String pair production in non homogeneous backgrounds

Stefano Bolognesi

Università di Pisa

Work in collaboration with E. Rabinovici and G.Tallarita

Napoli TFI 20/11/2015

Plan of the talk

Introduction to pair production, worldline formalism and non homogeneous backgrounds in field theory

Plan of the talk

Introduction to pair production, worldline formalism and non homogeneous backgrounds in field theory

String theory examples of pair production which are treatable with the *‘‘worldsheet instanton’’ technique*:

1) String suspended between D-branes
2) Holographic Schwinger effect

Work in collaboration with E. Rabinovici and G.Tallarita

Plan of the talk

Introduction to pair production, worldline formalism and non homogeneous backgrounds in field theory

String theory examples of pair production which are treatable with the *‘‘worldsheet instanton’’ technique*:

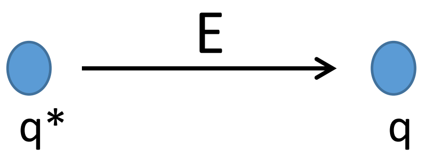
1) String suspended between D-branes
2) Holographic Schwinger effect

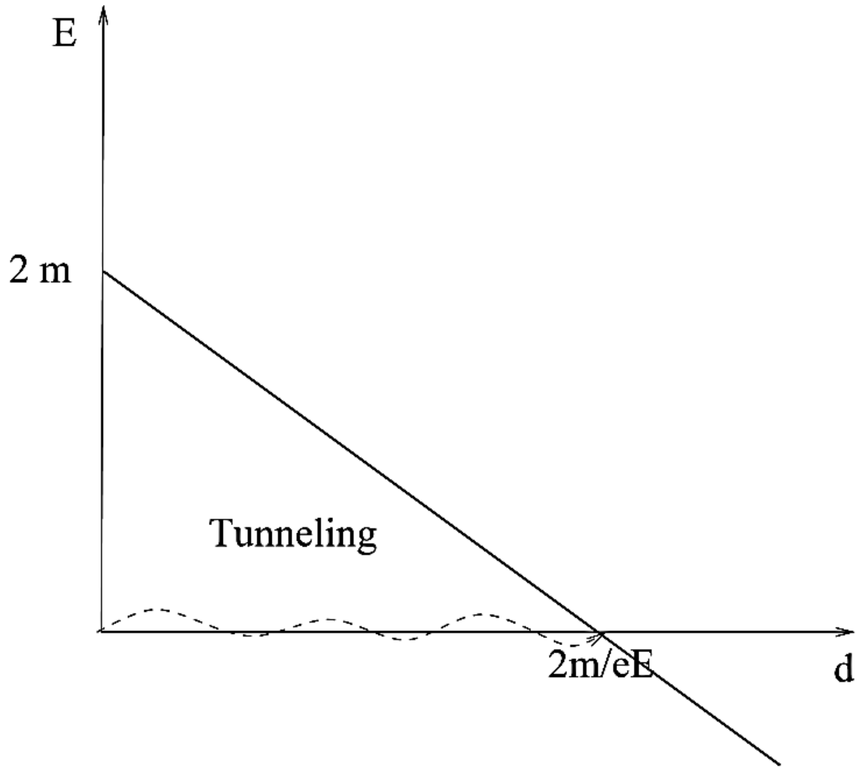
Effects of non-homogeneous backgrounds on string production

Work in collaboration with E. Rabinovici and G.Tallarita

Pair production

Schwinger effect for example, is non-perturbative pair production of q-q* due to electric background

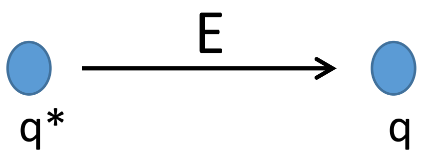




$$\mathrm{Im}\,\mathcal{L}[E] = \frac{e^2E^2}{16\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}\exp\left[-\frac{m^2\pi n}{eE}\right]$$

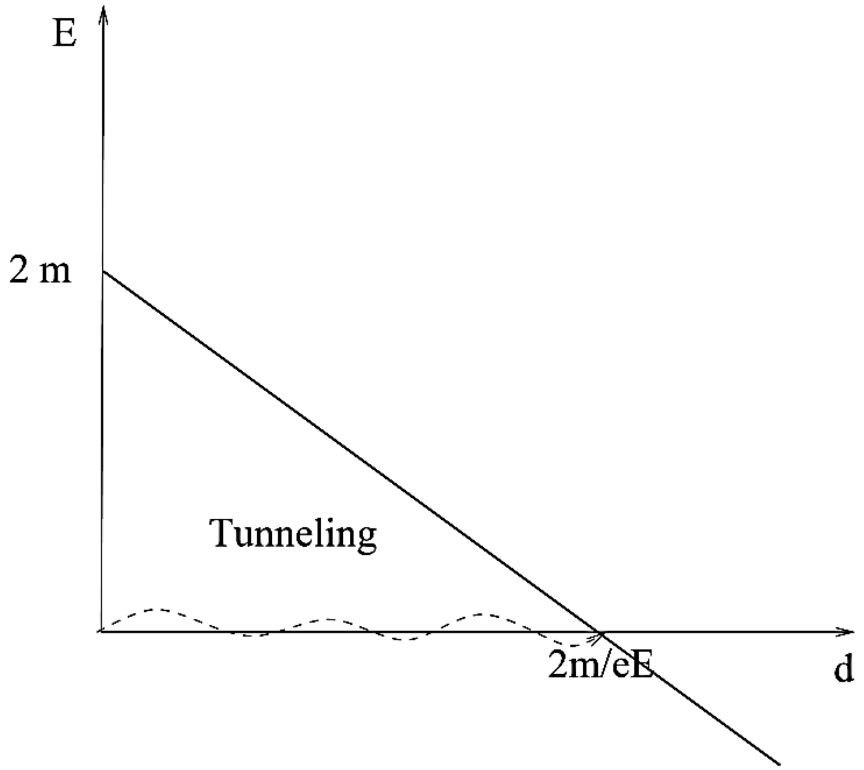
Pair production

Schwinger effect for example, is non-perturbative pair production of q-q*
due to electric background



This effect becomes significant at

$$E \simeq m^2/e$$



Worldline instanton

One searches for stationary solution of the Euclidean worldline action

$$S_E = m \int d\tau \sqrt{\dot{x}^\mu \dot{x}^\mu} + iq \int d\tau \dot{x}^\mu A^\mu$$

Same trajectories of a charged particle moving in a background magnetic field

$$\frac{m\ddot{x}^\mu}{2\sqrt{\dot{x}^\mu \dot{x}^\mu}} = iqF_{\mu\nu}\dot{x}^\nu$$

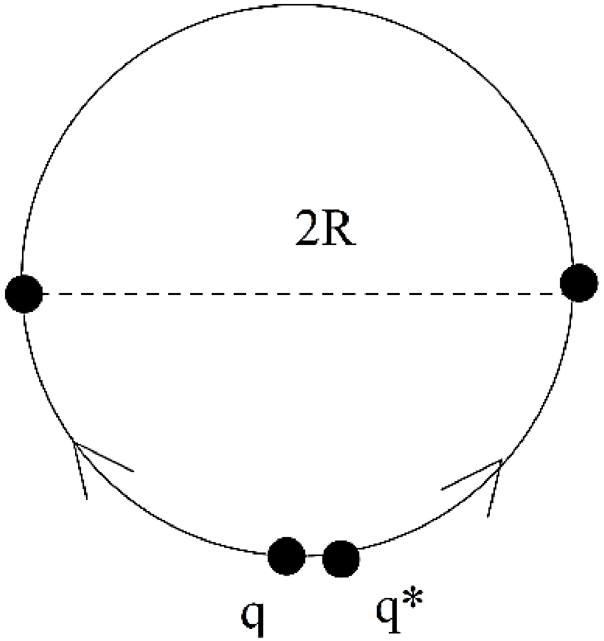
$$\dot{x}^\mu \dot{x}^\mu = \text{const} = L^2$$

Afflek-Alvarez-Manton 82

Worldline instanton

For a constant background the solution is given by a circular trajectory

$$x_3(\tau) = \frac{m}{qE}\cos(2\pi\tau) \qquad x_4(\tau) = \frac{m}{qE}\sin(2\pi\tau)$$



with action consistent with Schwinger formula

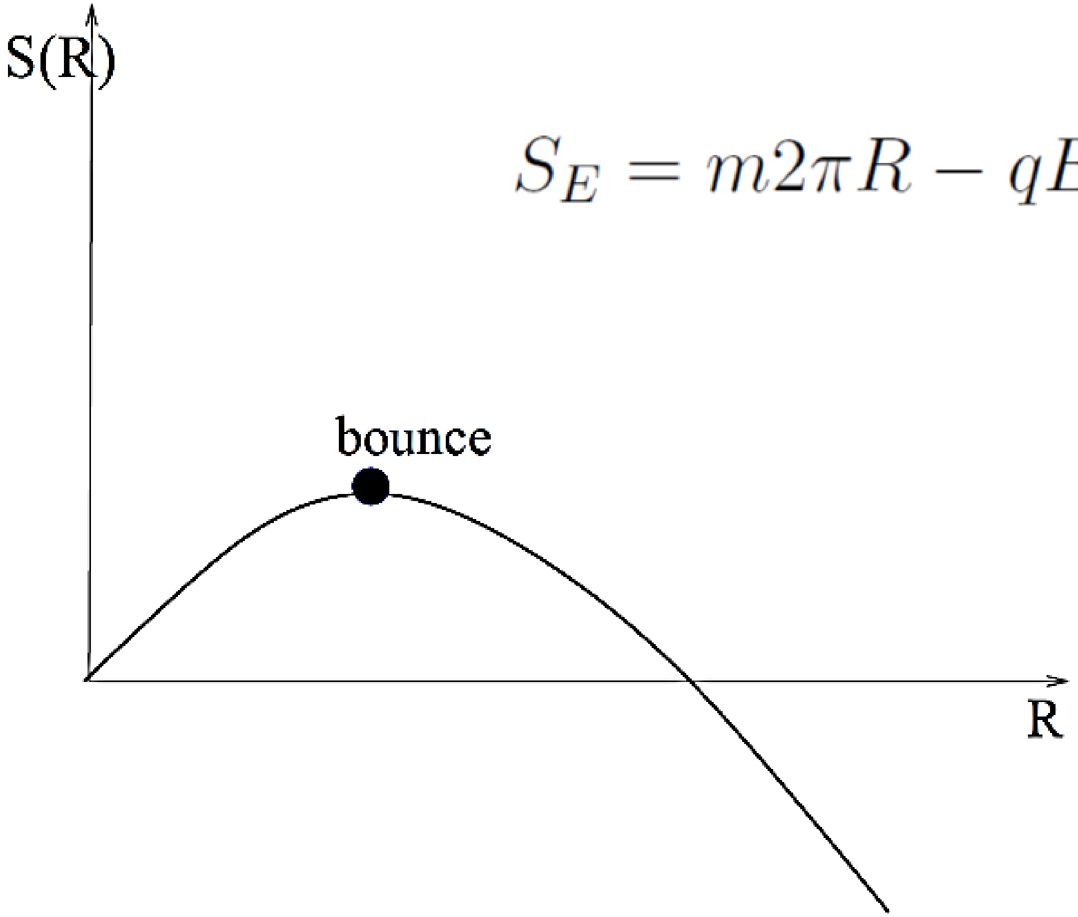
$$S_E = \frac{\pi m^2}{qE}$$

Afflek-Alvarez-Manton 82

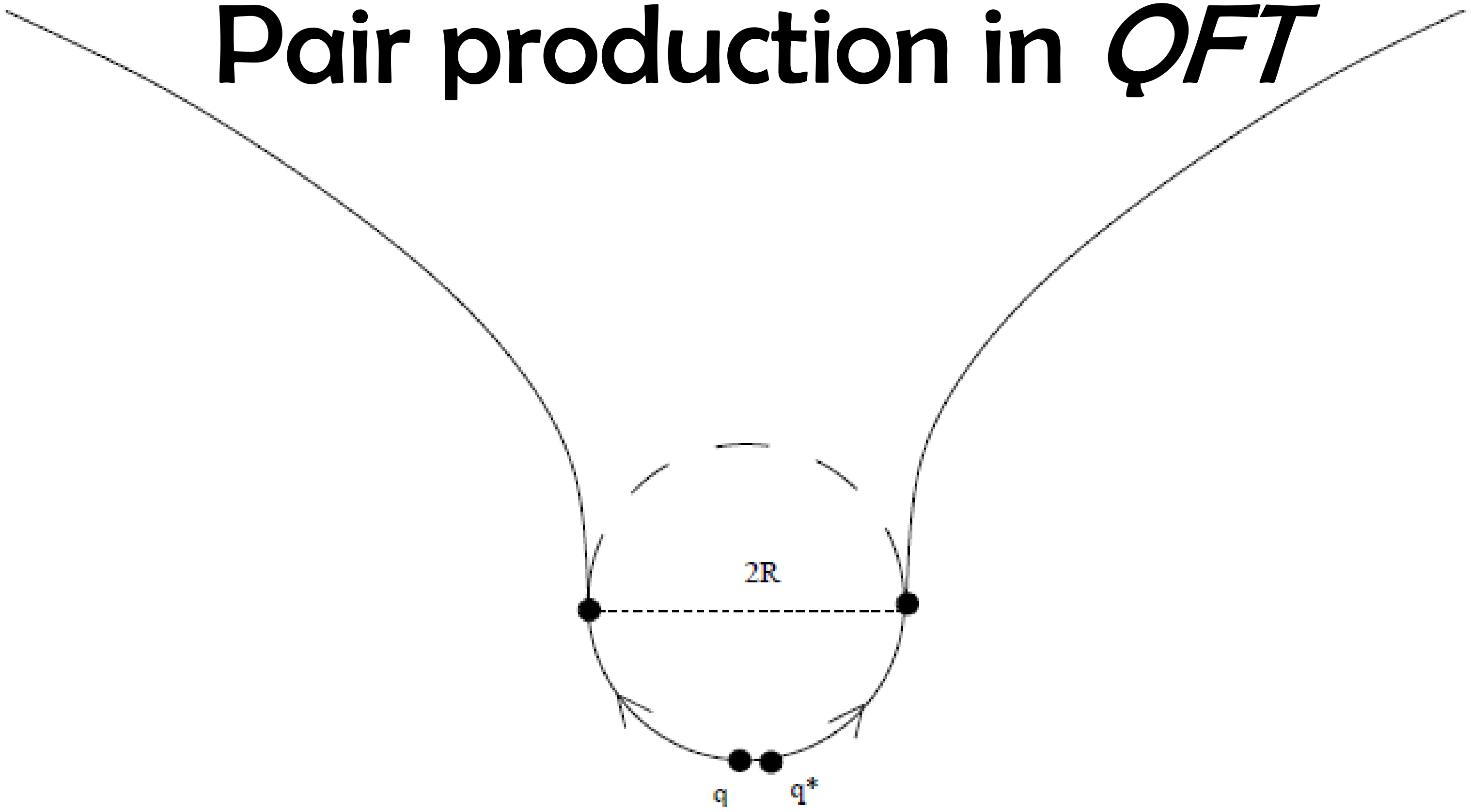
Pair production in *QFT*

Solutions are "extremal" circular orbits with radius R

$$S_E = m2\pi R - qE\pi R^2$$



Pair production in *QFT*



$$S_E = \frac{\pi m^2}{qE}$$

They describe the tunneling in Minkowsky

$$P \propto \exp\left(-S_E\right)$$

Non homogeneous background

Schwinger effect becomes significant at $E \simeq m^2/e$ 10^{18} V/m

Non homogeneous backgrounds can lower this value significantly!

