A Goldstino at the bottom of the cascade

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based on 1509.03594 (JHEP)

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[see also 1412.6499 (PRD) w/ Argurio, Musso, Porri & Redigolo
and 1310.6897 (JHEP) w/ Argurio, Di Pietro, Porri & Redigolo]

TFI 2015 - Naples

Motivations

In general...

- Understand (super)symmetry breaking in strongly coupled QFTs → Holography it's a powerful tool!

As far as supersymmetry...

- ~~SUSY~~ in String theory and existence of metastable vacua.

Specific to this paper...

- Contribute to ongoing debate on antiD-branes in warped throats from a different (but instructive!) perspective.

- Improve understanding of holographic renormalization for cascading gauge theories (little is known from QFT side).

Preliminaries

- Which SQFT can break SUSY? A necessary condition is that conformal invariance is explicitly broken:

$$E_{vac} = \langle T_{00} \rangle \sim \langle T^{\mu}_{\mu} \rangle \text{ at odds with operator identity } T^{\mu}_{\mu} = 0$$

From Lorentz invariance

The SCFT must be deformed by (marginally) relevant, SUSY-preserving, operators.

Note: This means that dual backgrounds cannot be AdS!

~~AdS~~

Should depart from AdS-ness...
and do it at enough pace!

- Strongly coupled ~~SUSY~~ dynamics diverse and complicated. But it exists a universal feature: the goldstino. This is what we search for, holographycally!

Preliminaries

- ***Recall***: in AdS/CFT different vacua of the same QFT are described by bulk solutions having the same asymptotic, up to normalizable modes (VEV). Non-normalizable modes correspond to deformations of the QFT.

- Suppose to have a bulk solution which breaks SUSY. There are two basic questions one should answer:
  - Q1: Is the solution gravitationally (meta)stable?
  - Q2: Is the bulk mode dual to the goldstino present?

  A positive answer to the first question guarantees that the solution describes an actual QFT vacuum. To the second, that SUSY is broken spontaneously.

Preliminaries

- It should be possible to answer these two questions independently:
  - The goldstino appears as massless pole in supercurrent 2-point function (in IR $\mathbf{S}_\mu = \sigma_\mu \bar{\mathbf{G}}$)

$$\langle S_{\mu\alpha}\,\bar{S}_{\nu\dot{\beta}}\rangle$$

Complicated structure; it depends on the vacuum one is considering!

  - In fact, we don't need it all! Information fully encoded in (quasi local) contact term implied by SUSY Ward identity

$$\langle \partial^\mu S_{\mu\alpha}(x)\,\bar{S}_{\nu\dot{\beta}}(0)\rangle = -2\sigma^\mu_{\alpha\dot{\beta}}\,\langle T_{\mu\nu}\rangle\,\delta^4(x)$$

Upon integration, it relates pole residue to vacuum energy.

→ WIs depend on UV data, only. Vacuum stability is an IR property.

~~SUSY~~ in String th. & Holography

- There are (at least) two different classes of theories one can consider in holographic set-ups which allow for ~~SUSY~~ →

Class 1: $\mathcal{L} = \mathcal{L}_{SCFT} + \sum_i \lambda_i \int d^2\theta\, \mathcal{O}^i \,, \quad 1 \leq \Delta_{\mathcal{O}^i} < 3$

relevant operators

AAdS backgrounds

[Argurio et al. '14]

Class 2: $\mathcal{L}$ without a UV fixed point (... !?)

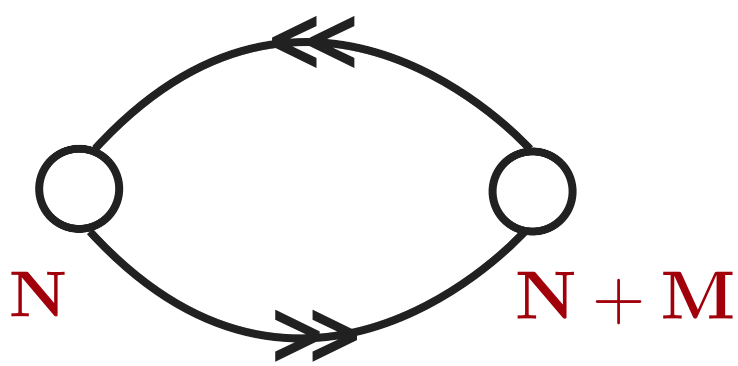
Ubiquitous in top-down models, i.e. quiver gauge theories arising from stacks of regular and fractional D-branes at CY singularities

Cascading backgrounds

log-divergent, not AAdS!

