Black holes and M-theory

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Introduction

Black holes: a great laboratory for gravity...

...and in particular for string theory!

- Microstate counting [Strominger, Vafa '95]
- AdS/CFT: unitarity of black hole evolution
- Bound states/multicenter solutions [Denef '00]
- Fuzzball conjecture [Mathur]

In this talk we will review some recent progress in supersymmetric AdS_4 black holes

[Cacciatori, Klemm '09]

[Dall'Agata, Gnecchi '10]
[Hristov, Vandoren '10]

Many of these can be naturally embedded in M/string theory

[Halmagyi, Petrini, Zaffaroni '11]

[Katmadas, AT '15]

They have interesting AdS/CFT interpretations, that can even lead to their microstate counting

[Hristov, AT, Zaffaroni '13]

[Benini, Hristov, Zaffaroni '15]

Plan

- Black holes in AdS_4
- AdS/CFT interpretation
- M-theory

I. Black holes in AdS

$$ds^2 = -U^2 dt^2 + U^{-2} dr^2 + r^2 d\Omega^2$$

gravitational
potential

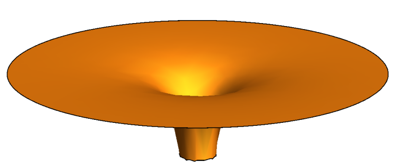
S^2 angular directions

black holes in flat space:

el.charge mag.charge

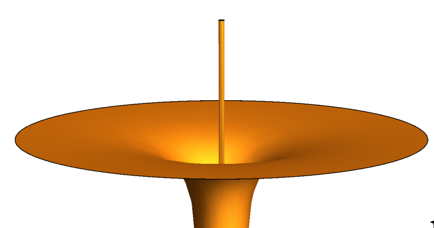
perfect square:

$$U^2 = 1 - \frac{2M}{r}$$



Schwarzschild

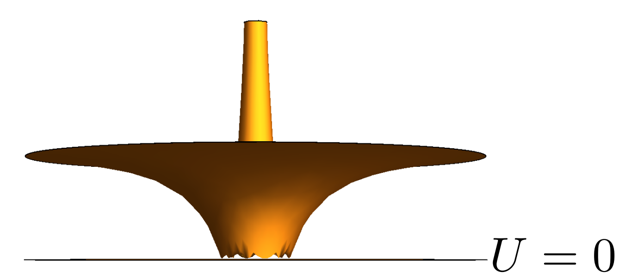
$$U^2 = 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{r^2}$$



two horizons

Reissner–Nordström

$$U^2 = (1 - \frac{M}{r})^2$$



extremal Reissner–Nordström

$$ds^2 = -U^2 dt^2 + U^{-2} dr^2 + r^2 d\Omega^2$$

the extreme case is

perhaps better known with $\rho \equiv r - M$

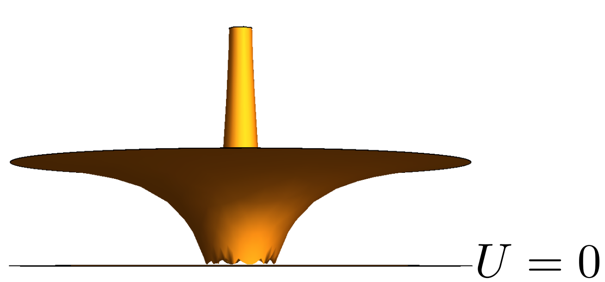
$$ds^2 = -\frac{1}{\left(1+\frac{M}{\rho}\right)^2} dt^2 + \left(1 + \frac{M}{\rho}\right)^2 \underbrace{(d\rho^2 + \rho^2 d\Omega^2)}_{\text{flat } \mathbb{R}^3!}$$

multicenter generalizations...

