FROM CLASSICAL TO QUANTUM THEORY OF THE EARTH-MOON SYSTEM?

Giampiero Esposito, INFN, Naples; October 2, 2015

Plan of seminar

- (i) History of the 3-body problem, and main achievements with hindsight: Birth of chaos theory (Poincaré); discovery of Trojan satellites of Jupiter (Lagrange); Apollo missions (Szebehely); relativistic celestial mechanics (Brumberg).
- (ii) Lagrangian points of the Earth-Moon system in General Relativity.
- (iii) Lagrangian points of the Earth-Moon system in effective field theories of gravity.

Henri Poincaré

Poincaré and the 3-body problem

Henri Poincaré and the Birth of Chaos Theory: An Introduction to the English Translation of *Les Méthodes nouvelles de la Mécanique céleste*

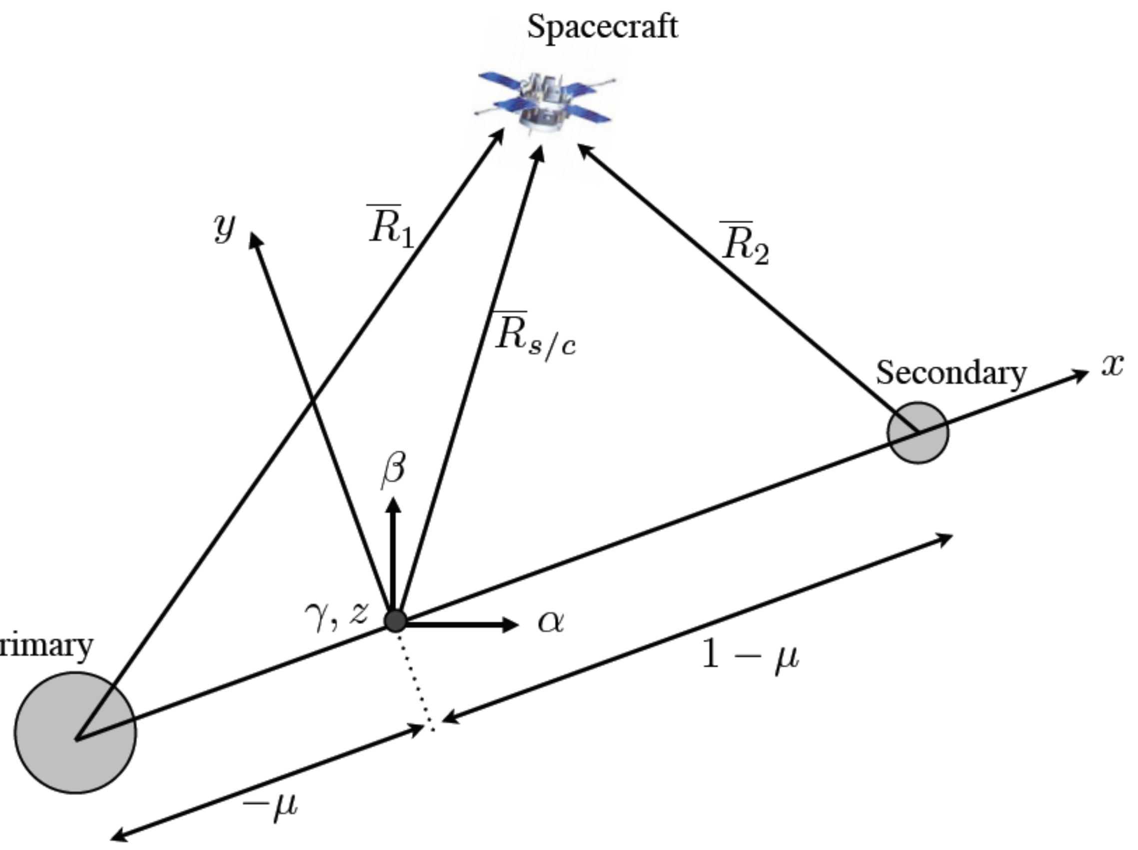
Daniel L. Goroff, Harvard University

"It is not an easy task to attempt to summarize the colossal work of Poincaré," wrote Henri Villat in a 1954 survey of Henri Poincaré's contri-

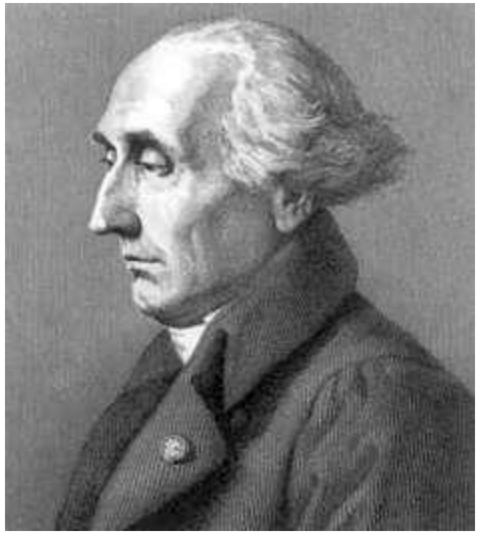
Some of his findings

- He discovered periodic solutions of first and second kind, studied linear stability, discovered two types of asymptotic solutions and proved the existence of doubly asymptotic solutions, and the non-existence of uniform integrals.

