

AdS₄ black holes and 3d Gauge Theories

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[work in collaboration with F. Benini, K. Hristov]

F. Benini-AZ; arXiv 1504.03698

F. Benini-K.Hristov-AZ; arXiv 1510.xxxxx

Introduction

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- ▶ I consider supersymmetric black holes in AdS_4
- ▶ I show how to count their micro-states with a field theory computation

The computation uses recent localization techniques that allow to evaluate exact quantities in supersymmetric gauge theories.

Introduction

It is an old result that there is striking similarity between the black hole mechanics and thermodynamics

$$dM = \frac{1}{8\pi G} \kappa dA \quad \Longleftrightarrow \quad dE = T dS$$

where A is the areas of the horizon.

The black hole has a temperature $T = \frac{\hbar \kappa}{2\pi}$ and an entropy

$$S = \frac{A}{4G\hbar}$$

What is the microscopical origin of the entropy? Extremal black holes have $T = 0$ and the entropy is just the number of ground states of the system.

Introduction

One of the success of string theory is the microscopic counting of micro-states for a class of asymptotically flat black holes entropy [Vafa-Strominger'96]

- ▶ The black holes are realized by putting together D-branes, extended objects that have gauge theories on the world-volume
- ▶ The entropy is obtained by counting states in the corresponding gauge theory

Introduction

No similar result for AdS black holes. But AdS should be simpler and related to holography:

- ▶ A gravity theory in AdS_{d+1} is the dual description of a CFT_d

The entropy should be related to the counting of states in the dual CFT. People failed for AdS_5 black holes (states in $N=4$ SYM).

AdS₄ black holes

There are many 1/4 BPS asymptotically AdS₄ static black holes

$$ds^2 = e^{\kappa(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\kappa(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\kappa(X)} r^2 ds_{S^2}^2$$

- ▶ solutions asymptotic to *magnetic AdS₄* and with horizon AdS₂ × S²
- ▶ characterized by a collection of magnetic charges $\int_{S^2} F$

[Cacciatori, Klemm; Gnechchi, Dall'agata; Hristov, Vandoren];

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Some arise in truncation of M theory on AdS₄ × S⁷

- ▶ four abelian vectors $U(1)^4 \subset SO(8)$ that come from the reduction on S⁷.
- ▶ vacua of a $N = 2$ gauged supergravity with 3 vector multiplets; one vector is the graviphoton.

AdS₄ black holes

There are many 1/4 BPS asymptotically AdS₄ static black holes

$$ds^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\mathcal{K}(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 ds_{S^2}^2$$

$$F = -2i\sqrt{X^0 X^1 X^2 X^3}$$

$$e^{-\mathcal{K}(X)} = i \left(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda \right) = \sqrt{16 X^0 X^1 X^2 X^3}$$

$$X^i = \frac{1}{4} - \frac{\beta_i}{r}, \quad X^0 = \frac{1}{4} + \frac{\beta_1 + \beta_2 + \beta_3}{r}$$

with arbitrary parameters $\beta_1, \beta_2, \beta_3$.

AdS₄ black holes

There are many 1/4 BPS asymptotically AdS₄ static black holes

$$ds^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\mathcal{K}(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 ds_{S^2}^2$$

The parameters are related to the magnetic charges supporting the black hole

$$n_1, n_2, n_3, n_4, \quad n_i = \frac{1}{2\pi} \int_{S^2} F^{(i)}, \quad \sum n_i = 2$$

by

$$n_1 = 8(-\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_2\beta_3),$$

$$n_2 = 8(-\beta_2^2 + \beta_1^2 + \beta_3^2 + \beta_1\beta_3),$$

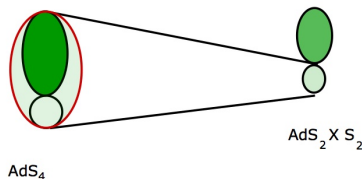
$$n_3 = 8(-\beta_3^2 + \beta_1^2 + \beta_2^2 + \beta_1\beta_2).$$