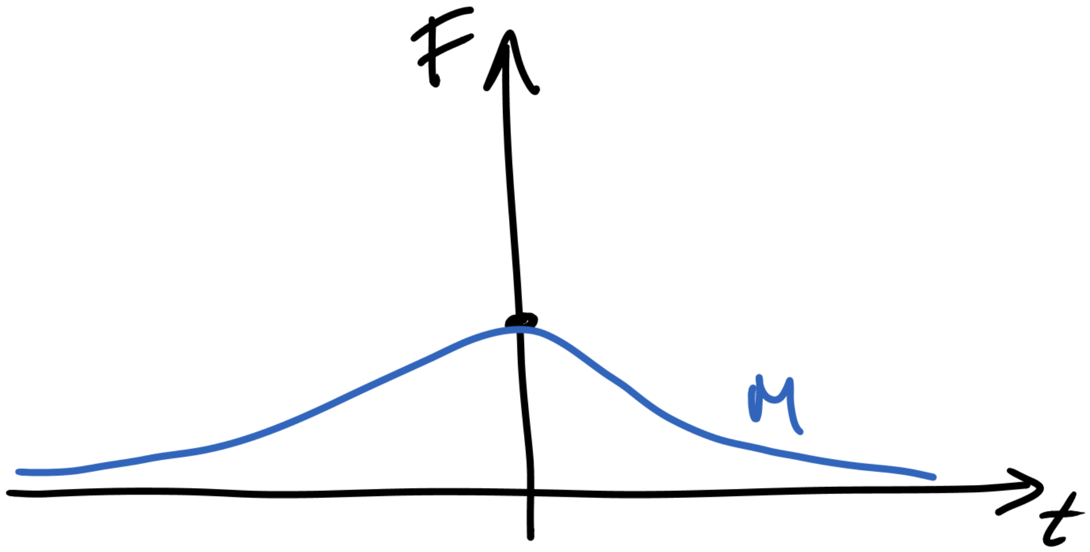
Direct observation of the Schwinger effect may possible in the near future with the use of strong laser pulses

Sauter
Brezin-Itzykson 70
…

Time-dependent pulse

The simplest case is a single pulse of electric field dependent on time only

$$E^3(t) = \frac{E}{\cosh^2(\omega t)}$$



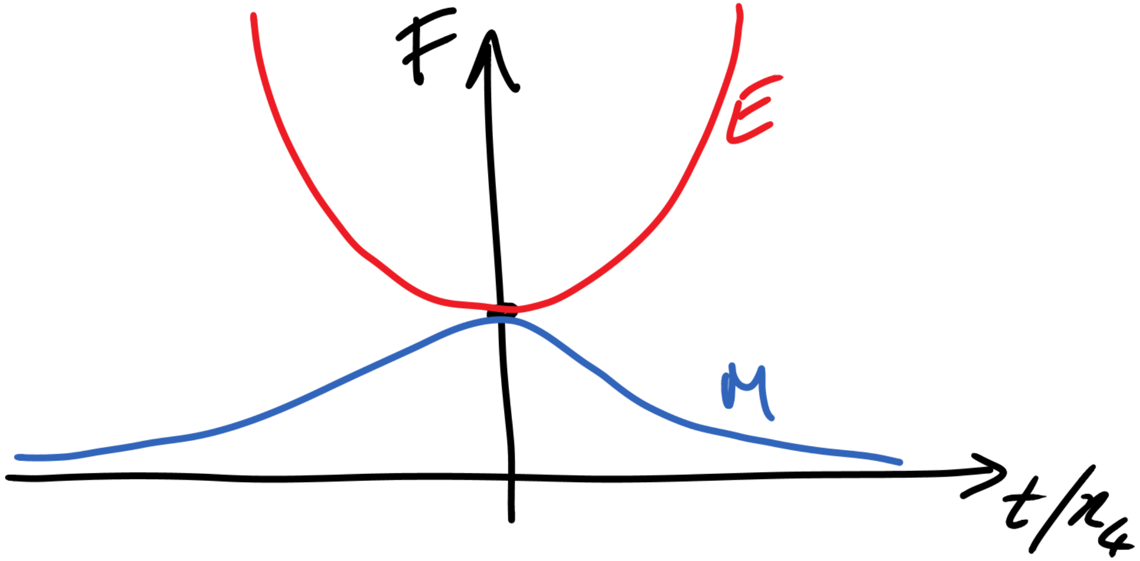
Time-dependent pulse

The simplest case is a single pulse of electric field dependent on time only

$$E^3(t) = \frac{E}{\cosh^2(\omega t)}$$

The Euclidean corresponding field is

$$F_{34} = \frac{-iE}{\cos^2(\omega x_4)}$$



We already see that there should be enhancement of pair production if the instanton is finite!

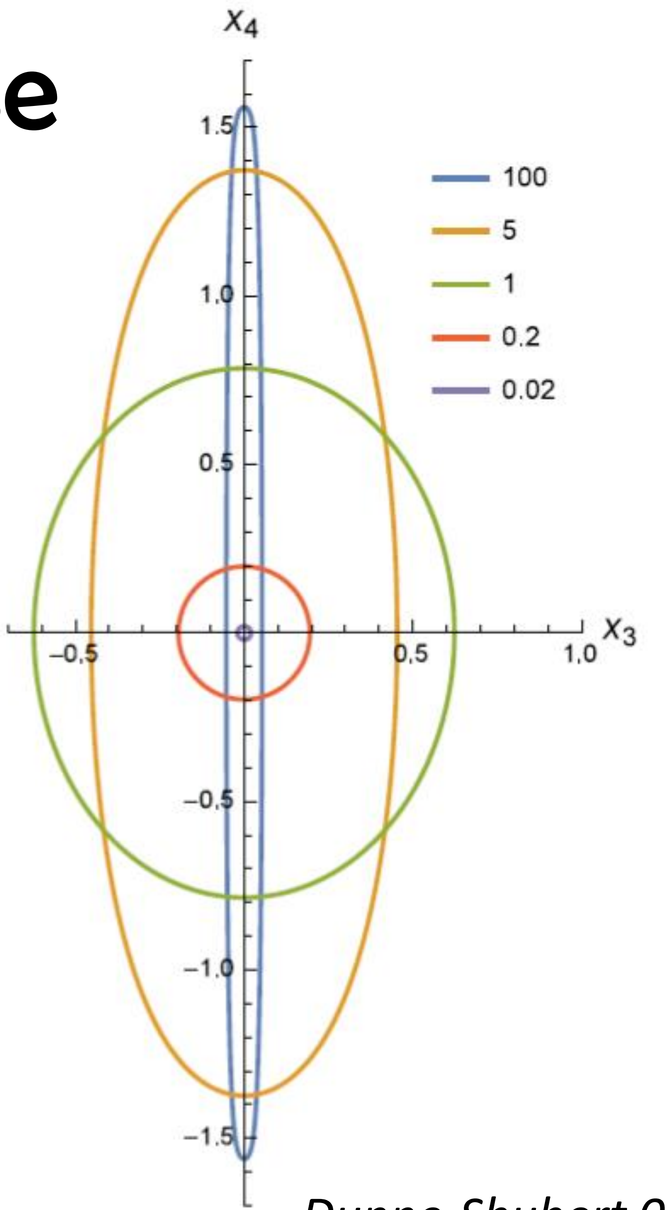
Time-dependent pulse

The exact solution is:

$$x_3(\tau) = \frac{1}{\omega}\frac{1}{\sqrt{1+\gamma^2}}\,\mathrm{arcsinh}(\gamma\cos(2\pi\tau))$$

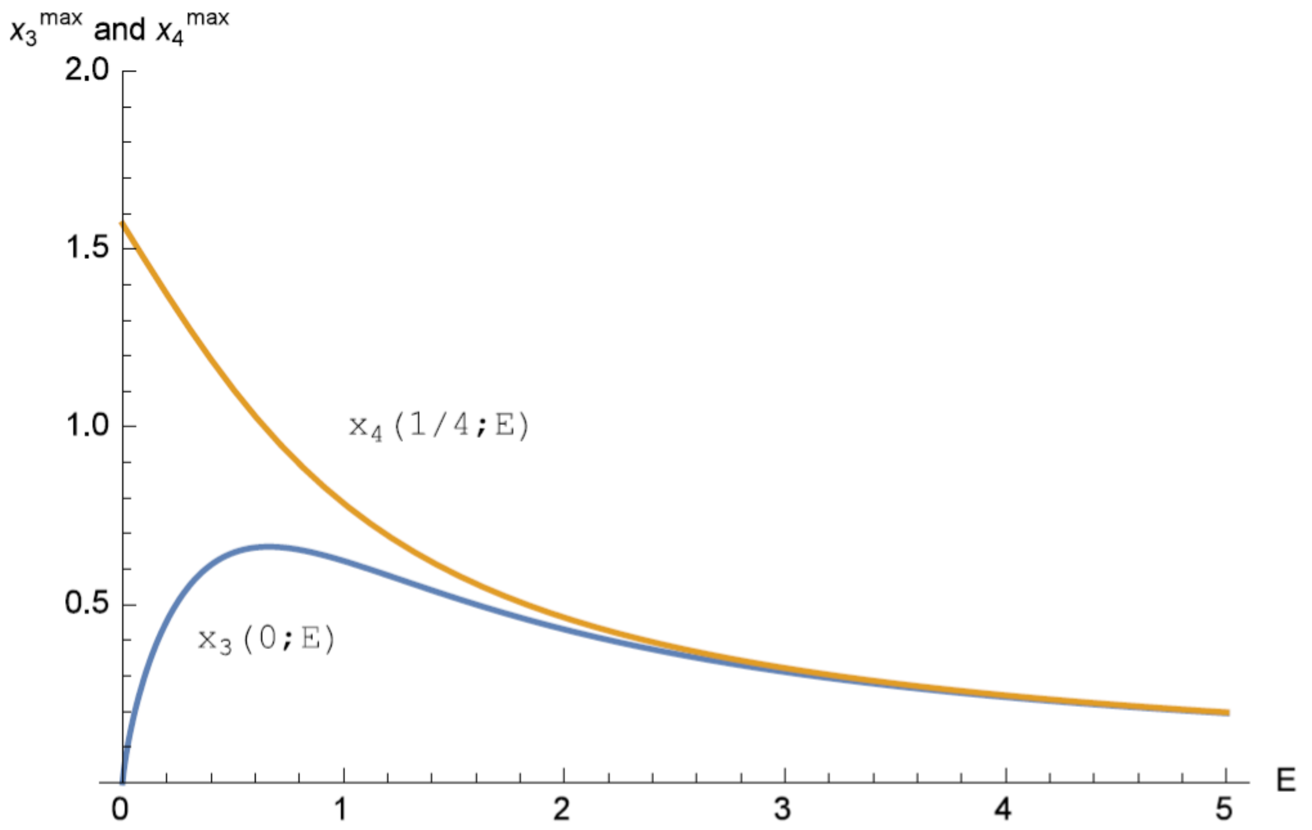
$$x_4(\tau) = \frac{1}{\omega}\arcsin\left(\frac{\gamma}{\sqrt{1+\gamma^2}}\sin(2\pi\tau)\right)$$

