Defects in Open String Field Theory:

Organizing the OSFT Landscape

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To Appear, (w/ ***Toshiko Kojita, Toru Masuda*** *and* ***Martin Schnabl)***

related works

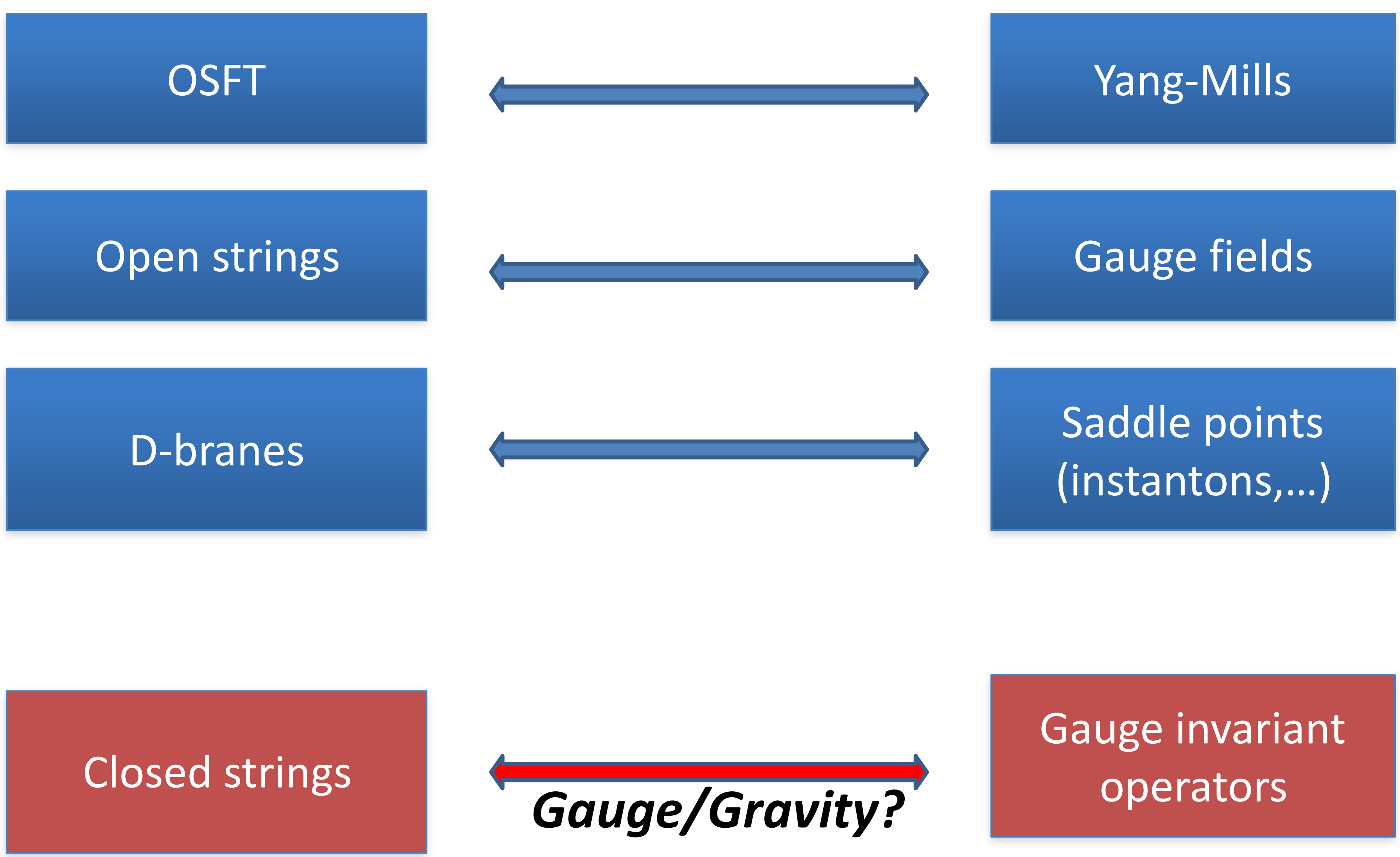
1406.3021, JHEP 1410 (2014) 029 *(w/* ***Ted Erler****)* *(solution for any D-brane system)*

