+ tm/OS)

[sea]

$$S_\ell^{sea} = a^4 \sum_x \bar{\psi}_\ell(x) \left\{ \frac{1}{2}\gamma_\mu \left(\nabla_\mu + \nabla_\mu^*\right) - i\gamma_5\tau^3 \left[M_{\rm cr} - \frac{a}{2}\sum_\mu \nabla_\mu^*\nabla_\mu \right] + \mu_\ell \right\} \psi_\ell(x)$$

$$S_h^{sea} = a^4 \sum_x \bar{\psi}_h(x) \left\{ \frac{1}{2}\gamma_\mu \left(\nabla_\mu + \nabla_\mu^*\right) - i\gamma_5\tau^1 \left[M_{\rm cr} - \frac{a}{2}\sum_\mu \nabla_\mu^*\nabla_\mu \right] + \mu_\sigma + \mu_\delta\tau^3 \right\} \psi_h(x)$$

$$\psi_\ell = (\psi_u \;\; \psi_d)^T$$
$$\psi_h = (\psi_s, \psi_c)^T$$

- Untwisted mass tuned to its critical value M_{cr}
- $m_\ell^R \equiv \mu_{\ell;\rm sea}^R = \frac{1}{Z_P}\mu_\ell$
- $m_{c,s;\rm sea}^R = \frac{1}{Z_P}(\mu_\sigma \pm \frac{Z_P}{Z_S}\mu_\delta)$

[valence]

$$S_q^{val,\rm OS} = a^4 \sum_x \sum_f \bar{q}_f \left\{ \frac{1}{2}\gamma_\mu \left(\nabla_\mu + \nabla_\mu^*\right) - i\gamma_5 r_f \left[M_{\rm cr} - \frac{a}{2}\sum_\mu \nabla_\mu^*\nabla_\mu \right] + \mu_f \right\} q_f(x)$$

- f : $u/d, s, h$ (c & heavier)
  $r_f = \pm 1$
- $m_f^R \equiv \mu_f^R = \frac{1}{Z_P}\mu_f$

- unitarity: match sea and val q-masses in terms of meson masses
- automatic O(a) improvement   [$O(a^2\mu_q)$ for mesons with $r_1 = -r_2$]

Frezzotti & Rossi, hep-lat/0306014, hep-lat/0311008, hep-lat/0407002

Baron et al. 1004.5284, 1005.2042, Carrasco et al. 1403.4504

Simulation parameters

a = 0.062 fm

0.082 fm

0.089 fm

β	V/a^4	$a\mu_{sea} = a\mu_\ell$	N_{cfg}	$a\mu_s$	$a\mu_c - a\mu_h$
1.90	$32^3 \times 64$	0.0030 0.0040 0.0050	150 150 150	0.0180, 0.0220, 0.0260	0.21256, 0.25000, 0.29404, 0.34583, 0.40675, 0.47840, 0.56267, 0.66178, 0.77836, 0.91546
1.90	$24^3 \times 48$	0.0040 0.0060 0.0080 0.0100	150 150 150 150		
1.95	$32^3 \times 64$	0.0025 0.0035 0.0055 0.0075	150 150 150 150	0.0155, 0.0190, 0.0225	0.18705, 0.22000, 0.25875, 0.30433, 0.35794, 0.42099, 0.49515, 0.58237 0.68495, 0.80561
1.95	$24^3 \times 48$	0.0085	150		
2.10	$48^3 \times 96$	0.0015 0.0020 0.0030	90 90 90	0.0123, 0.0150, 0.0177	0.14454, 0.17000, 0.19995, 0.23517, 0.27659, 0.32531, 0.38262, 0.45001, 0.52928, 0.62252

