Higher spin gauge theories: a glimpse into quantum gravity?

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Based on work with D. Francia, S. Fredenhagen, M. Henneaux,
J. Mourad, S. Pfenninger, J. Raeymaekers, A. Sagnotti, S. Theisen, …

Renato Musto Award 2015 for Theoretical Physics

The quest for quantum gravity

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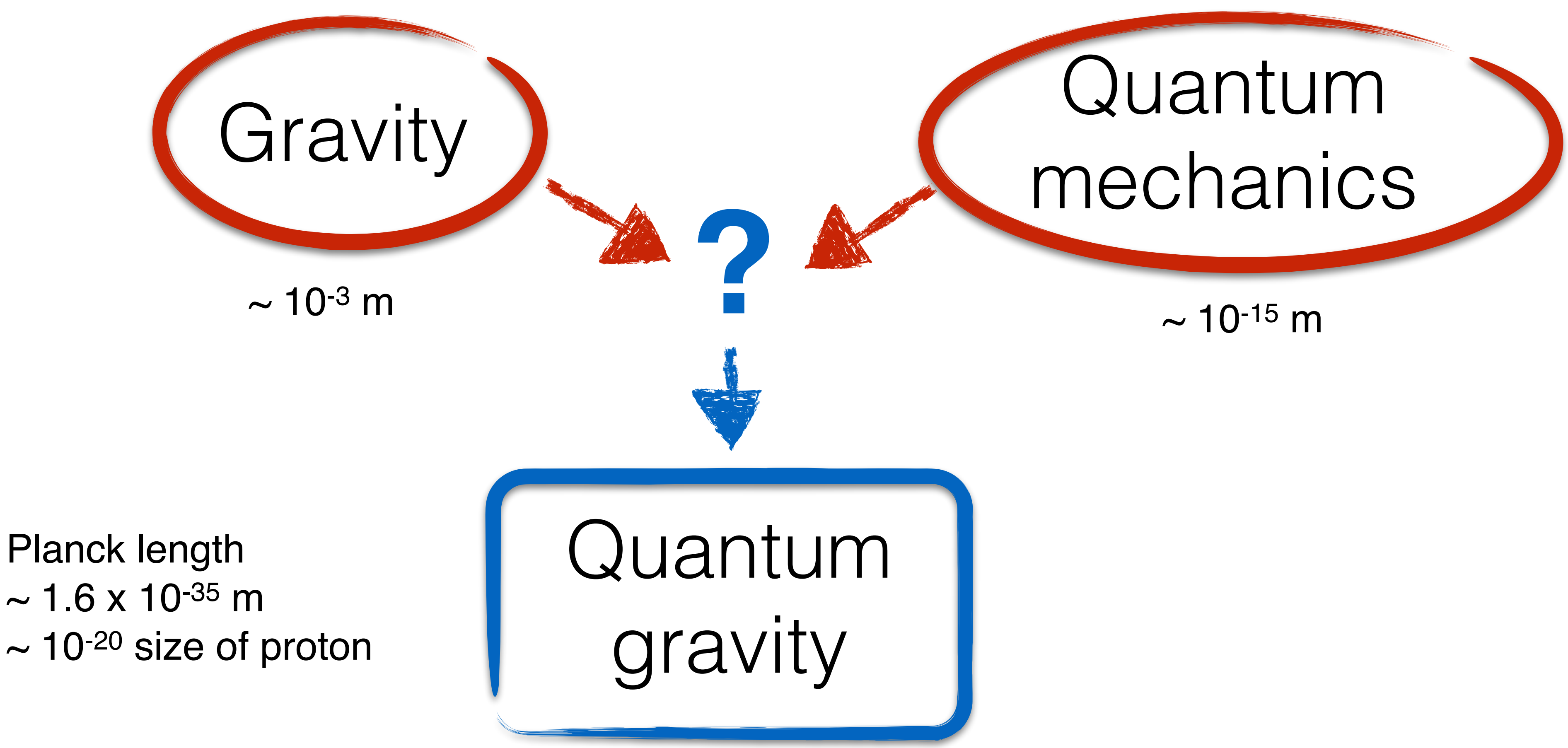

