One-loop Corrections to Holographic Wilson Loops

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Introduction

- Wilson Loops (WL) are an important class of gauge invariant non-local operators.

$$W_R(C) = \mathrm{Tr}_R P \exp\left(i \int_C A\right)$$

- The expectation value measures the effective action of an external particle; order parameter for confinement.
- The Lüscher term as quantum corrections to the QCD string (linear potential) $\mapsto$ Quantum Corrections in $\mathcal{N} = 4$.

Half BPS Wilson Loops in $\mathcal{N} = 4$ SYM

- Coupling of an external probe to the multiplet $\left(A_\mu, \phi^I, \lambda^A_\alpha\right)$:
  - Curve C in superspace, parameterized by $\left(x^\mu(s), y^I(s), \theta^\alpha_A(s)\right)$,
  - Representation R of $SU(N)$, corresponding to the charge of the particle.
- We consider only bosonic operators with $\theta^\alpha_A = 0$:

$$W_R(C) = \mathrm{Tr}_R P \exp\left(i \int_C ds \left(A_\mu \dot{x}^\mu + \phi_I \dot{y}^I\right)\right)$$

- SUSY forces C to be a timelike straight line with $y^2 = 1$.

$$W_R = \mathrm{Tr}_R P \exp\left(i \int dt \left(A_0 + \phi_1\right)\right)$$

Localization proves the Matrix Model Conjecture (Pestun)

- Want $\int \exp(S)$:

$$\int \exp(S) \quad \longrightarrow \quad \int \exp\left(S + tQV\right)$$

$$\frac{d}{dt}\int \exp\left(S + tQV\right) \quad = \quad 0 \tag{1}$$

- Independence of t, take $t \to \infty$ $\longrightarrow$ Classical plus one-loop.
- Localizes on a Gaussian action.

$$S = \frac{4\pi^2}{g_{YM}^2}\, r^2 a^2 \tag{2}$$

- a is a constant matrix coming from ϕ^I.
- r is the radius of S^4.
- Compare to the diagrammatic intuition of Drukker-Gross, Erickson-Semenoff-Zarembo.

Matrix Model: Fundamental

- The Matrix Model computation gives

$$
\begin{aligned}
\langle W_\Box \rangle_{\text{circle}} &= \frac{1}{N} L^1_{N-1}\left(-\frac{\lambda}{4N}\right) e^{\lambda/8N} \\
&\approx \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{\lambda}{38N^2} I_2(\sqrt{\lambda}) + \frac{\lambda^2}{1280N^4} I_4(\sqrt{\lambda}) + \ldots \\
&\approx \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \\
&\approx \exp\left(\sqrt{\lambda} - \frac{3}{4}\ln\lambda - \frac{1}{2}\ln\frac{\pi}{2} + \ldots\right)
\end{aligned}
$$

Gravity Side: Beyond the leading order

- Forste-Ghoshal-Theisen 9903042, Drukker-Gross-Tseytlin 0001204, Kruczenski-Tirziu 0803.0315, Buchbinder-Tseytlin 1404.4952.

$$\begin{aligned} \langle W \rangle &= \exp(-\Gamma), \qquad \Gamma = \Gamma_0 + \Gamma_1, \\ \Gamma_1 &= \frac{1}{2} \ln \frac{[\det(-\nabla^2 + 2)]^3 [\det(-\nabla^2)]^5}{[\det(-\nabla^2 + \frac{1}{4} R^{(2)} + 1)]^8} \end{aligned} \tag{3}$$

- Five massless modes (S^5); three massive modes $AdS_2 \subset AdS_5$.

$$\langle W_{\Box} \rangle_{\text{circle}} = \exp\left(\sqrt{\lambda} - \frac{1}{2} \ln(2\pi) + \ldots \right)$$

WL beyond the leading order: Problem/Opportunity

$$\langle W_\square \rangle_{\rm circle} = \exp\left(\sqrt{\lambda} - \frac{3}{4}\ln\lambda - \frac{1}{2}\ln\frac{\pi}{2} + \ldots\right)$$

$$\langle W_\square \rangle_{\rm circle} = \exp\left(\sqrt{\lambda} - \frac{1}{2}\ln(2\pi) + \ldots\right)$$

- Missing the $\ln(\lambda)$ term on the gravity side (zero modes, more later..).
- Numerical discrepancy is not an error: Drukker-Gross-Tseytlin 0001204, Kruczenski-Tirziu 0803.0315, Buchbinder-Tseytlin 1404.4952.

