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Theories of the Fundamental Interactions

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Born-Infeld/Gravity Duality

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Based on:

G. Grignani, T. Harmark, AM and M. Orselli – arXiv:1512.xxxx [hep-th]

Outline

Overview

- New duality between 4d Born-Infeld (BI) theory and 5d classical gravity
- manifestation of the open/closed string duality
- Born-Infeld side ⟶ low energy effective theory for open strings ending on a D3-brane in a slowly varying background Kalb-Ramond field
- gravity side ⟶ gravitational (closed string) description of D3-branes in the same background Kalb-Ramond field
- the duality is a correspondence between effective theories in a similar sense as in the fluid/gravity correspondence
- it can be used to derive higher derivative correction to the BI action

Setup

- N coincident D3-branes in 10d Minkowski background

Mink^{10}	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$

- flat embedding $\longrightarrow$ $x_a(\sigma) = \sigma_a$, for $a = 0, 1, 2, 3$, and $x_i(\sigma) = 0$ for $i = 4, 5, ..., 9$

- IIB SUGRA background has $\phi = 0$ and $H_{abc} = 0$

- turn on $B_{ab} \parallel$ D3 world-volume (while $B_{ij} = 0$)

- $\mathcal{F}_{ab} = B_{ab} \longleftrightarrow$ gauge invariant field strength on the D3

- IIB SUGRA EOMs $\longrightarrow$ $dB_{(2)} = 0 \longrightarrow \partial_{[a}\mathcal{F}_{bc]} = 0$

Setup

- Assume $\mathcal{F}_{ab}$ slowly varying over the D3

$$\frac{R}{\ell_s} \gg (g_s N)^{1/4} \quad \text{and} \quad \frac{R}{\ell_s} \gg 1$$

$R =$ minimal length scale of variation of $\mathcal{F}_{ab}$

- when $g_s N \ll 1 \longrightarrow$ open string description $\longrightarrow$ Born-Infeld action

- when $g_s N \gg 1 \longrightarrow$ closed string description $\longrightarrow$ type IIB SUGRA

Outline

Born-Infeld theory

BI action: low energy effective action for open strings ending on a single D3 valid when $g_s \ll 1$ (perturbative string theory)

$$S_{\mathsf{BI}} = -T_{\mathsf{D3}} \int d^4\sigma \sqrt{-\det\left(\eta_{ab} + \mathcal{F}_{ab}\right)}$$

$$\mathcal{F}_{ab} = B_{ab} + 2\pi\ell_s^2 F_{ab} \qquad \text{and} \qquad T_{\mathsf{D3}} = \left(g_s \ell_s^4 (2\pi)\right)^{-1}$$

$$\partial_{[a}\mathcal{F}_{bc]} = 0 \quad \longleftrightarrow \quad d\mathcal{F} = 0$$

- useful notation:

$$\mathcal{F}_a{}^b = \mathcal{F}_{ac}\eta^{cb} \quad G_a{}^b = \delta_a^b - \mathcal{F}_a{}^c\mathcal{F}_c{}^b \quad m^4 = \det(G_a{}^b)$$

$$\text{BI action} \quad \longrightarrow \quad S_{\mathsf{BI}} = -T_{\mathsf{D3}} \int d^4\sigma\, m^2$$

BI EOMs

- EOMs by varying the action

$$\partial^b (m G^{-1} \mathcal{F})_{ba} = 0$$

- Born-Infeld energy-momentum tensor

$$\tau_{ab} = -T_{\mathrm{D3}} m (G^{-1})_{ab}$$

- energy-momentum tensor conservation

$$\partial^b \tau_{ba} = 0 \quad \longrightarrow \quad \partial^b (m (G^{-1})_{ba}) = 0$$

$\longrightarrow$ these eq.s along with $d\mathcal{F} = 0$ $\longrightarrow$ constraints on how $\mathcal{F}_{ab}$ can vary along the D3

Outline

Gravity setup

- SUGRA description of N D3-branes in the background of a slowly varying Kalb-Ramond potential