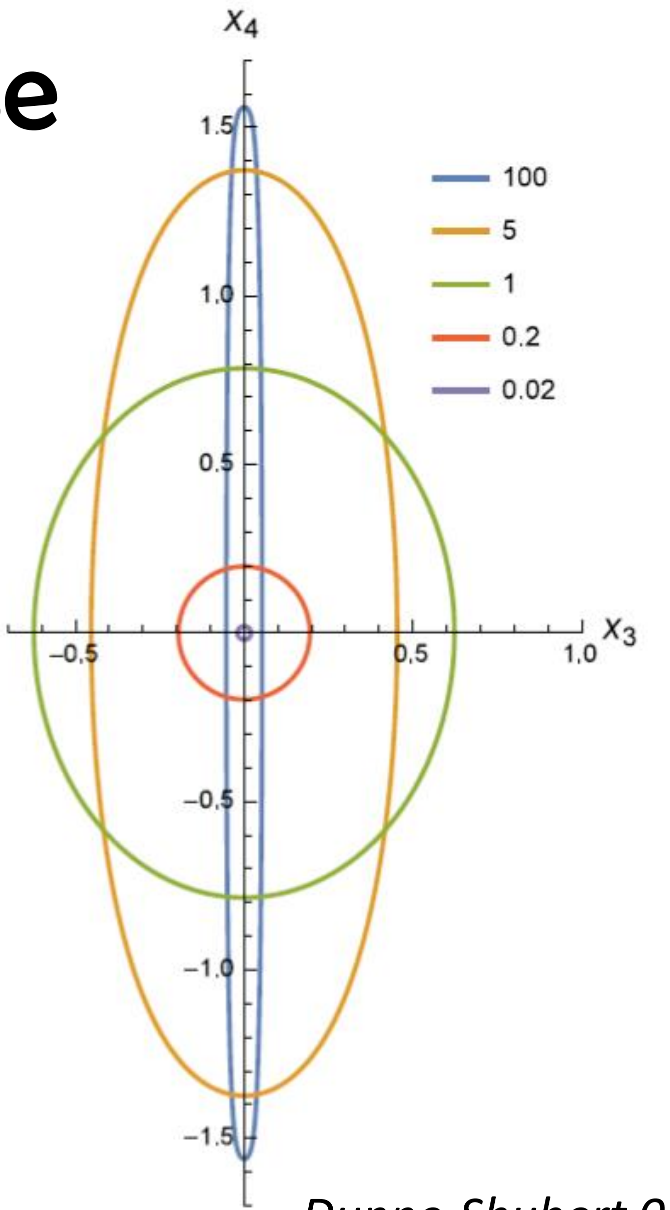
$$\gamma = \frac{m\omega}{qE}$$



Dunne-Shubert 05

Time-dependent pulse





Dunne-Shubert 05

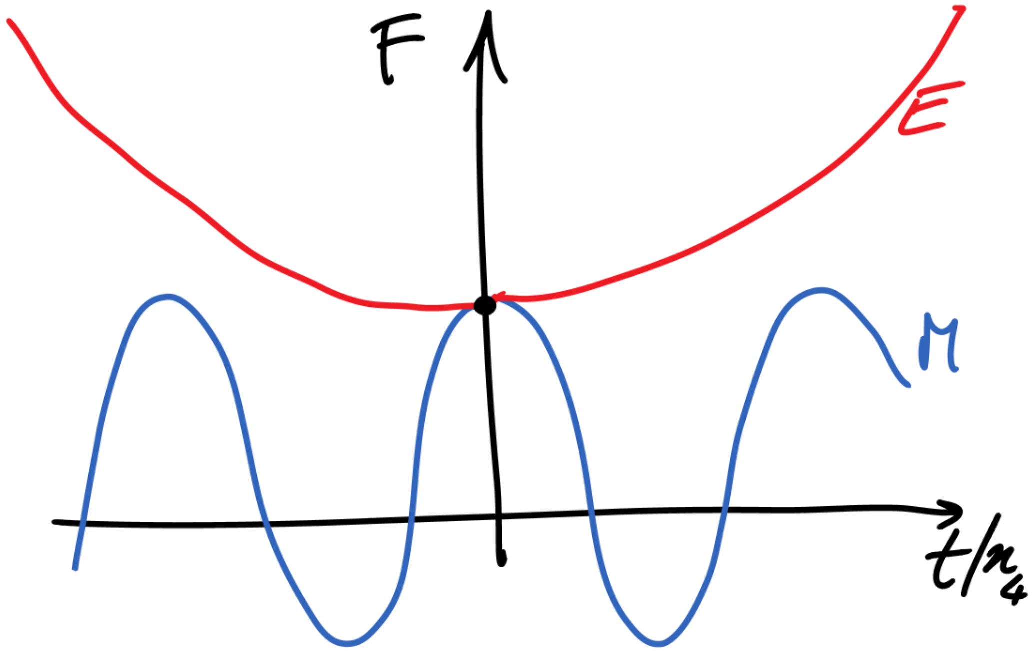
Time-dependent oscillation

Another important case is an oscillating electric field with fixed frequency

$$E^3(t) = E\cos(\omega t)$$

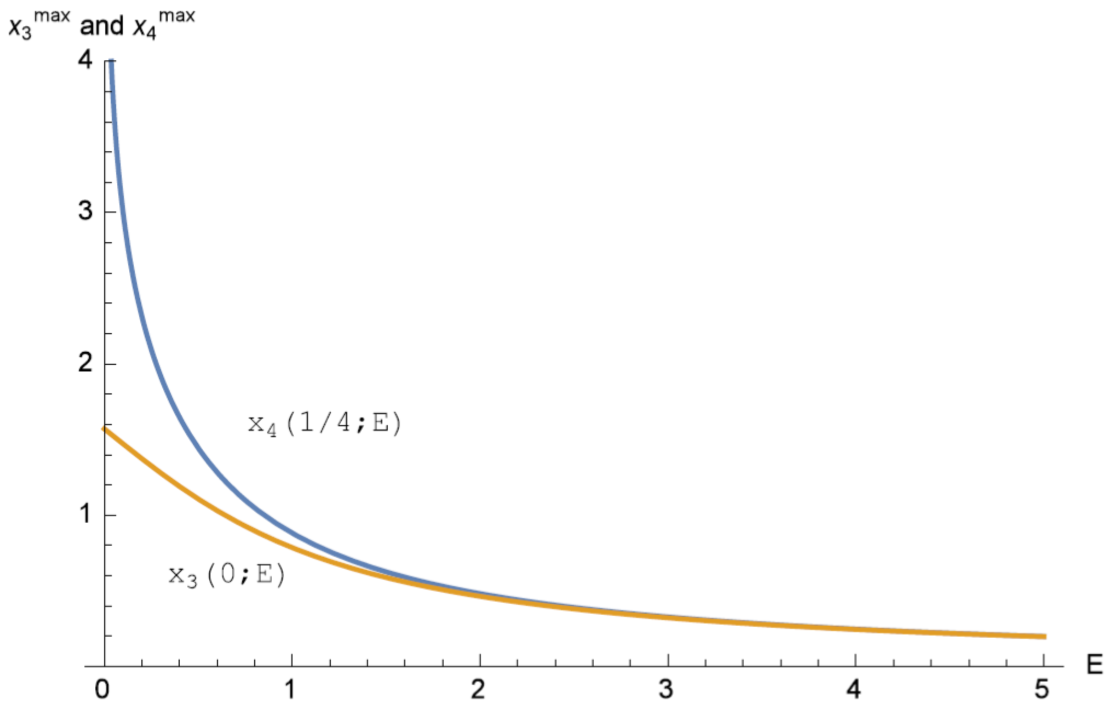
The Euclidean corresponding field is

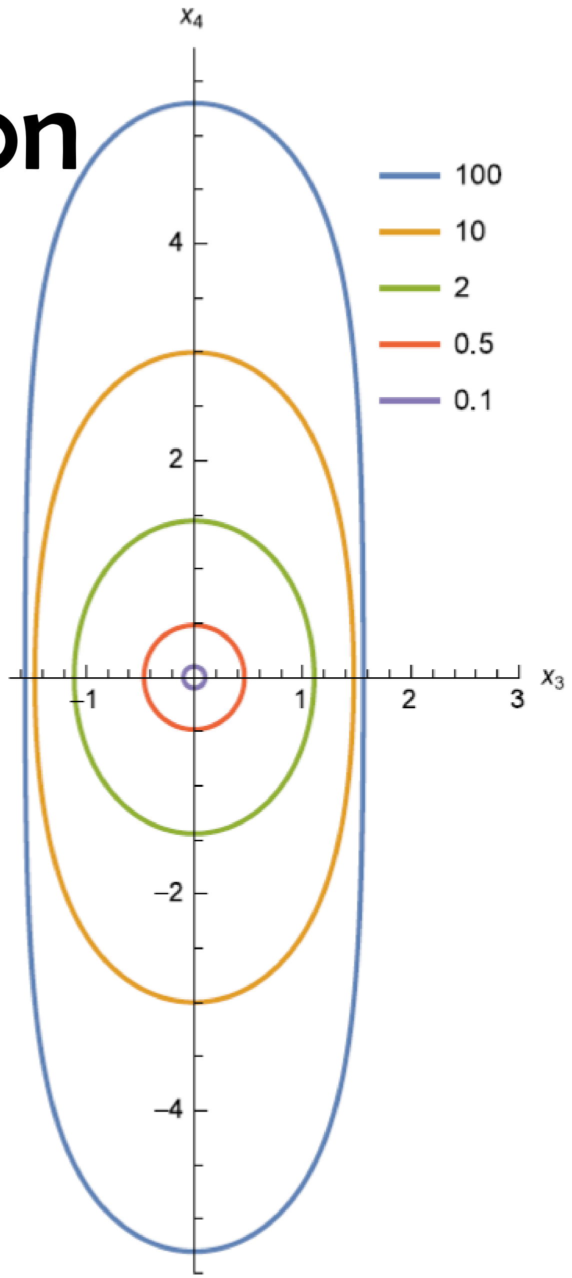
$$F_{34} = -iE\cosh(\omega x_4)$$



Time-dependent oscillation

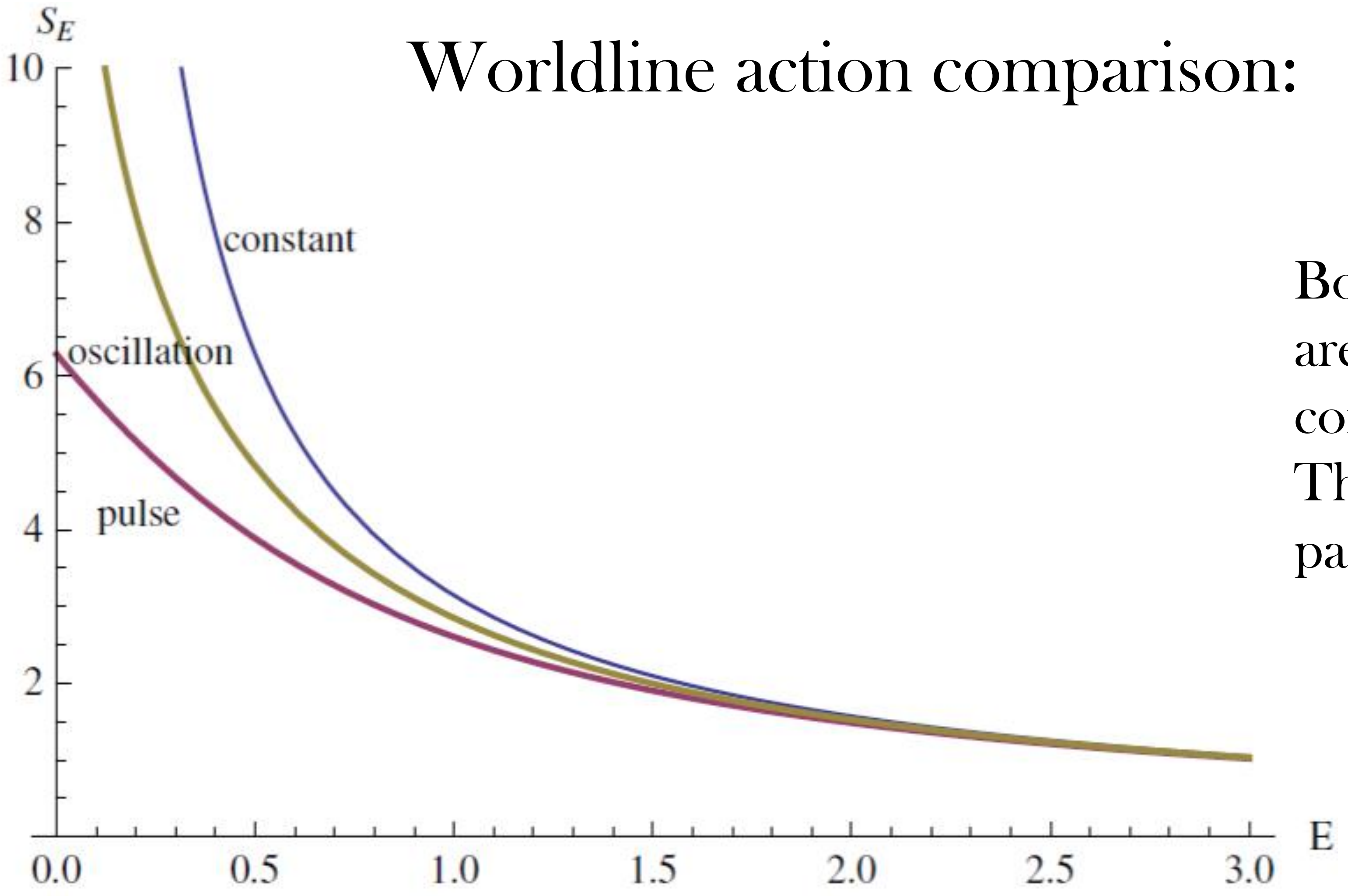
The exact solution is:





Summary of Results

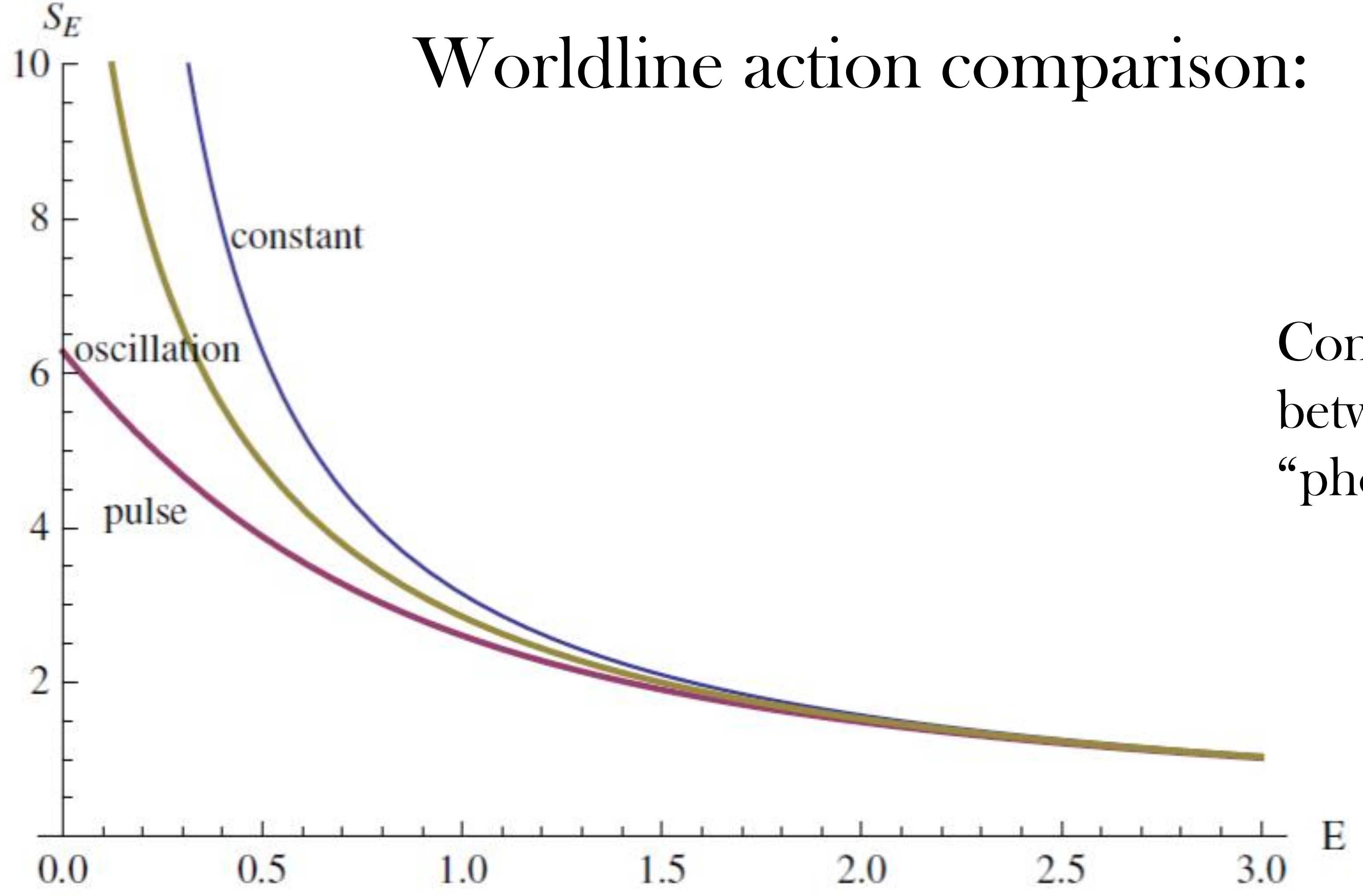
Worldline action comparison:



Both time dependent actions are lower than the “locally constant” approximation. This indicate enhancement of pair production

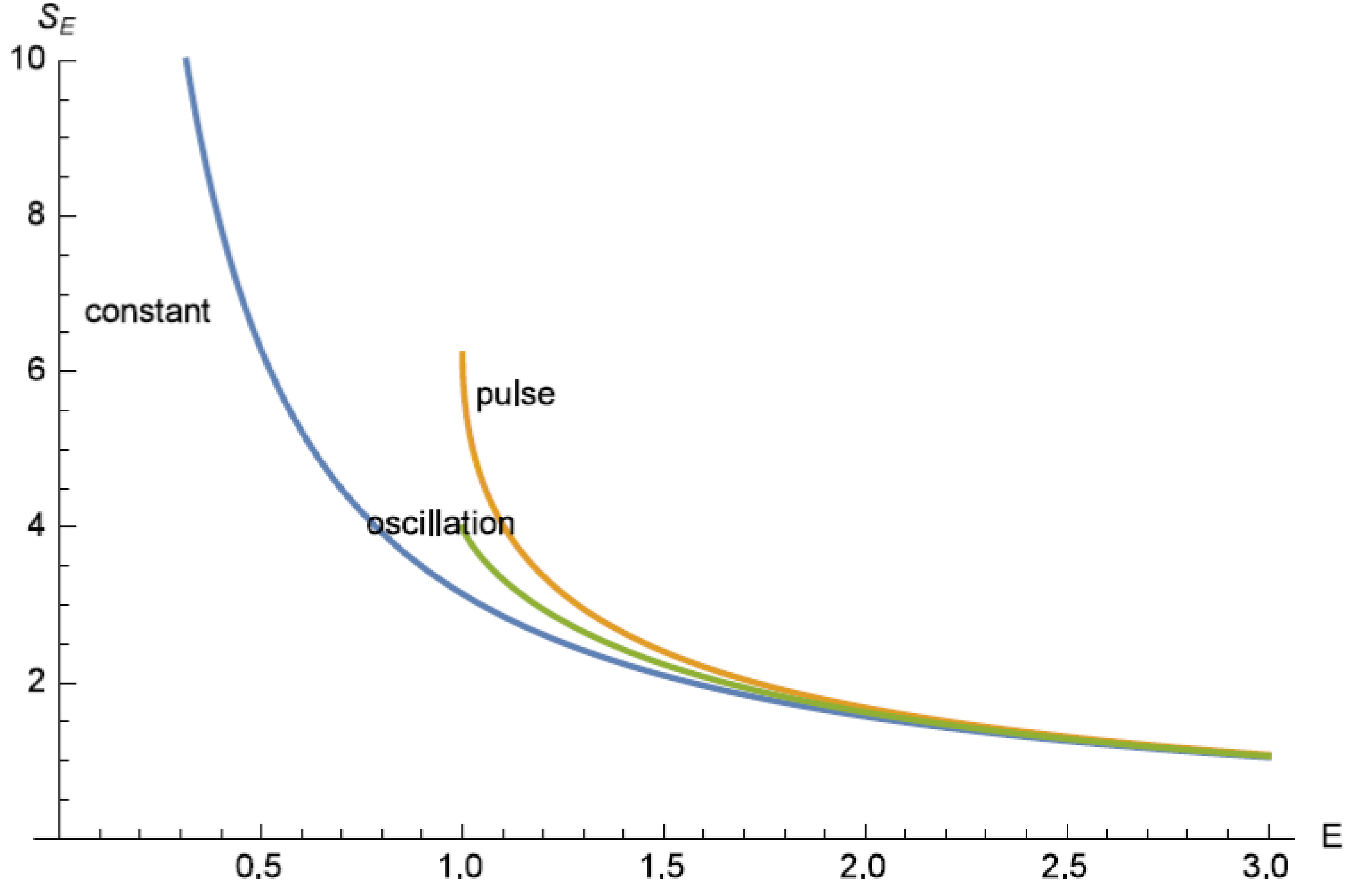
Summary of Results

Worldline action comparison:



Continuous interpolation between “tunnelling” and “photon absorption”

Summary of Results



For space dependent backgrounds is just the opposite

Pair production in string theory

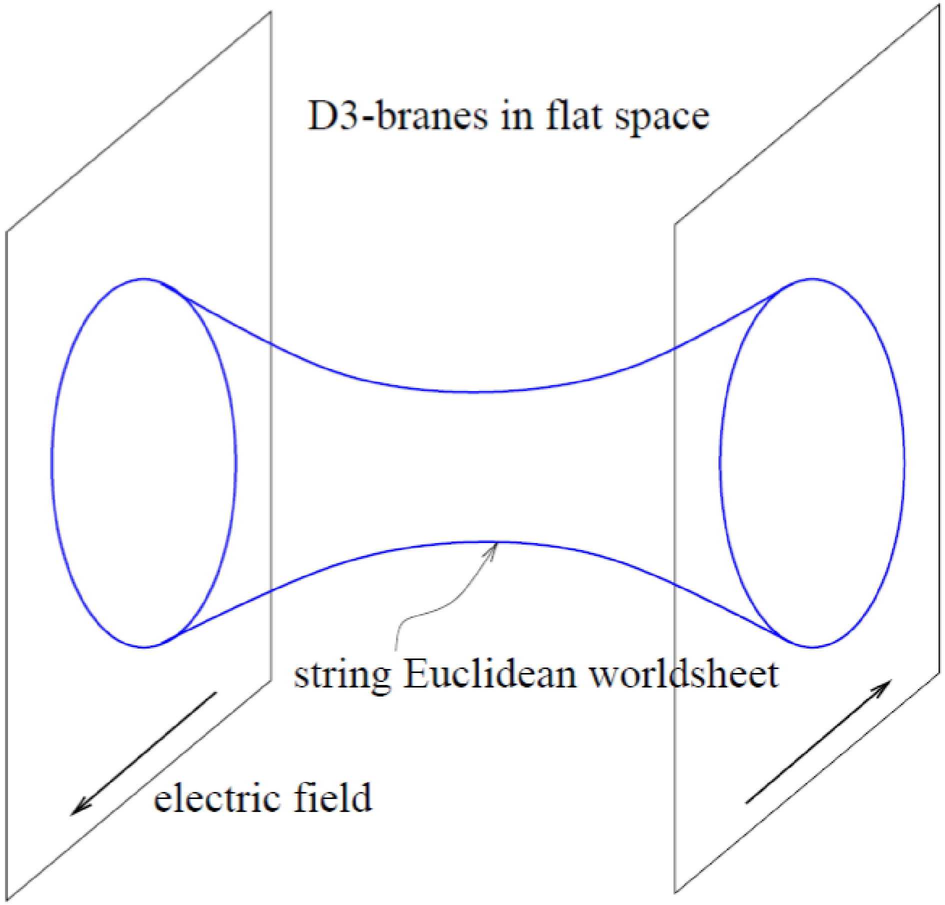
In string theory there is a critical value where the barrier for pair production disappears and the vacuum is "catastrophically" unstable

$$eE_{cr} = \frac{1}{2\pi\alpha'}$$

Fradkin-Tseytlin 85
Burgess 87
Bachas-Porrati 92
…

Not much is known about string pair production in non homogeneous backgrounds

String suspended between two D-branes



This is a case where pair production can be studied with the "worldsheet instanton" technique

$$m = Td$$

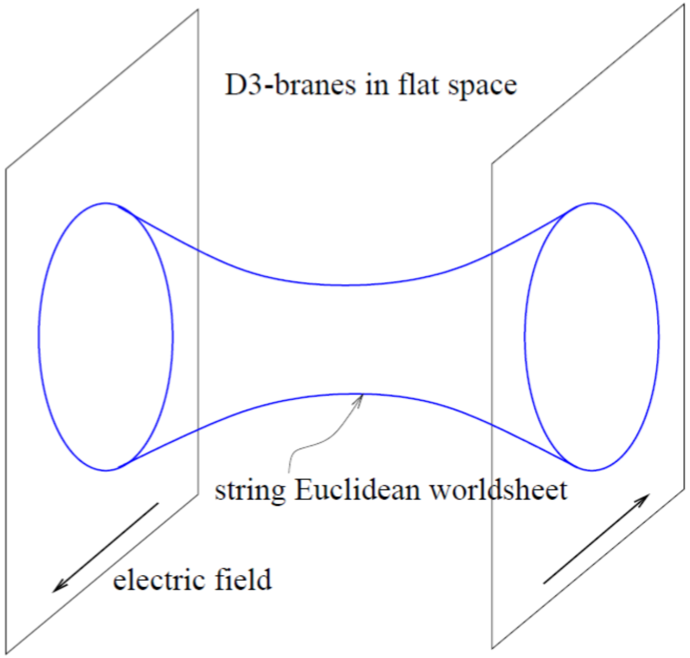
$$S = T\int d\sigma d\tau \sqrt{\det\ g_2(\sigma,\tau)} + iq\int_{boundary} dX^\mu A_\mu$$

Minimal surface solution

For constant background, and so circular symmetry:

$$S_E = T\int_{-d/2}^{d/2} dz 2\pi r(z)\sqrt{1+r'(z)^2} - 2qE\pi R^2$$

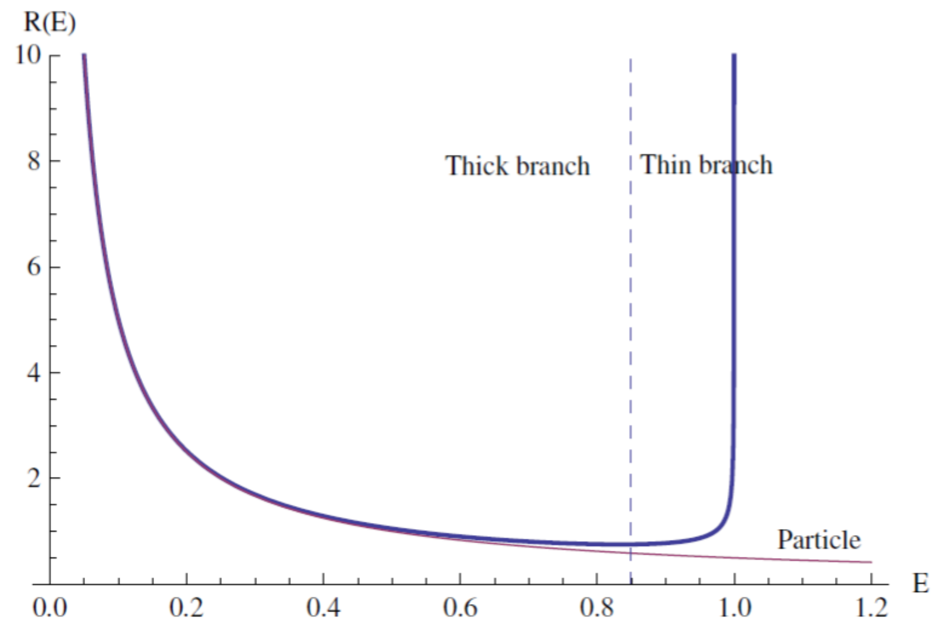
The solutions are the well known "catenaries" minimal s

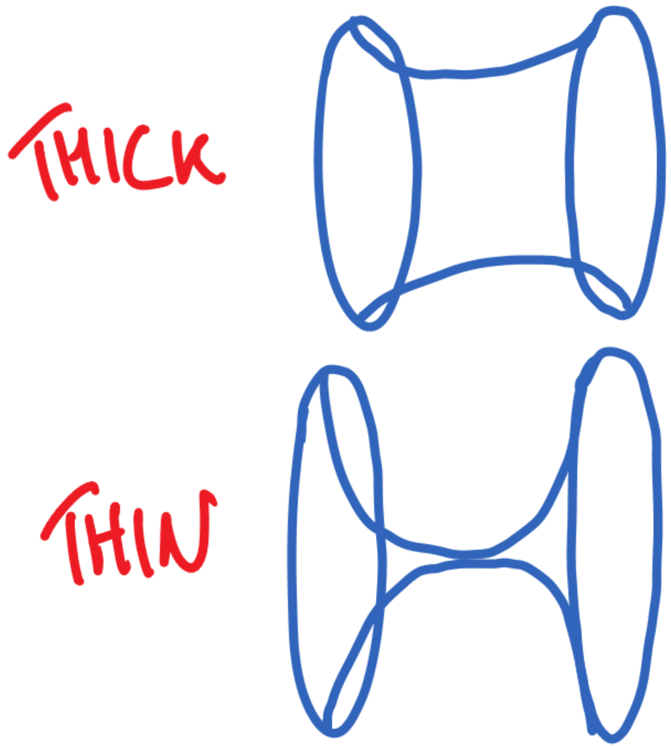


$$r(z) = \frac{1}{c}\cosh\left(cz\right) \qquad R = \frac{1}{c}\cosh\left(cd/2\right)$$

String suspended between two D-branes

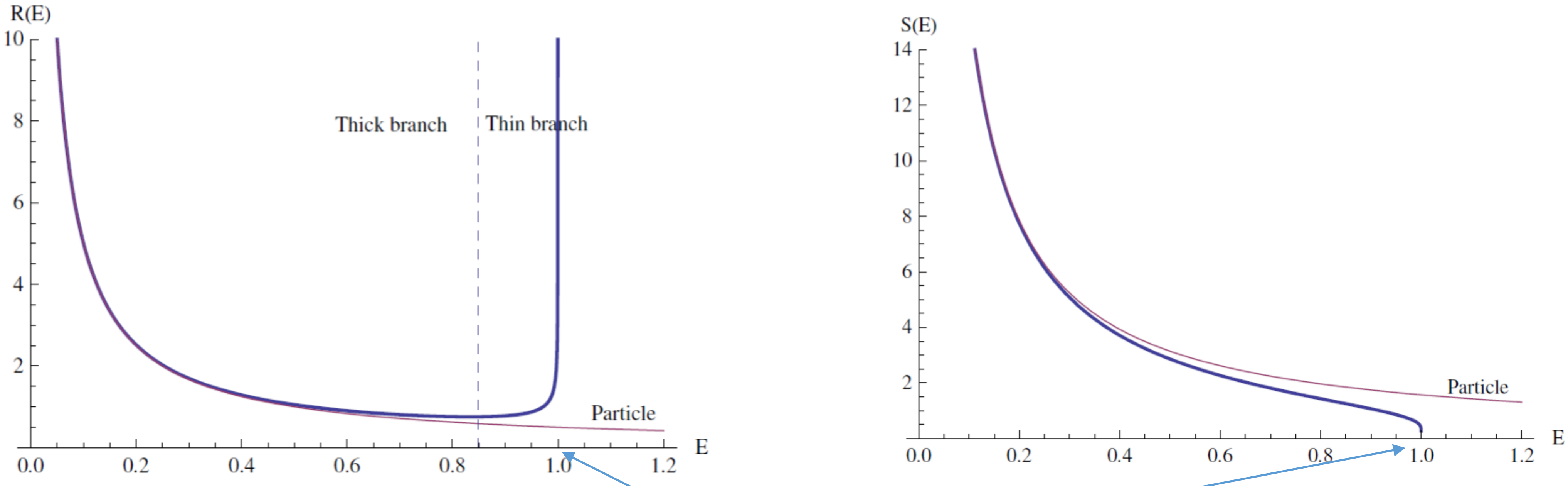
For R sufficiently large there are two solutions: “thick neck” and “thin neck”





String suspended between two D-branes

For R sufficiently large there are two solutions: “thick neck” and “thin neck”

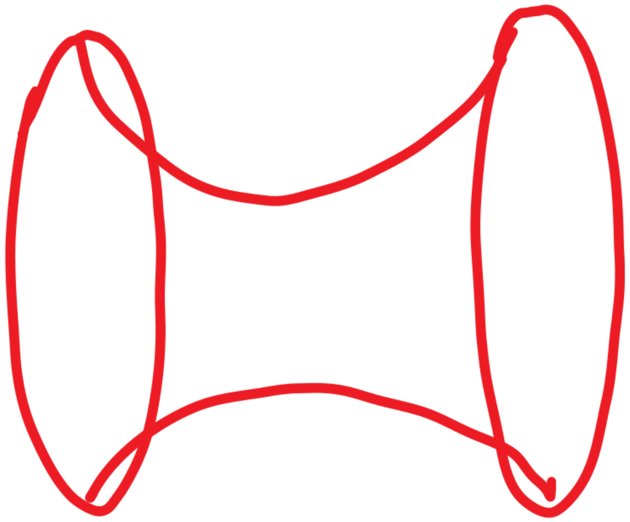


Critical field is as expected: $qE_{\rm cr} = T$

Time-dependent setup

We work in cylindrical cordinates (z, r, θ)

and have to find a function of two variables $r(z, \theta)$



$$S_E = T\int_{-d/2}^{d/2} dz \int_0^{2\pi} d\theta \sqrt{r^2(1+(\partial_z r)^2)+(\partial_\theta r)^2} - iq\int_0^{2\pi} d\theta (A_\theta + A_r \partial_\theta r)$$

Time-dependent setup

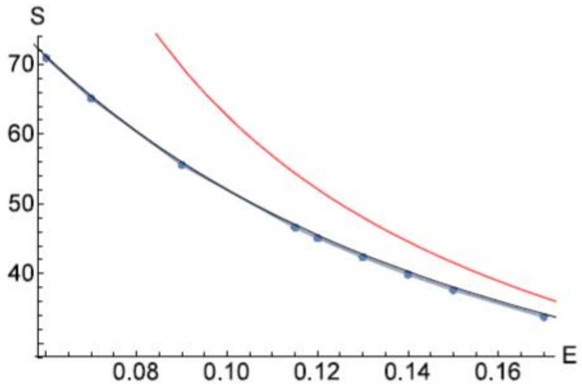
The boundary term is given by the specific choice of background electric field

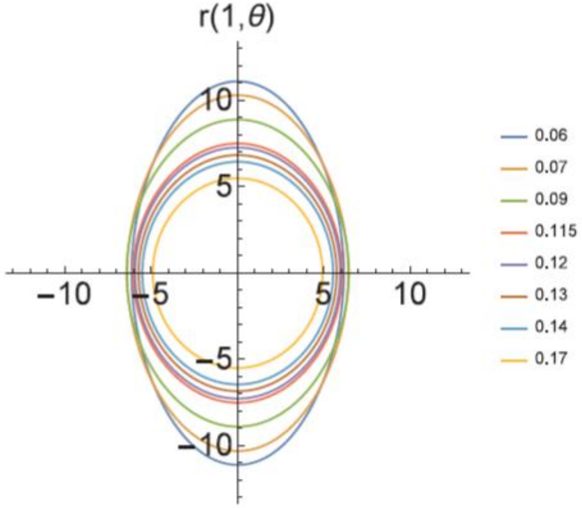
$$A_\theta = -iEf(r\sin\theta)r\sin\theta \qquad\qquad A_r = iEf(r\sin\theta)\cos\theta$$