Looking for a knob or an extra parameter

- Drukker & Fiol '05 recognized that the D3 captured the leading behavior of multiply wrapped WL.
- The WL in the antisymmetric representation of $SU(N)$ is described by a D5 brane (Yamaguchi '06).
- A complete dictionary was proposed by Gomis & Passerini '06 for all half-BPS WL in arbitrary representations.

Configuration	Representation of $SU(N)$
F1	Fundamental
D3	Symmetric
D5	Antisymmetric

Higher Representations: Beyond the leading order

$$Z = \int dM \exp\left(-\frac{2N}{\lambda}\mathrm{Tr}\left(M^2\right)\right) , \tag{4}$$

- M is a $N \times N$ matrix, λ is the 't Hooft coupling. Work in the eigenvalue basis: $M = \mathrm{diag}\{m_1, m_2, m_3, \ldots, m_N\}$.
- VEV for k-th symmetric and antisymmetric representations use the generating functions for the relevant polynomials: $F_A(t) = \prod_{i=1}^N (1 + te^{m_i})$ and $F_S(t) = \prod_{i=1}^N (1 - te^{m_i})^{-1}$ [Hartnoll-Kumar].

$$\langle F_{A,S}(t)\rangle = \frac{1}{Z}\int \prod_{j=1}^{N}[dm_j]\Delta^2(m)F_{A,S}\exp\left(-\frac{2N}{\lambda}\sum_{i=1}^{N} m_i^2\right) , \tag{5}$$

Large N

- Continuous distribution, Wigner semi-circle:

$$\rho(m) = \frac{2}{\pi\lambda}\sqrt{\lambda - m^2}, \qquad -\sqrt{\lambda} \leq m \leq \sqrt{\lambda}.$$

- VEV of WL in the rank-k representation using the residue theorem.

$$\begin{aligned}\langle W_{S,A}\rangle \quad &= \quad d_{S,A}^{-1}\frac{\sqrt{\lambda}}{2\pi i}\int_C dz \exp \\ \left(\mp N\left[\frac{2}{\pi}\int_{-1}^{1} \right.\right. & \left.\left. dx \quad \sqrt{1-x^2}\log\left(1 \mp e^{\sqrt{\lambda}(-x+z)}\right) \pm f\sqrt{\lambda}z\ \right]\right),\end{aligned}$$

Symmetric beyond the leading order

$$\kappa = \frac{k\sqrt{\lambda}}{4N}. \tag{6}$$

$$\langle W_{S_k} \rangle = \exp\left(2N\left[\kappa\sqrt{1+\kappa^2} + \sinh^{-1}\kappa\right] + \frac{1}{2}\ln\frac{\kappa^3}{\sqrt{1+\kappa^2}}\right). \tag{7}$$

Antisymmetric WL beyond the leading order

$$\theta_k : \qquad \pi \frac{k}{N} = \theta_k - \sin\theta_k \cos\theta_k. \tag{8}$$

$$\langle W_{A_k} \rangle = \exp\left(\frac{2N\sqrt{\lambda}}{3\pi} \sin^3\theta_k - \frac{1}{2} \ln \sin \theta_k \right),$$

Holographic Description of Wilson Loops

- According to the dictionary a Wilson Loop in the k-symmetric representation of $SU(N)$ is dual to a single D3 brane with k units of fundamental string charge dissolved in it.
- In the probe approximation the D3 brane is described by an $AdS_2 \times S^2$ geometry with k units of flux (Drukker & Fiol, '05).
- The bosonic D3 brane action is

$$S_B = T_{D3} \int d^4\sigma \sqrt{\det\left(g_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}\right)} - T_{D3} \int C_{(4)}$$

Tension $T_{D3} = \frac{N}{2\pi^2 L^4}$.

$AdS_5 \times S^5$

- The $AdS_5 \times S^5$ IIB background is described by

$$\begin{aligned} ds^2_{AdS_5\times S^5} &= ds^2_{AdS_5} + L^2 d\Omega_5^2 \\ F_{(5)} &\equiv (1+*)\, dC_{(4)} \\ &= -\frac{4}{L}\,(1+*)\,\mathrm{vol}\,(AdS_5) \end{aligned}$$

- We work in the following coordinate system:

$$ds^2_{AdS_5} = L^2\left(\cosh^2(u) ds^2_{AdS_2} + \sinh^2(u) d\Omega_2^2 + du^2\right)$$

$$C_{(4)} = 4L^4 f(u)\, \underbrace{e^0\wedge e^1}_{AdS_2} \wedge \underbrace{e^2 \wedge e^3}_{S^2} \qquad f(u) = \frac{1}{32}\sinh(4u) - \frac{u}{8}$$

- The solution dual to the BPS WL is

$$u = u_k \qquad \theta^{\hat{i}} = \theta_0^{\hat{i}} \qquad 2\pi\alpha' F = iL^2 \cosh(u_k) e^{\underline{0}} \wedge e^{\underline{1}}$$

$AdS_2 \times S^2$ worldvolume with electric flux

$$ds^2 = L^2 \left(\cosh^2(u_k) ds^2_{AdS_2} + \sinh^2(u_k) d\Omega_2^2\right),$$

$$\sinh(u_k) = \frac{k\sqrt{\lambda}}{4N} \equiv \kappa.$$

k is the fundamental string charge dissolved on the brane.