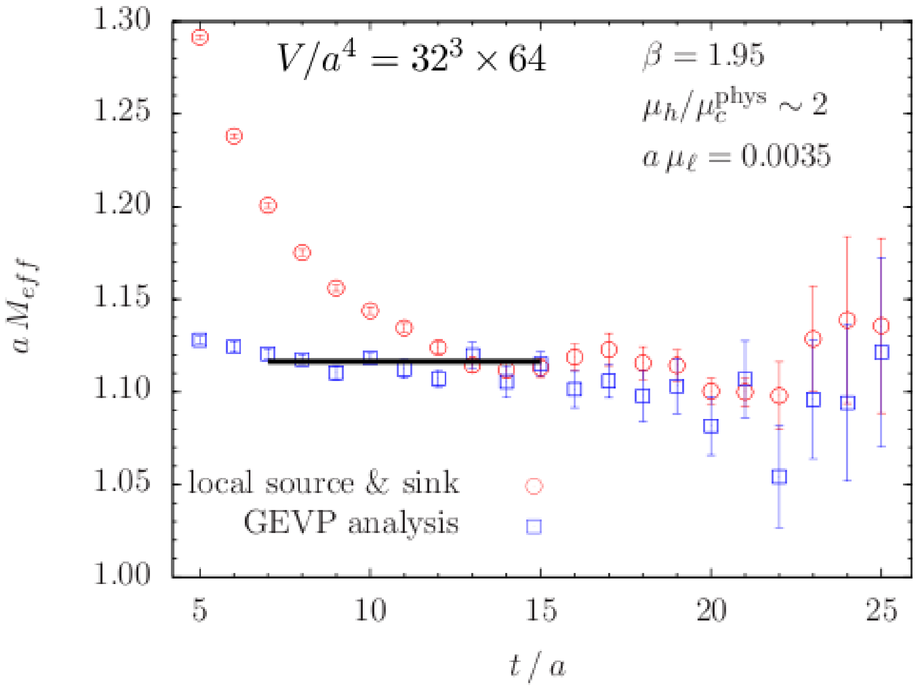
β	L(fm)	M_π(MeV)	$M_\pi L$
1.90	2.84	245 282 314	3.53 4.06 4.53
1.90	2.13	282 344 396 443	3.05 3.71 4.27 4.78
1.95	2.61	239 281 350 408	3.16 3.72 4.64 5.41
1.95	1.96	435	4.32
2.10	2.97	211 243 296	3.19 3.66 4.46

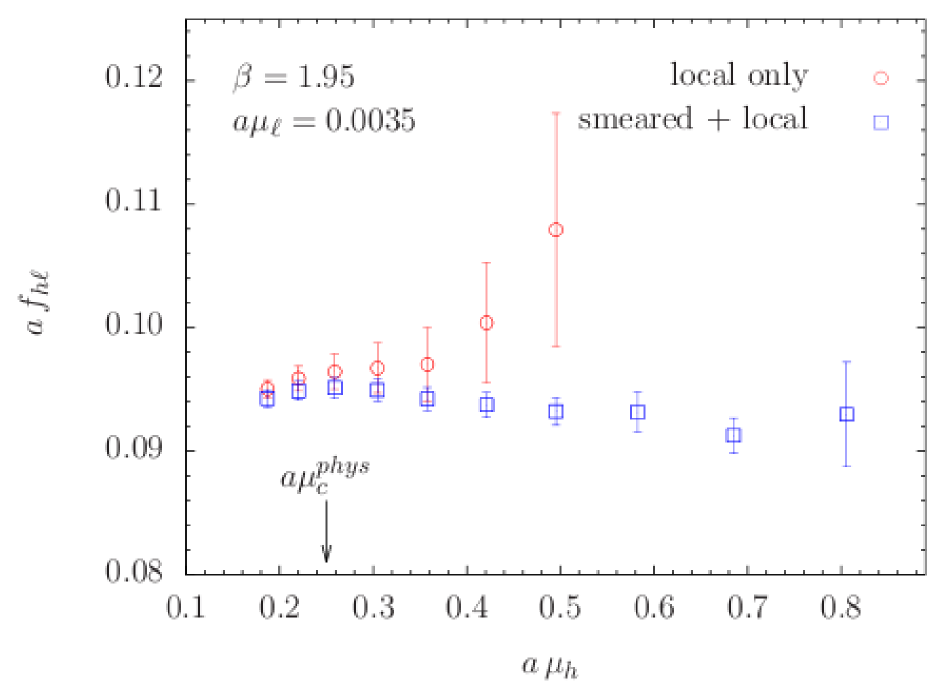
- measurements on 1:20 thermalised gauge conf + blocking
- error analysis: jackknife + bootstrap for fit cross correlations
- RCs from RI-MOM on separate Nf=4 ensembles

