Exact holography in N=2 theories and the enhançon mechanism

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work in progress with A. Zein-Assi

The holography paradox

- AdS/CFT: fantastic playground to understand gravity
- CFT evolution is unitary $\Rightarrow$ unitary theory of quantum gravity

In principle solves all black hole puzzles (information paradox, physics behind horizon, etc.)

In practice lots of confusion and debates (firewalls?)

Why?

Lack of explicit computations and examples

- weakly coupled gravity = strongly coupled field theory, hard to tackle explicitly
- but some progress in $\mathrm{AdS}_3/\mathrm{CFT}_2$ [Hartman, ...]

General setting

Main question

- take a large N 4d gauge theory, pick a state in the theory
- can one find holographic dual geometry (SUGRA + stringy corrections) from gauge theory side?

Interesting case thermal state in CFT → black hole in AdS

More modest goal vacuum state of supersymmetric CFT

↪ Simplest example $\mathcal{N} = 4$ SYM: Coulomb branch of vacua

$\updownarrow$

near-horizon multiple D3-brane SUGRA solutions

[Ferrari, MM, Rovai]

↪ More interesting $\mathcal{N} = 2$ gauge theories

$\mathcal{N} = 2$ SUSY and the enhançon

- SUGRA solutions dual to theories with 8 supercharges often singular in IR
- singularity = "repulson": massive particles experience repulsive force [Kallosh and Linde]
- singurality surrounded by region where some probe D-branes become tensionless ("enhançon radius")
  $\rightarrow$ new light d.o.f.'s not described by SUGRA
- enhançon mechanism: D-branes pile up at enhançon radius, inside solution must be excised and replaced by solution with vanishing fluxes [Johnson, Peet, Polchinski]

Our goal

- study exact holographic dual (finite N, finite $\lambda = g^2 N$) from gauge theory
- large N limit → large M_{pl}: find SUGRA result outside enhançon radius, enhançon geometry inside
- enhançon mechanism: non-perturbative ($O(e^{-1/\lambda})$, with $\sqrt{\lambda} = R^2_{\mathrm{AdS}}/\alpha'$) stringy corrections to SUGRA

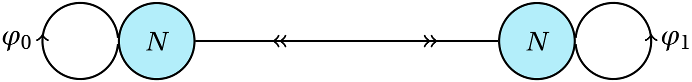
Outline

D3-branes on $\mathbb{C}^2/\mathbb{Z}_2$

- put N D3-branes transverse to $\mathbb{C} \times \mathbb{C}^2/\mathbb{Z}_2$ where $\mathbb{Z}_2$ acts by

$$(z^2, z^3) \to (-z^2, -z^3)$$

- field theory: $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge group, $\mathcal{N} = 2$ SCFT



- SUGRA dual $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_2$ with $\mathbb{Z}_2$ action defined by $\mathrm{S}^5 \hookrightarrow \mathbb{C}^3$ [Kachru, Silverstein]

A cascading repulson geometry

- add M fractional D3-branes [Bertolini, di Vecchia, Frau, Lerda, Marotta, Pesando; Polchinski]
  → gauge group $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$, non-conformal
- fractional Dp-branes can be seen as D$(p+2)$-branes wrapped on exceptional cycle Σ, couple to twisted supergravity scalar

$$\gamma = \frac{1}{2\pi \ell_s^2} \int_\Sigma \left(C_2 + \frac{i}{g_s} B_2 \right)$$

- equation of motion for γ: Poisson equation on orbifold fixed plane $\mathbb{C}$

$$\Delta \gamma(z) = 2i \sum_{j=1}^{M} \delta^2(z - z_j) \Rightarrow \gamma(z) = \gamma^{(0)} + \frac{i}{\pi} \sum_{j=1}^{M} \log \frac{z - z_j}{\mu}$$

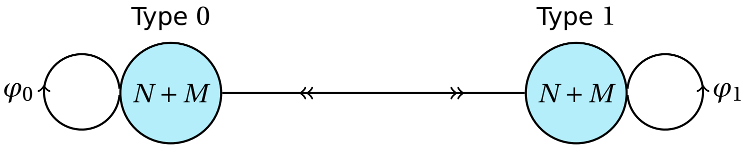
- warp factor H sourced by branes as usual, but also by γ, $\Delta H \sim |\partial_z \gamma|^2 + \cdots$

⇒ "cascading" solution like Klebanov-Strassler (but no Seiberg duality in $\mathcal{N} = 2$)

- IR of the cascade: repulson singularity
- enhançon radius ρ_1 s.t. $\gamma(\rho_1) = 0$

A field theory UV completion

- original solution: infinite cascade, not very tractable in field theory
- gauge group $\mathrm{SU}(N)_0 \times \mathrm{SU}(N+M)_1$ but towards the UV $\to \mathrm{SU}(N+2M) \times \mathrm{SU}(N+M) \to \mathrm{SU}(N+2M) \times \mathrm{SU}(N+3M) \to \cdots$
- UV complete as Coulomb branch of $\mathrm{SU}(N+M) \times \mathrm{SU}(N+M)$: $\mathrm{SU}(N+M) \times \mathrm{SU}(N+M) \to \mathrm{SU}(N) \times \mathrm{U}(1)^{M-1} \times \mathrm{SU}(N+M)$ by giving vev to φ_0 (complex scalar) [Benini, Bertolini, Closset, Cremonesi]