Type IIB Supergravity Lagrangian (bosonic part)

$$\mathcal{L} = \sqrt{-g}\Big(\mathcal{R}^{(10)} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{12}e^{-\phi}H_{\mu\nu\rho}H^{\mu\nu\rho} - \frac{1}{2}e^{2\phi}\partial_\mu\chi\partial^\mu\chi - \frac{1}{12}e^{\phi}F_{\mu\nu\rho}F^{\mu\nu\rho} - \frac{1}{4\cdot 5!}F_{\mu\nu\rho\lambda\sigma}F^{\mu\nu\rho\lambda\sigma}\Big) + \frac{1}{8\cdot 4!}\epsilon^{\mu_1\ldots\mu_{10}}A_{\mu_1\ldots\mu_4}\partial_{\mu_5}B_{\mu_6\mu_7}\partial_{\mu_8}A_{\mu_9\mu_{10}}$$

boundary conditions

$$\lim_{r\to 0} B_{ab} = 0 \qquad \lim_{r\to\infty} B_{ab} = \mathcal{F}_{ab} \quad \text{with } d\mathcal{F} = 0$$

$$\lim_{r\to\infty} g_{\mu\nu} = \eta_{\mu\nu}\ (\text{Mink}^{10}) \qquad \lim_{r\to\infty} \phi, \chi, A^{(2)} = 0$$

$r =$ radial coordinate in the transverse space of the D3 (D3 are at $r = 0$)

Gravity setup

- using symmetries and gauge freedom $\longrightarrow$ ten-dimensional metric can be written as

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = e^{\frac{1}{2}\eta}\left(h_{mn}dx^m dx^n + e^{2\rho}d\Omega_5^2\right) \qquad m,n = 0,\ldots,4$$

  with $\rho = \log r$ and h_{mn} and η only depends on x^m

- impose that we have N D3-branes $\longrightarrow$ set the RR 5-form field strength to

$$F^{(5)} = 4r_c^4(\omega_5 + *\omega_5) \qquad r_c^4 = \frac{N}{2\pi^2 T_{\mathsf{D3}}}$$

  r_c sets the length scale of the thickness of the D3

- only the metric and $F^{(5)}$ involve the S^5 directions $\longrightarrow$ 5d reduction

5d EOMs

$$\frac{\partial_m\left(\sqrt{-h}\,e^{2\phi+2\eta+5\rho}\,\partial^m\chi\right)}{e^{5\rho}\sqrt{-h}}=-\frac{1}{6}e^{\phi+\eta}F_{mnk}H^{mnk}$$

$$\frac{\partial_m\left(\sqrt{-h}\,e^{2\eta+5\rho}\,\partial^m\phi\right)}{e^{5\rho}\sqrt{-h}}=-\frac{1}{12}e^{\eta-\phi}H_{mnk}H^{mnk}+\frac{1}{12}e^{\eta+\phi}F_{mnk}F^{mnk}+e^{2\eta+2\phi}\partial_m\chi\partial^m\chi$$

$$\partial_m\left(\sqrt{-h}\,e^{\eta+5\rho}(e^{-\phi}H^{mnk}-\chi e^{\phi}F^{mnk})\right)=-\frac{4r_c^4}{6}\epsilon^{nklpq}(F_{lpq}+\chi H_{lpq})$$

$$\partial_m\left(\sqrt{-h}\,e^{\eta+5\rho}e^{\phi}F^{mnk}\right)=\frac{4r_c^4}{6}\epsilon^{nklpq}H_{lpq}$$

$$\begin{aligned}4e^{-2\rho}-5\partial^m\rho\partial_m\rho-\frac{1}{2}\partial^m\eta\partial_m\eta-\frac{13}{4}\partial^m\eta\partial_m\rho-D^mD_m\rho-\frac{1}{4}D^mD_m\eta\\=-\frac{1}{48}e^{-\phi-\eta}H_{mnk}H^{mnk}-\frac{1}{48}e^{\phi-\eta}F_{mnk}F^{mnk}+4r_c^8e^{-2\eta-10\rho}\end{aligned}$$