Correlators

- light & strange : one-end trick
- charm & heavier : gaussian smearing on sink / source

→ (LL, SL, LS, SS) → GEVP





$$f_{ps} = (\mu_1 + \mu_2)\frac{\langle 0|P^L|ps\rangle}{M_{ps}sinh(M_{ps})}$$

b-mass computation

$$Q_m = \frac{M_{hs}}{(M_{h\ell})^{\gamma}(M_{cs})^{(1-\gamma)}}$$

$$\lim_{m_h^{\rm pole}\to\infty}\left(\frac{Q_m}{(m_h^{\rm pole})^{(1-\gamma)}}\right) = {\rm const.}$$

(HQET asymptotic condition)

γ : parameter [0, 0.9]

no need to tune it.

used for better control of

- syst. discr. effects,
- ratio interpolation fit to b-mass

ratio

$$y_Q(m_h^{(n)}, \lambda; m_\ell, m_s, a) = \lambda^{(\gamma-1)} \frac{Q_m(m_h^{(n)}; m_\ell, m_s, a)}{Q_m(m_h^{(n)}/\lambda; m_\ell, m_s, a)} \left(\frac{\rho(m_h^{(n)}, \mu\ \ )}{\rho(m_h^{(n)}/\lambda, \mu\ \ )}\right)^{(\gamma-1)}$$