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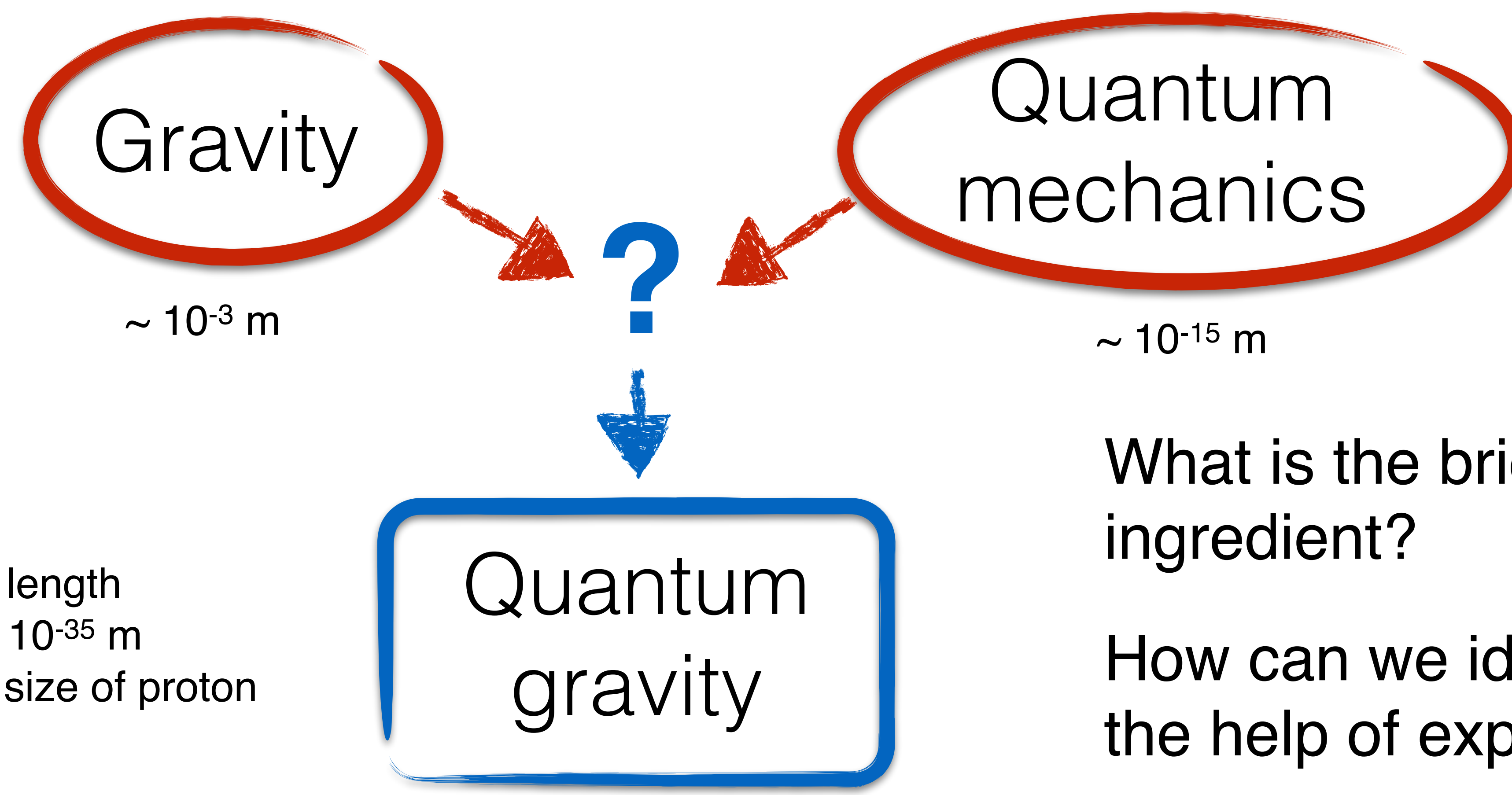

Planck length
~ 1.6×10^{-35} m
~ 10^{-20} size of proton

What is the bridging ingredient?

How can we identify it without the help of experiments?