~~SUSY~~ in String th. & Holography

- A prototype example is the KS model. [Klebanov-Strassler '00]



Gauge Group	$SU(N+M) \times SU(N)$
Global Symmetries	$SU(2) \times SU(2) \times U(1)_B \times Z_{2M}$
Bifundamental Matter	$A_i, B_k (i,k = 1,2)$
Superpotential	$W = \lambda Tr(A_i B_k A_j B_l)\epsilon^{ij}\epsilon^{kl}$

→ N regular and M fractional D3-branes at the tip of $\mathbf{C(T^{1,1})}$.

- For $\mathbf{M = 0}$ the theory is (super)conformal. [Klebanov-Witten '98]

$$\mathcal{O}_\phi \sim \frac{1}{g_1^2} + \frac{1}{g_2^2} \longleftrightarrow e^{-\phi} \quad , \quad \mathcal{O}_{\tilde{b}} \sim \frac{1}{g_1^2} - \frac{1}{g_2^2} \longleftrightarrow \tilde{b}^\Phi = e^{-\phi} b^\Phi$$

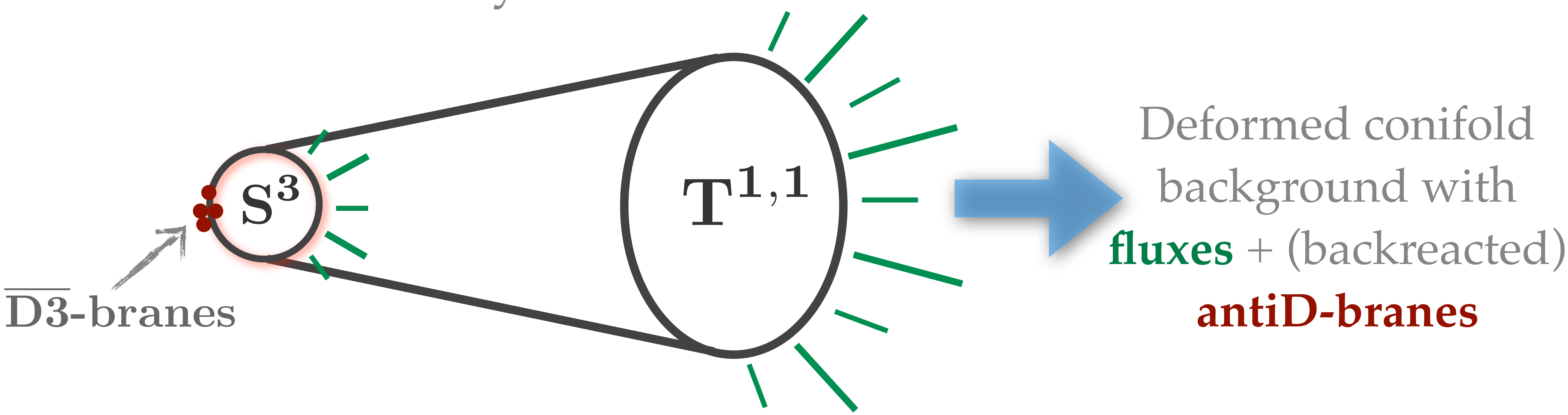
- For $\mathbf{M \neq 0}$ conformal invariance is broken: $\mathcal{O}_{\tilde{b}}$ becomes relevant and triggers an RG-flow → duality cascade.

~~SUSY~~ in String th. & Holography

- For $\mathbf{N = kM}$ there are both mesonic and baryonic branches of SUSY vacua.
- For $\mathbf{N = kM - p}$ with $\mathbf{p < M}$ the baryonic branch is lifted and only the mesonic branch survives.

[DYMARSKY-KLEBANOV-SEIBERG'05]

[KACHRU-PEARSON-VERLINDE '01] argued that there exist ~~SUSY~~ vacua on the would-be baryonic branch!



- *Question*: is there a goldstino there?

Cascading theories from 5d SUGRA

- Holographic dictionary (and machinery) defined in terms of 5d effective d.o.f. → need to compactify type IIB on $\mathbf{T^{1,1}}$. The resulting N=2 d=5 effective theory is very complicated. Two simplifications make our life simpler:
  - Focus on $\mathbf{SU(2)} \times \mathbf{SU(2)}$ invariant sector. [Cassani-Faedo '10]

    → graviton multiplet + 2 hypers
    + 1 massive vector multiplet

  - Need to look to UV asymptotic only, up to the order ~~SUSY~~ deformation appears.