$$\begin{aligned}(\mathcal{R}^{(5)})_{mn}=5\partial_m\rho\partial_n\rho+5D_mD_n\rho-\frac{1}{2}\partial_m\eta\partial_n\eta+2D_mD_n\eta+\frac{1}{2}h_{mn}\partial^k\eta\partial_k\eta+\frac{1}{4}h_{mn}D^kD_k\eta\\+\frac{5}{4}h_{mn}\partial^k\eta\partial_k\rho+\frac{1}{2}\partial_m\phi\partial_n\phi+\frac{1}{2}e^{2\phi}\partial_m\chi\partial_n\chi+\frac{1}{12}e^{-\phi-\eta}(3H_m{}^{kl}H_{nkl}\\-\frac{1}{4}h_{mn}H^{klp}H_{klp})+\frac{1}{12}e^{\phi-\eta}(3F_m{}^{kl}F_{nkl}-\frac{1}{4}h_{mn}F^{klp}F_{klp})-4r_c^8e^{-2\eta-10\rho}h_{mn}\end{aligned}$$

D3 with constant $\mathcal{F}_{ab}$

Known solutions

- D3 solution $\longrightarrow$ $\mathcal{F}_{ab} = 0$
- F1-D3 solution $\longrightarrow$ $\vec{E} \neq 0,\ \vec{B} = 0$
- D1-D3 solution $\longrightarrow$ $\vec{B} \neq 0,\ \vec{E} = 0$
- (F1||D1)-D3 solution $\longrightarrow$ $\vec{E} \times \vec{B} = 0$
- (F1⊥D1)-D3 solution $\longrightarrow$ $\vec{E} \cdot \vec{B} = 0$

$$\mathcal{F}_{ab} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}$$

$$\vec{E} = (E_1, E_2, E_3)$$

$$\vec{B} = (B_1, B_2, B_3)$$

Grignani, Harmark, AM, Orselli arXiv:1311.3834

D3 with constant $\mathcal{F}_{ab}$

New (general) solution

$\longrightarrow$ generalization of the previous solutions

$$h_{mn}dx^m dx^n = h_{ab}dx^a dx^b + dr^2 \quad (a,b=0,\ldots,3) \qquad \rho = \log r$$

$$h_{ab} = \{(I + m r_c^4 r^{-4} G^{-1})^{-1}\}_a{}^c \eta_{cb} \qquad \sqrt{-h} = e^{-2\eta}$$

$$e^{2\eta} = 1 + \frac{\mathrm{Tr}(G)}{2m}\frac{r_c^4}{r^4} + \frac{r_c^8}{r^8} \quad e^{2\phi} = e^{-2\eta}\left(1 + m\frac{r_c^4}{r^4}\right)^2 \quad \chi = \frac{\mathrm{Tr}(\mathcal{F}\,{}^*\mathcal{F})}{4}\frac{\frac{r_c^4}{r^4}}{1 + m\frac{r_c^4}{r^4}}$$

$$B_{ab} = \mathcal{F}_a{}^c h_{cb} \qquad A_{ab} = \frac{r_c^4}{r^4}\,{}^*\mathcal{F}_a{}^c h_{cb}$$

- low energy description of D3 dynamics $\longrightarrow r_c \gg \ell_s \longrightarrow g_s N \gg 1$
- low energy effective description of the strongly coupled Born-Infeld theory

Derivative expansion

- consider the D3 solution with general $\mathcal{F}_{ab}$
- let $\mathcal{F}_{ab}$ be a slowly varying function of the world-volume coordinates x_a $\longleftrightarrow$ analogy with fluid/gravity correspondence
- set up a perturbative procedure to build a corrected solution $\longrightarrow$ world-volume derivative expansion
- zeroth order $\longrightarrow$ D3 with constant $\mathcal{F}_{ab}$