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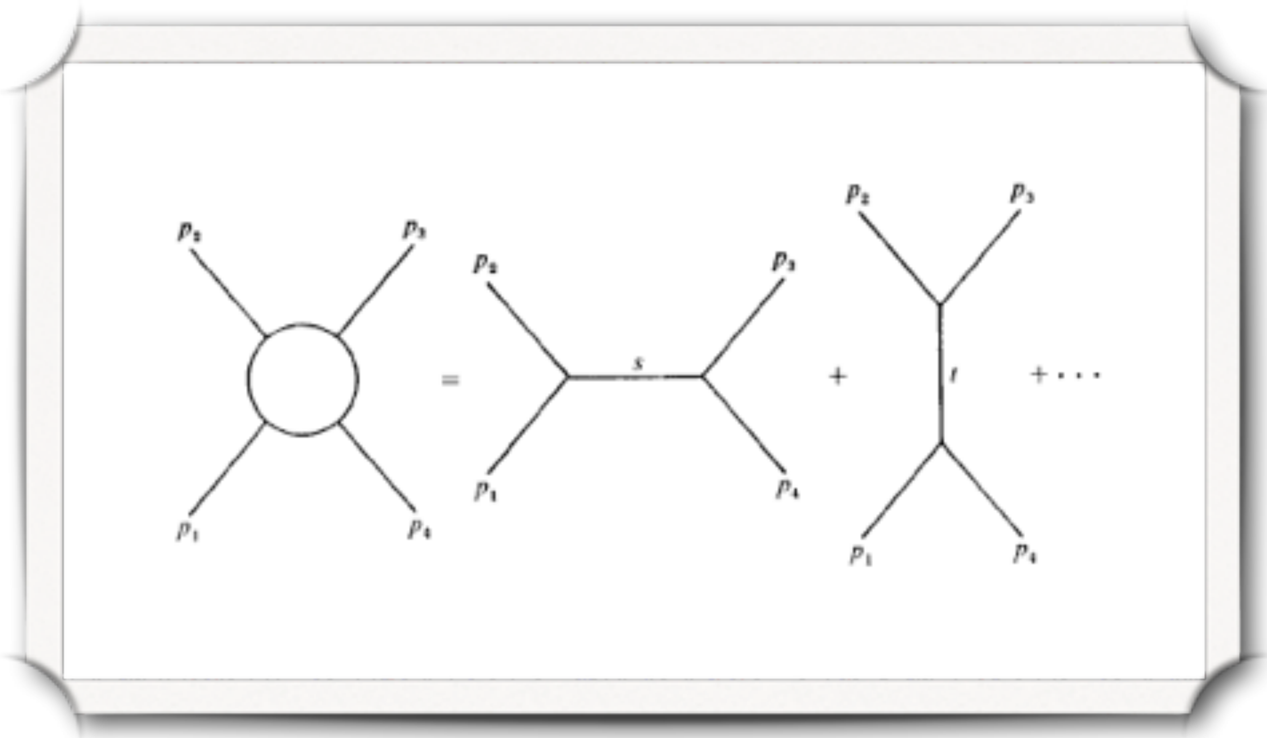
- String Theory: quantum completion of Einstein gravity?
  - Rough idea of the mechanism:



$$A_J(s,t) \sim -\frac{g^2(-s)^J}{t-M^2}$$

Higher spin
↓
worse UV divergence

Why higher spins?

- Einstein gravity is perturbatively non renormalisable
- Matter couplings can worsen (e.g. scalar) or improve the UV behaviour (e.g. supersymmetry)
- String Theory: quantum completion of Einstein gravity?
  - Veneziano amplitude and dual resonance models: Veneziano (1968)