    → look for solutions up to order $\mathbf{z^4}$!
- *Note*: Solutions should be domain-wall like (metric+scalars) and have all same asymptotics (KT soln [Klebanov-Tseytlin '00])!

Cascading theories from 5d SUGRA

- The most general solution compatible with KS-theory b.c. is

$$\begin{cases} ds^2 = \frac{1}{z^2}\left(e^{2Y(z)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2X(z)}dz^2\right) \\ e^{2Y} = h^{\frac{1}{3}}(z)\,h_2^{\frac{1}{2}}(z)\,h_3^{\frac{1}{2}}(z) \quad , \quad e^{2X} = h^{\frac{4}{3}}(z)\,h_2^{\frac{1}{2}}(z) \end{cases}$$

$$\begin{cases} e^{2U} = h^{\frac{5}{2}}(z)\,h_2^{\frac{3}{2}}(z) \quad , \quad e^{2V} = h_2^{-\frac{3}{2}}(z) \\ b^\Phi(z) = -\frac{9}{2}g_sM\ \log(z/z_0) \\ \qquad + z^4\left[\left(\frac{9\pi N}{4M} + \frac{99}{32}g_sM - \frac{27}{4}g_sM\log(z/z_0)\right)\mathcal{S} - \frac{9}{8}g_sM\varphi\right] + \mathcal{O}(z^8) \\ \phi(z) = \log g_s + z^4\left(3\,\mathcal{S}\log(z/z_0) + \varphi\right) + \mathcal{O}(z^8) \end{cases}$$

Two-parameter family of ~~SUSY~~ solutions.

$$h(z) = \frac{27\pi}{4g_s}\left(g_sN + \frac{3}{8\pi}(g_sM)^2 - \frac{3}{2\pi}(g_sM)^2\log(z/z_0)\right)$$

$$+\frac{z^4}{g_s}\left[\left(\frac{54\pi g_sN}{64} + \frac{81}{4}\frac{13}{64}(g_sM)^2 - \frac{81}{16}(g_sM)^2\log(z/z_0)\right)\mathcal{S} - \frac{81}{64}(g_sM)^2\varphi\right] + \mathcal{O}(z^8)$$

$$h_2(z) = 1 + \frac{2}{3}\mathcal{S}\,z^4 + \mathcal{O}(z^8) \quad , \quad h_3(z) = 1 + \mathcal{O}(z^8)$$

[de Wolfe-Kachru-Mulligan '08]

Holographic Ward identities

- Ward identities are relations among correlators of local operators, descending from global symmetries.
- Turn on sources for local operators, gauge the global symmetries and require invariance of generating functional under local gauge transformations.

→ Get relations between 1pt-functions (at finite source!) and in turn, upon differentiation, the WIs.

- ***Holography*** naturally adapted to this procedure:
  - global symmetries on the boundary ⟷ gauge symmetries in the bulk
  - bulk fields contain arbitrary sources for local operators, and transform under local symmetries in the bulk.

Holographic Ward identities

- *Strategy*: work at finite cut-off & identify sources with induced fields at cut-off shell. Remove cut-off at the end.
- Renormalized 1-point functions in the presence of sources

$$\langle T^{\mu\nu}\rangle = \frac{2}{\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial\tilde{\gamma}_{\mu\nu}} \qquad \langle \overline{S}^{-\mu}\rangle = \frac{-2i}{\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial\tilde{\Psi}^{+}_{\mu}}$$

$$\langle \mathcal{O}_{\phi}\rangle = \frac{1}{2\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial\phi} \qquad \langle \overline{\mathcal{O}}^{+}_{\xi_\phi}\rangle = \frac{1}{\sqrt{-\tilde{\gamma}}}\frac{i}{\sqrt{2}}\frac{\partial S_{ren}}{\partial\xi^{-}_{\phi}}$$

$$\langle \mathcal{O}_{\tilde{b}}\rangle = \frac{1}{2\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial\tilde{b}^{\Phi}} \qquad \langle \overline{\mathcal{O}}^{+}_{\tilde{\xi}_b}\rangle = \frac{1}{\sqrt{-\tilde{\gamma}}}\frac{i}{\sqrt{2}}\frac{\partial S_{ren}}{\partial\tilde{\xi}^{-}_{b}}$$

$\mathcal{S}_{\mathbf{ren}} = \mathcal{S}_{\mathbf{reg}} + \mathcal{S}_{\mathbf{ct}}$: renormalized action (at finite cut-off!)

Explicit expression of counter-terms not needed to derive Ward identities!