1506.03723, JHEP 1508 (2015)149 *(w/* ***Martin Schnabl****)* *(non-pertrurbative marginal deformations II)*
1402.3546, JHEP 1405 (2014) 004 *(non-pertrurbative marginal deformations I)*
1207.4785, JHEP 1307 (2013) 033 *(w/* ***Matej Kudrna*** *and* ***Martin Schnabl****)* *(boundary state from OSFT)*
1201.5122, JHEP 1206 (2012) 084 *(w/* ***Ted Erler****)* *(pure gauge form of OSFT solutions II)*
1201.5119 , JHEP 1204 (2012) 107 *(w/* ***Ted Erler****)* *(pure gauge form of OSFT solutions I)*

Theory of Fundamental Interactions, Napoli
16/09/2015

Open String Field Theory (OSFT) is a microscopic theory for D-branes, formulated as a ***field theoretic*** description of ***open strings***

Helpful analogy



OPEN STRING FIELD THEORY

- Fix a bulk CFT (closed string background)
- Fix a reference BCFT_0 (open string background, D-brane's system)
- The string field is a state in BCFT_0

$$|\psi\rangle = \sum_i t_i \, \psi^i(0)|0\rangle_{SL(2,R)}$$

- There is a non-degenerate inner product (BPZ)

$$\langle \psi, \phi \rangle = \langle \psi(-1)\phi(1) \rangle^{Disk}_{\mathrm{BCFT}_0}$$

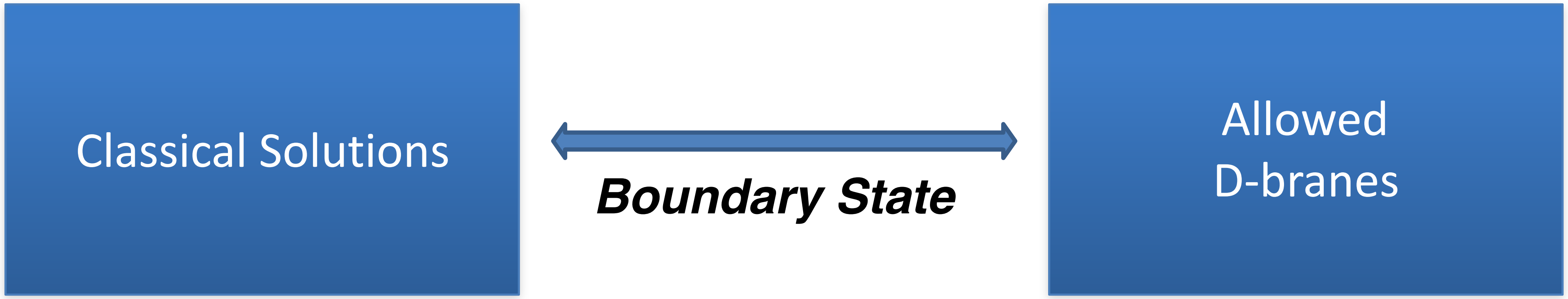
- The bpz-inner product allows to write a target-space action

$$S[\psi] = -\frac{1}{2}\langle \psi, Q\psi \rangle_{\mathrm{BCFT}_0} - \frac{1}{3}\langle \psi, \psi * \psi \rangle_{\mathrm{BCFT}_0} = S_{eff}[t_i]$$

- Witten product *: associative product between states (OPE+conf. map
- ***Equation of motion***

$$Q\Psi + \Psi * \Psi = 0$$

OSFT CONJECTURE (*once known as Sen's Conjectures*)



- Key tool for connecting the two sets is the OSFT construction of the boundary state (**Kiermaier, Okawa, Zwiebach (2008), Kudrna, CM, Schnabl (2012)**)

- The (KMS) boundary state is constructed from gauge invariant quantities starting from a given solution

$$Q\Psi_* + \Psi_*^2 = 0 \quad \Longrightarrow \quad \begin{aligned} &|B_*\rangle = \sum_\alpha n_*^\alpha \, |V_\alpha\rangle\rangle \\ &n_*^\alpha = \langle V^\alpha | B_* \rangle = \langle V^\alpha(0) \rangle_{\rm disk}^{\rm BCFT_*} \;=\; W_{V^\alpha}[\Psi_* - \boxed{\Psi_{\rm tv}}] \end{aligned}$$