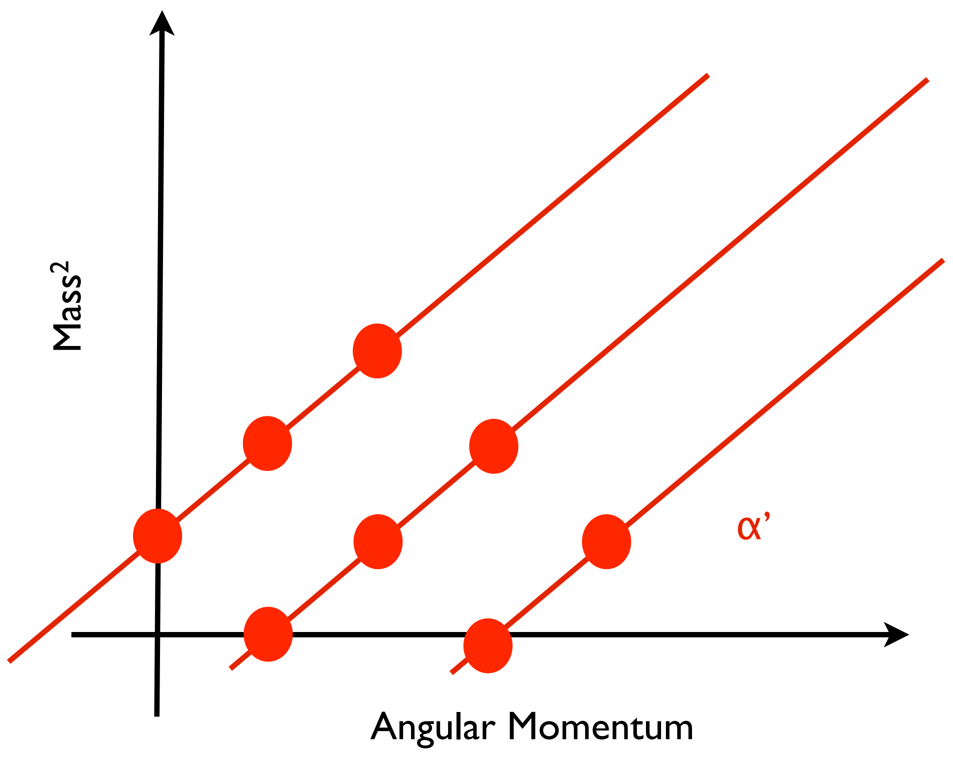
$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = \sum_{n=0}^{\infty} \frac{Re(\alpha(s))}{\alpha(t)-n}$$