First order

- neglect in the EOMs all the terms with more than one world-volume derivative

- equations with one world-volume derivative correction are

$$\partial_b \left(e^{-\eta-\phi} H^{bar} - e^{-\eta+\phi} \chi F^{bar} \right) = \frac{4r_c^4}{6r^5} \epsilon^{abcd} (F_{bcd} + \chi H_{bcd}) \tag{1}$$

$$\partial_b \left(e^{-\eta+\phi} F^{bar} \right) = -\frac{4r_c^4}{6r^5} \epsilon^{abcd} H_{bcd} \tag{2}$$

$$\frac{1}{2}\partial_b(h^{bc}\partial_r h_{ca}) - \frac{1}{2}\Gamma^c_{ab} h^{bd}\partial_r h_{dc} = -\frac{1}{2}\partial_a\eta\partial_r\eta + \frac{1}{2}\partial_a\phi\partial_r\phi + \frac{1}{2}e^{2\phi}\partial_a\chi\partial_r\chi + \frac{1}{4}e^{-\phi-\eta} H_a{}^{bc} H_{bcr} + \frac{1}{4}e^{\phi-\eta} F_a{}^{bc} F_{bcr} \tag{3}$$

First order

Consider the above equations to leading order at large r

- Eq. (1) $\longrightarrow$ $\partial_b H^{bar} = 0$ $\longrightarrow$ $\partial^b (mG^{-1}\mathcal{F})_{ba} = 0$
- Eq. (2) $\longrightarrow$ $\partial_b F^{bar} = 0$ $\longrightarrow$ $\partial^b ({}^*\mathcal{F})_{ba} = 0$ $\longleftrightarrow$ $d\mathcal{F} = 0$
- Eq. (3) $\longrightarrow$ $\partial^b \partial_r h_{ba} = 0$ $\longrightarrow$ $\partial^b (mG^{-1})_{ba} = 0$

these are the constraints on the variation of $\mathcal{F}_{ab}$

$\longrightarrow$ Exactly the same eqs we got from the BI!

Higher-derivative correction

- starting from second order $\longrightarrow$ corrections to the zeroth order solution

- general corrected solution

$$Y_I(x^a, r) = Y_I^{(0)}(x^a, r) + Y_I^{(2)}(x^a, r) + Y_I^{(4)}(x^a, r) + \mathcal{O}(\partial^6)$$

$$Y_I = (\chi, \phi, A_{ab}, B_{ab}, \eta, h_{ab})$$

- no corrections with an odd number of world-volume derivatives $\longleftrightarrow$ no covariant expression in terms of $\mathcal{F}_{ab}$

Second order

- Eq.s that determine $Y_I^{(2)}$ involve two world-volume derivatives

$$H_{(0)}^{m,I} Y_I^{(2)} + H_{(1)}^{m,I} \partial_r Y_I^{(2)} + H_{(2)}^{m,I} \partial_r^2 Y_I^{(2)} = K^{m,ab,I} \partial_a \partial_b Y_I^{(0)} + L^{m,ab,IJ} \partial_a Y_I^{(0)} \partial_b Y_I^{(0)}$$

- solution in a large r expansion has the form

$$Y_I^{(2)} = \sum_{n=0}^{\infty} \frac{C_{I,n}^{(2)}}{r^{2+4n}}$$

- to leading order at large r

$$\chi^{(2)} = \frac{r_c^4}{16 r^2} \partial_a \partial^a (\mathcal{F}\, {}^*\mathcal{F}) + \mathcal{O}(r^{-6}) \qquad \phi^{(2)} = \frac{r_c^4}{4 r^2} \partial_a \partial^a \left(m - \frac{\mathrm{Tr} G}{4m} \right) + \mathcal{O}(r^{-6})$$

$$B_{ab}^{(2)} = -\frac{r_c^4}{4 r^2} 3 \partial^c \partial_{[c} (m G^{-1} \mathcal{F})_{ab]} + \mathcal{O}(r^{-6}) \qquad A_{ab}^{(2)} = \frac{r_c^4}{4 r^2} 3 \partial^c \partial_{[c} {}^*\mathcal{F}_{ab]} + \mathcal{O}(r^{-6})$$

$$\eta^{(2)} = \frac{r_c^4}{16 r^2} \partial^c \partial_c \left(\frac{\mathrm{Tr} G}{m} \right) + \mathcal{O}(r^{-6}) \qquad h_{ab}^{(2)} = -\frac{r_c^4}{4 r^2} \partial^c \partial_c (m G^{-1})_{ab} + \mathcal{O}(r^{-6})$$