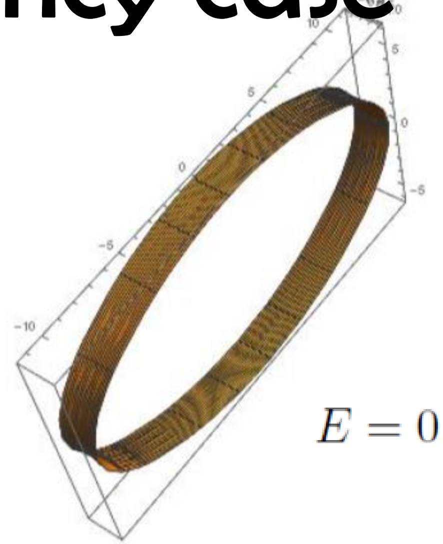
$$f(x_4) = \frac{\tan(\omega x_4)}{\omega} \qquad \text{pulse}$$

$$f(x_4) = \frac{\sinh(\omega x_4)}{\omega} \qquad \text{oscillating}$$

Solution for small frequency case

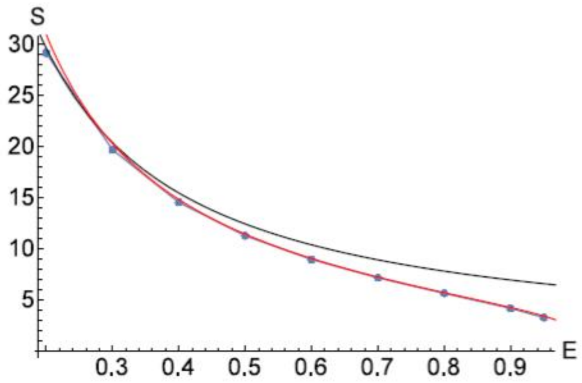


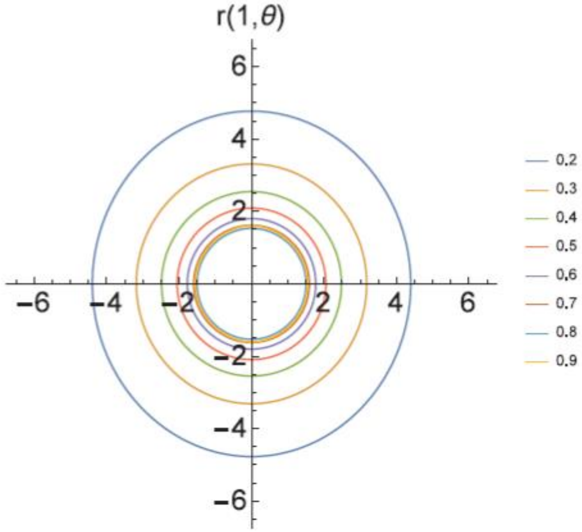


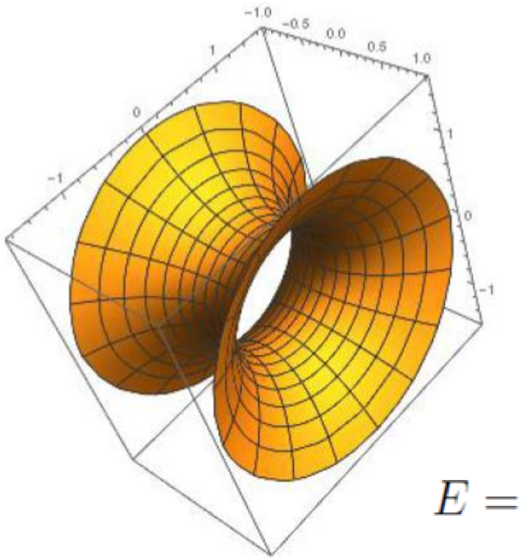


$E = 0.06$

Pulse
with $\omega = 0.1$

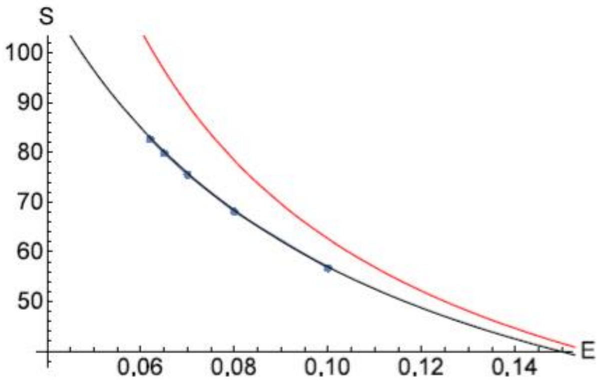


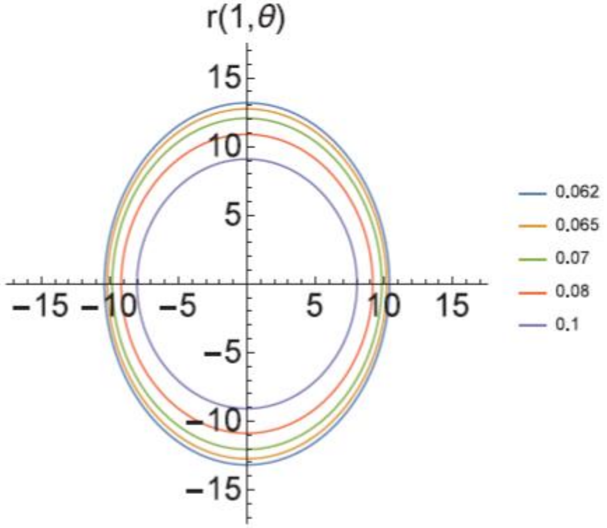


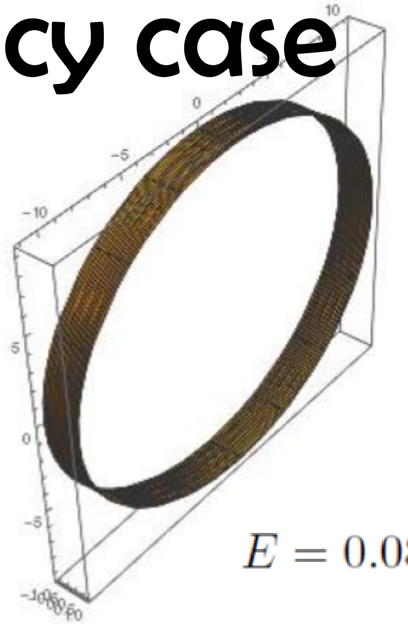


$E = 0.9$

Solution for small frequency case

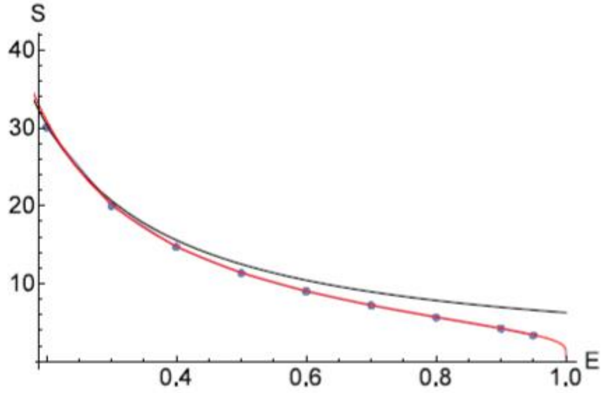


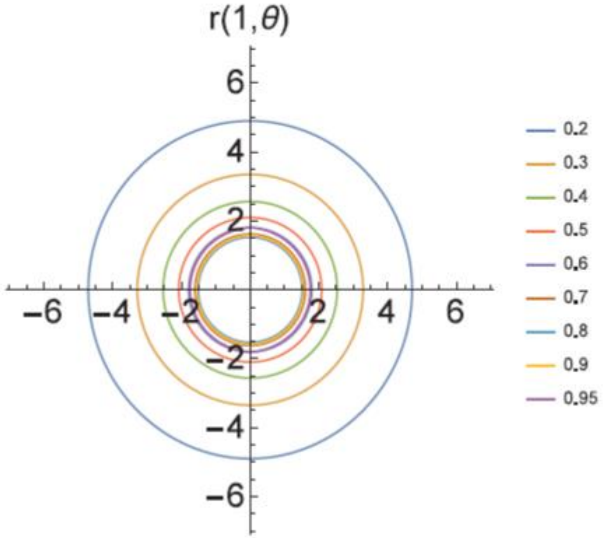


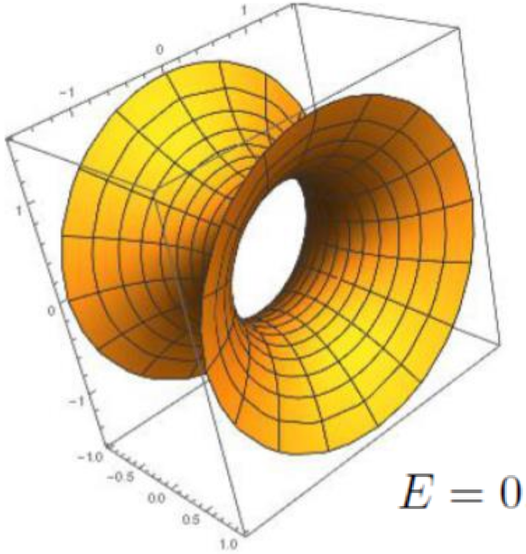


$E = 0.08$

Oscillation with $\omega = 0.1$







$E = 0.9$

Solution for higher frequency

String effects are expected to be important when E is big enough

$$qE \simeq T.$$

Non-homogeneity effects are expected to be important when E is small enough

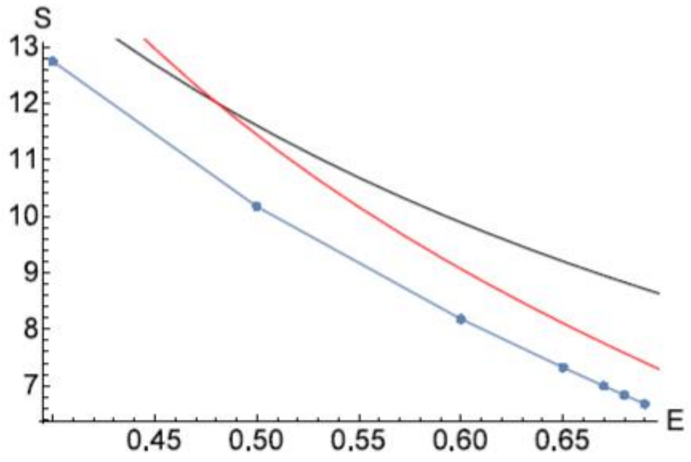
$$qE < m\omega$$

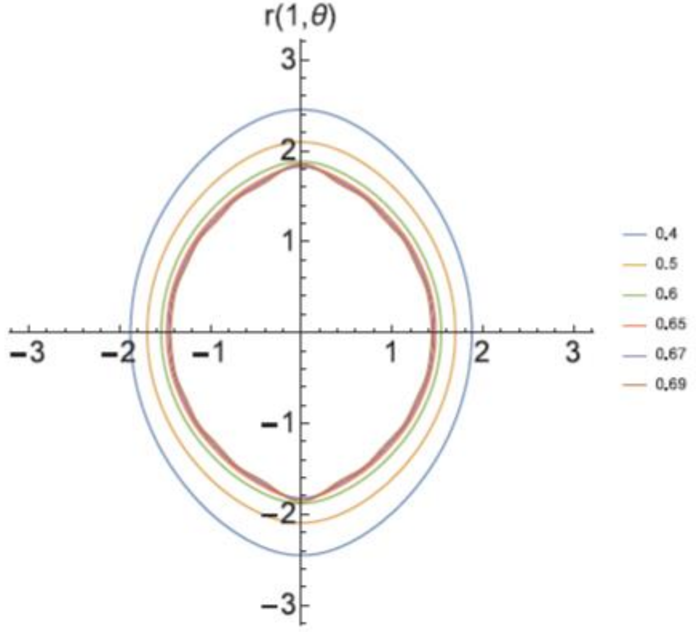
So we need to increase omega to see effects that are both "stringy"
and related to non-homogeneity