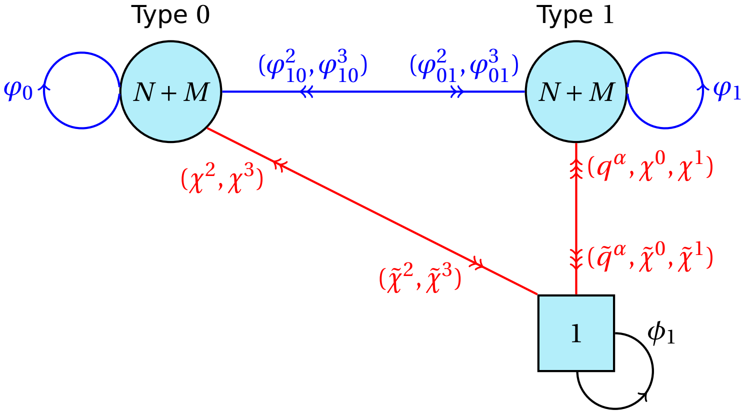


- Classically $\langle \varphi_0 \rangle = \mathrm{diag}(\underbrace{0,\ldots,0}_{\text{N times}}, z_0\,\omega, z_0\,\omega^2, \ldots, z_0\,\omega^M)\,, \quad \langle \varphi_1 \rangle = 0$

  where $\omega^M = -1$, enhançon vacuum, $\mathbb{Z}_M$ symmetry ($\sim$ smearing)
- Quantum Theory [Seiberg, Witten] vacuum parameterized by polynomials $T_0(z) = z^N(z^M + z_0^M)$, $T_1(z) = z^{N+M}$

Adding a probe fractional D(-1)-brane

- now add a probe fractional D(-1)-brane of type 1
- action for the system can be found by going through orbifold procedure in field theory [Douglas, Moore]



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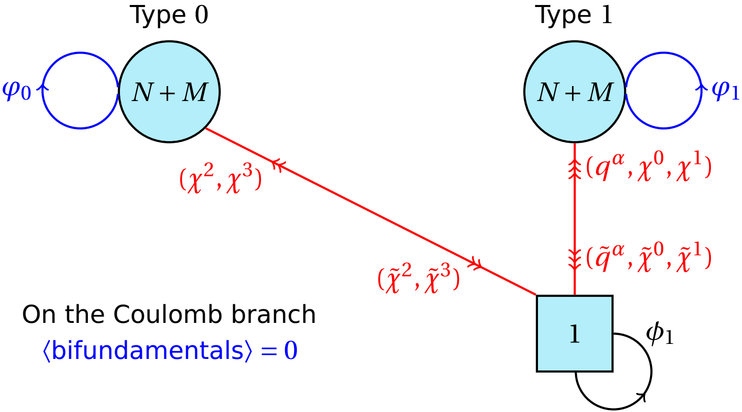

On the Coulomb branch
$\langle \text{bifundamentals} \rangle = 0$

Two different interpretations of D(-1)-brane

Gauge theory	looking at 1-instanton sector for gauge group 1
Bulk theory	probe brane in holographic dual geometry wrapped D1-brane $\Rightarrow S_{\text{probe}} = S_{\text{DBI}} + S_{\text{WZ}} = -2\pi i \gamma$
Holography	both descriptions equivalent [Maldacena; Ferrari]
$\Rightarrow$	dynamics of D3-branes = background geometry for bulk probe
In practice	integrate out D3 fields and mixed D3/D(-1) moduli
Yields	bulk probe action $S_{\text{probe}}(z) = -2\pi i \gamma(z)$ with $z = \ell_s^2 \phi_1$

The field γ in terms of a field theory correlator

- rescale adjoint gauge theory scalars $Z_0 = \ell_s^2 \varphi_0$, $Z_1 = \ell_s^2 \varphi_1$
- action is quadratic in mixed D3/D(-1) moduli
  $\rightarrow$ integration gives superdeterminant
- gauge theory integration: take its expectation value in gauge theory
- Result: $e^{2\pi i \gamma} = e^{2\pi i \tau_1} \left\langle \dfrac{\det(z - Z_0)^2}{\det(z - Z_1)^2} \right\rangle = e^{2\pi i \tau_1 + \langle \mathrm{tr} \log(z - Z_0) \rangle^2 - \langle \mathrm{tr} \log(z - Z_1) \rangle^2}$

same result as [Billó, Frau, Fucito, Giacone, Lerda, Morales, Ricci Pacifici] obtained by more involved argument