- The solution preserves the same bosonic symmetries as the field theory operator:
  - $SL(2,\mathbb{R}) \times SO(3)$ are realized as isometries of the worldvolume.
  - $SO(5)$ corresponds to rotations around a fixed point on S^5.
- It also preserves half of the $AdS_5 \times S^5$ supersymmetries $OSp(4^*|4) \supset SL(2,\mathbb{R}) \times SO(3) \times SO(5)$.

Comparison with Matrix Model

- For the circle we find, after properly considering boundary terms,

$$S_{on-shell} = -2N\left(\kappa\sqrt{1+\kappa^2} + \sinh^{-1}\kappa\right) \qquad \text{circle}$$

- This correctly reproduces the matrix model calculation in the large λ limit.

D3 brane action

- Bosonic action:

$$S_B = T_{D3}\int d^4\sigma\,\sqrt{\det\left(g+2\pi\alpha' F\right)} - T_{D3}\int P[C_4].$$

- Fermionic action (Martucci et al. '05):

$$S_F = \frac{T_{D3}}{2}\int d^4\sigma\,\sqrt{\det\left(g+2\pi\alpha' F\right)}\,\overline{\Theta}\left(1-\Gamma_{D3}\right)\tilde{M}^{\alpha\beta}\Gamma_\beta D_\alpha \Theta.$$

$$\tilde{M}_{\alpha\beta} = g_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}\tilde{\Gamma} \qquad \tilde{\Gamma} = \Gamma^{11}\otimes\sigma_3$$

$$D_m = \nabla_m + \frac{1}{16}\not{F}_{(5)}\Gamma_m \otimes \left(i\sigma_2\right).$$

Γ_{D3}: κ-symmetry projector.

Action for the fluctuations

- Expanding to quadratic order we find:
  - Bosons

$$S_\phi^{(2)} = \frac{T_{D3}\coth(u_k)}{2}\int d^4\sigma\sqrt{\hat{g}}\hat{g}^{\alpha\beta}\left(\partial_\alpha\phi^{\hat{\underline{4}}}\partial_\beta\phi_{\hat{\underline{4}}} + \partial_\alpha\phi^{\hat{\underline{i}}}\partial_\beta\phi_{\hat{\underline{i}}}\right),$$

$$S_a^{(2)} = \frac{T_{D3}\coth(u_k)}{4}\int d^4\sigma\sqrt{\hat{g}}\hat{g}^{\alpha\beta}\hat{g}^{\gamma\delta}f_{\alpha\gamma}f_{\beta\delta}.$$

Six massless scalars and a massless gauge field in $AdS_2 \times S^2$.

  - Fermions

$$S_\Theta^{(2)} = \frac{T_{D3}\coth(u_k)}{2}\int d^4\sigma\sqrt{\hat{g}}\,\overline{\Theta}\hat{\nabla\!\!\!/}\Theta.$$

Four massless Weyl fermions in $AdS_2 \times S^2$.

- Putting everything together,

$$\underbrace{6\times\left(\frac{1}{180}\right)}_{6\text{ scalars}}+\underbrace{1\times\left(-\frac{13}{90}\right)}_{1\text{ vector}}+\underbrace{4\times\frac{1}{2}\times\left(\frac{11}{180}\right)}_{4\text{ Weyl fermions}}-\underbrace{2\times\left(\frac{1}{180}\right)}_{\text{ghosts}}=0$$

- The contribution to the partition function from non-zero modes vanishes! Must consider zero modes.
  - There are only vector zero modes on $AdS_2\times S^2$.
- This results coincides with an alternative calculation of Buchbinder-Tseytlin [1404.4952].

$$\int\exp\left(-S_{\mathrm{D3}}\right) = \exp\Bigg(2N\left[\kappa\sqrt{1+\kappa^2}+\sinh^{-1}\kappa\right] -\frac{1}{2}\ln\frac{\kappa^3}{\sqrt{1+\kappa^2}}\Bigg). \quad (9)$$

Holographic k-antisymmetric WL

- D5 on $AdS_2 \times S^4$ with flux agreement with matrix model at leading order [Yamaguchi '06]. The flux k fixes the angle θ_k:

$$\pi \frac{k}{N} = \theta_k - \sin\theta_k \cos\theta_k. \tag{10}$$

- The spectrum for the $AdS_2 \times S^4$ D5 brane; agrees with expected $OSp(4^*|4)$ structure [A.Faraggi, W. Mück LPZ 1112.5028].
- Heat kernels on $AdS_2 \times S^4$ [A.Faraggi, W. Mück and LPZ 1112.5028].