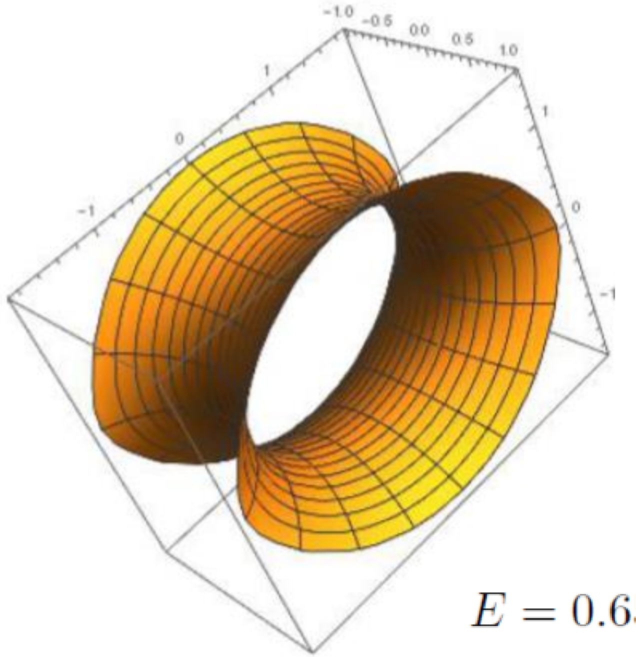
Solution for higher frequency

Pulse

with $\omega = 0.3$



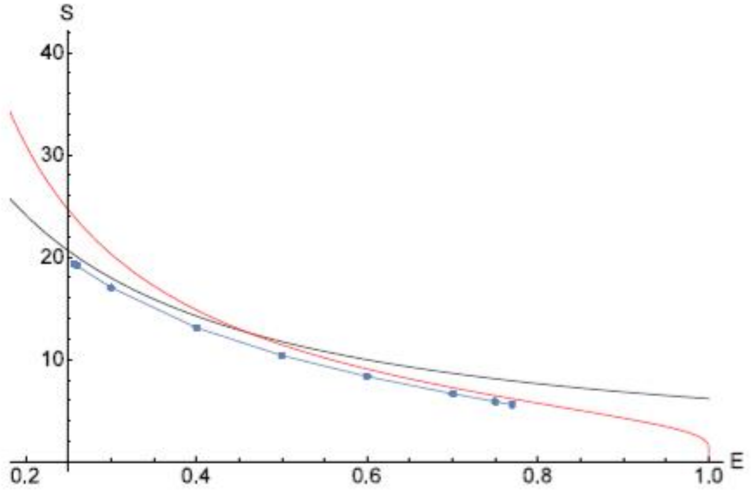


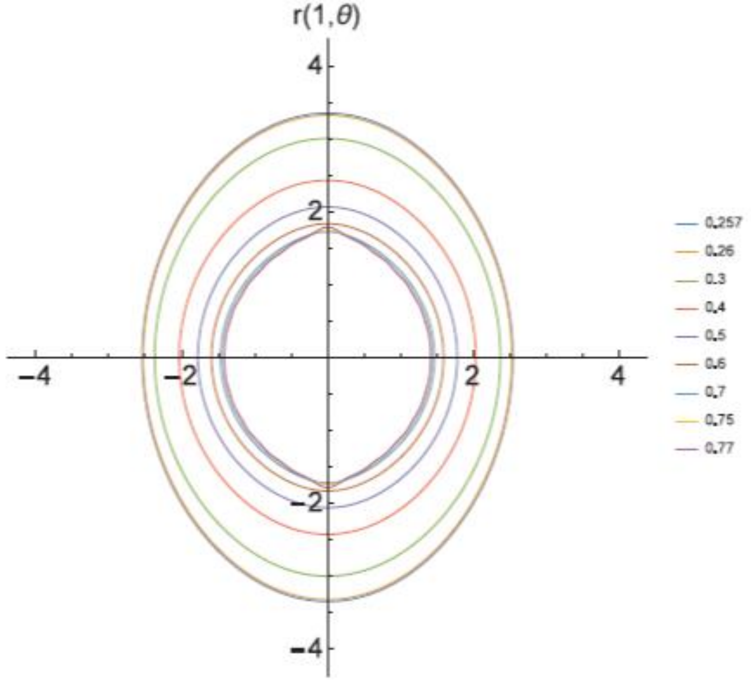


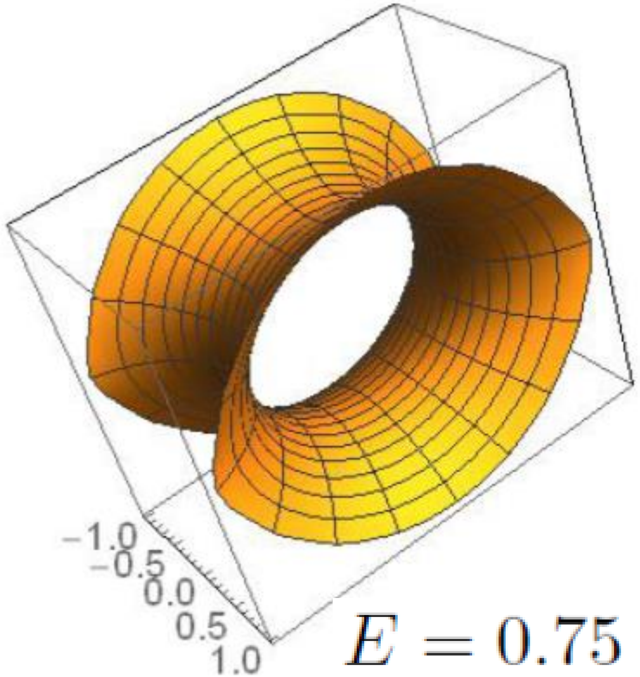
$E = 0.65$

Solution for higher frequency

Oscillation
with $\omega = 0.4$







$E = 0.75$

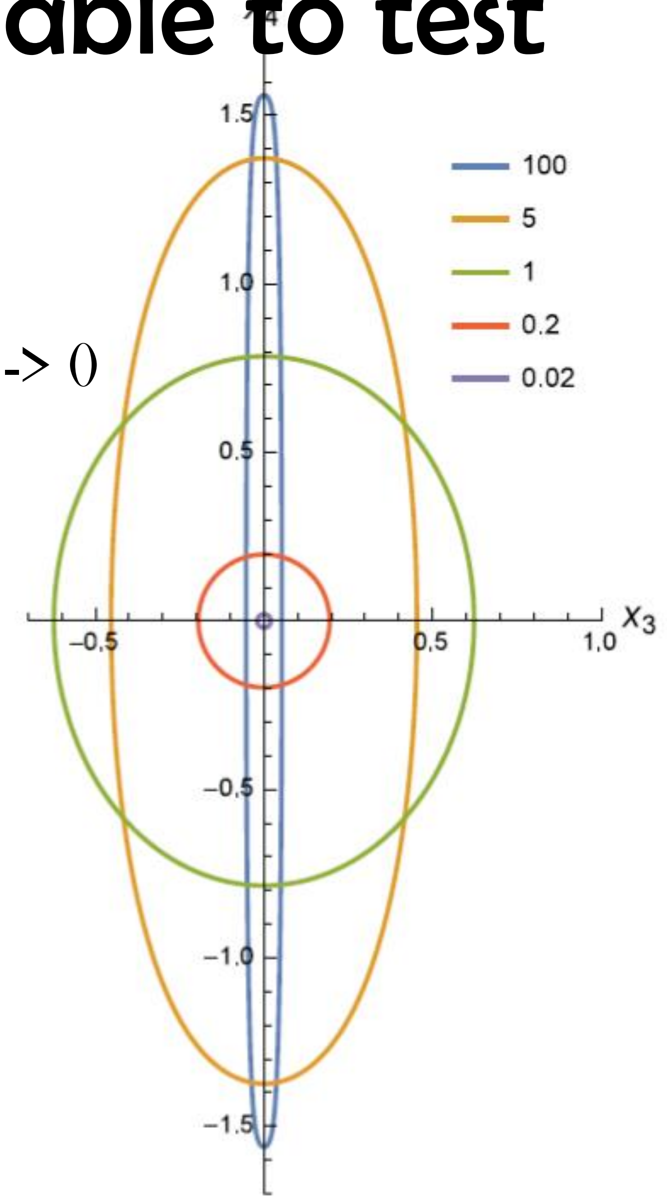
Some expectations we are not able to test yet...

For the pulse electric field, the instanton area goes to zero as E -> 0

$$A_L \simeq 4x_3^{\max} x_4^{\max} = \frac{2\pi q E}{m\omega^3} \log\left(\frac{2m\omega}{qE}\right)$$

We thus "always" expect some string effect at ***small*** E

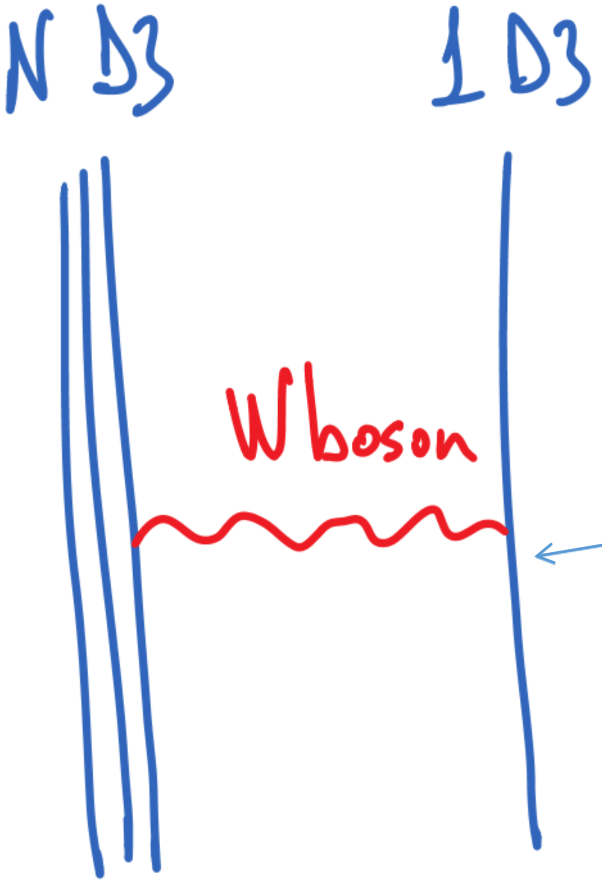
$$\frac{TqE_{\rm low}}{\pi m^2\omega^2} \log\left(\frac{2m\omega}{qE_{\rm low}}\right) \simeq 1$$



Holographic Schwinger effect

We consider N=4 SYM in the Coulomb phase

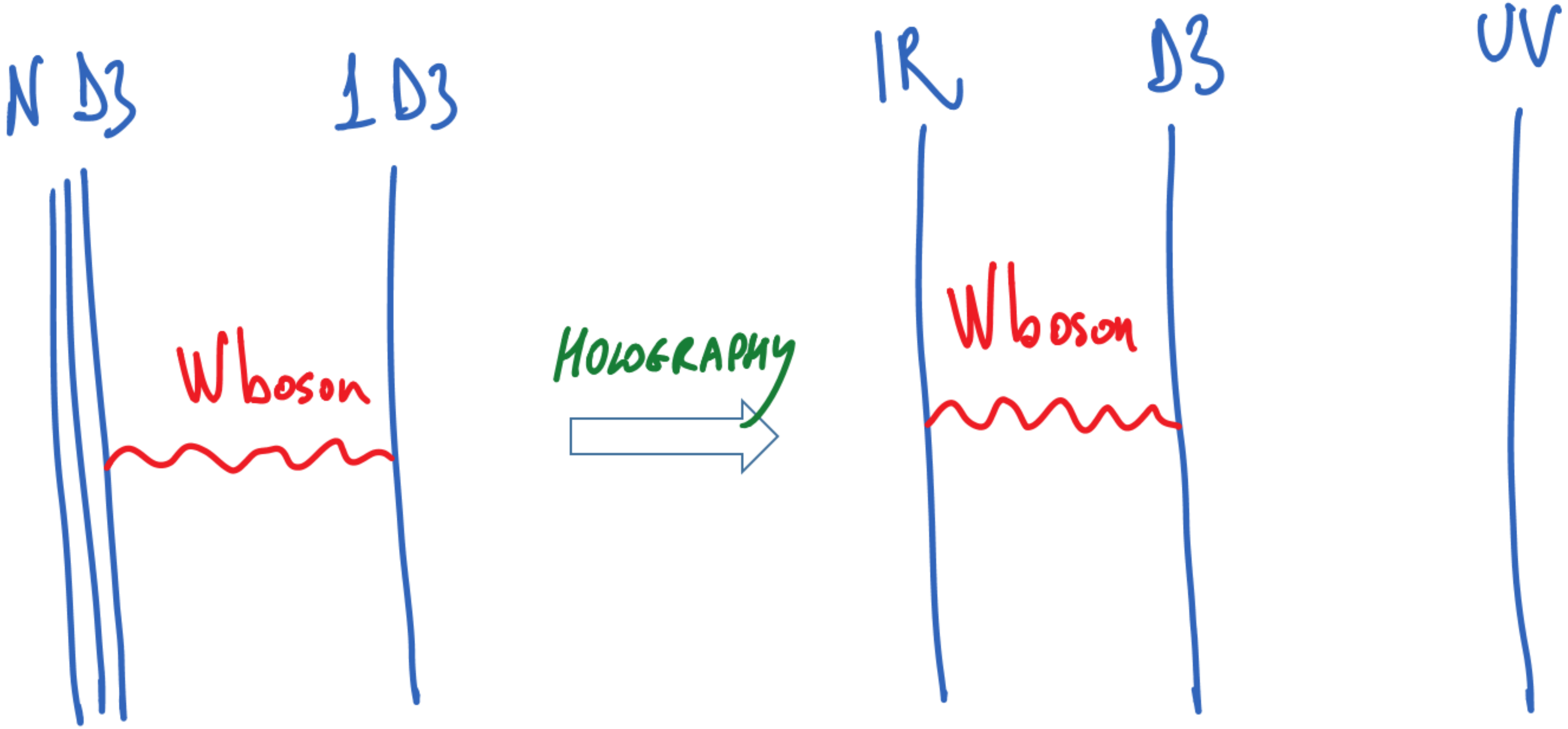
$$SU(N+1) \rightarrow SU(N) \times U(1)$$



This is the U(1) of the Schwinger effect

Holographic Schwinger effect

We consider N=4 SYM in the Coulomb phase



Holographic Schwinger effect

The dictionary is the usual

$$g_s = g^2/4\pi \qquad L^2/l_s^2 = \sqrt{\lambda}/2\pi$$

plus the relation for the W mass

$$\frac{1}{l_s^2}\int_0^{r_0}\sqrt{-\det h_{ab}} = \frac{L^2 r_0}{l_s^2} = \frac{\sqrt{\lambda}r_0}{2\pi}$$

$$m = \frac{TL^2}{z_0}$$

Holographic setting

The DBI action predicts the string to break down at "local" string scale

This is translated into a genuine QFT scale

$$S_{DBI} = \frac{1}{g_s l_s^4} \int d^4x' \sqrt{-\det(\eta_{\mu\nu} - l_s^2 F_{\mu\nu,\ loc})}$$

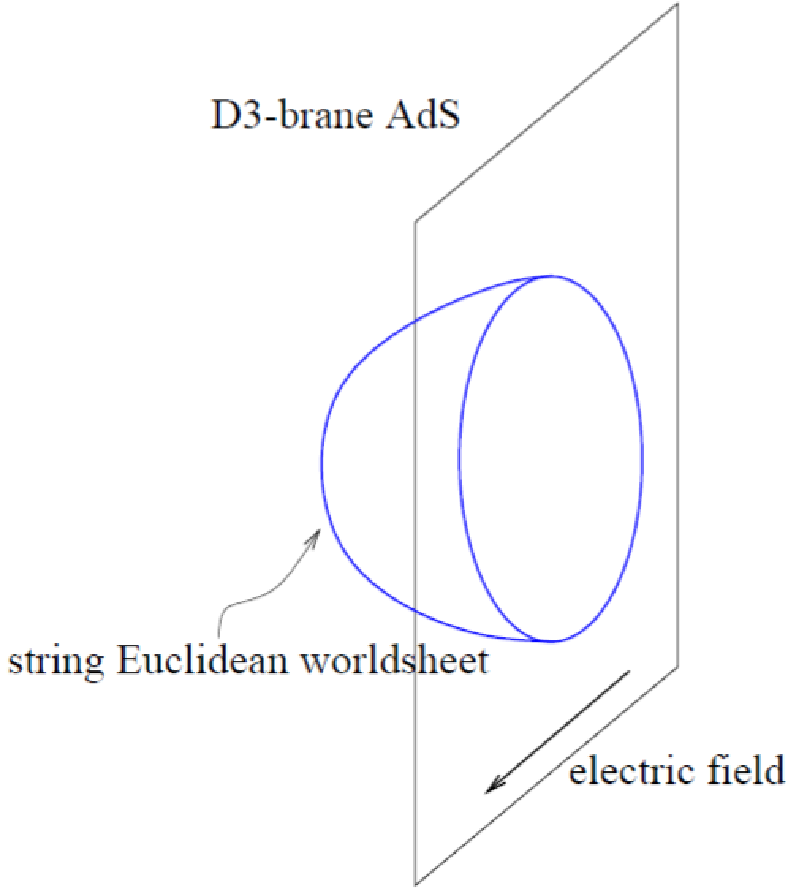
This is the value where the barrier for the pair production drops to zero in the QFT!

$$E_{cr} = \frac{r_0^2 L^2}{l_s^2} = \frac{2\pi m^2}{\sqrt{\lambda}}$$

Holographic worldsheet instanton

In holography, the instanton is a Wilson loop attached to the D3 brane

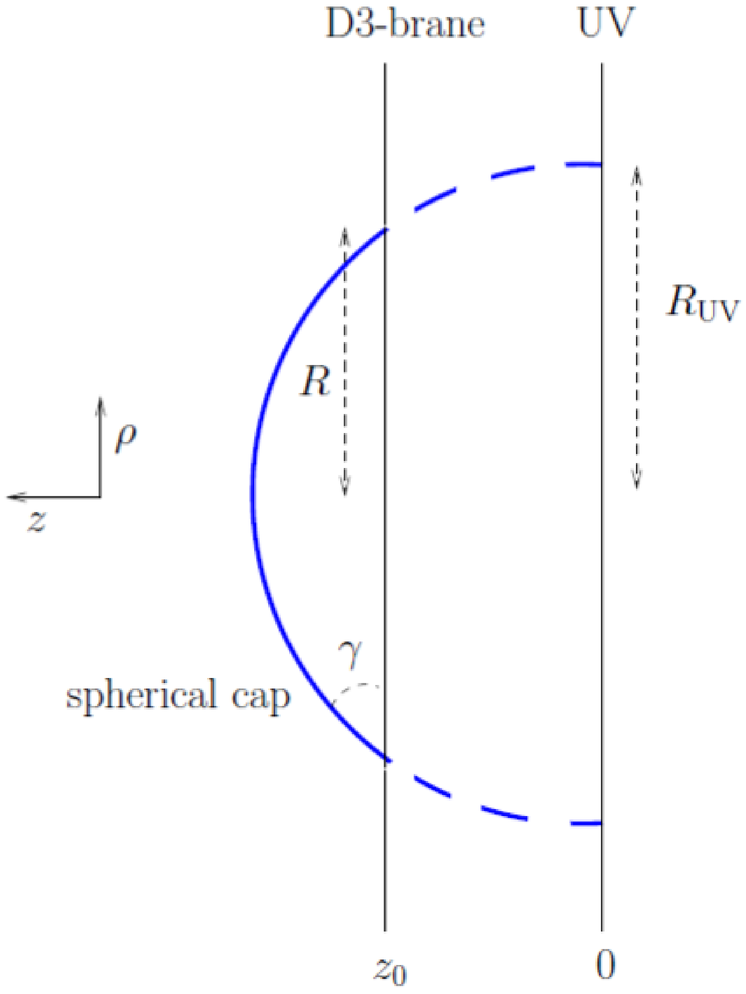
$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + dx_\mu dx^\mu\right)$$