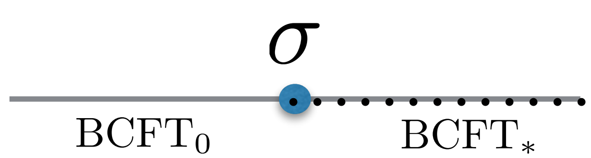
Tachyon Vacuum
Sen-Zwiebach 1999
Schnabl 2005

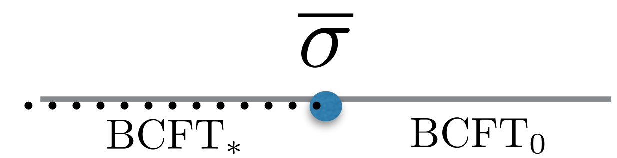
- ***Intriguing possibility of relating BCFT consistency conditions (Cardy-Lewellen, Pradisi-Sagnotti-Stanev) with OSFT equation of motion!***

SOLUTION FOR ANY BACKGROUND

Erler, CM (2014)

- A change in boundary conditions is encoded in a bcc operator (*Cardy, '86-'89*)



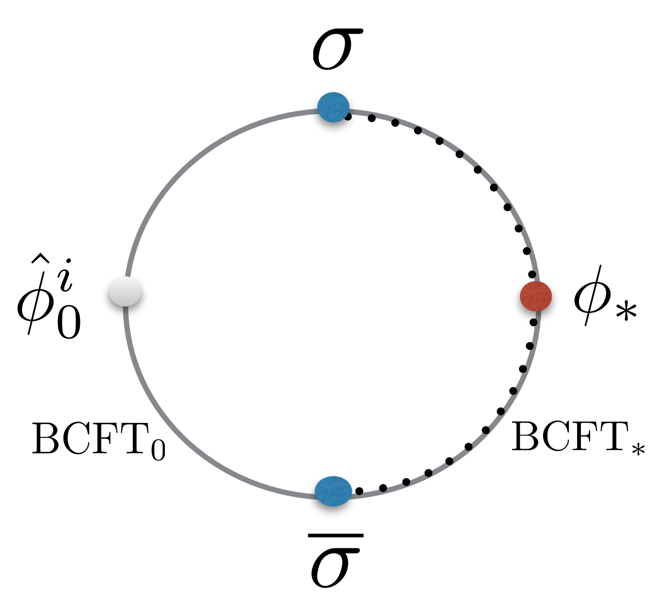


- OSFT: describe the dof of a target **BCFT*** using the dof of a reference **BCFT$_0$**

$$\phi_*(0)|0\rangle_* \to \sigma(1)\phi_*(0)\overline{\sigma}(-1)|0\rangle_0 = \sum_i t_i \phi_0^i(0)|0\rangle_0$$

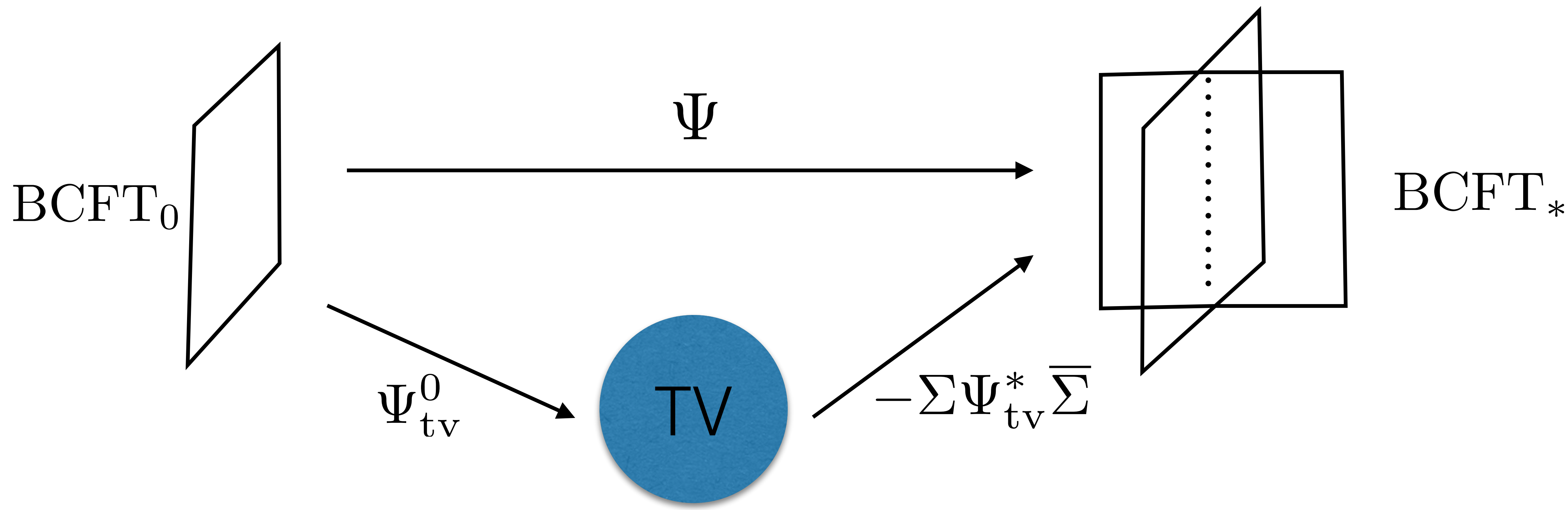
Idea back from Vacuum SFT (Rastelli, Sen, Zwiebach,2000)