$$ds^2 = -H^{-2} dt^2 + H^2 ds^2_{\mathbb{R}^3}$$

$$H = 1 + \sum_i \frac{M_i}{\rho - \rho_i}$$

$$U^2 = (1 - \frac{M}{r})^2$$



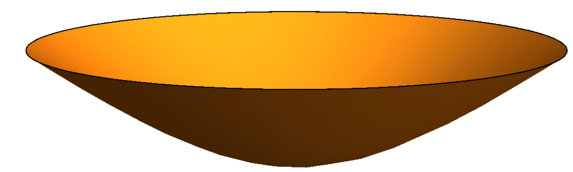
extremal Reissner–Nordström

How do we put them in AdS?

$$ds^2 = -U^2 dt^2 + U^{-2} dr^2 + r^2 d\Omega^2$$

$$U^2 = 1 - \Lambda r^2$$

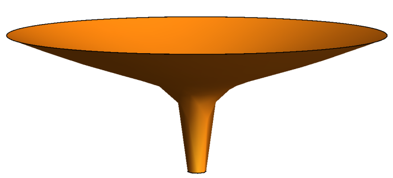
AdS4 itself looks nice
in these coordinates:

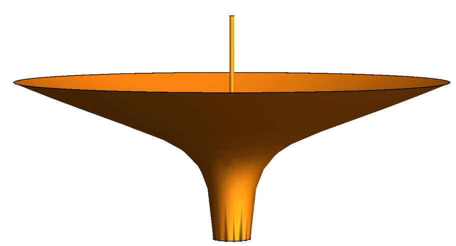


black holes in AdS4:

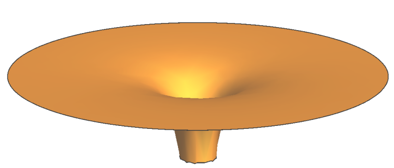
$$U^2 = 1 - \frac{2M}{r} - \Lambda r^2 \qquad U^2 = 1 - \frac{2M}{r} + \frac{Q^2+P^2}{r^2} - \Lambda r^2$$

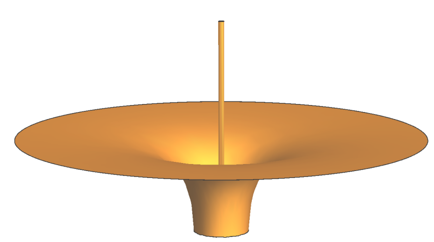
$\Lambda < 0$





$\Lambda = 0$





Schwarzschild

Reissner–Nordström

what happens if we impose supersymmetry?

$$ds^2 = -U^2 dt^2 + U^{-2} dr^2 + r^2 d\Omega^2$$

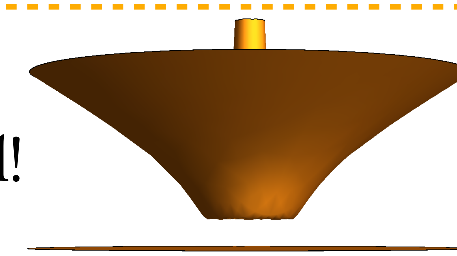
let's try in minimal supergravity

[Romans '92]

[gauged, so that it has an AdS4 vacuum]

either BPS or extremal

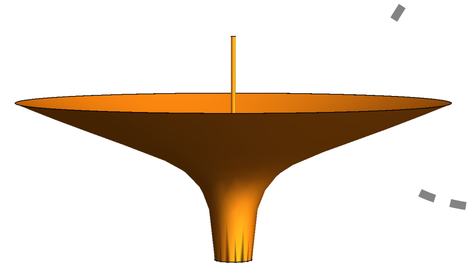
1/2 BPS
non-extremal!