AdS₄ black holes

There are many $1/4$ BPS asymptotically AdS₄ static black holes

$$ds^2 = e^{\kappa(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\kappa(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\kappa(X)} r^2 ds_{S^2}^2$$

Asymptotic to AdS₄ for $r \gg 1$ and with horizon AdS₂ $\times$ S² at some $r = r_h$



$$ds^2 \sim \frac{dr^2}{r^2} + r^2(-dt^2 + d\theta^2 + \sin^2 \theta d\phi^2)$$

$$ds^2 \sim -(r - r_h)^2 dt^2 + \frac{dr^2}{(r - r_h)^2} + (d\theta^2 + \sin^2 \theta d\phi^2)$$

AdS₄ black holes

There are many 1/4 BPS asymptotically AdS₄ static black holes

$$ds^2 = e^{\kappa(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\kappa(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\kappa(X)} r^2 ds_{S^2}^2$$

The entropy is the area of S^2

$$S = 8r_h^2 \sqrt{X^0(r_h)X^1(r_h)X^2(r_h)X^3(r_h)}$$

for example, for $n_1 = n_2 = n_3$

$$\sqrt{-1 + 6n_1 - 6n_1^2 + (-1 + 2n_1)^{3/2} \sqrt{-1 + 6n_1}}$$

AdS₄ black holes

There are many **1/4 BPS** asymptotically AdS₄ static black holes

$$ds^2 = e^{\kappa(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\kappa(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\kappa(X)} r^2 ds_{S^2}^2$$

supersymmetry is preserved by a twist

$$(\nabla_\mu - iA_\mu)\epsilon = \partial_\mu \epsilon \quad \implies \quad \epsilon = \text{const}$$

AdS₄ black holes

General vacua of a bulk effective action

$$\mathcal{L} = -\frac{1}{2}\mathcal{R} + F_{\mu\nu}F^{\mu\nu} + V\ldots$$

with a metric

$$ds_{d+1}^2 = \frac{dr^2}{r^2} + (r^2 ds_{M_d}^2 + O(r)) \quad A = A_{M_d} + O(1/r)$$

and a gauge fields profile, correspond to CFTs on a d-manifold M_d and a non trivial background field for the symmetry

$$L_{CFT} + J^\mu A_\mu$$

AdS₄ black holes

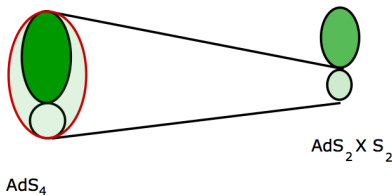
In the case of the AdS₄ black holes

- ▶ the boundary is $S^2 \times R$ (or $S^2 \times S^1$ after Wick rotation)
- ▶ bulk gauge fields induce **magnetic backgrounds** for R and global symmetries in the CFT
- ▶ bulk supersymmetry induce boundary susy (twist)

$$(\nabla_\mu - iA_\mu)\epsilon = \partial_\mu\epsilon = 0$$

AdS₄ black holes

AdS black holes are dual to a topologically twisted CFT on $S^2 \times S^1$ with background magnetic fluxes for the global symmetries



Entropy of black holes
Counting of microstates

Partition function of twisted
3d CFT on $S_2 \times S_1$

QM fixed point

The background

Consider an $\mathcal{N} = 2$ gauge theory on $S^2 \times S^1$

$$ds^2 = R^2 (d\theta^2 + \sin^2 \theta d\varphi^2) + \beta^2 dt^2$$

with a background for the R-symmetry proportional to the spin connection:

$$A^R = -\frac{1}{2} \cos \theta d\varphi = -\frac{1}{2} \omega^{12}$$

so that the Killing spinor equation

$$D_\mu \epsilon = \partial_\mu \epsilon + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon - i A_\mu^R \epsilon = 0 \quad \implies \quad \epsilon = \text{const}$$

The background

This is just a topological twist. [Witten '88]

The result becomes interesting when supersymmetric backgrounds for the flavor symmetry multiplets (A_μ^F, σ^F, D^F) are turned on:

$$u^F = A_t^F + i\sigma^F, \quad q^F = \int_{S^2} F^F = iD^F$$

and the path integral, which can be exactly computed by localization, becomes a function of a set of magnetic charges q^F and chemical potentials u^F .