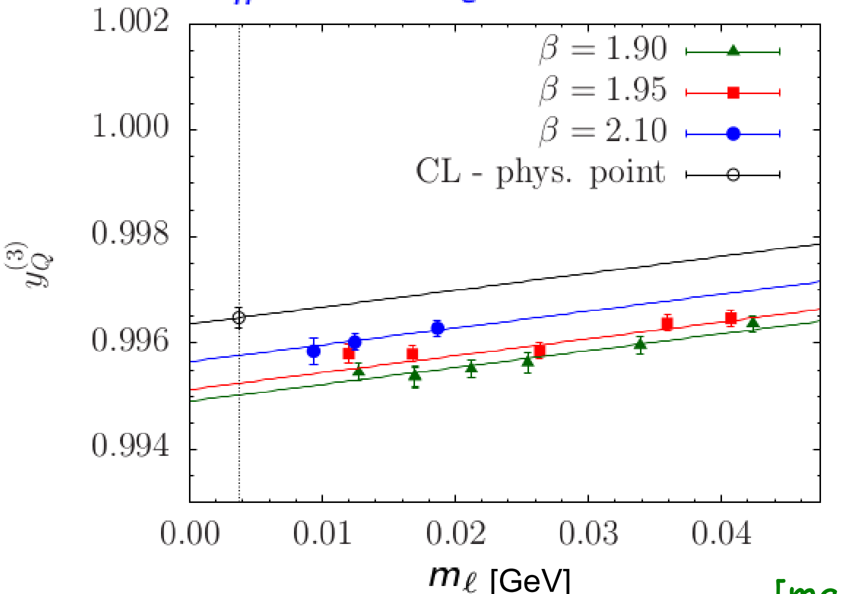
$$m_h^{(n)} = \lambda\ m_h^{(n-1)}$$

$$\left(m_h^{pole} = \rho(m_h, \mu) m_h(\mu)\right)$$

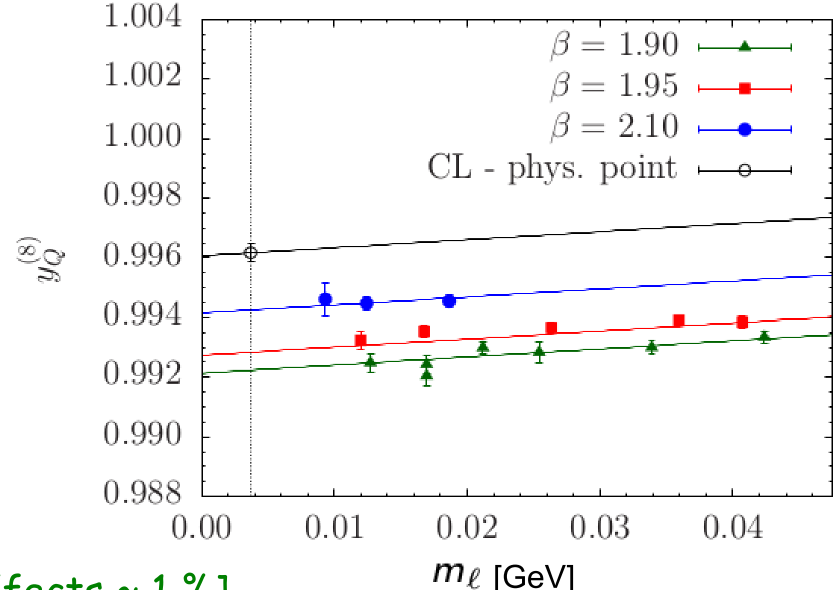
(extrapolate in CL + phys. light quark)

(known to N3LO;
strong cancellations in ratios →
sub % effect to final results)

$m_h \sim 1.4 m_c$



$m_h \sim 3.5 m_c$



[max. discr. effects ~ 1 %]

Ratios in the B-decay constants computation

$$\mathcal{F}_{hq} = f_{hq}/M_{hq}, \quad q = \ell, s$$

$$\lim_{m_h^{\rm pole} \to \infty} \mathcal{F}_{hq}\, (m_h^{\rm pole})^{3/2} = \text{const.} \qquad \lim_{m_h^{\rm pole} \to \infty} \left(\mathcal{F}_{hs}/\mathcal{F}_{h\ell}\right) = \text{const.}$$

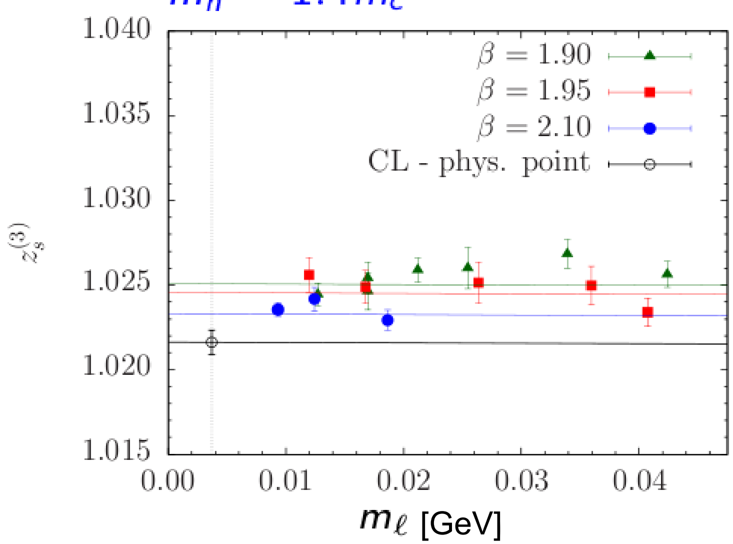
(HQET asymptotic conditions)

ratios

$$z_s(m_h, \lambda; m_\ell, m_s, a) = \lambda^{3/2} \frac{\mathcal{F}_{hs}(m_h, m_\ell, m_s, a)}{\mathcal{F}_{hs}(m_h/\lambda, m_\ell, m_s, a)} \cdot \frac{C_A^{stat}(\mu^*, m_h/\lambda)}{C_A^{stat}(\mu^*, m_h)} \frac{[\rho(m_h, \mu)]^{3/2}}{[\rho(m_h/\lambda, \mu)]^{3/2}}$$