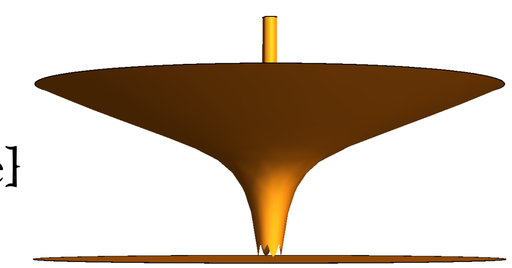
$U > 0$
no horizons

$$U^2 = (1 - \frac{M}{r})^2 - \Lambda r^2$$

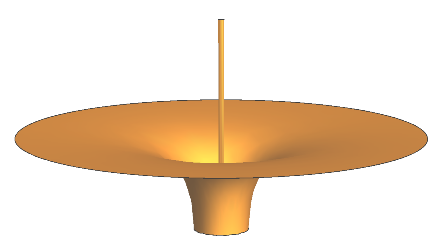
$\Lambda < 0$



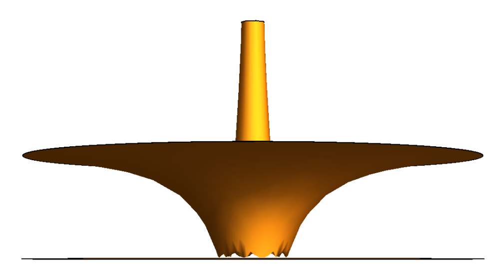
extremal
[horizons coincide]
but non-BPS



$\Lambda = 0$



Reissner–Nordström



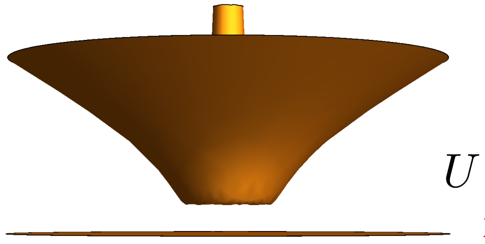
$U = 0$

extremal Reissner–Nordström

there is in fact a third possibility

$Q = M, P = 0$

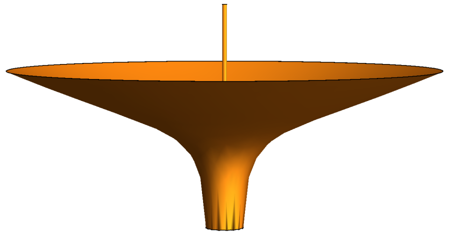
1/2 BPS
non-extremal!



$U > 0$
no horizons

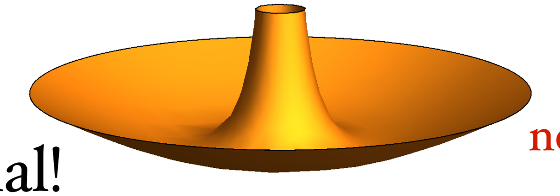
$$U^2 = (1 - \frac{M}{r})^2 - \Lambda r^2$$

both of them have
naked singularities



magnetically charged:
$M = 0$, $P = \frac{1}{2\sqrt{-\Lambda}}$

1/4 BPS
non-extremal!



$U > 0$
no horizons

$$U^2 = \left(\frac{r}{\sqrt{-\Lambda}} + \frac{2\sqrt{-\Lambda}}{r}\right)^2 + \frac{Q^2}{r^2}$$