Computing the correlators

- chiral correlator, computable thanks to exact results in $\mathcal{N}=2$ quiver theories [Nekrasov, Pestun; Fucito, Morales, Ricci Pacifici]
- define for $a=0,1$ $\mathfrak{q}_a = e^{2\pi i \tau_a}$ and

$$y_a(z) = \exp\langle \operatorname{tr}\log(z-Z_a)\rangle = z^{N+M}\exp\left[-\sum_{k=2}^{\infty}\frac{1}{kz^k}\langle \operatorname{tr} Z_a^k\rangle\right]$$

- if Z_a ordinary matrices, $y_a(z)$ polynomials
- not true in the quantum theory $\Rightarrow y_a \neq T_a$
- rather $T_0(z) \propto y_0(z)\,\theta_3\left(\frac{y_1(z)^2}{\mathfrak{q}_1 y_0(z)^2};\mathfrak{q}^2\right)$ $T_1(z) \propto y_1(z)\,\theta_2\left(\frac{y_1(z)^2}{\mathfrak{q}_1 y_0(z)^2};\mathfrak{q}^2\right)$
- argument of elliptic θ-functions is $e^{-2\pi i\gamma} \rightarrow$ need to invert relations
- can be done explicitly $\rightarrow \gamma(z)$ as elliptic integral ($T_r = T_0/T_1$)

$$2\pi i\left(\gamma(z)-\tau_1\right) = \beta(\mathfrak{q}_a)\int_z^{\infty}\mathrm{d}x\,\frac{T_r'(x)}{\sqrt{T_r(x)^2-\alpha_1(\mathfrak{q}_a)^2}\sqrt{T_r(x)^2-\alpha_2(\mathfrak{q}_a)^2}}$$

- Valid for finite M, n, finite couplings, any Coulomb branch vacuum!

Taking limits

Check ($\mathrm{SU}(M)$ super-QCD with $N_f = 2M$)

- possible to turn off one of the gauge couplings, say $\mathfrak{q}_0 \to 0$
- integral becomes elementary: $2\pi i\,\gamma(z) = \log\left(\frac{1-\sqrt{1-T_r(z)^2/\alpha_2^2}}{1+\sqrt{1-T_r(z)^2/\alpha_2^2}}\right)$
- agrees with [Billó, Frau, Fucito, Giacone, Lerda, Morales, Ricci Pacifici; Martucci, Morales, Ricci Pacifici]

Large M limit $\mathfrak{q}_a^{1/M} \sim e^{-\#/\lambda}$ fixed

- particularize to enhançon vacuum, $T_r = T_0/T_1 = 1 + (\frac{z_0}{z})^M$
- for generic z, reduces to SQCD result

The enhançon mechanism

Three different regions (with $\rho_1 = |z_0|\mathfrak{q}_1^{1/2M}$)

Region $|z| > |z_0|$ above Higgsing scale,

$$\gamma(z) = \tau_1 \quad \text{constant}$$

Region $\rho_1 < |z| < |z_0|$ during the flow, but outside of enhançon

$$\gamma(z) = \tau_1 + \frac{iM}{\pi} \log \frac{z}{z_0} \quad \text{SUGRA behavior}$$

Region $|z| < \rho_1$ inside the enhançon

$$\gamma(z) = -\frac{\pi}{2} \quad \text{constant (but} \neq 0\text{)}$$

Note discontinous jump, already seen in special case [Cremonesi]

The $\mathcal{N} = 2^*$ theory

- break $\mathcal{N} = 4 \to \mathcal{N} = 2$ by giving mass m to adjoint hypermultiplet
- superpotential $W_{\text{mass}} = m(\varphi_2^2 + \varphi_3^2)$
- holographically, turning on non-normalizable mode
- dual SUGRA solution known for 1-dimensional subspace of Coulomb branch [Pilch, Warner]
- deep IR is singular, expected to be resolved by enhançon mechanism [Buchel, Peet, Polchinski]
- enhançon radius: vanishing of axio-dilaton $\tau = C_0 + ie^{-\phi}$
  $\Rightarrow$ tensionless D-branes

Adding an D(-1) probe

- bulk description: probe D(-1)-brane with action $S_{\text{probe}} = -2\pi i\tau$
- gauge theory description: 1-instanton sector
- again, "integrate out D3-branes"
  - mixed moduli → superdeterminant
  - gauge dynamics → expectation value
- restricting probe to plane $\phi_1 = \phi_2 = 0$ (∼ Coulomb branch): chiral correlator

$$e^{2\pi i\tau(z)} = e^{2\pi i\tau_0}\left\langle \frac{\det\left[(z/\ell_s^2 - \varphi)^2 - m^2\right]}{\det(z/\ell_s^2 - \varphi)^2} \right\rangle$$

- can be computed in principle using results of [Nekrasov, Okounkov; Nekrasov, Pestun], not yet completely solved

Summary & outlook

- D-brane probes: useful entry to holographic dictionary
- allowed translate exact results in gauge theory to exact results in bulk
- derived non-perturbative stringy effect: enhançon mechanism at work

- Future directions
  - complete $\mathcal{N} = 2^*$ analysis
  - 3d $\mathcal{N} = 4$ theories (see [Cottrell, Hanson, Hashimoto])
  - less SUSY ($\mathcal{N} = 1$ in 4d)
  - extremal black holes