  - Infinitely many bad contributions can conspire to give a nice answer

HIGHER SPINS ⇔ QUANTUM GRAVITY

Why higher-spin *gauge* theories

- String Theory: exchange of infinitely many massive particles



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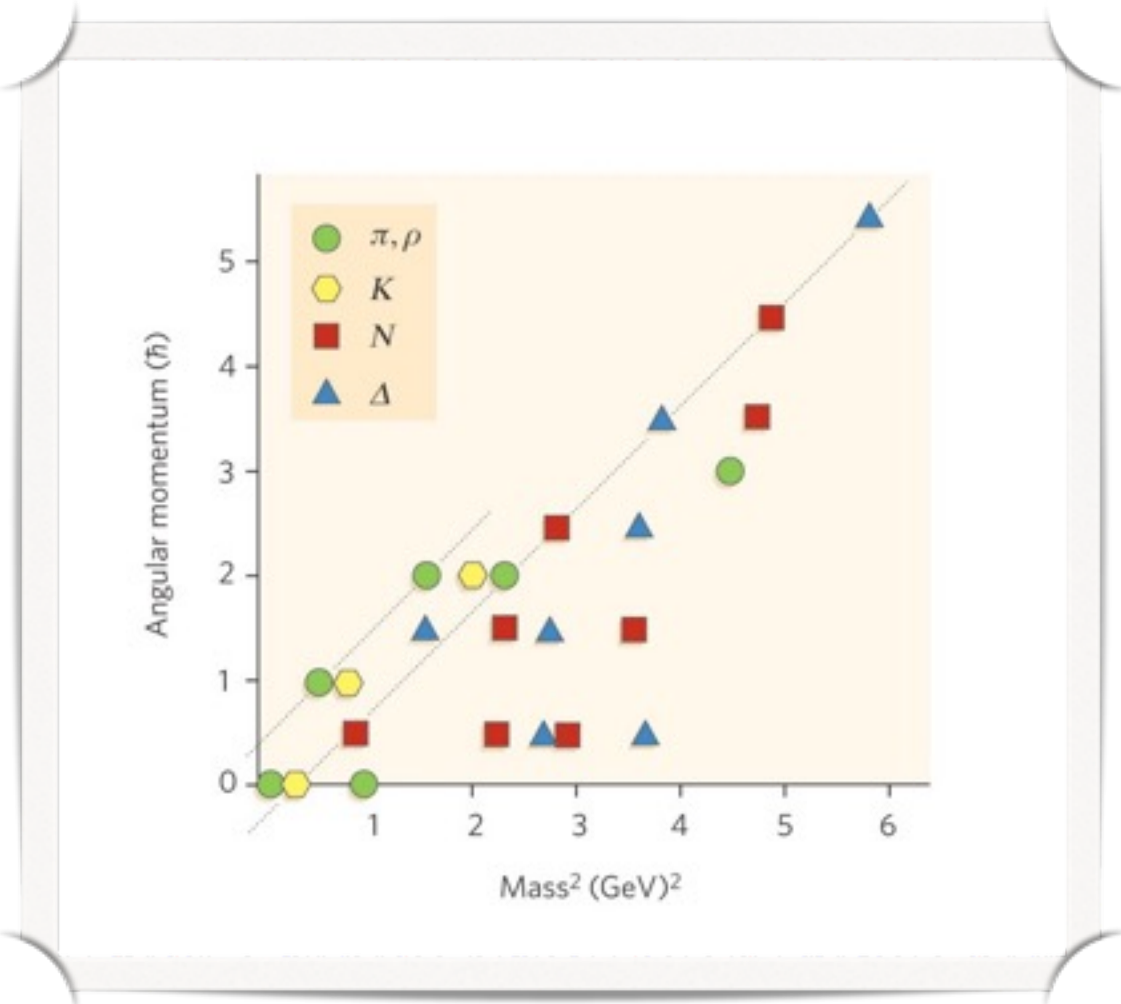
- String Theory has been originally built to describe strong interactions, not gravity!



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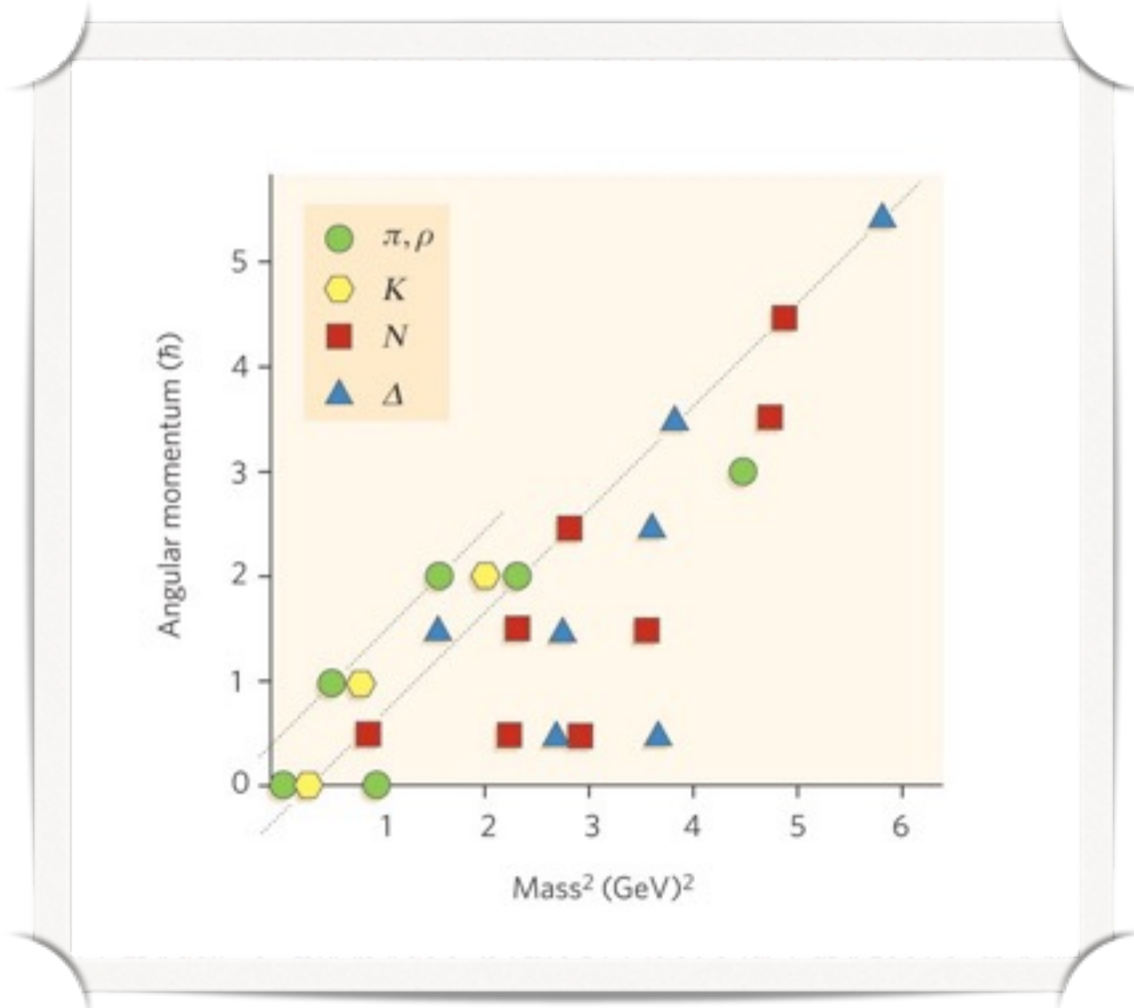