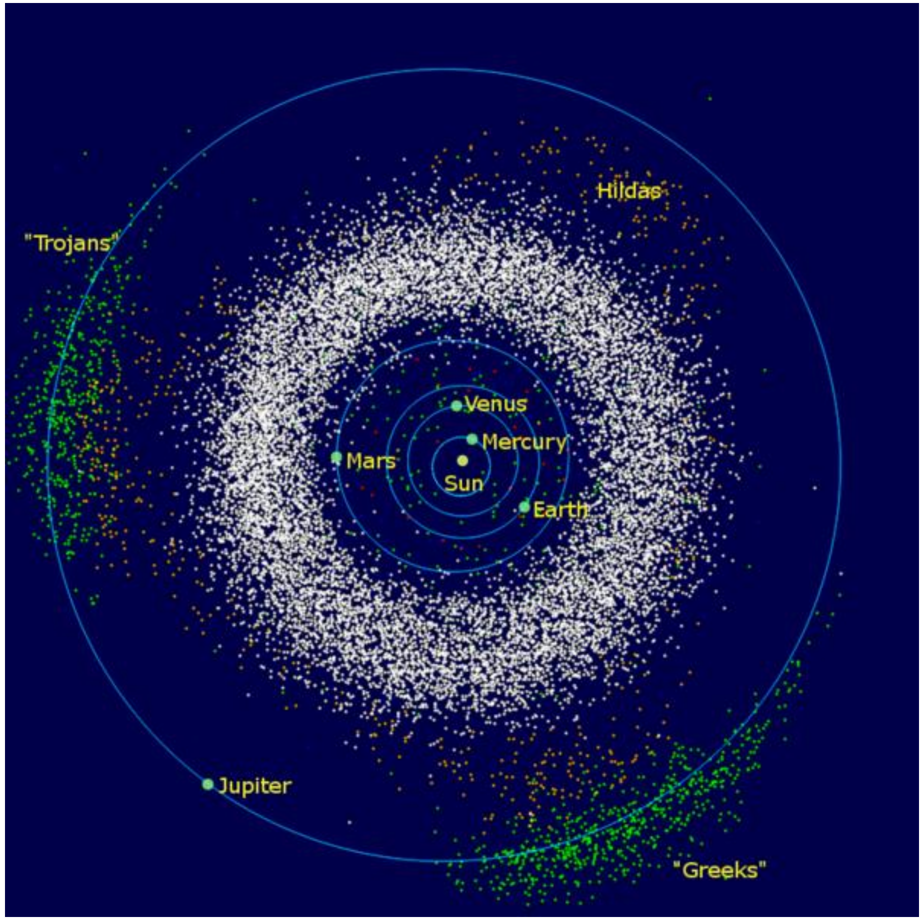
Restricted planar three-body problem



Joseph Lagrange



Trojan satellites



V. Szebehely

He was among the theoretical leaders of the Apollo missions in the sixties

From V. Brumberg

To S. Kopeikin, i.e.

The Science of Relativistic Celestial Mechanics.

Introduction for a layman.

