BPS black holes in $\mathcal{N} = 2\,, d = 4$ supergravity

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- non-homogeneous deformation of the stu model;
- results and a black hole solution.

$\mathcal{N} = 2$, $d = 4$ supergravity: field content

- supergravity multiplet $\rightarrow$

$g_{\mu\nu}$	graviton
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- matter multiplets
  - n_V vector multiplets $\longrightarrow$

A^α_μ	n_V vector fields
z^α	n_V scalar fields
$\chi^{i\alpha}$	$2n_V$ fermions, gauginos

  - n_H hypermultiplets $\longrightarrow$

q^u	$4n_H$ scalar fields, hyperscalars
ξ^A	$2n_H$ fermions, hyperinos

$\mathcal{N} = 2$, $d = 4$ supergravity: classical solutions

Configurations of the bosonic fields only, the fermionic fields vanish.
Choose $n_H = 0$,

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e^a_\mu & \\
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$$\begin{array}{ll} e^a_\mu & \\ A^\Lambda & \Lambda = 0, \cdots, n_V \\ z^\alpha & \alpha = 1, \cdots, n_V \end{array}$$

z^α parametrize a special Kähler manifold, base of a symplectic bundle with sections

$$\mathcal{V} = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} = \mathrm{e}^{\mathcal{K}/2} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}, \qquad D_{\bar{\imath}} \mathcal{V} = 0$$

which obey the constraint $i \langle \mathcal{V}, \bar{\mathcal{V}} \rangle = 1$.

$\mathcal{N}=2$, $d=4$ supergravity: Lagrangian

$$\mathscr{L} = \frac{R}{2} - g_{i\bar{\jmath}}\,\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{\jmath}} - V_g + \frac{1}{4} I_{\Lambda\Sigma}\, F^\Lambda_{\mu\nu}\, F^{\Sigma\mu\nu} + \frac{1}{8\sqrt{-g}}\,\epsilon^{\mu\nu\rho\sigma} R_{\Lambda\Sigma}\, F^\Lambda_{\mu\nu}\, F^\Sigma_{\rho\sigma}$$

The Lagrangian is determined by geometry

$$g_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} \mathcal{K}\,,$$

$$N_{\Lambda\Sigma} = R_{\Lambda\Sigma} + i I_{\Lambda\Sigma} \quad s.t. \quad M_\Lambda = N_{\Lambda\Sigma} L^\Sigma\,,\; D_{\bar{\imath}}\,\bar{M}_\Lambda = N_{\Lambda\Sigma} D_{\bar{\imath}}\,\bar{L}^\Sigma\,.$$

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$$+ \frac{1}{4}\, I_{\Lambda\Sigma}\, F^\Lambda_{\mu\nu}\, F^{\Sigma\mu\nu} + \frac{1}{8\sqrt{-g}}\,\epsilon^{\mu\nu\rho\sigma} R_{\Lambda\Sigma}\, F^\Lambda_{\mu\nu}\, F^\Sigma_{\rho\sigma}$$

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If a prepotential F exists,

$$\mathcal{V} = \mathrm{e}^{\mathcal{K}/2} \begin{pmatrix} X^\Lambda \\ \partial_\Lambda F \end{pmatrix}\,, \quad N_{\Lambda\Sigma} = \bar{F}_{\Lambda\Sigma} + 2i\, \frac{\mathrm{Im} F_{\Lambda\Lambda'}\, X^{\Lambda'}\, \mathrm{Im} F_{\Sigma\Sigma'}\, X^{\Sigma'}}{X^\Omega\, \mathrm{Im} F_{\Omega\Omega'}\, X^{\Omega'}}$$

$\mathcal{N} = 2$, $d = 4$ supergravity: potential

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- $U(1) \subset SU(2)_R$, using Fayet-Iliopoulos terms.

$$V_g = g^{i\bar{\jmath}} \, D_i\mathcal{L} \, D_{\bar{\jmath}}\bar{\mathcal{L}} - 3\,|\mathcal{L}|^2 \ ,$$

$$\mathcal{L} = \langle \mathcal{G}, \mathcal{V} \rangle = e^{\mathcal{K}/2} \left(X^\Lambda g_\Lambda - F_\Lambda \, g^\Lambda \right)$$

where $\mathcal{G} = \left(g^\Lambda, g_\Lambda \right)$ is a symplectic vector

and g^Λ , g_Λ are the FI parameters.