[Benini-AZ; arXiv 1504.03698]

A topologically twisted index

The path integral can be re-interpreted as a **twisted index**: a trace over the Hilbert space $\mathcal{H}$ of states on a sphere in the presence of a magnetic background for the R and the global symmetries,

$$\mathrm{Tr}_{\mathcal{H}} \left((-1)^F e^{iJ_F A^F} e^{-\beta H} \right)$$

$$Q^2 = H - \sigma^F J_F$$

holomorphic in u^F

where J_F is the generator of the global symmetry.

Localization

Exact quantities in supersymmetric theories with a charge $Q^2 = 0$ can be obtained by a saddle point approximation

$$Z = \int e^{-S} = \int e^{-S+t\{Q,V\}} \underset{t \gg 1}{=} e^{-\bar{S}|_{class}} \times \frac{\det_{fermions}}{\det_{bosons}}$$

Very old idea that has become very concrete recently, with the computation of partition functions on spheres and other manifolds supporting supersymmetry.

The partition function

The path integral for an $\mathcal{N} = 2$ gauge theory on $S^2 \times S^1$ with gauge group G localizes on a set of BPS configurations specified by data in the vector multiplets $V = (A_\mu, \sigma, \lambda, \lambda^\dagger, D)$

- ▶ A magnetic flux on S^2 , $\mathfrak{m} = \frac{1}{2\pi} \int_{S^2} F$ in the co-root lattice
- ▶ A Wilson line A_t along S^1
- ▶ The vacuum expectation value σ of the real scalar

Up to gauge transformations, the BPS manifold is

$$(u = A_t + i\sigma, \mathfrak{m}) \in \mathcal{M}_{\text{BPS}} = (H \times \mathfrak{h} \times \Gamma_{\mathfrak{h}}) / W$$

The partition function

The path integral reduces to a the saddle point around the BPS configurations

$$\sum_{\mathfrak{m} \in \Gamma_{\mathfrak{h}}} \int du d\bar{u} \mathcal{Z}^{\text{cl} + 1\text{-loop}}(u, \bar{u}, \mathfrak{m})$$

- ▶ The integrand has various singularities where chiral fields become massless
- ▶ There are fermionic zero modes

The two things nicely combine and the path integral reduces to an r -dimensional contour integral of a meromorphic form

$$\frac{1}{|W|} \sum_{\mathfrak{m} \in \Gamma_{\mathfrak{h}}} \oint_{\mathcal{C}} Z_{\text{int}}(u, \mathfrak{m})$$

The partition function

- In each sector with gauge flux $\mathfrak{m}$ we have a meromorphic form

$$Z_{\text{int}}(u, \mathfrak{m}) = Z_{\text{class}} Z_{1\text{-loop}}$$

$$Z_{\text{class}}^{\text{CS}} = x^{k\mathfrak{m}}$$

$$x = e^{iu}$$

$$Z_{1\text{-loop}}^{\text{chiral}} = \prod_{\rho \in \mathfrak{R}} \left[\frac{x^{\rho/2}}{1 - x^{\rho}} \right]^{\rho(\mathfrak{m}) - q + 1}$$

$q = R$ charge

$$Z_{1\text{-loop}}^{\text{gauge}} = \prod_{\alpha \in G} (1 - x^{\alpha}) (i du)^r$$

- Supersymmetric localization selects a particular contour of integration C and picks some of the residues of the form $Z_{\text{int}}(u, \mathfrak{m})$.