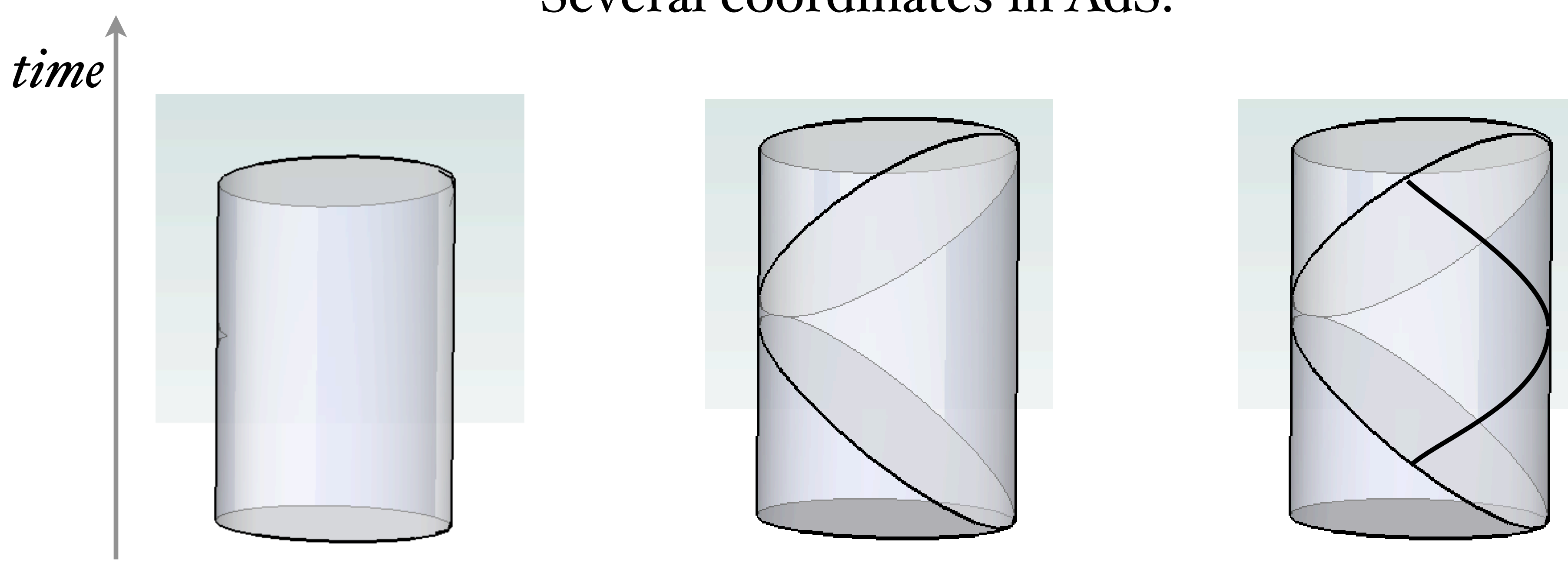
Can we get genuine AdS black holes [not naked singularities!] which are supersymmetric?

- Still in minimal supergravity:

[Caldarelli, Klemm '98; Kostelecky, Perry '95]

  - make them rotate
  - dirty trick: hyperbolic horizon!

Several coordinates in AdS:

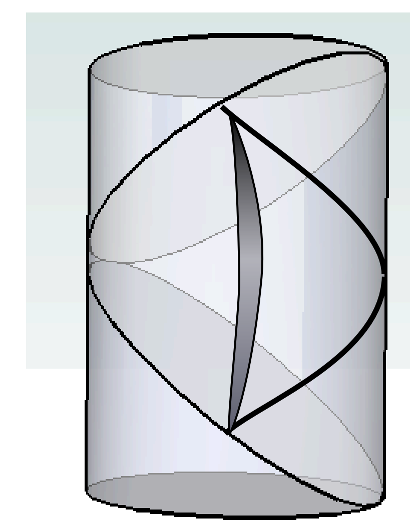


	global	Poincaré	hyperbolic
	[the ones we used so far]		
at ∞: $\mathbb{R}_{time} \times$	S^2	$\mathbb{R}^2$	$\mathbb{H}_2$
		$\mathbb{R}^2/\mathbb{Z}^2 = T^2$	$\mathbb{H}_2/\Gamma = \Sigma_{g\geq 2}$

in this coordinate patch
one can have a
1/4 BPS, magnetic black hole

the horizon and spatial ∞ are
a Riemann surface

fine, but a bit funny...



hyperbolic

$\mathbb{H}_2$

$\mathbb{H}_2/\Gamma = \Sigma_{g\geq 2}$

- Include more matter: situation does not improve — eg.[Sabra '99]

- Finally, missing ingredient: making scalars flow — [Cacciatori, Klemm '09]

original example: minimal + 3 vector multiplets — see also [Dall'Agata, Gnecchi '10] [Hristov, Vandoren '10]

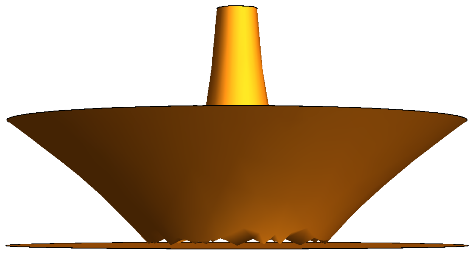
prepotential $\mathcal{F} = \sqrt{X^0 X^1 X^2 X^3}$

Fayet–Iliopoulos gauging: gravitinos charged under graviphoton

nontrivial potential: $\exists$ AdS vacuum

Ansatz: $X^I = \alpha^I + \frac{\beta^I}{r}$

horizon!