$t_i =$



$$\langle \hat{\phi}_0^i | \phi_0^j \rangle = \delta^{ij}$$

- Connect two generic backgrounds by passing through the ***tachyon vacuum*** (***simplest universal solution: no D-branes***)



$$\Psi = \Psi^0_{\mathrm{tv}} - \Sigma\Psi^*_{\mathrm{tv}}\overline{\Sigma}$$

$$\Sigma \in \mathcal{H}_{0*}$$

$$\overline{\Sigma} \in \mathcal{H}_{*0}$$

$$\overline{\Sigma}\Sigma = 1$$

$$Q\Psi + \Psi^2 = 0$$

$$Q_{\mathrm{tv}}\overline{\Sigma} = Q_{\mathrm{tv}}\Sigma = 0$$

- **The Sigma's can be constructed due to the trivial cohomology at the Tachyon Vacuum, using bcc's**

$$Q_{\rm tv}A \equiv QA + [\Psi_{\rm tv}, A] = 1$$

No open strings at TV

$$Q_{\rm tv}\overline{\Sigma} = Q_{\rm tv}\Sigma = 0 \longrightarrow \begin{array}{l} \Sigma = Q_{\rm tv}(A\sigma) \\ \overline{\Sigma} = Q_{\rm tv}(A\overline{\sigma}) \end{array}$$

$$\overline{\Sigma}\Sigma = Q_{\rm tv}\left(A\overline{\sigma}\sigma\right) = 1$$ **IF** $$\overline{\sigma}\sigma = 1$$

- **Convenient universal choice**

$$\sigma = e^{i\sqrt{h}X^0}\sigma_*^{(c=25)}$$

$$\overline{\sigma} = e^{-i\sqrt{h}X^0}\overline{\sigma}_*^{(c=25)}$$

Explicitly possible for time independent backgrounds! (this adds a pure gauge time-like Wilson line)

Chan Paton's factors gets DYNAMICALLY GENERATED!

So we are now in a new phase for OSFT

- ***Tantalizing conjecture***

OSFT EOM implies BCFT constraints (bootstrap)

This is why EM works, essentially!

- ***Can we find the most generic solution to OSFT?***
- ***All known D-branes give rise to solutions, can we reverse the argument to DISCOVER new D-branes?***
- ***Long-standing problem in CFT!***

…As a first step in this challenge let's see how to generate new solutions from known ones:

Topological Defects in OSFT

Open string defect operators in OSFT

(Kojita, Masuda, CM, Schnabl, 2015)

$$\mathcal{D} : H_{\rm open} \to H'_{\rm open}$$

$$[\mathcal{D}, Q] = 0$$

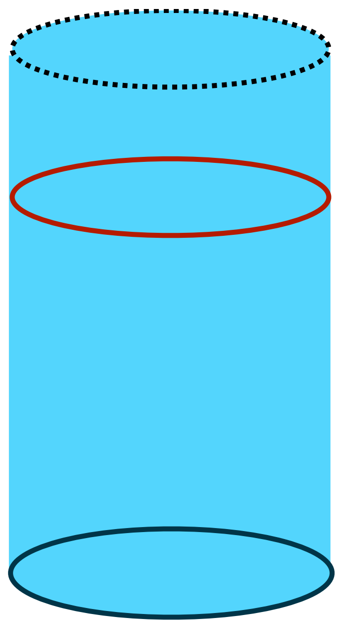
$$\mathcal{D}(\Psi_1 * \Psi_2) = (\mathcal{D}\Psi_1) * (\mathcal{D}\Psi_2)$$

- They map solutions to solutions

$$Q\Psi + \Psi * \Psi = 0 \qquad \to \qquad Q(\mathcal{D}\Psi) + (\mathcal{D}\Psi) * (\mathcal{D}\Psi) = 0$$

- ***Generalization of symmetries*** (which are ***group-like defects***)

- An operator $\mathcal{D}$ can be explicitly constructed starting from a ***closed*** topological defect line D_{cl} .