Holographic Ward identities

- ***Recipe***: 1. Fix bulk gauge redundancy by picking a radial gauge, 2. Study transformations of the sources under residual local symmetries preserving the gauge.
- Gauge fixing condition (Fefferman-Graham gauge):

$$ds^2 = dr^2 + \gamma_{\mu\nu} dx^\mu dx^\nu \quad , \quad \Psi_r = 0$$

- Bulk diff preserving this gauge:

$$\dot{\xi}^r = 0 \, , \, \dot{\xi}^\mu + \gamma^{\mu\nu}(r,x)\partial_\nu \xi^r = 0$$

Solution: $\xi^r = \sigma(x) \, , \, \xi^\mu = \xi_0^\mu(x) - \int^r dr' \gamma^{\mu\nu}(r',x)\partial_\nu \sigma(x)$

- Bulk SUSY transformations preserving this gauge:

$$(\nabla_r + \frac{1}{6}\mathcal{W}\Gamma_r)\epsilon = 0 \quad \text{Solution:} \quad \begin{cases} \epsilon^+(z,x) = z^{-1/2} h(z)^{1/12} \epsilon_0^+(x) + \mathcal{O}(z^4) \\ \epsilon^-(z,x) = z^{1/2} h(z)^{-1/12} \epsilon_0^-(x) + \mathcal{O}(z^4) \end{cases}$$

$$\epsilon = \epsilon^+ + \epsilon^-$$

Holographic Ward identities

- $\sigma(\mathbf{x})$ parametrizes Weyl transformations: $\delta_\sigma \mathcal{S}_{\mathbf{ren}} = \mathbf{0}$

$$\langle T^\mu_\mu \rangle + 9M \langle \mathcal{O}_{\tilde{b}} \rangle + \left[\frac{i}{4} \langle \overline{S}^{-\mu} \rangle \tilde{\Psi}^+_\mu + \frac{i}{\sqrt{2}} \langle \overline{\mathcal{O}}^+_{\xi^\phi} \rangle \xi^-_\phi + \frac{i}{\sqrt{2}} \langle \overline{\mathcal{O}}^+_{\tilde{\xi}^b} \rangle \tilde{\xi}^-_b + \text{h.c.} \right] = 0$$

- $\epsilon^-(\mathbf{x})$ parametrizes superWeyl transformation: $\delta_{\epsilon^-} \mathcal{S}_{\mathbf{ren}} = \mathbf{0}$

$$\frac{i}{2} \langle \overline{S}^{-\mu} \tilde{\Gamma}_\mu \rangle = \frac{9M}{\sqrt{2}} \langle \overline{\mathcal{O}}^+_{\tilde{\xi}_b} \rangle$$

- $\epsilon^+(\mathbf{x})$ parametrizes SUSY transformations: $\delta_{\epsilon^+} \mathcal{S}_{\mathbf{ren}} = \mathbf{0}$

$$\frac{i}{2} e^{-\frac{2}{15}U} \langle \partial_\mu \overline{S}^{-\mu} \rangle = -\frac{1}{2} \langle T^{\mu\nu} \rangle \overline{\tilde{\Psi}}^+_\mu \tilde{\Gamma}_\nu + i \langle \mathcal{O}_\phi \rangle \overline{\xi}^-_\phi + i \langle \mathcal{O}_{\tilde{b}} \rangle \overline{\tilde{\xi}}^-_b$$

- Ward identities can be obtained differentiating the above relations wrt covariant sources, then removing the cut-off (and set sources to 0, when needed).

at the cut-off w/ non-zero sources!

Holographic Ward identities

- From ϵ^+ identity, further differentiating wrt gravitino/ hyperini we get:

$$\langle \partial^\mu S_{\mu\alpha}(x)\ \bar{S}_{\nu\dot\beta}(0)\rangle = -2\,\sigma^\mu_{\alpha\dot\beta}\langle T_{\mu\nu}\rangle\ \delta^4(x)$$

$$\langle \partial^\mu S^\alpha_\mu(x)\ \mathcal{O}_{\xi_\phi\alpha}(0)\rangle = -\sqrt{2}\ \langle \mathcal{O}_\phi\rangle\ \delta^4(x)$$

$$\langle \partial^\mu S^\alpha_\mu(x)\ \mathcal{O}_{\tilde\xi_b\alpha}(0)\rangle = -\sqrt{2}\ \langle \mathcal{O}_{\tilde b}\rangle\ \delta^4(x)$$

Higher-components operators VEVs → break SUSY!