Third order

Third order eq.s to leading order at large r

- Eq. (1) $\longrightarrow \partial_b H^{bar} = 0 \longrightarrow \partial_b \partial^c \partial_{[c}(mG^{-1}\mathcal{F})_{ab]} = 0$
- Eq. (2) $\longrightarrow \partial_b F^{bar} = 0 \longrightarrow \partial_b \partial^c \partial_{[c}{}^*\mathcal{F}_{ab]} = 0$
- Eq. (3) $\longrightarrow \partial^b \partial_r h_{ba} = 0 \longrightarrow \partial^b \partial^c \partial_c (mG^{-1})_{ba} = 0$

$\longrightarrow$ these are trivially satisfied assuming the first order constraints

Fourth order

- fourth order corrections at large r

$$Y_I^{(4)} = C_{I,0}^{(4)} \log r + \frac{C_{I,4}^{(4)}}{r^4} + \mathcal{O}\left(\frac{1}{r^8}\right)$$

- one can easily compute $C_{I,0}^{(4)}$

$$C_{\chi,0}^{(4)} = -\frac{r_c^4}{64}\partial^2\partial^2(\mathcal{F}\,{}^*\mathcal{F}) \qquad C_{\phi,0}^{(4)} = -\frac{r_c^4}{16}\partial^2\partial^2\left(m - \frac{\mathrm{Tr}G}{4m}\right)$$

$$C_{B_{ab},0}^{(4)} = \frac{r_c^4}{16}3\partial^2\partial^c\partial_{[c}(mG^{-1}\mathcal{F})_{ab]} \qquad C_{A_{ab},0}^{(4)} = -\frac{r_c^4}{16}3\partial^2\partial^c\partial_{[c}{}^*\mathcal{F}_{ab]}$$

$$C_{\eta,0}^{(4)} = -\frac{r_c^4}{64}\partial^2\partial^2\left(\frac{\mathrm{Tr}G}{m}\right) \qquad C_{h_{ab},0}^{(4)} = \frac{r_c^4}{16}\partial^2\partial^2(mG^{-1})_{ab}$$

where $\partial^2 = \partial_a\partial^a$

- the next-to-leading coefficient $C_{I,4}^{(4)}$ cannot be fixed

Comparison with the BI

- the constraints we got at first and third order are consistent with the BI ones
- what about the corrections to the fields at second and fourth order?
- corrections to the metric $\longrightarrow$ corrections to EM tensor
- compare this corrected EM tensor with BI EM tensor $\longrightarrow$ BI higher derivative corrections

Reading off the energy-momentum tensor

- EM tensor can be read off from the metric in the asymptotic region
- in our approximation the asymptotic region is $r_c \ll r \ll R$
- linearize the metric

$$h_{ab} = \eta_{ab} + \bar{h}_{ab} \qquad \eta = \bar{\eta} \quad \text{with } |\bar{h}_{ab}| \ll 1\,, |\bar{\eta}| \ll 1$$

- make the gauge choice $\longrightarrow$ $\eta^{ab}\bar{h}_{ab} = -4\bar{\eta}$ and $\partial^b \bar{h}_{ba} = 0$
- imagine an infinitely thin 3-brane sitting at $r = 0$ with EM tensor $\tau_{ab}(x^c)$ on the brane
- linearized Einstein equations in the region $r_c \ll r \ll R$

$$\left[\partial_r^2 + \frac{5}{r}\partial_r + \partial^c\partial_c\right]\bar{h}_{ab} = -16\pi G\,\delta^6(r)\tau_{ab}$$

Linearized Einstein eq.s

- assume τ_{ab} varies slowly along the world-volume
- expand τ_{ab} in higher-derivative corrections

$$\tau_{ab} = \tau_{ab}^{(0)} + \tau_{ab}^{(2)} + \tau_{ab}^{(4)} + \mathcal{O}(\partial^6)$$

- expand $\bar{h}_{ab}$ in higher-derivative contributions

$$\bar{h}_{ab} = \bar{h}_{ab}^{(0)} + \bar{h}_{ab}^{(2)} + \bar{h}_{ab}^{(4)} + \mathcal{O}(\partial^6)$$