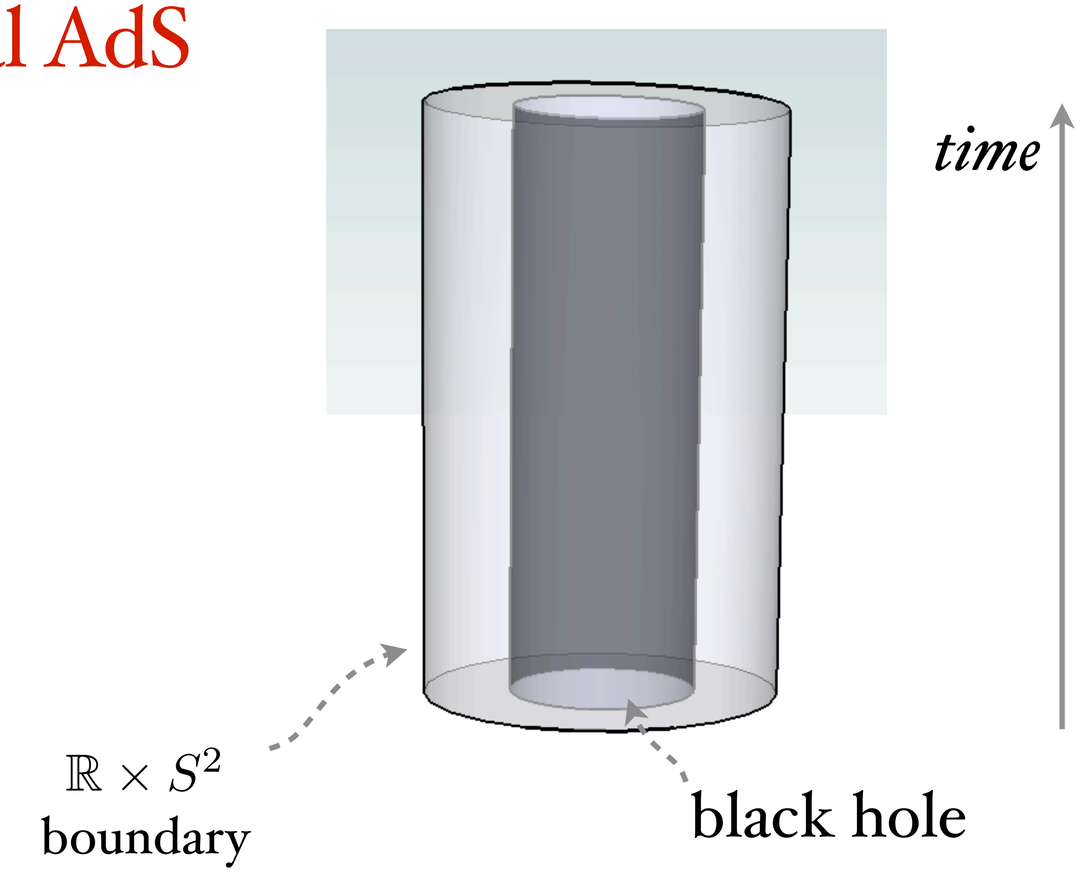


$$U^2 = e^{\mathcal{K}} \left(\frac{r}{\sqrt{-\Lambda}} - \frac{2\sqrt{-\Lambda}}{r} \right)^2$$

$(\mathrm{Im} X^I \bar{\mathcal{F}}_I)^{-1}$

II. AdS/CFT interpretation

So we have a genuine black hole in global AdS



what is its holographic interpretation?