- From ϵ^- and σ identities:

$$\langle \sigma^\mu_{\alpha\dot\beta}\bar{S}^{\dot\beta}_\mu\rangle = -9\sqrt{2}M\ \langle \mathcal{O}_{\tilde\xi_b\,\alpha}\rangle \quad , \quad \langle T^\mu_\mu\rangle = -9M\langle \mathcal{O}_{\tilde b}\rangle$$

In perfect agreement with QFT answer: $\boldsymbol{\Delta}(\mathcal{O}_{\tilde{\mathbf{b}}}) < \mathbf{4}$, $\boldsymbol{\Delta}(\mathcal{O}_\phi) = \mathbf{4}$!

→ Goldstino eigenstate: $\mathbf{G} \sim \langle \mathcal{O}_{\tilde b}\rangle\, \mathcal{O}_{\tilde\xi_b}$!!

Goldstino & antiD-branes

- Finally, we evaluate the bosonic 1pt-fnc on bulk solutions

$$\langle T^{\mu}_{\mu} \rangle = -12\,\mathcal{S} \quad , \quad \langle \mathcal{O}_\phi \rangle = \frac{(3\mathcal{S}+4\varphi)}{2} \quad , \quad \langle \mathcal{O}_{\tilde{b}} \rangle = \frac{4}{3\,M}\mathcal{S}$$

[Aharony et al. '05, De Wolfe et al. '08]

- SUSY case $\mathcal{S} = \varphi = \mathbf{0}$: 0 vacuum energy, no goldstino.

- Branch $\mathcal{S} = \mathbf{0}\,,\ \varphi \neq \mathbf{0}$: 0 vacuum energy, no goldstino but ~~SUSY~~!? Not a vacuum of KS theory: explicit breaking! *Note*: don't need to know full solution (found in [Kuperstein et al. '14]).

- Branch $\mathcal{S} \neq \mathbf{0}\,,\ \varphi = \mathbf{0}$: $\neq 0$ vacuum energy, goldstino mode, ~~SUSY~~ VEV, trace identity fulfilled. In DKM, normalizable mode $\mathcal{S}$ matched with perturbation by ($\mathbf{p}$) antiD3-branes!

Goldstino & antiD-branes

- *Comment*: in our (asymptotic) solutions $\mathcal{S}$ and φ are free parameters. In a full solution, they get fixed by IR b.c.

$$\mathcal{S} = \mathcal{S}(M, N, p, \epsilon)\,,\ \varphi = \varphi(M, N, p, \epsilon)$$

  - A solution with $\mathcal{S} = \varphi = \mathbf{0}$ (for $\mathbf{p} = \mathbf{0}$ theory) known: KS.
  - A solution w/ $\mathcal{S} = \mathbf{0}\,,\ \varphi \neq \mathbf{0}$ was found [Kuperstein et al. '14]. It is singular, but it is plausible the singularity can be resolved (it is KT-like). Still, our analysis shows that no matter IR b.c. for φ, it could not be a KS theory vacuum.
  - A solution w/ $\mathcal{S} \neq \mathbf{0}\,,\ \varphi = \mathbf{0}$ would correspond to a ~~SUSY~~ vacuum of KS theory w/ $\mathbf{p} \neq \mathbf{0}$. In KPV $\mathbf{p}$ is # of antiD3 and IR b.c. fix $\mathcal{S} \sim (\mathbf{p}/\mathbf{N})\,\mathbf{e}^{-\frac{8\pi N}{3 g_s M^2}}$ → *if* antiD solution exists, it would be a KS theory vacuum.

Summary

- Holographic ***derivation*** of Ward identities for cascading backgrounds.

Note: insensitive to the IR, but give constraints on solutions that correspond to given vacua (e.g. ~~SUSY~~ vacua in KS theory).

- Worked at finite cut-off in terms of induced fields. It seems promising for systematics of Holographic Renormalization in ***generic*** set-ups.

- Provided a ***necessary*** consistency check for the existence of metastable vacua in the KS cascading gauge theory.

- Evidence for 1-1 ***correspondence*** between spontaneous ~~SUSY~~ and antiD-brane states in quiver gauge theories.

OUTLOOK

- On with the program of holographic renormalization, e.g. systematic derivation of fermionic counter-terms.
- Extension of present analysis to the full background (including conifold deformation parameter). [BENA ET AL. '11]
- (By use of induced field formalism) Derive SUSY-preserving counter-terms in generic setups (curved manifolds!).
- Towards effective theory for strongly coupled Goldstone bosons and fermions.

Thank you!