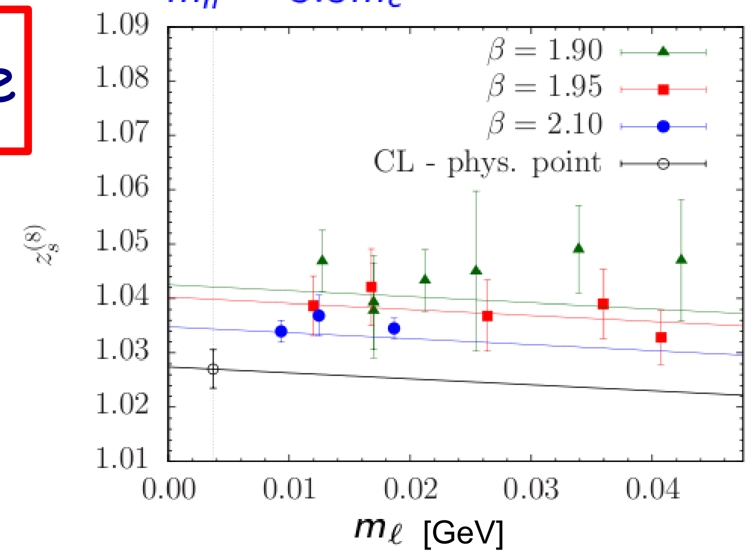
$$z_d(m_h, \lambda; m_\ell, a) = \lambda^{3/2} \frac{\mathcal{F}_{h\ell}(m_h, m_\ell, a)}{\mathcal{F}_{h\ell}(m_h/\lambda, m_\ell, a)} \cdot \frac{C_A^{stat}(\mu^*, m_h/\lambda)}{C_A^{stat}(\mu^*, m_h)} \frac{[\rho(m_h, \mu)]^{3/2}}{[\rho(m_h/\lambda, \mu)]^{3/2}}$$