Sergei Kopeikin

Mizzou

University of Missouri-Columbia

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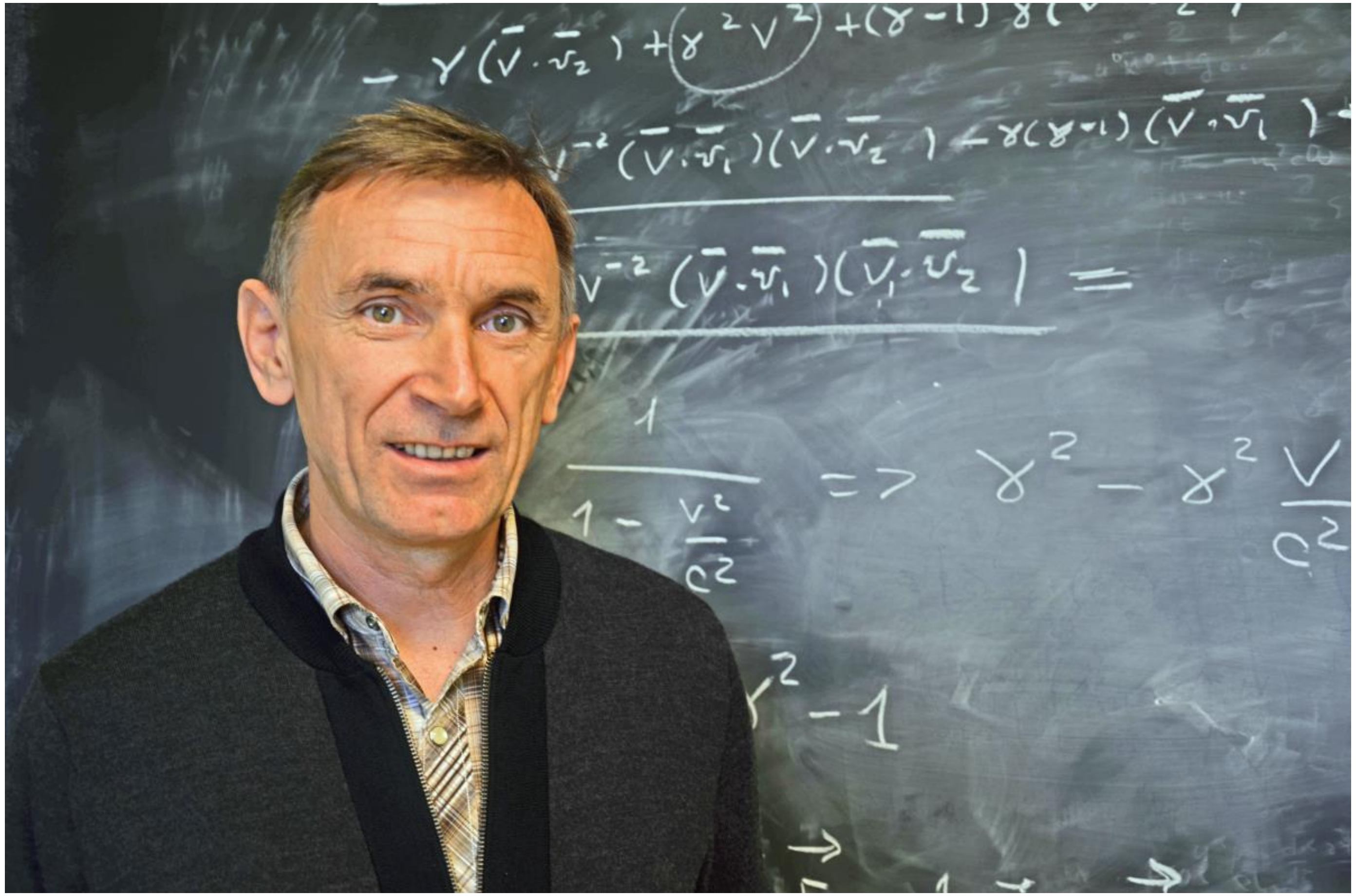
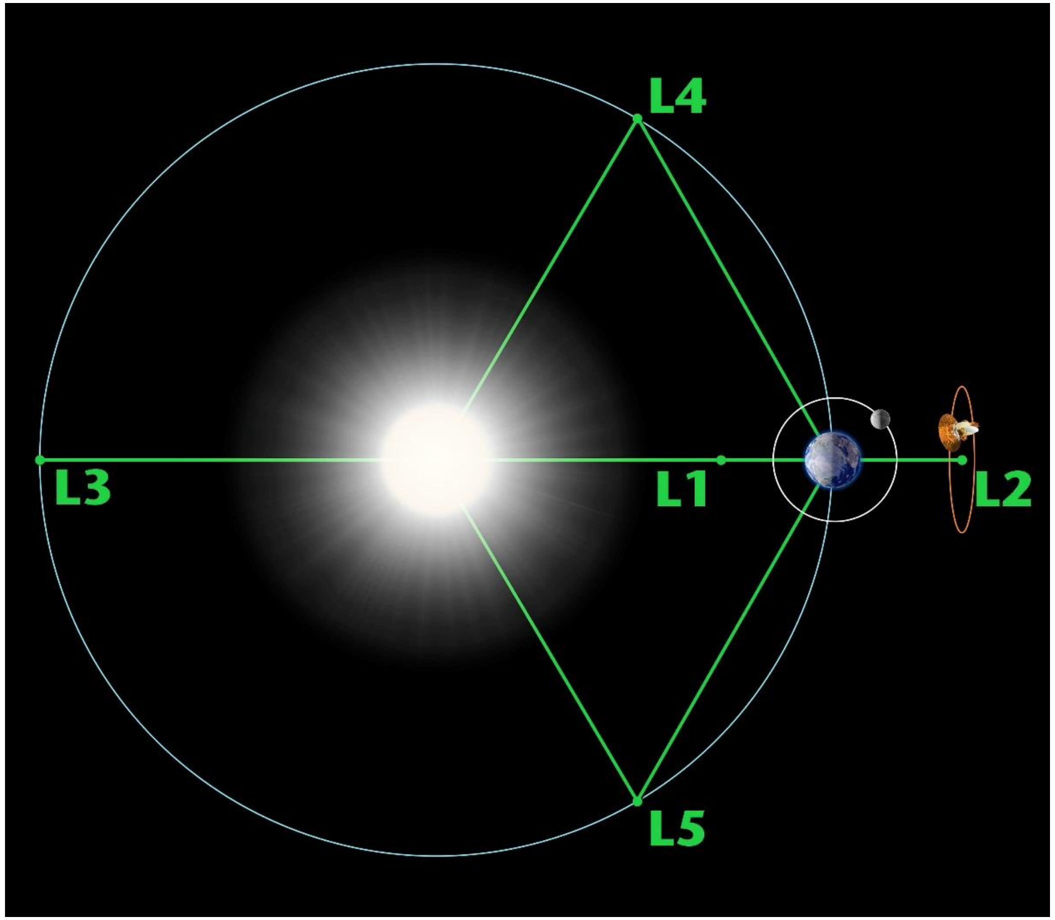
S. Kopeikin



Earth-Moon Lagrangian points



Relativistic restricted 3-body problem

- We study equations of motion of the planetoid in a synodic (i.e. co-rotating, non-inertial) coordinate system with origin at the centre of mass of the primaries (for two bodies only, Newtonian and relativistic centre of mass turn out to coincide).