- Crucial element: magnetic fields at ∞

$$F_I \propto g_I \mathrm{vol}_{S^2}$$

They become background external fields
for some global symmetries in the CFT3

$$\langle e^{A_I^0 J_I} \rangle_{\mathrm{CFT}_3} = Z_{\mathrm{bulk}}(A_I \to A_I^0)$$

- In particular, the graviphoton always has charge -1

R-symmetry current

twisting

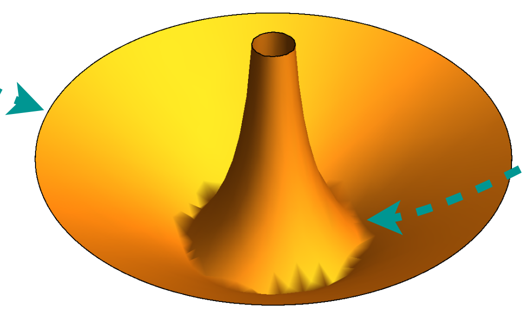
supercharge ϵ satisfies $0 = \nabla_\mu^A \epsilon = \partial_\mu \epsilon$

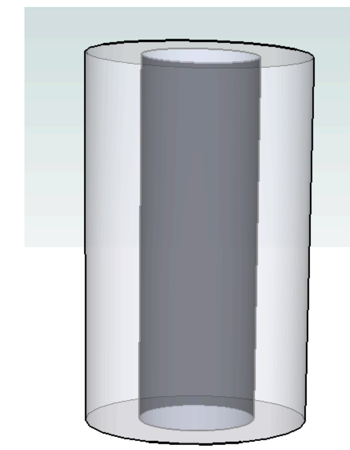
'gauge field A cancels spin connection'

The 1/4 BPS solutions look like

$\mathrm{AdS}_4 \dashrightarrow \mathrm{AdS}_2 \times S^2$

[infinity] [near-horizon]





the field theory dual should be

twisted SCFT_3 on $\mathbb{R} \times S^2$ $\dashrightarrow$ SCFT_1 on $\mathbb{R}$

[UV] [IR]

More generally:

[Hristov, AT, Zaffaroni '13]

SCFT$_3$ on curved M_3 preserves susy $\Longleftrightarrow$ M_3 admits null or timelike conformal Killing vector

[bilinear of supercharge ϵ]

For $M_3 = \mathbb{R} \times S^2$:

$$F = 0$$

$z =$ complicated in general

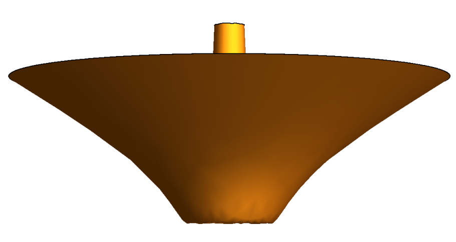
[one simple ex.: $z = \partial_t + \partial_\phi$]

$$F = -\frac{1}{2}\mathrm{vol}_{S^2}$$

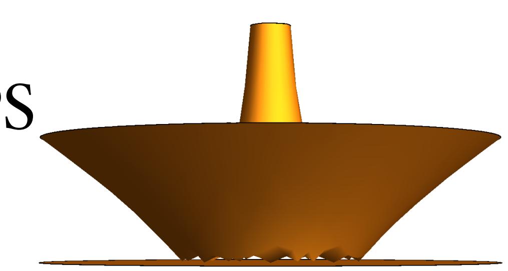
$$z = \partial_t$$

[interpolating family: $z = \partial_t + a\partial_\phi$]

1/2 BPS
naked sing.



1/4 BPS



[interpolating: rotating BH]

the field theory dual should be

twisted SCFT_3 on $\mathbb{R} \times S^2$ ⇢ SCFT_1 on $\mathbb{R}$

[UV] [IR]

- similar to Maldacena-Nuñez twisted compactifications
- the SCFT_3 can be obtained in some cases using the M-theory lift

[next part of this talk!]