$m_h \sim 1.4 m_c$



f_{Bs} case

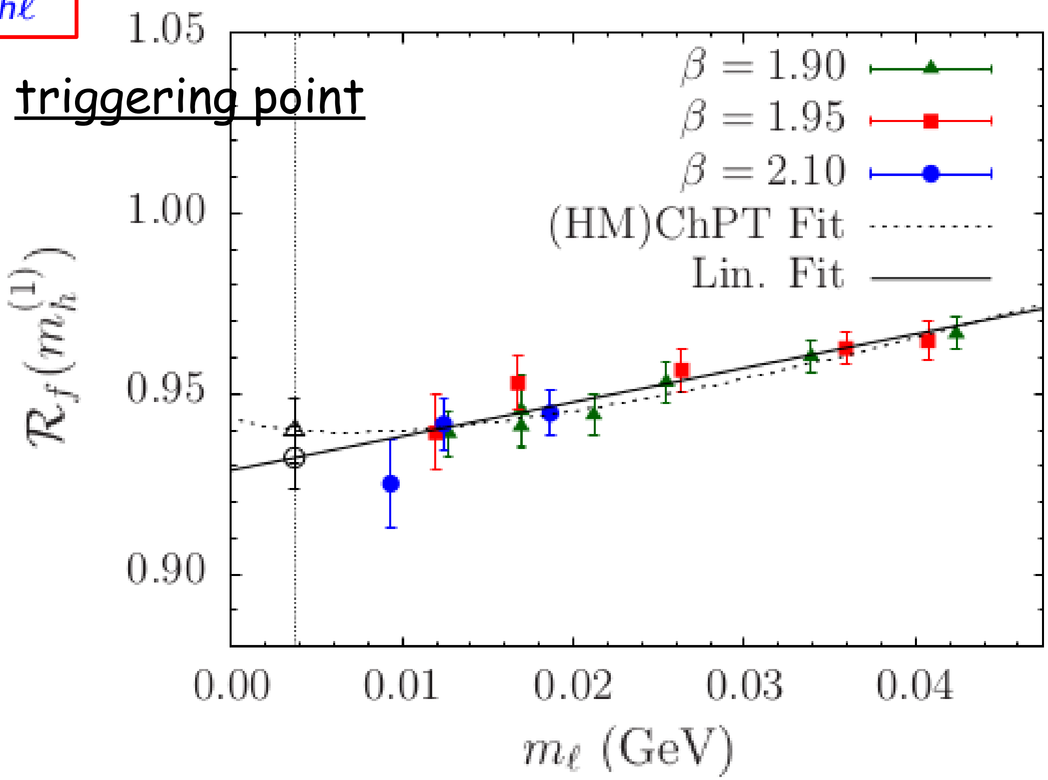
$m_h \sim 3.5 m_c$



[max. discr. effects ~ 1-2 %]

f_{Bs} / f_B

$$\frac{\mathcal{F}_{hs}}{\mathcal{F}_{h\ell}} = \frac{f_{hs} / M_{hs}}{f_{h\ell} / M_{h\ell}}$$



$$\mathcal{R}_f = \frac{\mathcal{F}_{hs} / \mathcal{F}_{h\ell}}{f_{s\ell} / f_{\ell\ell}}$$

For better control of the chiral fit (suppressed log contribution)

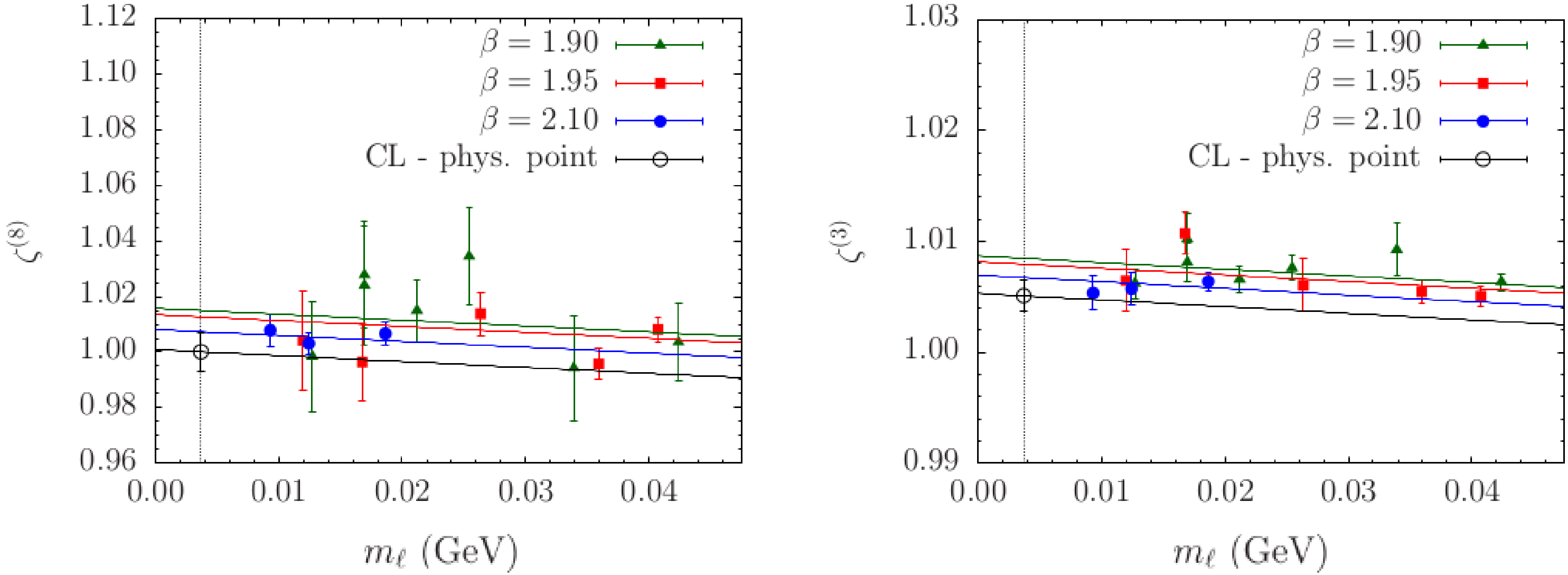
Fit ansatze: polinomial & SU(2) ChPT+HMChPT