[Jeffrey-Kirwan residue - similar to Benini, Eager, Hori, Tachikawa '13; Hori, Kim, Yi '14]

A Simple Example: SQED

The theory has gauge group $U(1)$ and two chiral Q and $\tilde{Q}$

$$Z = \sum_{m \in \mathbb{Z}} \int \frac{dx}{2\pi i x} \left(\frac{x^{\frac{1}{2}} y^{\frac{1}{2}}}{1 - xy} \right)^{m+n} \left(\frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{1 - x^{-1}y} \right)^{-m+n}$$

	$U(1)_{\tilde{g}}$	$U(1)_A$	$U(1)_R$
Q	1	1	1
$\tilde{Q}$	-1	1	1

Consistent with duality with three chirals with superpotential XYZ

$$Z = \left(\frac{y}{1 - y^2} \right)^{2n-1} \left(\frac{y^{-\frac{1}{2}}}{1 - y^{-1}} \right)^{-n+1} \left(\frac{y^{-\frac{1}{2}}}{1 - y^{-1}} \right)^{-n+1}$$

Aharony and Giveon-Kutasov dualities

The twisted index can be used to check dualities: for example, $U(N_c)$ with $N_f = N_c$ flavors is dual to a theory of chiral fields M_{ab} , T and $\tilde{T}$, coupled through the superpotential $W = T\tilde{T} \det M$

$$Z_{N_f=N_c} = \left(\frac{y}{1-y^2} \right)^{(2n-1)N_c^2} \left(\frac{\xi^{\frac{1}{2}} y^{-\frac{N_c}{2}}}{1-\xi y^{-N_c}} \right)^{N_c(1-n)+t} \left(\frac{\xi^{-\frac{1}{2}} y^{-\frac{N_c}{2}}}{1-\xi^{-1} y^{-N_c}} \right)^{N_c(1-n)-t}$$

Aharony and Giveon-Kutasov dual pairs for generic (N_c, N_f) have the same partition function.

Refinement and other dimensions

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- ▶ In a $(2,2)$ theory in 2d on S^2 we are computing amplitudes in gauged linear sigma models [also Cremonesi-Closset-Park '15]
- ▶ In a $\mathcal{N} = 1$ theory on $S^2 \times T^2$ we are computing an elliptically generalized twisted index

[also Closset-Shamir '13; Nishioka-Yaakov '14; Yoshida-Honda '15]

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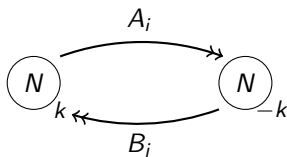
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[also Closset-Shamir '13; Nishioka-Yaakov '14; Yoshida-Honda '15]

The index adds to and complete the list of existing tools (superconformal indices, sphere partition functions) for testing dualities.

Going back to the black hole

The dual field theory to $\text{AdS}_4 \times S^7$ is known: is the ABJM theory with gauge group $U(N) \times U(N)$



with quartic superpotential

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

defined on twisted $S^2 \times \mathbb{R}$ with magnetic fluxes n_i for the R /global symmetries

$$SU(2)_A \times SU(2)_B \times U(1)_B \times U(1)_R \subset SO(8)$$

The dual field theory

The ABJM twisted index is

$$\begin{aligned}
 Z = & \frac{1}{(N!)^2} \sum_{\mathbf{m}, \tilde{\mathbf{m}} \in \mathbb{Z}^N} \int \prod_{i=1}^N \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} x_i^{k m_i} \tilde{x}_i^{-k \tilde{m}_i} \times \prod_{i \neq j}^N \left(1 - \frac{x_i}{x_j}\right) \left(1 - \frac{\tilde{x}_i}{\tilde{x}_j}\right) \times \\
 & \times \prod_{i,j=1}^N \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}} y_1}{1 - \frac{x_i}{\tilde{x}_j} y_1} \right)^{m_i - \tilde{m}_j - n_1 + 1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}} y_2}{1 - \frac{x_i}{\tilde{x}_j} y_2} \right)^{m_i - \tilde{m}_j - n_2 + 1} \\
 & \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}} y_3}{1 - \frac{\tilde{x}_j}{x_i} y_3} \right)^{\tilde{m}_j - m_i - n_3 + 1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}} y_4}{1 - \frac{\tilde{x}_j}{x_i} y_4} \right)^{\tilde{m}_j - m_i - n_4 + 1}
 \end{aligned}$$

where $\mathbf{m}, \tilde{\mathbf{m}}$ are the gauge magnetic fluxes and y_i are fugacities for the three independent $U(1)$ global symmetries ($\prod_i y_i = 1$)