The GR effective potential

$$
\begin{aligned}
W &= \frac{\Omega^2}{2}(\xi^2+\eta^2) + c^2\left[\frac{R_\alpha}{r} + \frac{R_\beta}{s} - \frac{1}{2}\left(\frac{(R_\alpha)^2}{r^2} + \frac{(R_\beta)^2}{s^2}\right)\right] \\
&+ \frac{1}{8c^2} f^2(\xi,\eta,\dot{\xi},\dot{\eta}) + \frac{3}{2}\left(\frac{R_\alpha}{r} + \frac{R_\beta}{s}\right) f(\xi,\eta,\dot{\xi},\dot{\eta}) \\
&+ \frac{R_\beta}{(1+\rho)}\Omega l\left(4\dot{\eta} + \frac{7}{2}\Omega\xi\right)\left(\frac{1}{r} - \frac{1}{s}\right) \\
&+ \frac{R_\beta}{(1+\rho)}\Omega^2 l^2\left[-\frac{\eta^2}{2(1+\rho)}\left(\frac{\rho}{r^3} + \frac{1}{s^3}\right) - \frac{l}{rs} + \frac{(\rho-2)}{2(1+\rho)}\frac{1}{r} + \frac{(1-2\rho)}{2(1+\rho)}\frac{1}{s}\right]
\end{aligned}
$$

Our notation

$\rho \equiv \frac{\beta}{\alpha}$ $\qquad R_\alpha \equiv \frac{G\alpha}{c^2}$ $\qquad R_\beta \equiv \frac{G\beta}{c^2}$

$$f(\xi, \eta, \dot{\xi}, \dot{\eta}) \equiv \dot{\xi}^2 + \dot{\eta}^2 + 2\Omega(\xi\dot{\eta} - \eta\dot{\xi}) + \Omega^2(\xi^2 + \eta^2)$$

$$\omega \equiv \sqrt{\frac{G(\alpha+\beta)}{l^3}}$$

$$\Omega \equiv \omega\left[1 - \frac{3}{2}\frac{(R_\alpha + R_\beta)}{l}\left(1 - \frac{1}{3}\frac{\rho}{(1+\rho)^2}\right)\right]$$

Planar equations of motion of the planetoid

$$\ddot{\xi} - 2\Omega\dot{\eta} = \frac{\partial W}{\partial \xi} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial W}{\partial \dot{\xi}}\right)$$

$$\ddot{\eta} + 2\Omega\dot{\xi} = \frac{\partial W}{\partial \eta} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial W}{\partial \dot{\eta}}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial W}{\partial \dot{\xi}}\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial W}{\partial \dot{\eta}}\right) = 0 \text{ if } \dot{\xi} = \dot{\eta} = \ddot{\xi} = \ddot{\eta} = 0$$

Computational strategy

- It is enough to evaluate the zeroes of the gradient of W, to find the equilibrium points, despite W is much more cumbersome than in Newtonian gravity.

Planar coordinates of non-collinear Lagrangian points

$$\xi^{(GR)} - \xi^{(N)} = 2.73 \text{ mm}, \ \eta^{(GR)} - \eta^{(N)} = -1.59 \text{ mm}$$

Distances of the planetoid from the Earth at collinear Lagrangian points

$$R_1^{(GR)} - r_1^{(N)} = R_1 - r_1 = -7.61 \text{ m at } L_1,$$

$$R_2^{(GR)} - r_2^{(N)} = R_2 - r_2 = 9.40 \text{ m at } L_2,$$

$$R_3^{(GR)} - r_3^{(N)} = R_3 - r_3 = -1.13 \text{ m at } L_3.$$

Metric tensor components in a synodic frame for the relativistic restricted planar 3-body problem