Quadratic fluctuations of D5

$$
\begin{aligned}
S_B &= \frac{T_{\rm D5}}{2\sin\theta_k}\int d^6\xi\sqrt{g}\Big[\chi^i(\nabla_a\nabla^a-\frac{2}{L^2})\chi_i \\
&\quad +\chi^5(\nabla_a\nabla^a+\frac{4}{L^2})\chi_5-\frac{1}{2}f^{\mu\nu}f_{\mu\nu}-f^{\mu\alpha}f_{\mu\alpha} \\
&\quad -\frac{1}{2}f^{\alpha\beta}f_{\alpha\beta}-\frac{4i}{L}\chi^5\epsilon^{\alpha\beta}f_{\alpha\beta}\Big], \\
S_F &= \frac{T_{\rm D5}}{2\sin\theta_k}\int d^6\xi\sqrt{g}\,\bar\Theta\Big[\Gamma^a\nabla_a+\frac{1}{L}\Gamma^{6789}\Big]\Theta,
\end{aligned} \tag{11}
$$

- χ^i – triplet: $AdS_2\subset AdS_5$; a χ^5 – singlet $(S^4\subset S^5)$;
- f_{ab} gauge field.
- Evil coupling $\chi^5\epsilon^{\alpha\beta}f_{\alpha\beta}$.

Holographic k-antisymmetric at one loop

$$\int \exp\left(-S_{\mathrm{D5}}\right) = \exp\left(\frac{2N\sqrt{\lambda}}{3\pi}\sin^3\theta_k - \frac{1}{6}\ln\sin\theta_k\right).$$

k-symmetric at one loop: a factor of -1

$$\int \exp\left(-S_{\rm D3}\right) = \exp\left(2N\left[\kappa\sqrt{1+\kappa^2}+\sinh^{-1}\kappa\right] -\frac{1}{2}\ln\frac{\kappa^3}{\sqrt{1+\kappa^2}}\right). \qquad (12)$$

$$\langle W_{S_k}\rangle = \exp\left(2N\left[\kappa\sqrt{1+\kappa^2}+\sinh^{-1}\kappa\right] +\frac{1}{2}\ln\frac{\kappa^3}{\sqrt{1+\kappa^2}}\right). \qquad (13)$$

k-antisymmetric at one loop: a factor of 3

$$\int \exp\left(-S_{\mathrm{D5}}\right) = \exp\left(\frac{2N\sqrt{\lambda}}{3\pi}\sin^3\theta_k - \frac{1}{6}\ln\sin\theta_k\right).$$

$$\langle W_{A_k}\rangle_{Saddle} = \exp\left(\frac{2N\sqrt{\lambda}}{3\pi}\sin^3\theta_k - \frac{1}{2}\ln\sin\theta_k\right),$$

Back to the detective story about the fundamental ...

- The AdS/CFT formula:

$$< W(C) >_{CFT} = < V(\Sigma \to C) >_{String} \tag{14}$$

- What we are doing:

$$< Sting - Correlator >_{String} \approx \exp(-S_{class}) \text{``Det''} \tag{15}$$

- We are missing aspects of string perturbation theory: Ghost zero modes, etc.
- $\Longrightarrow$ Compare configurations with the same world sheet topology!
- The $1/4$ BPL Wilson beyond the leading order.

The 1/4 BPS WL at One Loop

$$W(C) = \frac{1}{N}\mathrm{Tr}P\exp\oint(iA_\mu\dot{x}^\mu + |\dot{x}|\Theta_I\Phi_I)d\tau. \tag{16}$$

- Contour

$$\begin{aligned} x^\mu(\tau) &= (a\cos\tau, a\sin\tau, 0, 0), \\ \Theta_I &= (\sin\theta_0\cos\tau, \sin\theta_0\sin\tau, \cos\theta_0, 0, 0, 0). \end{aligned} \tag{17}$$

- In the planar limit

$$\langle W(C)\rangle = \frac{2}{\sqrt{\lambda'}}I_1(\sqrt{\lambda'}), \tag{18}$$

where $\lambda' = \lambda\cos^2\theta_0$.