The dual field theory

Strategy:

- ▶ Re-sum geometric series in $\mathfrak{m}, \tilde{\mathfrak{m}}$.

$$Z = \int \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} \frac{f(x_i, \tilde{x}_i)}{\prod_{j=1}^N (e^{iB_i} - 1) \prod_{j=1}^N (e^{i\tilde{B}_j} - 1)}$$

- ▶ Find the zeros of denominator $e^{iB_i} = e^{i\tilde{B}_j} = 1$ at large N
- ▶ Evaluate the residues at large N

$$Z \sim \sum_I \frac{f(x_i^{(0)}, \tilde{x}_i^{(0)})}{\det \mathbb{B}}$$

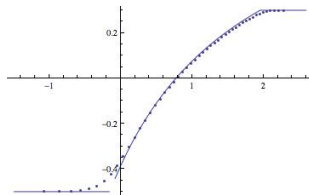
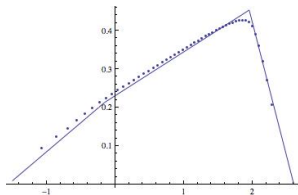
The large N limit

Step 2: solve the large N Limit of algebraic equations giving the positions of poles

$$1 = x_i^k \prod_{j=1}^N \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})} = \tilde{x}_j^k \prod_{i=1}^N \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})}$$

with an ansatz

$$\log x_i = i\sqrt{N}t_i + v_i, \quad \log \tilde{x}_i = i\sqrt{N}t_i + \tilde{v}_i$$



The large N limit

Step 3: plug into the partition function. The final result is surprisingly simple

$$\mathbb{R}e \log Z = -\frac{1}{3} N^{3/2} \sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4} \sum_a \frac{n_a}{\Delta_a} \quad y_i = e^{i\Delta_i}$$

This function can be extremized with respect to the Δ_i and

$$\boxed{\mathbb{R}e \log Z|_{crit}(n_i) = \text{BH Entropy}(n_i)}$$

The large N limit

The twisted index depends on Δ_i because we are computing the trace

$$\mathrm{Tr}_{\mathcal{H}}(-1)^F e^{i\Delta_i J_i} \equiv \mathrm{Tr}_{\mathcal{H}}(-1)^R$$

where $R = F + \Delta_i J_i$ is a possible R-symmetry of the system.

Here an extremization is at work: symmetry enhancement at the horizon AdS_2

$$\mathrm{QM}_1 \rightarrow \mathrm{CFT}_1$$

- ▶ R is the exact R-symmetry at the superconformal point
- ▶ Natural thing to extremize: in even dimensions central charges are extremized, in odd partition functions...

The large N limit

The extremization reflects exactly what's going on in the bulk. The graviphoton field strength depends on r

$$T_{\mu\nu} = e^{K/2} X^\Lambda F_{\Lambda,\mu\nu}$$

suggesting that the R-symmetry is different in the IR and indeed

$$\Delta_i|_{crit} \sim X^i(r_h)$$

Conclusions

The main message of this talk is that you can related the entropy of a class of AdS_4 black holes to a microscopic counting of states.

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But don't forget that we also gave a general formula for the topologically twisted path integral of 2d $(2,2)$, 3d $\mathcal{N} = 2$ and 4d $\mathcal{N} = 1$ theories.

- ▶ Efficient quantum field theory tools for testing dualities.

Conclusions

With many field theory questions/generalizations

- ▶ Higher genus $S^2 \rightarrow \Sigma$? Include Witten index
- ▶ 2d theories, learn about Calabi-Yau's and sigma-models?
- ▶ Extremization of the index is a general principle?