$$
\begin{aligned}
g_{00} = & \; 1 - 2\frac{R_\alpha}{r} - 2\frac{R_\beta}{s} - \frac{\Omega^2}{c^2}(\xi^2+\eta^2) + 2\left[\left(\frac{R_\alpha}{r}\right)^2 + \left(\frac{R_\beta}{s}\right)^2\right] \\
& - 2\frac{(R_\alpha+R_\beta)}{l^3}\left(\frac{R_\alpha}{r}+\frac{R_\beta}{s}\right)(\xi^2+\eta^2) + 4\frac{R_\alpha}{r}\frac{R_\beta}{s} \\
& + \frac{(2-\rho)}{(1+\rho)}\frac{R_\alpha}{r}\frac{R_\beta}{l} + \frac{(2\rho-1)}{(1+\rho)}\frac{R_\beta}{s}\frac{R_\alpha}{l} - 7\frac{\xi}{l^2}\left(\frac{R_\alpha}{r}R_\beta - \frac{R_\beta}{s}R_\alpha\right) \\
& + (1+\rho)^{-1}\frac{\eta^2}{l}\left[\rho\left(\frac{R_\alpha}{r}\right)^3\frac{R_\beta}{(R_\alpha)^2} + \left(\frac{R_\beta}{s}\right)^3\frac{R_\alpha}{(R_\beta)^2}\right],
\end{aligned}
$$

$$
2cg_{01} = \left(1 + 2\frac{R_\alpha}{r} + 2\frac{R_\beta}{s}\right)2\Omega\eta,
$$

Remaining metric components

$$2cg_{02} = -\left(1 + 2\frac{R_\alpha}{r} + 2\frac{R_\beta}{s}\right)2\Omega\xi - 8\frac{\Omega^2 l}{(1+\rho)}\left(\rho\frac{R_\alpha}{r} - \frac{R_\beta}{s}\right),$$

$$g_{03} = 0,$$

$$g_{ij} = -\left(1 + 2\frac{R_\alpha}{r} + 2\frac{R_\beta}{s}\right)\delta_{ij},\ i,j = 1,2,3.$$

Lagrangian for the equations of motion

$$L = \frac{1}{2} \sum_{\mu,\nu=0}^{3} g_{\mu\nu} \frac{\mathrm{d}x^\mu}{\mathrm{d}t} \frac{\mathrm{d}x^\nu}{\mathrm{d}t}.$$

Standing on the shoulders of giants: T. Levi-Civita

and J. F. Donoghue

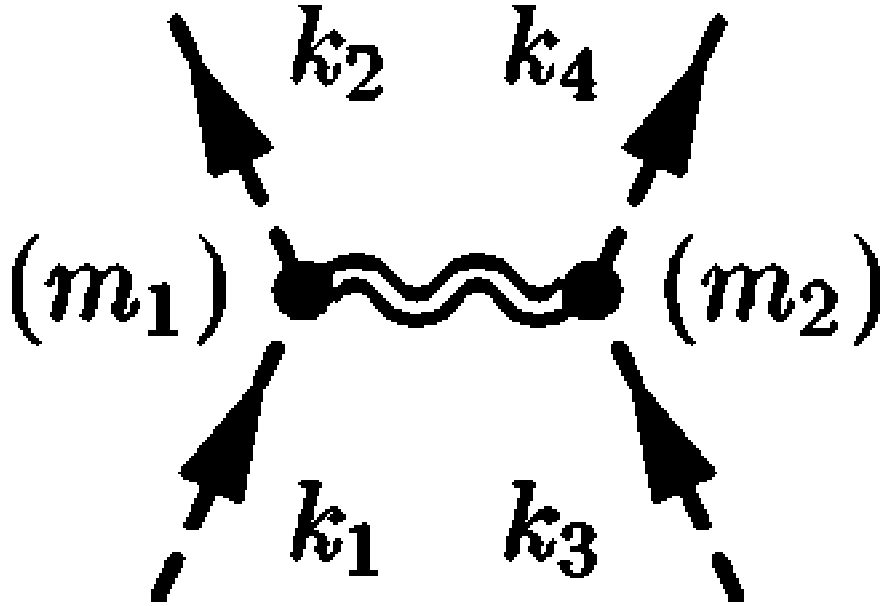
Logical chain:

- Levi-Civita proved in 1938-41 the cancellation theorem for the Lagrangian of the N-body problem in GR. The effects of extension and structure of the bodies are encoded in a family of parameters, which differ only by a tiny amount from the masses. Thus, it is unnecessary to assume point masses, because the effects of size get eventually cancelled exactly, and it is as if we were dealing with material points which do not affect at all their centre of gravity. This can be applied also to the solar system.
- Donoghue proves in between 1994 and 2003 that the long-distance Newtonian potential among two heavy masses receives quantum corrections in integer powers of G. To linear order in G, gravitational radii and Planck length squared contribute, and only them.