- linearized Einstein eq.s become

$$\left[\partial_r^2 + \frac{5}{r}\partial_r\right] h_{ab}^{(0)} = -16\pi G\, \delta^6(r)\tau_{ab}^{(0)}$$
$$\left[\partial_r^2 + \frac{5}{r}\partial_r\right] h_{ab}^{(2)} + \partial^c\partial_c h_{ab}^{(0)} = -16\pi G\, \delta^6(r)\tau_{ab}^{(2)}$$
$$\left[\partial_r^2 + \frac{5}{r}\partial_r\right] h_{ab}^{(4)} + \partial^c\partial_c h_{ab}^{(2)} = -16\pi G\, \delta^6(r)\tau_{ab}^{(4)}$$

EM tensor

- Solution of the linearized Einstein eq.s

$$\bar{h}^{(0)}_{ab} = \frac{r_c^4}{r^4}\frac{\tau^{(0)}_{ab}}{NT_{\mathrm{D3}}}$$

$$\bar{h}^{(2)}_{ab} = \frac{r_c^4}{r^4}\frac{\tau^{(2)}_{ab}}{NT_{\mathrm{D3}}} + \frac{r_c^4}{4r^2}\frac{\partial^c\partial_c\tau^{(0)}_{ab}}{NT_{\mathrm{D3}}}$$

$$\bar{h}^{(4)}_{ab} = \frac{r_c^4}{r^4}\frac{\tau^{(4)}_{ab}}{NT_{\mathrm{D3}}} + \frac{r_c^4}{4r^2}\frac{\partial^c\partial_c\tau^{(2)}_{ab}}{NT_{\mathrm{D3}}} - \frac{r_c^4}{16}\log r\,\frac{\partial^c\partial_c\partial^d\partial_d\tau^{(0)}_{ab}}{NT_{\mathrm{D3}}}$$

Zeroth order EM tensor

- solution of linearized Einstein $\longrightarrow \bar{h}^{(0)}_{ab} = \frac{r_c^4}{r^4} \frac{\tau^{(0)}_{ab}}{N T_{\mathrm{D3}}}$

- leading correction to flat space metric at zeroth order

$$\bar{h}^{(0)}_{ab} = -\frac{r_c^4}{r^4} m (G^{-1})_{ab}$$

- plug $\bar{h}^{(0)}_{ab}$ into above solution of linearized Einstein eq.

$$\longrightarrow \tau^{(0)}_{ab} = -N T_{\mathrm{D3}} m (G^{-1})_{ab}$$

- this is the leading order EM-tensor for N D3-branes $= N\times$ Born-Infeld EM-tensor

Second order EM tensor

- solution of linearized Einstein $\longrightarrow$ $\bar{h}^{(2)}_{ab} = \frac{r_c^4}{r^4}\frac{\tau^{(2)}_{ab}}{NT_{\mathrm{D3}}} + \frac{r_c^4}{4r^2}\frac{\partial^c\partial_c\tau^{(0)}_{ab}}{NT_{\mathrm{D3}}}$
- zeroth order EM tensor $\longrightarrow$ $\tau^{(0)}_{ab} = -NT_{\mathrm{D3}}m(G^{-1})_{ab}$
- second order large r correction to the metric

$$\bar{h}^{(2)}_{ab} = -\frac{r_c^4}{4r^2}\partial^c\partial_c(mG^{-1})_{ab} + \mathcal{O}\left(\frac{1}{r^6}\right)$$

- plug $\tau^{(0)}_{ab}$ and $\bar{h}^{(2)}_{ab}$ into above sol. of linearized Einstein eq.

$$-\frac{r_c^4}{4r^2}\partial^c\partial_c(mG^{-1})_{ab} + \mathcal{O}\left(\frac{1}{r^6}\right) = \frac{r_c^4}{r^4}\frac{\tau^{(2)}_{ab}}{NT_{\mathrm{D3}}} - \frac{r_c^4}{4r^2}\partial^c\partial_c(mG^{-1})_{ab}$$

- second order EM tensor $\longrightarrow$ $\tau^{(2)}_{ab} = 0$
- Born-Infeld is not corrected at second order

Andreev, Tseytlin (1988)

Fourth order EM tensor?