D_{cl}

$|\Psi\rangle_{\mathrm{closed}}$

$$[D_{\mathrm{cl}}, T(z)] = [D_{\mathrm{cl}}, \bar{T}(\bar{z})] = 0$$

$$D^d_{\mathrm{cl}} = \sum_{i,\bar{i} \in H_{\mathrm{cl}}} D^d_{i\bar{i}} \, P_{i\bar{i}}$$ *Petkova-Zuber (2000)*

- In (diagonal) RCFT: as many fundamental defects as irreps. The fusion rules govern their composition and the action on boundary states

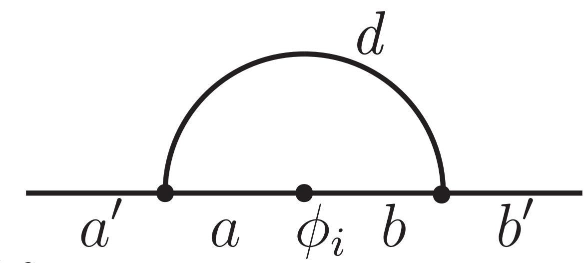
$$D^d_i = \frac{S_{di}}{S_{1i}}$$

$$[\phi_a][\phi_b] = \sum_i N^i_{ab}[\phi_i] \qquad D^d D^c = \sum_i N^i_{cd} \, D^i$$

$$D^c||a\rangle\rangle = \sum_i N^i_{ca} \, ||i\rangle\rangle$$ *Graham-Watts (2003)*

- In the open string sector we must have *(Diagonal Minimal models from now on!)*

$$\mathcal{D}^d\, \psi_i^{(ab)} = \sum_{\substack{a' \in d \times a \\ b' \in d \times b}} X^{dab}_{ia'b'}\, \psi_i^{(a'b')} = \sum_{\substack{a' \in d \times a \\ b' \in d \times b}}$$



- Determine *X* coeff. imposing the star algebra homomorphism

$$\mathcal{D}^d \left(\phi_i^{(ab)}(x)\phi_j^{(bc)}(y)\right) = \left(\mathcal{D}^d \phi_i^{(ab)}(x)\right)\left(\mathcal{D}^d \phi_j^{(bc)}(y)\right)$$

$$X^{dac}_{ka'c'} C^{(abc)k}_{ij} = \sum_{b' \in d \times b} C^{(a'b'c')k}_{ij} X^{dab}_{ka'b'} X^{dbc}_{kb'c'},$$

- In explicit case of diag. minimal models we find (pentagon identity)

$$X^{dab}_{ia'b'} = F_{di}\begin{bmatrix} a & b \\ a' & b' \end{bmatrix} \frac{\sqrt{F_{1a'}\begin{bmatrix} a & d \\ a & d \end{bmatrix} F_{1b'}\begin{bmatrix} b & d \\ b & d \end{bmatrix}}}{F_{1i}\begin{bmatrix} a & b \\ a & b \end{bmatrix}}$$

Generalizes Graham and Watts (2003)

- The composition (fusion) is trickier than in the bulk case. Naively we would expect

$$\mathcal{D}^d \mathcal{D}^c \psi = \bigoplus_e {N_{dc}}^e \mathcal{D}_e \psi \qquad (?)$$

- However explicit computation reveals a similarity transformation !