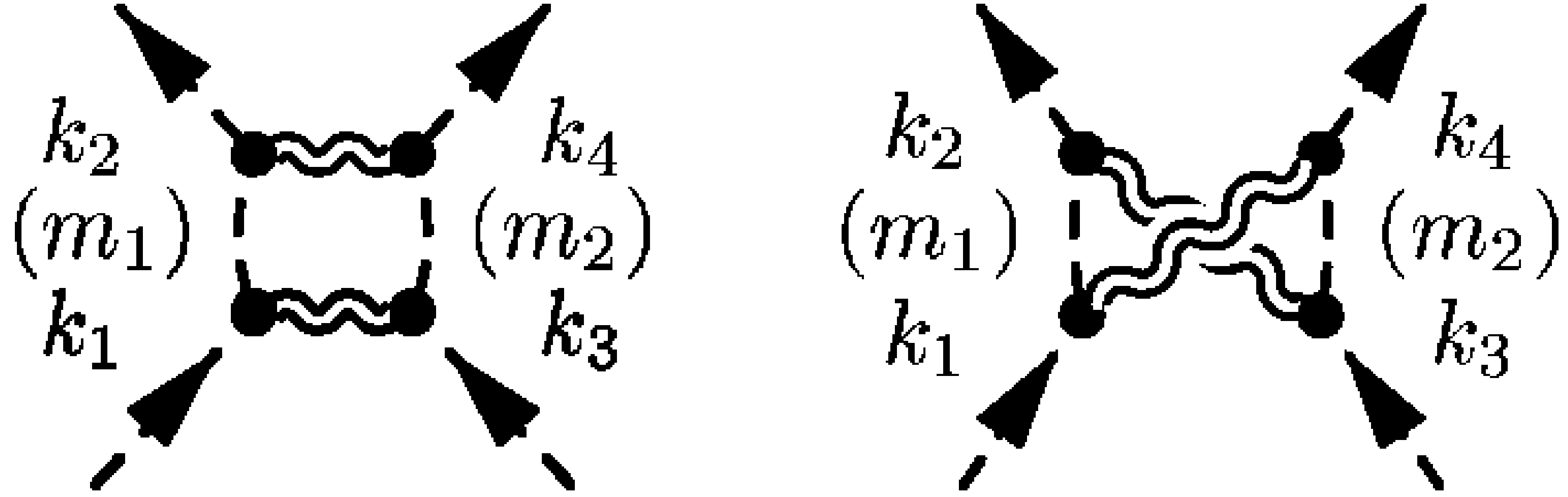
Logical chain (continued)

- One has to consider all one-loop diagrams that can contribute to the scattering of two masses: box, crossed box, triangle and double-seagull diagrams.
- The bound-state case is more relevant for celestial mechanics, and is obtainable in a simple way from the scattering calculation. It can also be justified a posteriori, as we are going to see.