Squaring of the action and BPS equations

BPS equations: motivations

Aim: new black hole solutions in gauged supergravity

- insight in fundamental questions of gravity, at classical and quantum level:
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Aim: new black hole solutions in gauged supergravity

- insight in fundamental questions of gravity, at classical and quantum level:
  - uniqueness theorem;
  - microstates counting;
  - attractor mechanism;
- many known solutions, but the general structure is not understood;
- solutions with AdS asymptotics $\mapsto$ gauge/gravity duality:
  - to address the issues of black hole physics;
  - to study strongly coupled condensed matter systems, duals to black holes.

BPS equations: motivations

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Action as a sum of squares

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supersymmetry conditions

$\Rightarrow$ supersymmetric solutions of the equations of motion.

BPS equations:

To obtain first order equations[1], we assume:

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}(dr^2 + e^{2\psi(r)}d\Omega_\kappa^2)\,,$$

$$d\Omega_\kappa^2 = d\theta^2 + f_\kappa^2(\theta)\,d\phi^2\,, \qquad f_\kappa(\theta) = \begin{cases} \sin\theta\,, & \kappa = 1 \\ \theta\,, & \kappa = 0 \\ \sinh\theta\,, & \kappa = -1 \end{cases}\,,$$

$$z^i = z^i(r)\,,$$

$$\mathcal{Q} = \left(p^\Lambda, q_\Lambda\right)\,, \qquad p^\Lambda = \frac{1}{\mathrm{vol}(\Sigma)}\int_\Sigma F^\Lambda\,, \qquad q_\Lambda = \frac{1}{\mathrm{vol}(\Sigma)}\int_\Sigma G_\Lambda\,.$$

[1] (arXiv:1012.3756; G. Dall'Agata, A. Gnecchi.)

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Their solutions satisfy:

- the equations of motion,
- the supersymmetry conditions;

$\Rightarrow$ they are the BPS equations we were looking for.

BPS equations:

The ensuing equations are:

$$2e^{2\psi}\left(e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{V})\right)' + e^{2(\psi-U)}\Omega\mathcal{M}\mathcal{G}+ \\ + 4e^{-U}(\alpha' + \mathcal{A}_r)\mathrm{Re}(e^{-i\alpha}\mathcal{V}) + \mathcal{Q} = 0\,,$$

$$\psi' = 2e^{-U}\mathrm{Im}(e^{-i\alpha}\mathcal{L})\,,$$

$$\alpha' + \mathcal{A}_r = -2e^{-U}\mathrm{Re}(e^{-i\alpha}\mathcal{L})\,.$$

where

$$\mathcal{A}_\mu = \mathrm{Im}(\partial_\mu z^i(\partial_i\mathcal{K}))\,,\quad e^{2i\alpha} = \frac{\mathcal{Z} - ie^{2(\psi-U)}\mathcal{L}}{\bar{\mathcal{Z}} + ie^{2(\psi-U)}\bar{\mathcal{L}}}\,,$$

$$\mathcal{Z} = \langle\mathcal{Q},\mathcal{V}\rangle\,,\quad \mathcal{L} = \langle\mathcal{G},\mathcal{V}\rangle\,.$$

Non-homogeneous deformation of the stu model

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- $n_H = 0\,,\; n_V = 3\,,\; U(1)$ FI gauging ;
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Motivations:

- the corresponding very special Kähler manifold is neither symmetric nor homogeneous $\Rightarrow$ nh-stu;
  - first black hole solution in a nh model;
- the prepotential has been constructed in Type II A string theory compactified on Calabi-Yau;
- A as a parameter to compare nh-stu $\leftrightarrow$ stu.