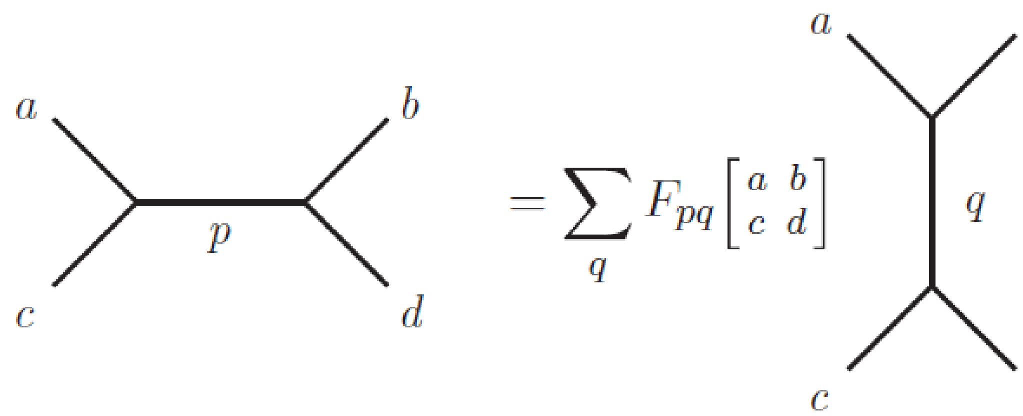
$$\mathcal{D}^d \mathcal{D}^c \psi = U_{dc} \left(\bigoplus_e {N_{dc}}^e \mathcal{D}_e \psi \right) U_{dc}^{-1}$$

$$(U_{dc})^T = (U_{dc})^{-1}$$

$$(U_{dc})^{\{aa'a''\}[e;a,a'']} = \sqrt{\frac{F_{1a''}\begin{bmatrix} d & a' \\ d & a' \end{bmatrix} F_{1a'}\begin{bmatrix} c & a \\ c & a \end{bmatrix}}{F_{1a''}\begin{bmatrix} e & a \\ e & a \end{bmatrix} F_{1e}\begin{bmatrix} d & c \\ d & c \end{bmatrix}}}\, F_{a'e}\begin{bmatrix} d & c \\ a'' & a \end{bmatrix}$$

These objects stay invariant under Moore-Seiberg F-matrix “gauge” symmetry (rescaling of CVO’s)

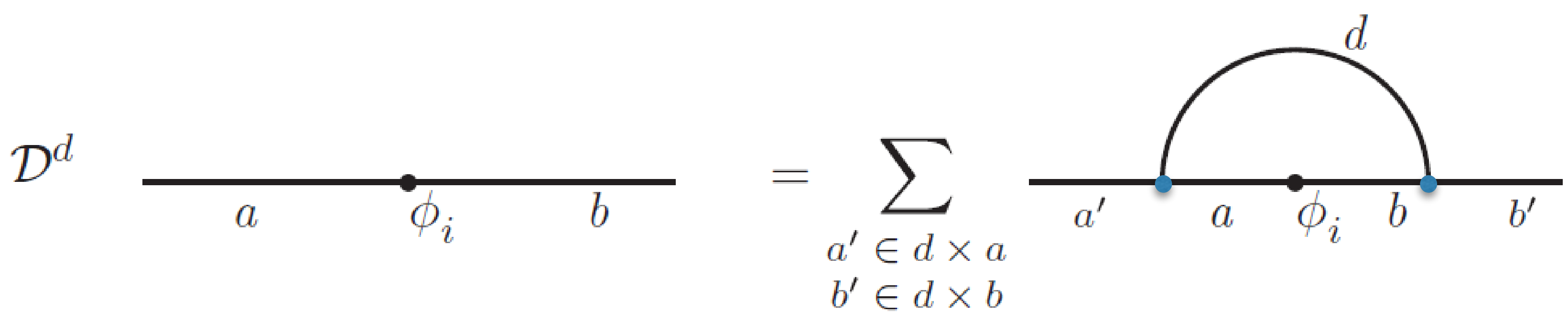
- The need for ***U*** is transparent using defect-network manipulations



See also Frolich, Fuchs, Runkel , Schwieger (TFT) (2006)