B.-Kiefer-Rabinovici 12

Holographic worldsheet instanton

$$S_E = T \int_0^R d\rho 2\pi\rho \left(\frac{L}{z(\rho)}\right)^2 \sqrt{1+z'(\rho)^2} - qE\pi R^2$$



Minimal surface is a spherical cap

$$z(\rho) = \sqrt{R_{\rm UV}^2 - \rho^2}$$

Boundary condition $\quad T\cos(\gamma) = qF_{\rm loc}$

Holographic worldsheet instanton

The action $S_E = \frac{\pi}{qEz_0^2}\left(\left(\frac{TL^2}{qEz_0^2}\right) - 1\right)^2$ vanishes when the spherical cap is tangent to the D-brane

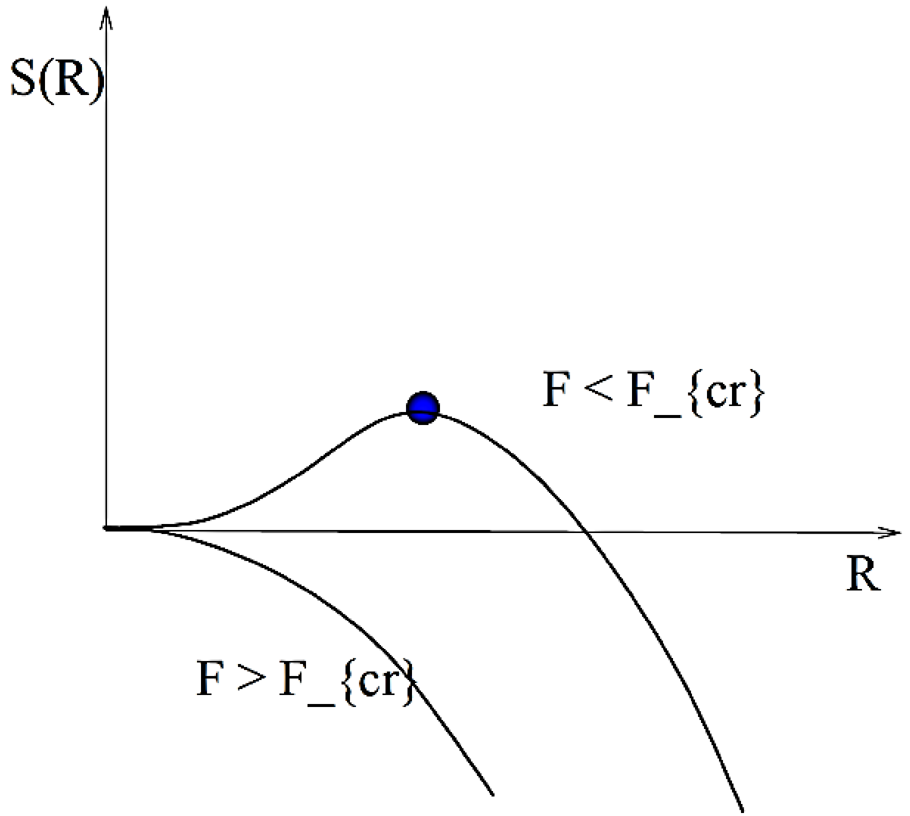
$$qE_{\text{cr}} = \frac{TL^2}{z_0^2}$$

Holographic worldsheet instanton

Tunneling barrier exists only below a certain critical field

$$S_E = \frac{\mathcal{T}\pi L^2 R^2}{z_0^2} - qF\pi R^2 + \dots$$

$$qF_{cr} = \frac{\mathcal{T} L^2}{z_0^2}$$



Time-dependence setup

As before:

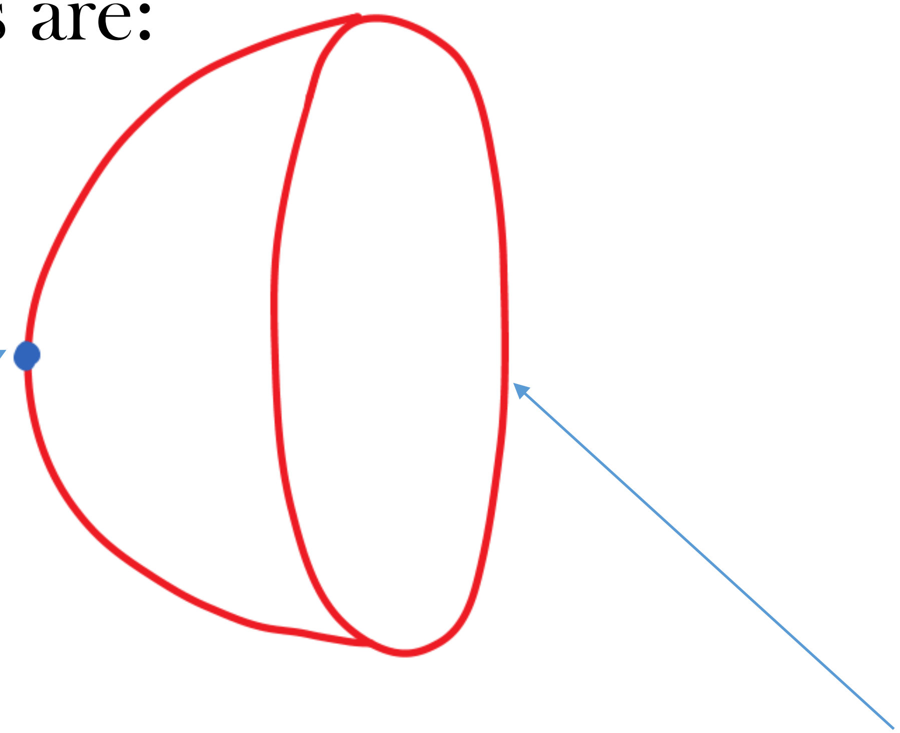
we work in cylindrical cordinates (z, r, θ)

and have to find a function of two variables $r(z, \theta)$

$$S_E = T \int_{z_0}^{Z_0} dz \int_0^{2\pi} d\theta \frac{L^2}{z^2} \sqrt{r^2(1 + (\partial_z r)^2) + (\partial_\theta r)^2} - iq \int_0^{2\pi} d\theta (A_\theta + A_r \partial_\theta r)$$

Time-dependence setup

Boundary conditions are:

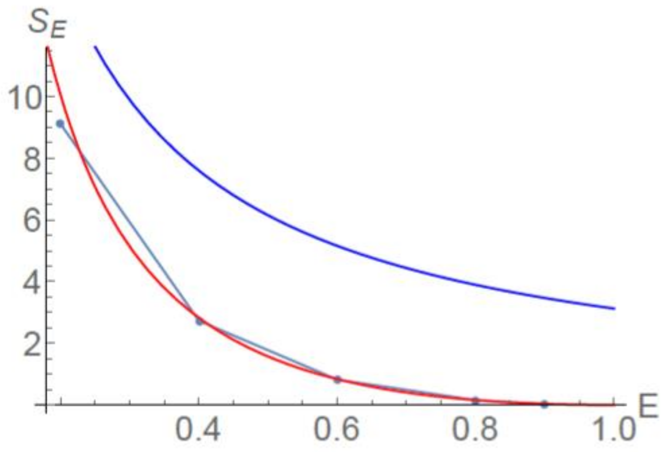


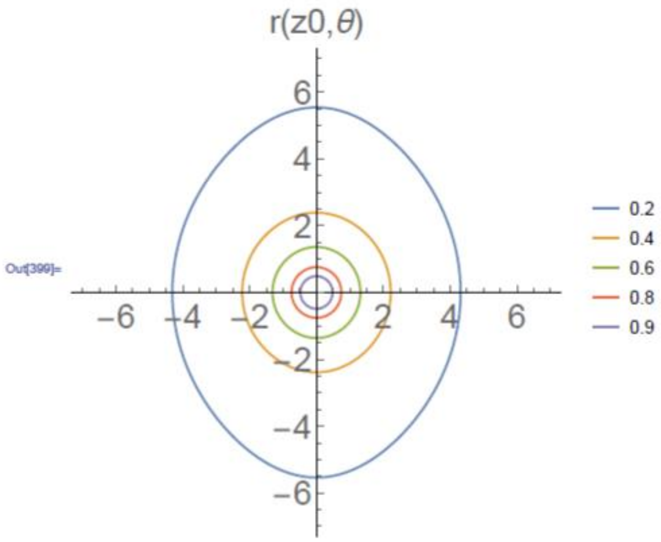
$$r(Z_0, \theta) = 0, \qquad r'(Z_0, \theta) = \infty$$

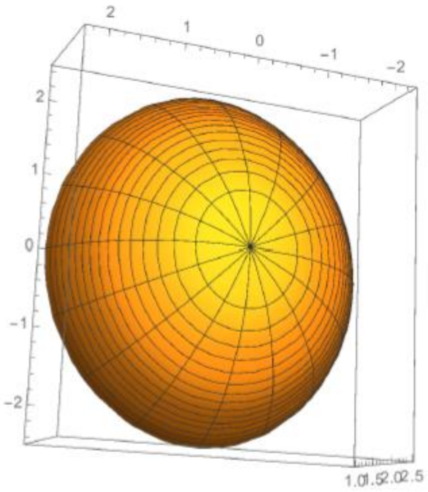
$$T\frac{L^2}{z_0^2}\frac{r^2\partial_z r}{\sqrt{r^2(1+(\partial_z r)^2)+(\partial_\theta r)^2}} = qEf'\left(r - \partial_\theta r \sin\theta\cos\theta\right)$$