$$\mathcal{R}_f^{(1)} = a_h^{(1)} + b_h^{(1)} m_\ell + D_h^{(1)} a^2$$

$$\mathcal{R}_f^{(2)} = a_h^{(2)} \left[1 + b_h^{(2)} m_\ell + \left[\frac{3(1+3\hat{g}^2)}{4} - \frac{5}{4}\right] \frac{2B_0 m_\ell}{(4\pi f_0)^2} \log\left(\frac{2B_0 m_\ell}{(4\pi f_0)^2}\right)\right] + D_h^{(2)} a^2$$

f_{Bs} / f_B

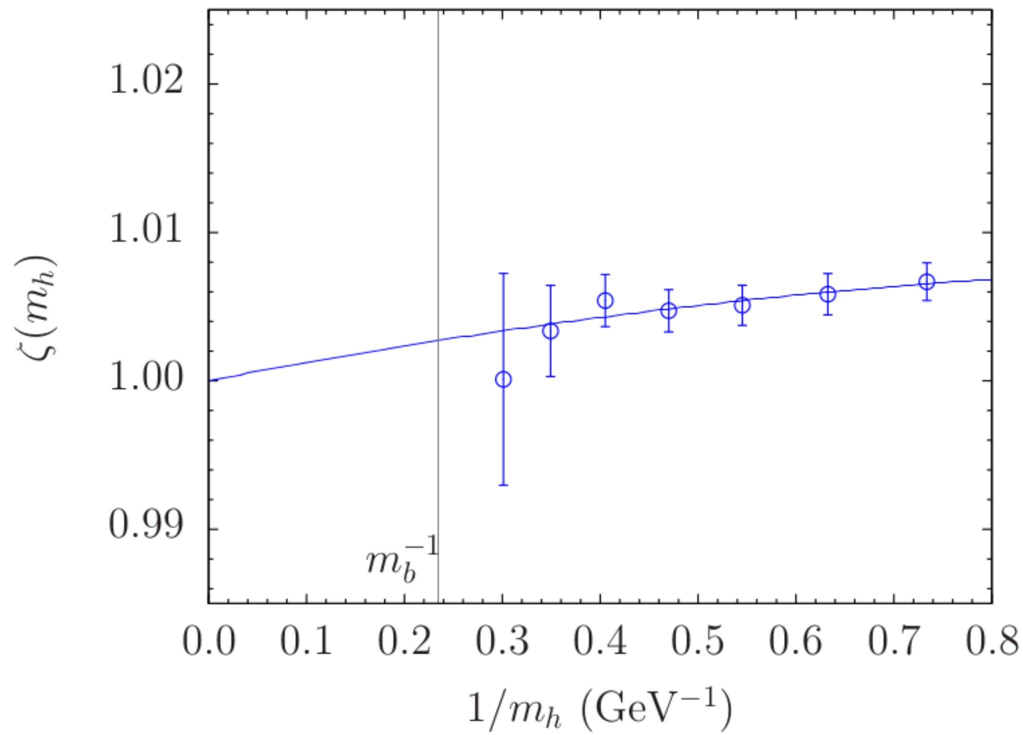
(Combined continuum & chiral fits for ratios in the f_{Bs}/f_B computation)



f_{Bs} / f_B

$$\frac{\mathcal{F}_{hs}}{\mathcal{F}_{h\ell}} = \frac{f_{hs}/M_{hs}}{f_{h\ell}/M_{h\ell}}$$

interpolation in the b-region



from the chain ratio procedure we get $\left(\frac{\mathcal{F}_{bs}}{\mathcal{F}_{bu(d)}} \frac{f_\pi}{f_K}\right)$

multiplication with $\frac{M_B}{M_{Bs}}\Big|^{expt} \frac{f_K}{f_\pi}\Big|^{latt.}$ $\longrightarrow$ f_{Bs}/f_B