- obtaining the SCFT_1 is hard
- however, localization computation in SCFT_3 $\sim$ index of SCFT_1
- This reproduces the entropy [Benini, Hristov, Zaffaroni '15]

in the original Cacciatori-Klemm black hole

Completely different from AdS_5 black holes

[Gutowski, Reall '04]

- have to rotate
- correspond to states in the CFT
- counting microstates is hard [index ≠ partition function]

[Kinney, Maldacena, Minwalla, Raju '05]

Also different from non-susy black holes interpreted as introducing temperature.

[Witten '98]

III. M-theory

[Cacciatori, Klemm '09]

original example: minimal + 3 vector multiplets

prepotential $\mathcal{F} = \sqrt{X^0 X^1 X^2 X^3}$ $\subset$ gauged $\mathcal{N} = 8$ supergravity [de Wit, Nicolai '82]

Fayet–Iliopoulos gauging

M-theory reduction on S^7

4 vectors: $\mathrm{U}(1)^4 \;\subset\; \mathrm{SO}(8)$

- the vacuum at infinity is $\mathrm{AdS}_4 \times S^7$
- the SCFT3 in this case is ABJM

[Aharony, Bergman, Jafferis, Maldacena'08]

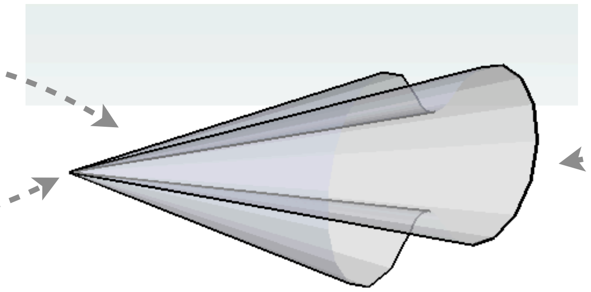
one can also use gauged supergravities obtained from other 7d spaces

- some cosets give rise to AdS_4 vacua $\quad \frac{SU(2)^3}{U(1)^2}, \frac{U(3)}{U(1)}, \frac{SO(5)}{SO(2)}$
- and to consistent truncations [Cassani, Koerber, Varela '12]
- so black hole solutions in these models can be lifted to M-theory solutions. [Halmagyi, Petrini, Zaffaroni '13]

But there are many more AdS_4 solutions...

8d cone

near-horizon limits
of M2-branes



M_7

$\mathrm{AdS}_4 \times M_7$

- For example: M_7 Sasaki-Einstein $\Rightarrow \mathcal{N} = 2$

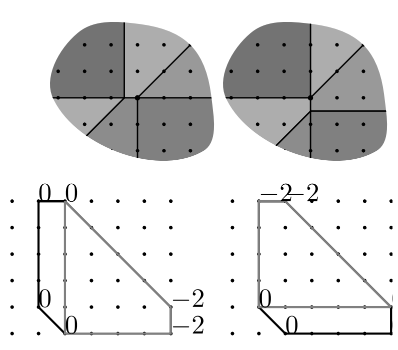
many examples!

$$\begin{array}{ccc} S^1 & \hookrightarrow & M_7 \\ & & \downarrow \\ & & M_6 \end{array}$$

Kähler-Einstein

many are being found right now
because of recent proof of "stability condition"

[Chen, Donaldson, Sun '12]



e.g. a recent one: 3folds with 2-torus action

[Süß '13; Datar,Székelyhidi '15]

- but for these no reduction has been worked out [let alone a consistent one]

In 4d ‘symmetric models’, elegant reformulation of scalar BPS equations

$[\mathcal{M}_{\text{scalars}} = G/H]$

[Katmadas ’14]

[schematically]

$$\underbrace{\partial_r \mathcal{H}}_{\text{scalars}} \propto I_4'(\mathcal{H}, \mathcal{H}, \underbrace{P}_{\text{gaugings}}) + \underbrace{\Gamma}_{\text{charges}}$$

- I_4: quartic invariant of G
- This formalism can be used to find analytic solutions

[Katmadas ’14, Halmagyi ’14]