Non-homogeneous deformation of the stu model:

We assumed:

- no axions, $z^i = -i\lambda^i\,,\ \lambda^i > 0\,;$
- projective coordinates, $\frac{X^i}{X^0} = -i\lambda^i\,;$

$$e^{-\mathcal{K}} = 8\left(\lambda^1\lambda^2\lambda^3 - \frac{A}{3}\left(\lambda^3\right)^3\right)\,,\quad g_{i\bar{\jmath}} = \partial_i\partial_{\bar{\jmath}}\mathcal{K}\,,$$

$$\mathcal{N}_{\Lambda\Sigma} = R_{\Lambda\Sigma} + iI_{\Lambda\Sigma}\,,\quad I_{\Lambda\Sigma} = -\frac{1}{8}\,e^{-\mathcal{K}}\begin{pmatrix} 1 & 0 \\ 0 & 4g_{i\bar{\jmath}} \end{pmatrix}\,;$$

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- gauging parameters $\mathcal{G} = \left(0, g^1, g^2, g^3, g_0, 0, 0, 0\right)$:
  - potential V_g;
  - dyonic charges $\mathcal{Q} = \left(p^0, 0, 0, 0, 0, q_1, q_2, q_3\right)$.

Non-homogeneous deformation of the stu model: results

Results:

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  - one breaks supersymmetry and exists only in nh-stu;
- the BPS equations are solved implicitly in the near horizon limit, $AdS \times \Sigma$ geometry;
- a class of new, full black hole solutions.

Non-homogeneous deformation of the stu model: black hole

BPS equations

↳ coupled system of differential equations in terms of

$$H^0 = \frac{e^{-U}}{\sqrt{2}} \left(\lambda^1 \lambda^2 \lambda^3 - \frac{A}{3} \left(\lambda^3\right)^3 \right)^{-1/2} ,$$

$$H_1 = \lambda^2 \lambda^3 H^0 , \quad H_2 = \lambda^1 \lambda^3 H^0 , \quad H_3 = \left(\lambda^3\right)^2 H^0 .$$

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Ansatz

$$\begin{aligned} \psi &= \log \left(a\, r^2 + c \right) , \\ H^0 &= e^{-\psi} \left(\alpha^0 r + \beta^0 \right) , \\ H_i &= e^{-\psi} \left(\alpha_i\, r + \beta_i \right) , \qquad i = 1, 2, 3 . \end{aligned}$$

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System of algebraic conditions on the parameters

$$\{\alpha^0, \alpha_i\,, \beta^0, \beta_i\,, a\,, c\,, p^0, q_i\} \ ;$$

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a scaling symmetry fixes a;

$\hookrightarrow$ recover the physical fields $\mapsto$ the metric $\psi\,, U$,
$\mapsto$ the scalars λ^i .

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  - $AdS_2 \times \Sigma$ if $r \to r_h$;
- although $\lambda^2 \propto \lambda^3$, the solution is not trivial (nh-st$^2 \neq$ st^2).

Non-homogeneous deformation of the stu model: black hole

- the entropy
  - depends only on gaugings and charges:

$$S_{BH} = \frac{A_h}{4} = S_{BH}\left(g_0\,, g^i, p^0, q_i\right)$$

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  - the product of the areas at the horizons

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- if $A \to 0$, the known solution in the st^2 model is recovered.

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$\mapsto$ possible extensions:

$\mapsto$ involving non Abelian gauge fields;
$\mapsto$ non extremal solutions;
$\mapsto$ non BPS solutions.

$\mapsto$ systematic study of the attractor mechanism in gauged supergravity.