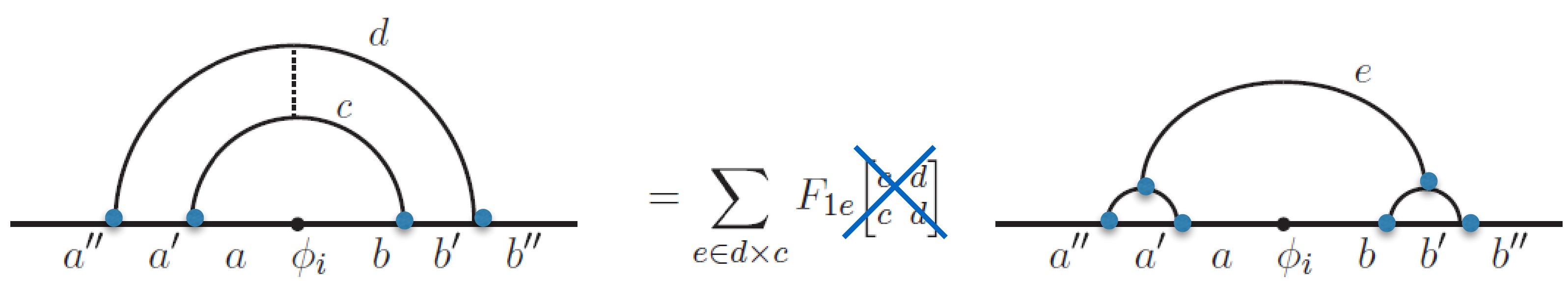
[These are defect networks, not (a priori) conformal blocks!]

- Defect action



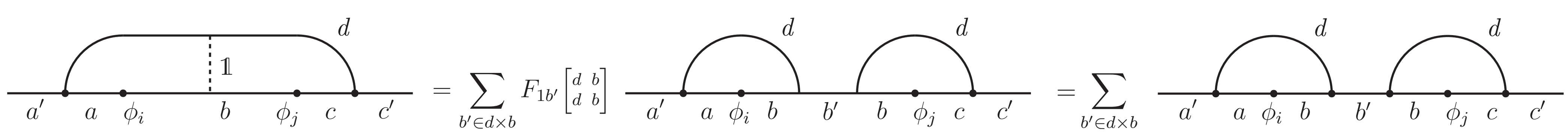
$= \mathcal{D}^d \phi_i^{ab}$

- Defect fusion



$= U_{dc}\left(\bigoplus_{e\in d\times c} \mathcal{D}^e \phi_i^{ab}\right) U_{dc}^T$

- Important to keep track of ***defect junctions*** (from which the square roots of F originates), example:



$\mathcal{D}^d(\phi_i^{ab} * \phi_j^{bc})$ $\qquad\qquad$ $(\mathcal{D}^d\phi_i^{ab}) * (\mathcal{D}^d\phi_j^{ab})$

OSFT observables and defects

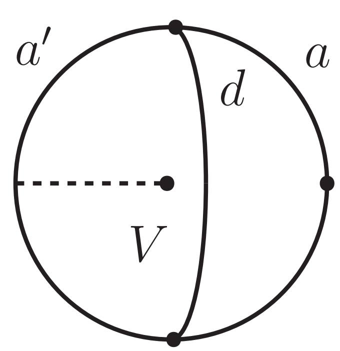
- Change in the (off-shell) action

$$S_{\rm OSFT}(\mathcal{D}^d\Psi) = \frac{g_d}{g_1} S_{\rm OSFT}(\Psi) \qquad g_a \equiv \langle\!\langle a \| 0\rangle_{SL(2,C)} = \langle 1\rangle^{\rm BCFT_{\it a}}_{\rm disk}$$

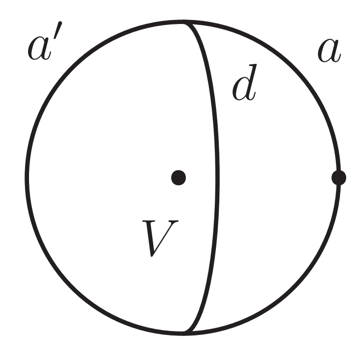
- Change in the gauge-invariant coupling to closed strings (Ellwood invariant)

$$\mathrm{Tr}_V[\mathcal{D}^d\Psi] = \mathrm{Tr}_{D^d_{\rm cl}V}[\Psi]$$

$\mathrm{Tr}_V\left[\mathcal{D}^d\Psi\right] = \sum_a \sum_{a'\in d\times a}$

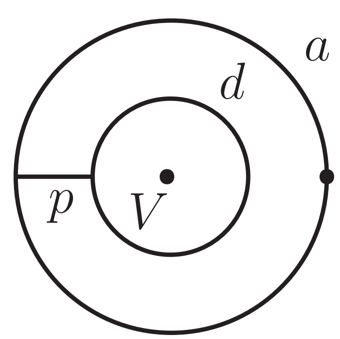


$f^{(1)}\circ\Psi^{aa} = \sum_a \sum_{a'\in d\times a} F_{1a'}\begin{bmatrix} d & a \\ d & a \end{bmatrix}$