Feynman’s diagram giving Newton’s law

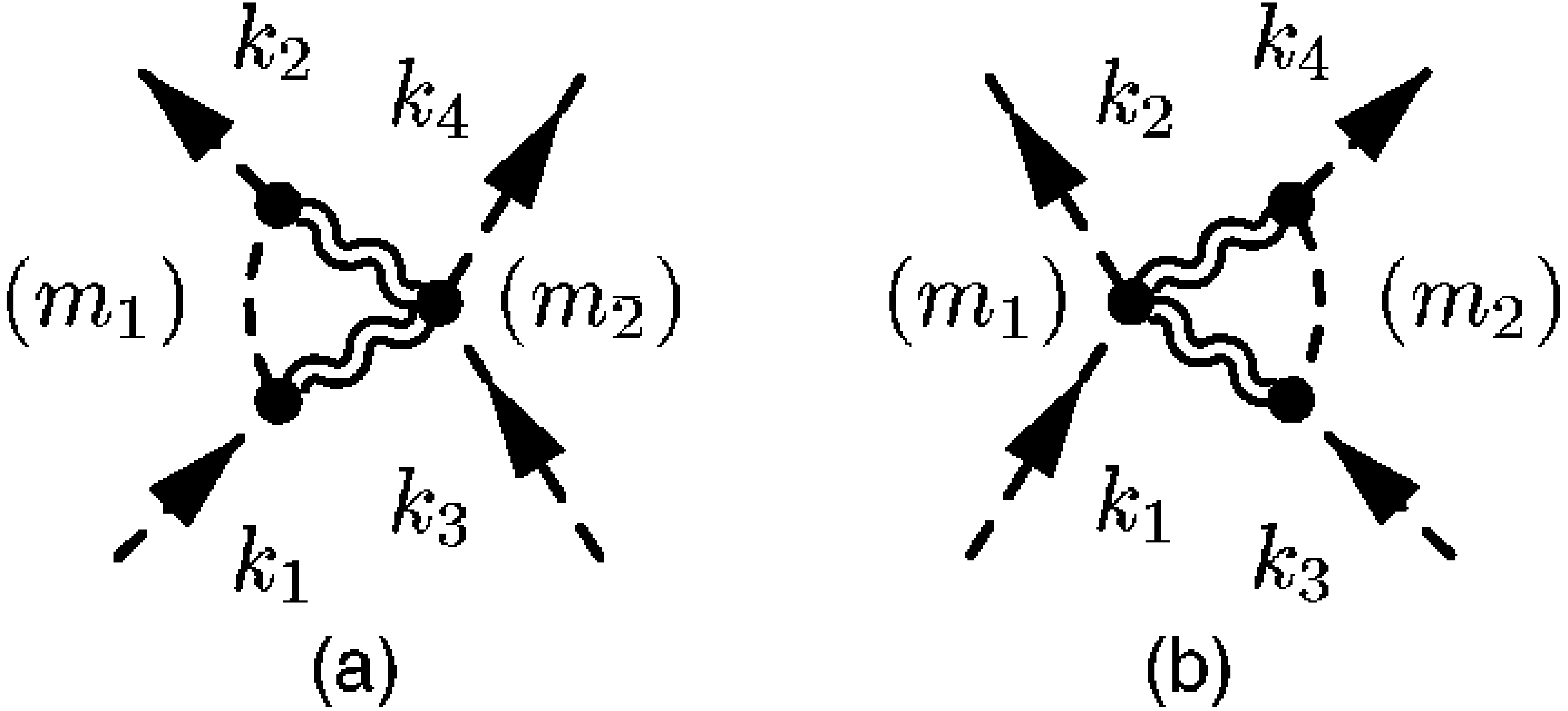


Box and crossed-box diagrams

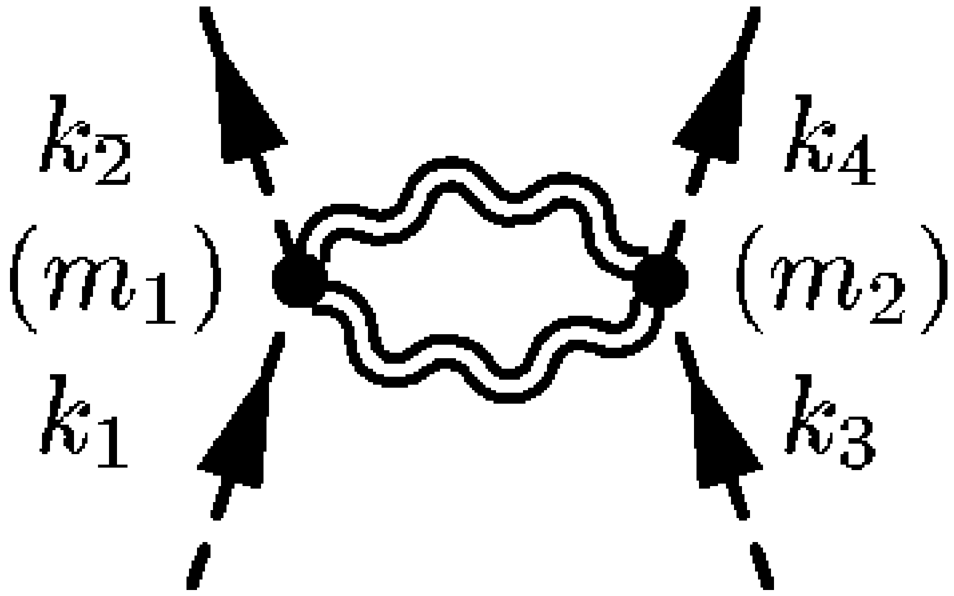


(a) (b)

Triangle diagrams for the scattering potential



Double-seagull diagram for the scattering potential



One-loop long distance Newtonian potential

$$V_E(r) \sim -\frac{G m_A m_B}{r}\left[1 + \left(\kappa_1 \frac{(R_A + R_B)}{r} + \kappa_2 \frac{(l_P)^2}{r^2} + \mathrm{O}(G^2)\right)\right]$$

$$\Longrightarrow \frac{V_E(r)}{c^2 m_B} \sim -\frac{R_A}{r}\left[1 + \left(\kappa_1 \frac{(R_A + R_B)}{r} + \kappa_2 \frac{(l_P)^2}{r^2} + \mathrm{O}(G^2)\right)\right],$$

Important remark

- The term quadratic in the Planck length does not affect the numerical values we find, being so small, but the effective-gravity weight of gravitational radii depends on the existence of such a term, because the term linear in G is a sum of the two.

Key terms: the dimensionless ratios

$$U_\alpha(r) \equiv \frac{R_\alpha}{r} = U_\alpha$$

$$U_\beta(s) \equiv \frac{R_\beta}{s} = U_\beta$$

New dimensionless ratios in effective gravity

$$V_\alpha(r) \sim \left[1+\kappa_2\frac{(l_P)^2}{r^2}\right]U_\alpha(r)+\kappa_1\left(1+\frac{R_m}{R_\alpha}\right)(U_\alpha(r))^2+\mathrm{O}(G^3)$$
$$\sim \left[1+\kappa_2\frac{(l_P)^2}{r^2}\right]U_\alpha(r)+\kappa_1(U_\alpha(r))^2,$$

$$V_\beta(s) \sim \left[1+\kappa_2\frac{(l_P)^2}{s^2}\right]U_\beta(s)+\kappa_1\left(1+\frac{R_m}{R_\beta}\right)(U_\beta(s))^2+\mathrm{O}(G^3),$$
$$\sim \left[1+\kappa_2\frac{(l_P)^2}{s^2}\right]U_\beta(s)+\kappa_1(U_\beta(s))^2,$$