Some solutions

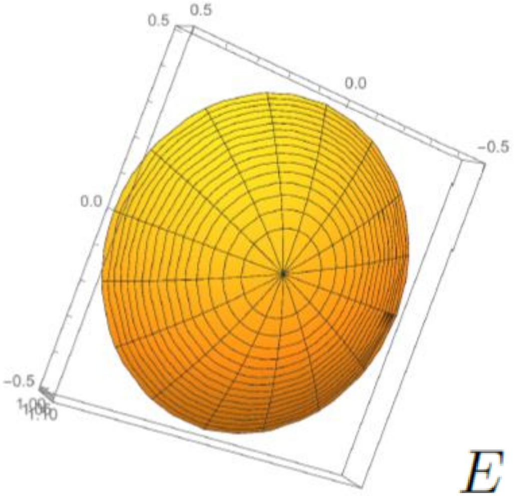
Pulse
with $\omega = 0.15$







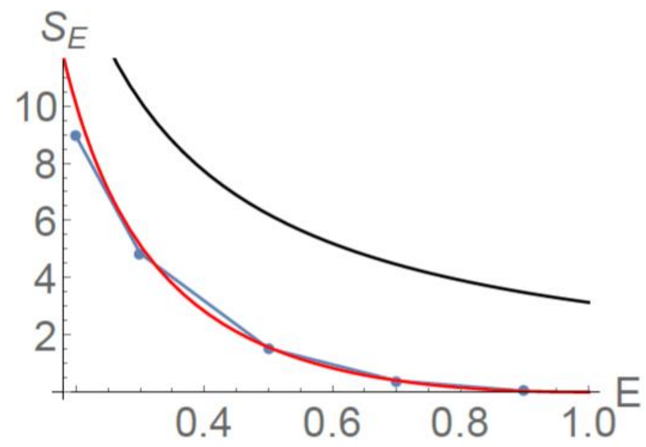
$E = 0.4$

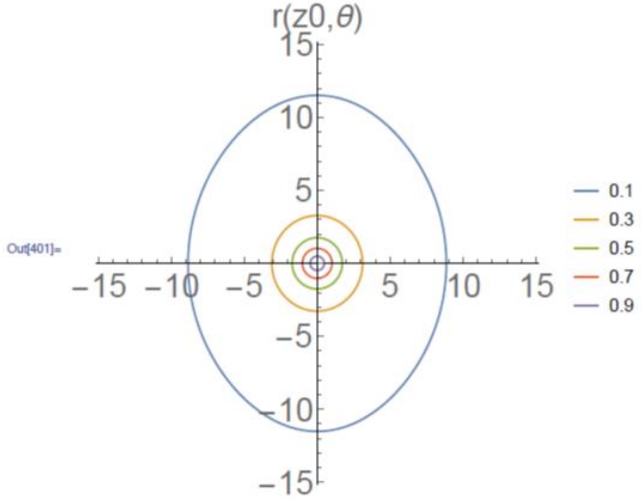


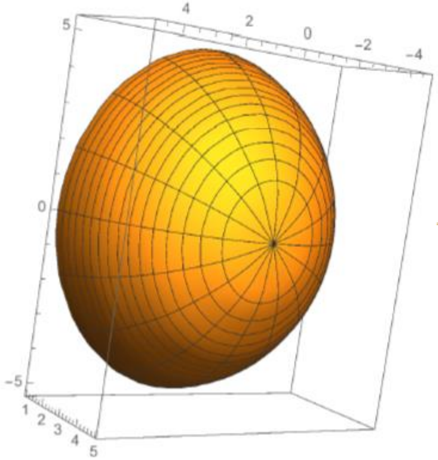
$E = 0.9$

Some solutions

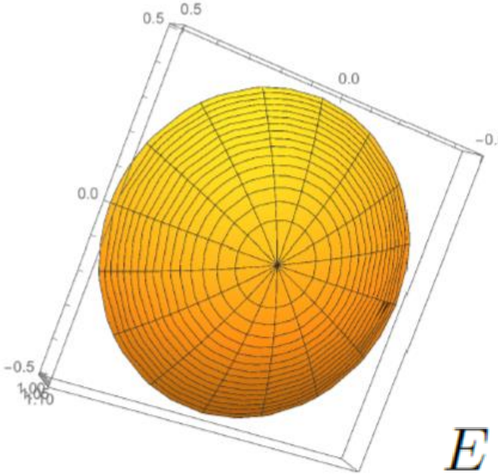
Oscillation
with $\omega = 0.15$







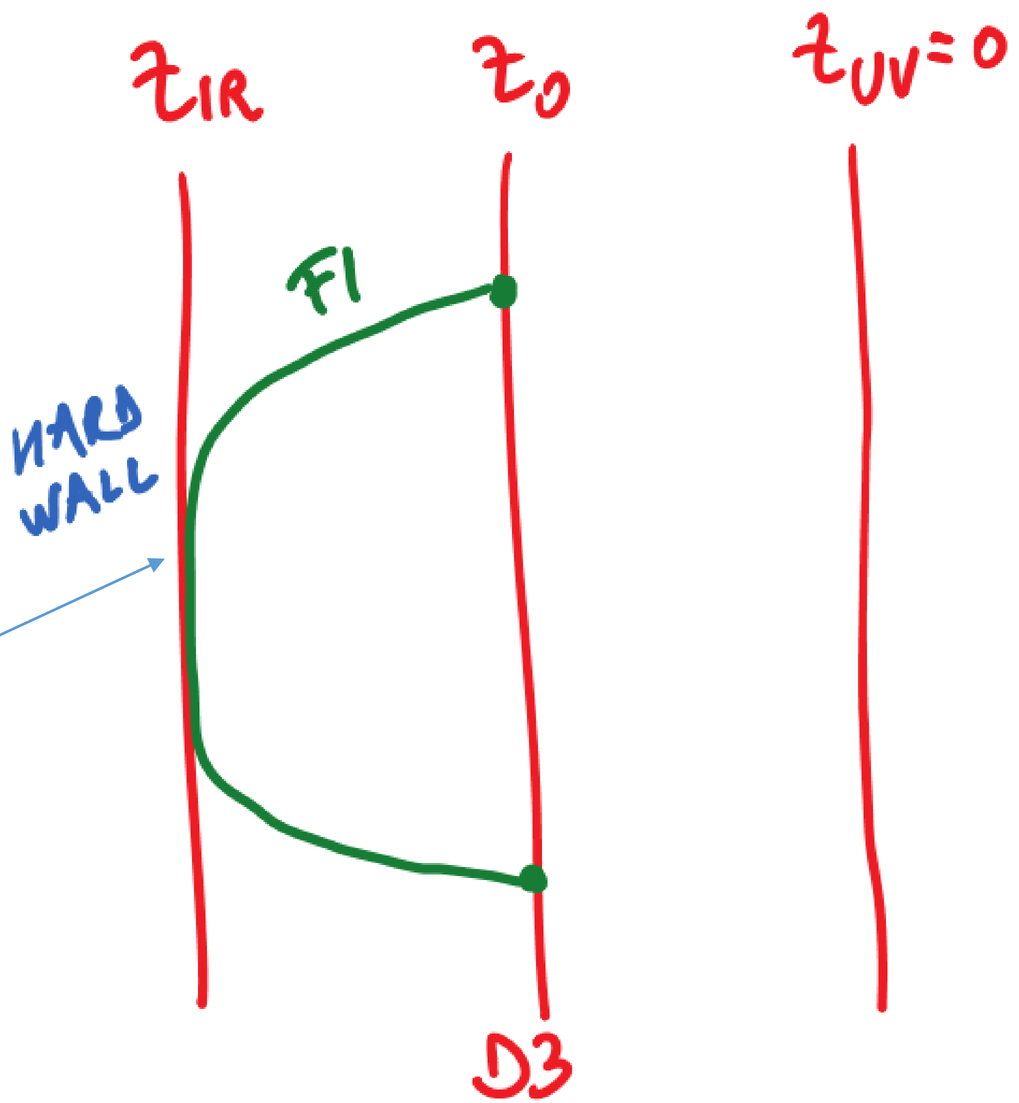
$E = 0.4$



$E = 0.9$

Confining background

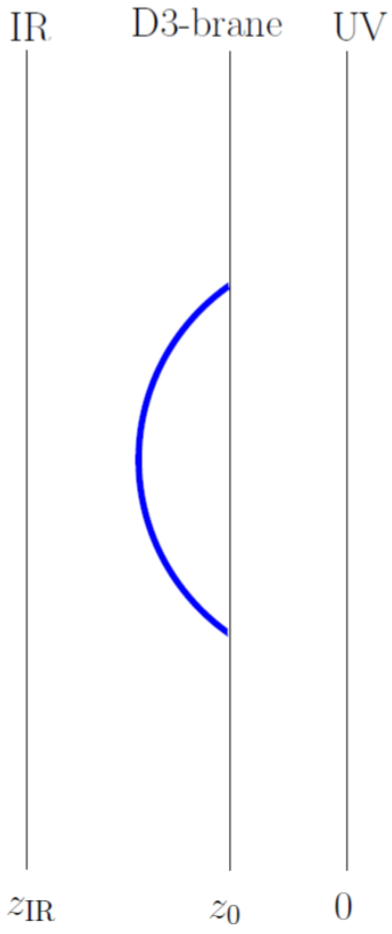
Hard-wall geometry:



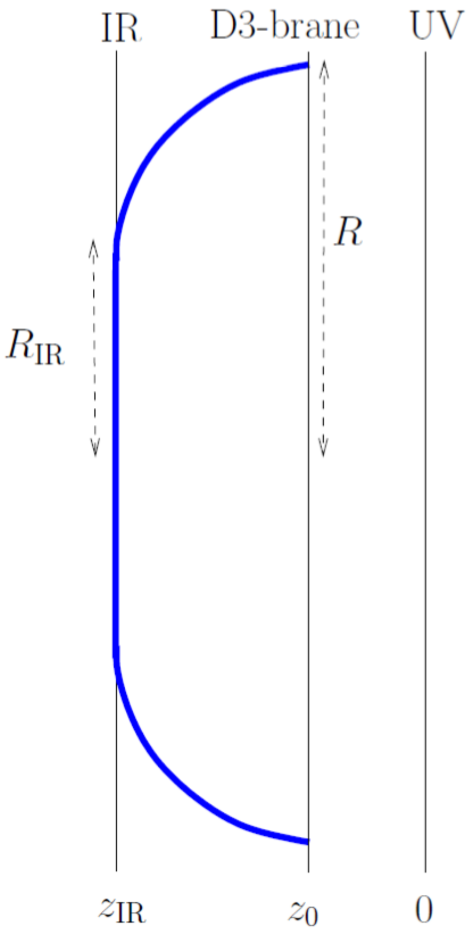
Confining string

$$T_c = T\frac{L^2}{z_{\rm IR}^2}$$

Confining background



$E > E'$

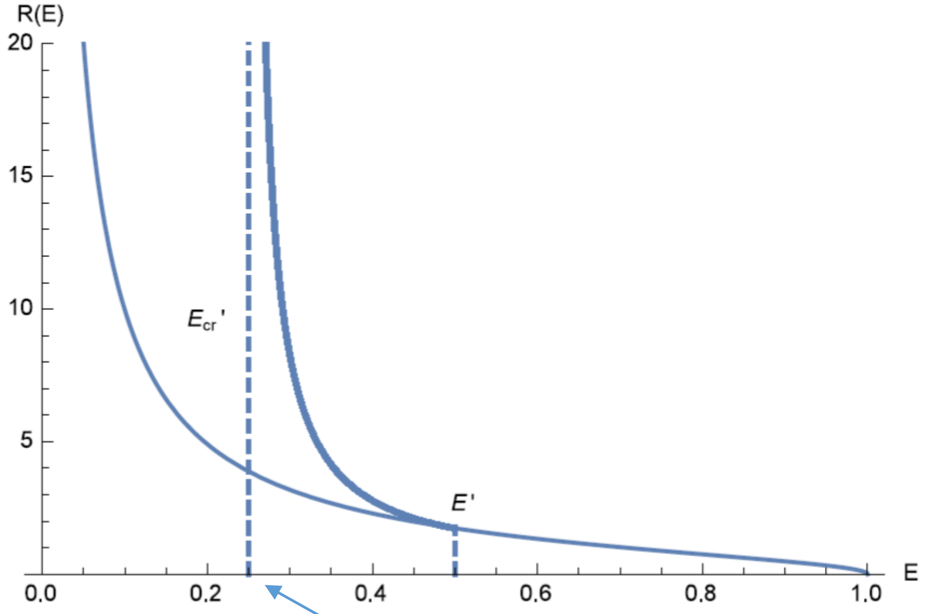


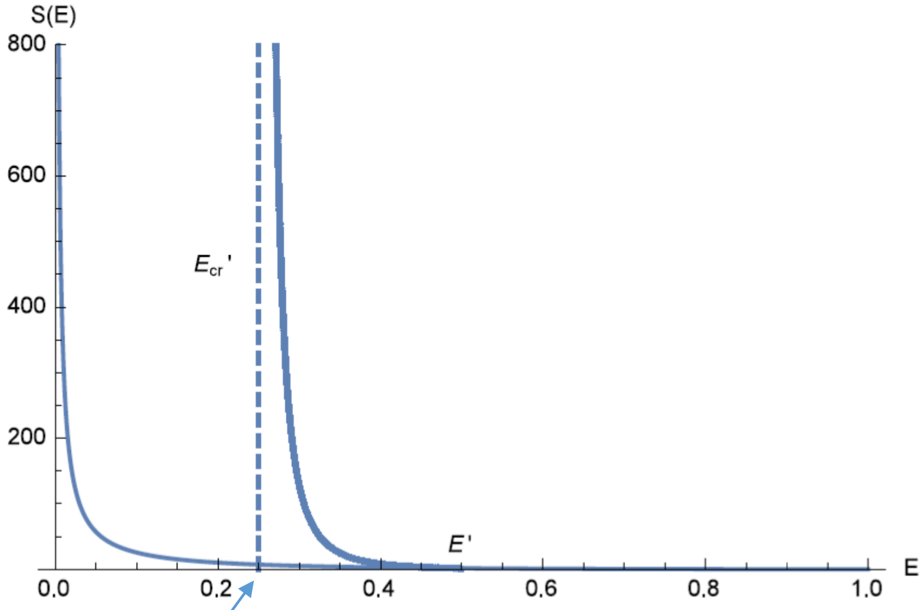
$E < E'$

At the field $\quad qE' = \dfrac{TL^2}{z_0 z_{\rm IR}}$

we have a transition in the type of worldsheet solution:

Low critical field in confing background





$$qE'_{\mathrm{cr}} = \frac{TL^2}{z_{\mathrm{IR}}^2} = T_{\mathrm{c}}$$

Low critical field in confining background

The interpretation of the “low” critical field in Minkowski:

$$V(R) = TL^2\left(\frac{1}{z_0} - \frac{1}{z_{\mathrm{IR}}}\right) + T_{\mathrm{c}}R - qER$$

$$\boxed{\text{When } qE > T_{\mathrm{c}} \text{ the electric field is not strong enough to overcome confinement}}$$

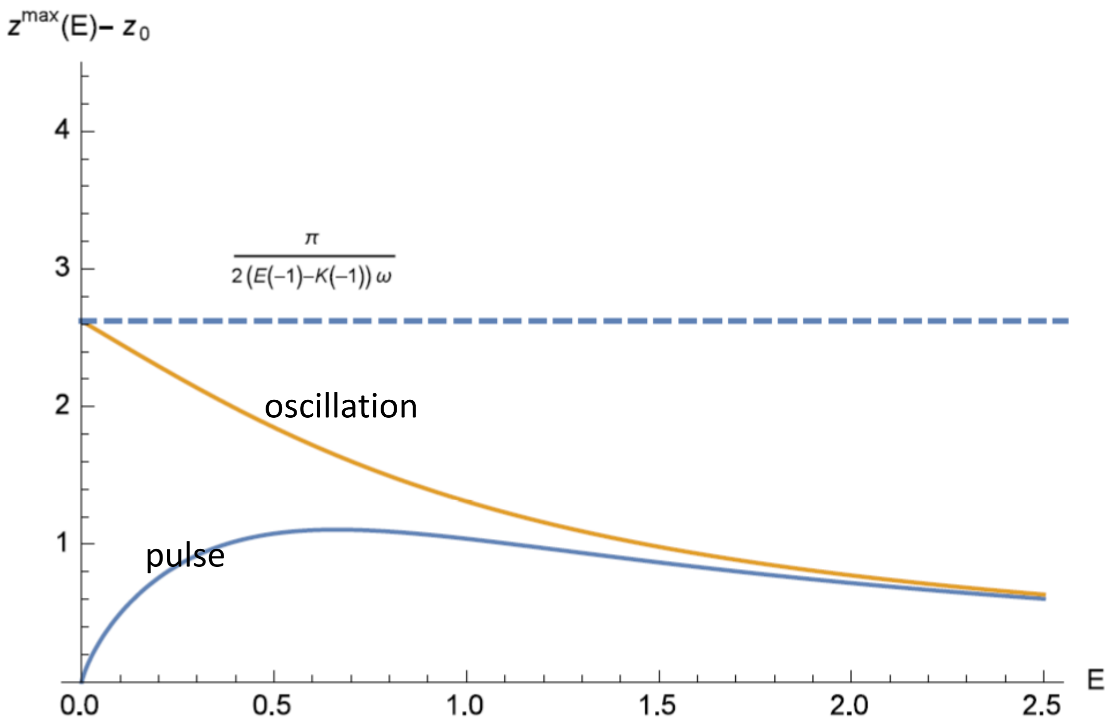
Low critical field and time-dependence

The critical low field $E'_{\rm cr}$ is in general modified by the frequency omega.

We show a clear example in which it vanishes!

Low critical field and time-dependence

Maximum holographic z for low E:

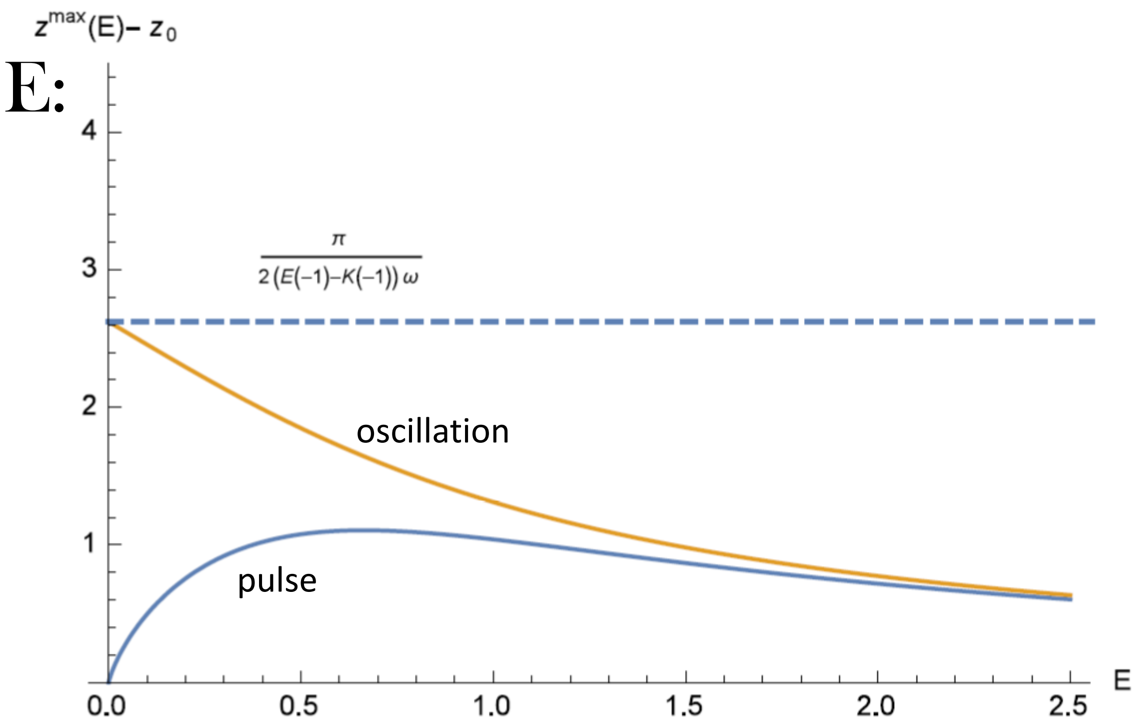


Unlike the constant background case, z max is bounded from above:

$$z_{\max} = z_0 + \frac{c\pi}{2\omega} \simeq \frac{2.62}{\omega}$$

Low critical field and time-dependence

Maximum holographic z for low E:



If the confinement scale is so that

$$z_{\rm IR} > z_{\rm max}$$

The Euclidean worldsheet is always contained inside the physical space.
Pair production happen <u>irrespectively</u> on how small is E.

Photons energy is enough to produce glueballs and charged particles are intermediary states

Conclusion

- We studied two cases of string pair production with the technique of Euclidean worldsheet instanton
- With this technique we can define the problem on non homogeneous backgrounds and solve PDE numerically
- String nature and non homogeneity work together when omega is big enough; for time dependent backgrounds they enhance even further the pair production
- The nature of pair production in confining background is highly modified by the non-homogeneity. In particular the low critical field can disappear