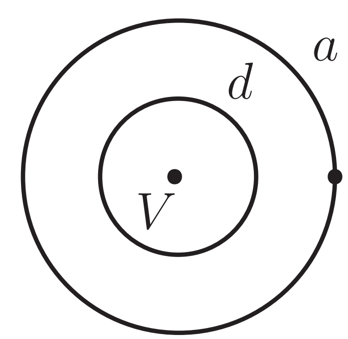


$f^{(1)}\circ\Psi^{aa}$

$= \sum_{a,p} \sum_{a'\in d\times a} F_{1a'}\begin{bmatrix} d & a \\ d & a \end{bmatrix} F_{a'p}\begin{bmatrix} a & a \\ d & d \end{bmatrix}$



$f^{(1)}\circ\Psi^{aa} = \sum_a$



$f^{(1)}\circ\Psi^{aa} = \mathrm{Tr}_{D^d V}[\Psi]$

OSFT boundary state and defects

- Given a solution $\Psi_{X\to Y}$, we can compute its boundary state

$$|B(\Psi_{X\to Y})\rangle_{\rm OSFT} = \|Y\rangle\rangle_{\rm BCFT}$$

Kiermaier, Okawa, Zwiebach (2008)
Kudrna, CM, Schnabl (2012)

- Previous slide computation has the important consequence that

$$|B(\mathcal{D}^d\Psi_{X\to Y})\rangle_{\rm OSFT} = D^d_{\rm cl}\|Y\rangle\rangle_{\rm BCFT}$$

Kojita, Masuda, CM, Schnabl (2015)

$$\mathcal{D}\Psi_{X\to Y} = \Psi_{DX\to DY}$$

Example: Ising Model OSFT

- 3 Virasoro labels, 3 fundamental boundary condtns, 3 topological defects

$$1\,, \qquad \epsilon\,, \qquad \sigma\,.$$

$$\epsilon \times \epsilon = 1$$
$$\epsilon \times \sigma = \sigma$$
$$\sigma \times \sigma = 1 + \epsilon$$

- Solutions are organized by the defects action (***group*** *and* ***duality*** *defects)*

$$\begin{tikzcd}
\Psi_{\sigma\to 1} \arrow[r, "\mathcal{D}_\sigma"] \arrow[d, shift left, "\mathcal{D}_\varepsilon"] & \Psi_{1+\varepsilon\to\sigma} \arrow[r, "\mathcal{D}_\sigma"] \arrow[loop above, "\mathcal{D}_\varepsilon"] & \Psi_{2\sigma\to 1+\epsilon} \arrow[r, "\mathcal{D}_\sigma"] \arrow[d, shift left, "\mathcal{D}_\varepsilon"] & \cdots \\
\Psi_{\sigma\to\varepsilon} \arrow[r, "\mathcal{D}_\sigma"'] \arrow[u, shift left, "\mathcal{D}_\varepsilon"] & \Psi'_{1+\varepsilon\to\sigma} \arrow[r, "\mathcal{D}_\sigma"'] \arrow[loop below, "\mathcal{D}_\varepsilon"] & \Psi_{2\sigma\to\varepsilon+1} \arrow[r, "\mathcal{D}_\sigma"'] \arrow[u, shift left, "\mathcal{D}_\varepsilon"] & \cdots
\end{tikzcd}$$

- All solutions and their BCFT observables explicitly generated starting from a single solution (for example)

$$\Psi_{\sigma\to 1} = \Psi_{\rm tv}^{\sigma\sigma} - \Sigma^{\sigma 1}\,\Psi_{\rm tv}^{11}\,\bar{\Sigma}^{\sigma 1}$$

CONCLUSIONS

- OSFT is (among other things) a dynamical field theory for BCFT.
- All known (time ind.) BCFT's remarkably give exact analytic solutions of OSFT. The string field is indeed "big enough"!
- Topological defects give rise to new operators in the open string algebra which map solutions to solutions.
- First explicit example of solution generating operators in OSFT: ***they must play an important role in the classification of OSFT solutions.***

★ *Away from diagonal/minimal/rational*

★ *Open **Super**string Field Theory Vs Boundary **S**CFT??*

Thank you.