Effective gravity Lagrangian

$$
\begin{aligned}
L_V \;=\; & \frac{c^2}{2}\Bigg\{1 - 2(V_\alpha + V_\beta) - \frac{\Omega^2}{c^2}(\xi^2 + \eta^2) + 2\left[(V_\alpha)^2 + (V_\beta)^2\right] \\
- \; & 2\frac{(R_\alpha + R_\beta)}{l^3}(\xi^2 + \eta^2)(V_\alpha + V_\beta) + 4V_\alpha V_\beta \\
+ \; & \frac{(2-\rho)}{(1+\rho)}\frac{R_\beta}{l}V_\alpha + \frac{(2\rho - 1)}{(1+\rho)}\frac{R_\alpha}{l}V_\beta - 7\frac{\xi}{l^2}(R_\beta V_\alpha - R_\alpha V_\beta) \\
+ \; & (1+\rho)^{-1}\frac{\eta^2}{l}\left[\rho\frac{R_\beta}{(R_\alpha)^2}(V_\alpha)^3 + \frac{R_\alpha}{(R_\beta)^2}(V_\beta)^3\right]\Bigg\} \\
- \; & \frac{1}{2}\left(\dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2\right)\Big[1 + 2(V_\alpha + V_\beta)\Big] + \Omega\eta\dot{\xi}\Big[1 + 2(V_\alpha + V_\beta)\Big] \\
- \; & \Omega\xi\dot{\eta}\Big[1 + 2(V_\alpha + V_\beta)\Big] - 4\frac{\Omega^2 l}{(1+\rho)}\dot{\eta}(\rho V_\alpha - V_\beta),
\end{aligned}
$$

Euler-Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L_V}{\partial \dot{\xi}}\right) - \frac{\partial L_V}{\partial \xi} = 0,\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L_V}{\partial \dot{\eta}}\right) - \frac{\partial L_V}{\partial \eta} = 0$$

Map from U to V dimensionless ratios in Newtonian gravity leads to

$$W_{\text{eff}} = \frac{\omega^2}{2}(\xi^2 + \eta^2) + c^2\left[(U_\alpha + U_\beta) + \kappa_1((U_\alpha)^2 + (U_\beta)^2)\right] + \mathrm{O}(G^2)$$

Map from U to V dimensionless ratios in GR leads to

$$
\begin{aligned}
W_{\text{eff}} &= \frac{\Omega^2}{2}(\xi^2+\eta^2) + c^2\Big[(U_\alpha + U_\beta) - \frac{1}{2}((U_\alpha)^2 + (U_\beta)^2)\Big] + \mathrm{O}(G^2) \\
&\sim \frac{\omega^2}{2}(\xi^2+\eta^2) + c^2\Big[(U_\alpha + U_\beta) - \frac{1}{2}((U_\alpha)^2 + (U_\beta)^2)\Big] + \mathrm{O}(G^2),
\end{aligned}
$$

Quantum corrections on non-collinear Lagrangian points

$$\xi_4 - \xi_4^{(GR)} = -1.46 \text{ mm} \Longrightarrow \xi_4 - \xi_4^{(N)} = 1.27 \text{ mm},$$

$$\eta_4 - \eta_4^{(GR)} = -0.86 \text{ mm} \Longrightarrow \eta_4 - \eta_4^{(N)} = -2.45 \text{ mm}.$$

Quantum corrections on collinear Lagrangian points

$$R_1' - R_1 = -0.62 \text{ mm},$$

$$R_2' - R_2 = -0.39 \text{ mm},$$

$$R_3' - R_3 = -1.48 \text{ mm}.$$

Summary of GR and quantum corrections on Lagrangian points

L_i	General Relativity-Newton	Quantum-General Relativity	Quantum-Newton
L_1	-7.61 m	-0.62 mm	-7.61 m
L_2	9.40 m	-0.39 mm	9.40 m
L_3	-1.13 m	-1.48 mm	-1.13 m
L_4	(2.73 mm, -1.59 mm)	(-1.46 mm, -0.86mm)	(1.27 mm, -2.45 mm)
L_5	(2.73 mm, -1.59 mm)	(-1.46 mm, -0.86mm)	(1.27 mm, -2.45 mm)

Final remarks

- The 7.61 m correction looks measurable, hence encouraging
- It is possible to conceive a new laser ranging test of general relativity by measuring this correction to the L1 Lagrangian point, an observable never used before in the Sun-Earth-Moon system. Such an experiment requires controlling the propulsion to precisely reach L1, an instrumental accuracy comparable to the measurement of the lunar geodesic precession, understanding systematic effects resulting from solar radiation pressure and heating of the satellite that leads to photon emission, and multi-body gravitational perturbations.
- Much harder task: to measure the tiny deviations of effective gravity from GR in the Sun-Earth-Moon system.

Our work

- E. Battista and G. Esposito, Phys. Rev. D89, 084030 (2014).
- E. Battista and G. Esposito, Phys. Rev. D90, 084010 (2014).
- E. Battista, S. Dell'Agnello, G. Esposito, J. Simo, Phys. Rev. D91, 084041 (2015).
- E. Battista, S. Dell'Agnello, G. Esposito, L. Di Fiore, J. Simo, A. Grado, Phys. Rev. D92, 064045 (2015).

E. Battista

S. Dell’Agnello, INFN-NASA agreement

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