- solution of linearized Einstein eq.

$$\bar{h}^{(4)}_{ab} = \frac{r_c^4}{r^4}\frac{\tau^{(4)}_{ab}}{NT_{\mathrm{D3}}} + \frac{r_c^4}{4r^2}\frac{\partial^c\partial_c\tau^{(2)}_{ab}}{NT_{\mathrm{D3}}} - \frac{r_c^4}{16}\log r\,\frac{\partial^c\partial_c\partial^d\partial_d\tau^{(0)}_{ab}}{NT_{\mathrm{D3}}}$$

- zeroth and second order EM tensors

$$\tau^{(0)}_{ab} = -NT_{\mathrm{D3}}m(G^{-1})_{ab} \qquad \tau^{(2)}_{ab} = 0$$

- fourth order large r correction to the metric

$$\bar{h}^{(4)}_{ab} = \frac{r_c^4}{16}\log r\,\partial^c\partial_c\partial^d\partial_d(mG^{-1})_{ab} + \mathcal{O}\left(\frac{1}{r^4}\right)$$

- plug $\tau^{(0)}_{ab}$, $\tau^{(2)}_{ab}$ and $\bar{h}^{(2)}_{ab}$ into above sol. of linearized Einstein eq.

$$\cancel{\frac{r_c^4}{16}\log r\,\partial^c\partial_c\partial^d\partial_d(mG^{-1})_{ab}} + \mathcal{O}\left(\frac{1}{r^4}\right) = \frac{r_c^4}{r^4}\frac{\tau^{(4)}_{ab}}{NT_{\mathrm{D3}}} + \cancel{\frac{r_c^4}{16}\log r\,\partial^c\partial_c\partial^d\partial_d(mG^{-1})_{ab}}$$

Outline

Conclusions

- explored a new duality between BI theory and 5d gravity
- open/closed string duality for D3-branes in a slowly varying background Kalb-Ramond field $\mathcal{F}_{ab}$
- world-volume derivative expansion to build the gravity solution for D3 with a slowly varying $\mathcal{F}_{ab}$ $\longrightarrow$ dual to BI theory (including derivative corrections)
- proved that the BI action is not corrected up to the second order in the derivative expansion
- what about the fourth order?

Extra slides

Duality chain generating constant $\mathcal{F}_{ab}$ D3 solution

- start with the D3 solution

$$h_{ab} = \frac{\eta_{ab}}{1 + \frac{r_c^4}{r^4}} \qquad e^{\eta} = 1 + \frac{r_c^4}{r^4} \qquad \phi = \chi = 0 \quad A_{ab} = B_{ab} = 0$$

↓ T-duality along x_1

- D2-branes smeared along x_1

↓ rotation in the 12-plane + T-duality along x_1

- D1-D3 bound state with D-strings along x_3

Duality chain... cont'd

- D1-D3 bound state with D-strings along x_3

  ↓ T-duality along x_3

- D0-D2 bound state smeared along x_3

  ↓ rotation in the 23-plane + boost along x_3 + T-duality along x_1

- F1-D1-D3 bound state with F-strings along x_3 and D-strings in the 13-plane

Second order solution

- Eqs that determine $Y_I^{(2)}$

$$H^{m,I}_{(0)} Y_I^{(2)} + H^{m,I}_{(1)} \partial_r Y_I^{(2)} + H^{m,I}_{(2)} \partial_r^2 Y_I^{(2)} = K^{m,ab,I} \partial_a \partial_b Y_I^{(0)} + L^{m,ab,IJ} \partial_a Y_I^{(0)} \partial_b Y_I^{(0)}$$

- particular solution $\longrightarrow$ $Y_I^{(2)} = \sum_{n=0}^{\infty} \frac{C_{I,n}^{(2)}}{r^{2+4n}}$

- solution of the homogeneous part $\longrightarrow$ $\delta Y_I = \sum_{n=1}^{\infty} \frac{C_{I,n}^{(2)}}{r^{4n}}$

- general second order solution $\longrightarrow$ $Y_I^{(2)} + \delta Y_I$

- demand δY_I to be a small perturbation of $Y_I^{(0)}$ and whole solution to be asymptotically flat $\longrightarrow$ $\delta Y_I = 0$