- experience from Minkowski suggests:

$$I_4 \dashrightarrow I_4^{\mathrm{Hit}}$$

'Hitchin functional'

- For Minkowski black holes, entropy $S \sim \sqrt{I_4(\Gamma, \Gamma, \Gamma, \Gamma)}$

- redoing the computation directly in IIA/IIB: [Hsu, Maloney, AT '14]

$$F_{\mathrm{RR}} = \mathrm{vol}_{S^2} \wedge f$$ [eg. even form in IIA]

represents charges Γ

$$S \sim \sqrt{I_4^{\mathrm{Hit}}(f, f, f, f)}$$

Hitchin functional

[Hitchin '02]

for even forms $(\alpha, \beta) \equiv \begin{matrix} *(\alpha_0\beta_6 - \alpha_2 \wedge \beta_4 \\ +\alpha_4 \wedge \beta_2 - \alpha_6\beta_0) \end{matrix}$

$$\mathcal{Q}^{AB} \equiv (f, \Gamma^{AB} f) \qquad \left[\begin{array}{c:c} (f, dx^m \wedge dx^n \wedge f) & (f, \iota_m dx^n \wedge f) \\ \hdashline (f, dx^m \wedge \iota_n f) & (f, \iota_m \iota_n f) \end{array} \right]$$

[wedge]

$$\Gamma^A = \begin{cases} dx^m \wedge & 1 \leq A \leq 6 \\ \iota_m & 7 \leq A \leq 12 \end{cases}$$

[contraction]

$$I_4^{\text{Hit}}(f, f, f, f) \equiv \mathcal{Q}^{AB} \mathcal{Q}_{AB}$$

- introduced by Hitchin precisely to solve an attractor-like equation
- $f = \text{Re}($'pure spinor'$) \Rightarrow \mathcal{Q} \propto$ 'gen. complex structure'
- example: $(T^2)^3$;

$$f = f_0 + f_i j_i + \tilde{f}_i \tilde{j}_i + f_6 \text{vol}$$

$j_1 = dx^1 \wedge dx^2$ etc.

$\tilde{j}_1 = * dx^1 \wedge dx^2$ etc.

$$I_4 = 4 f_6 f_1 f_2 f_3 + 4 \tilde{f}_6 \tilde{f}_1 \tilde{f}_2 \tilde{f}_3 + (f_0 f_6 - f_i \tilde{f}_i)^2 - \epsilon_{ijk} f^j f^k \epsilon_{ilm} \tilde{f}^l \tilde{f}^m$$

EoM + supersymmetry directly in M-theory

[Katmadas, AT '15]

- Ansatz:

$$\begin{tikzcd} S^1 \arrow[r, hook] & M_7 \arrow[d] \\ & M_6 \end{tikzcd}$$

internal space

[its S^1 also fibred over spacetime]

||

AdS black hole

- flux almost completely determined using [Gauntlett, Pakis '02]

- reduced everything to flow equations on M_6

flux+ fibration data

no legs along S^2 legs along S^2

$$\partial_r \mathcal{H} \propto (I_4^{\mathrm{Hit}})'(\mathcal{H}, \mathcal{H}, P) + \Gamma$$

e^{B+iJ}

on M_6

- no coset structure was assumed on M_6

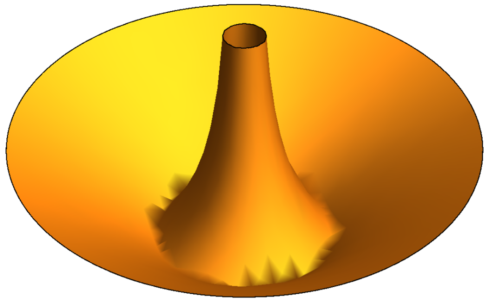
- we also went back to 4d...

we were able to show the flow equations
beyond 'symmetric models'

this seems to indicate that AdS black holes should be ubiquitous

Conclusions

AdS$_4$ black holes
are a new and exciting arena



They can be lifted to M-theory quite naturally

They have a "twisted compactification" CFT
interpretation that leads in some cases to
microstate counting