Holographic $1/4$ BPS WL

- The $EAdS_5$ metric is written as a foliation over $EAdS_2 \times S^2$:

$$\frac{ds^2_{AdS_5}}{L^2} = \cosh^2(u)\left(d\rho^2 + \sinh^2\rho\, d\psi^2\right) + \sinh^2(u)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right) + du^2\,.$$

- The metric on S^5 is taken to be

$$\frac{d\Omega_5^2}{L^2} = d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\left(d\chi^2 + \cos^2\chi\, d\alpha^2 + \sin^2\chi\, d\beta^2\right)\,.$$

Holographic 1/4 BPS WL

- Classical solution – Ansatz:

$$\psi = \tau, \qquad \sinh\rho = \frac{1}{\sinh\sigma},$$
$$u = 0$$
$$\phi = \tau, \qquad \sin\theta = \frac{1}{\cosh(\sigma_0 + \sigma)},$$

- Classical solution – World-sheet metric:

$$ds^2 = \left(\frac{1}{\sinh^2\sigma} + \frac{1}{\cosh^2(\sigma_0 - \sigma)}\right)(d\tau^2 + d\sigma^2). \tag{19}$$

Fluctuations

- A. Faraggi, LPZ, G. Silva and D. Trancanelli; see also Forini-Giangreco-Griguolo-Seminaro-Vescovi.
- The quadratic action for the bosonic fluctuations is

$$S_{(2,3,4)} = \frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \left(\delta^{ab}\partial_a\Phi\partial_b\Phi + 2\sinh^2\rho\,\Phi^2\right), \quad (20)$$

$$S_{(5,6)} = \frac{\sqrt{\lambda}}{2\pi} \int d\tau d\sigma \left(\delta^{ab}D_a\Phi(D_b\Phi)^\dagger - 2m^2(\sigma)|\Phi|^2\right), \quad (21)$$

$$S_{(7,8,9)} = \frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \left(\delta^{ab}\partial_a\Phi\partial_b\Phi - 2\sin^2\theta\,\Phi^2\right), \quad (22)$$

- Note the less symmetric splitting with respect to $1/2$ BPS $(3,5) \mapsto (3,2,3)$.

Symmetries of fluctuations

- The $SO(2) \times SO(3) \times SO(3) \subset SU(2|2)$ invariance of the spectrum is obvious.
- The fluctuations $\chi^{\underline{2}}$, $\chi^{\underline{3}}$ and $\chi^{\underline{4}}$ can be written as a scalar field action in AdS_2 with mass term $m^2 = 2$.
- The "mass" terms for $\chi^{\underline{5}}$ and $\chi^{\underline{6}}$, and $\chi^{\underline{7}}$, $\chi^{\underline{8}}$ and $\chi^{\underline{9}}$ all vanish in the limit $\theta_0 \to 0$,
- The $\theta_0 \to 0$ limit recovers the $SL(2,\mathbb{R}) \times SO(3) \times SO(5) \subset OSp(4^*|4)$ bosonic symmetry of the half-BPS solution.

- Stay tuned for the final comparison (A. Faraggi, LPZ, G. Silva and D. Trancanelli)!
- What other precision tests are there?
  - ABJM theories: $C = 2/(\pi^2 k)$

$$\langle W_{\Box}^{1/2} \rangle = \frac{1}{4} \csc\left(\frac{2\pi}{k}\right) \mathrm{Ai}[C^{-1/3}(N - \frac{k}{24} - \frac{7}{3k})] \qquad (23)$$

  - Fundamental representations with the same topology and different susy: $1/2$ versus $1/4$
  - Considering higher representations in ABJM holographically.
  - D2 (symmetric) and D6 (antisymmetric) branes with fluxes beyond the leading term (Faraggi, Mück, PZ, Rathee)

Parting words

- Gauge theory can, in principle, consider different orders of limits: (N, λ, k)
- Saddle point always takes $N \to \infty$ first. Corrections to Wigner distribution.
- Gravity is more rigid in its expansion: The gravitational description expansion parameters are: $1/\sqrt{\lambda}$ (F1), $1/N\sqrt{\lambda}$ (D5) and $1/N$ (D3)
- String theory zero modes and powers of λ; we will learn something important about quantum string theory on $AdS_5 \times S^5$.
- Take this lessons to computing corrections in QCD-like models (Bigazzi, Cotrone, Martucci, PZ '04).