- Why are we interested in higher-spin gauge theories?
  - Gauge symmetry ⇔ massless particles
  - Example: photon described by A_μ with $\delta A_\mu = \partial_\mu \lambda$

Why higher-spin *gauge* theories

- At high energies the masses become negligible

- Signals of a huge *gauge symmetry* emerging in the limit α’→ ∞

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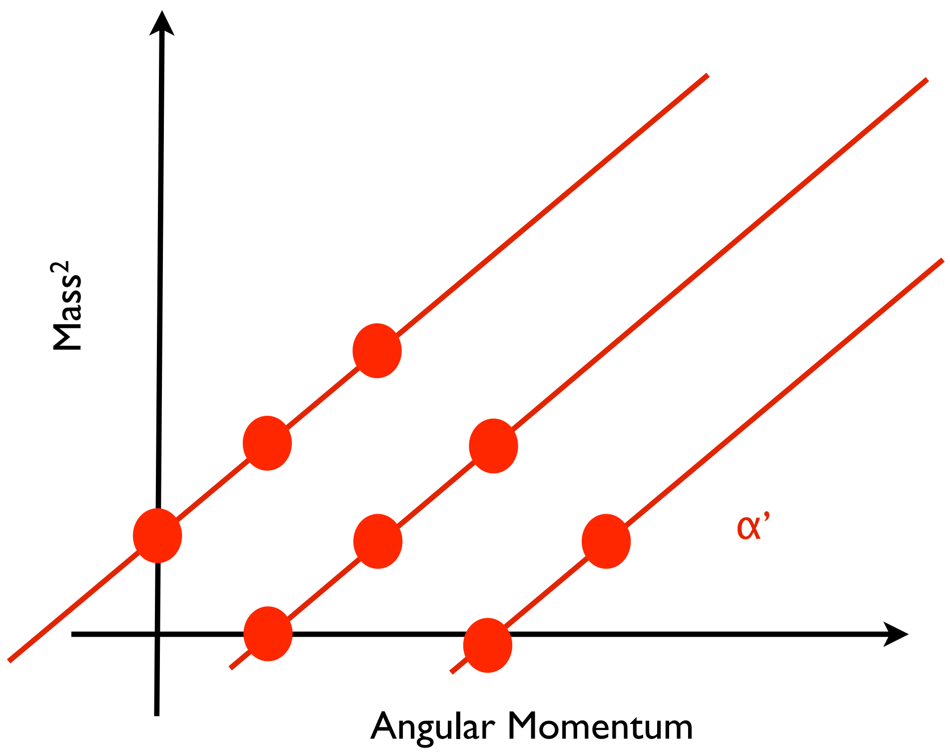

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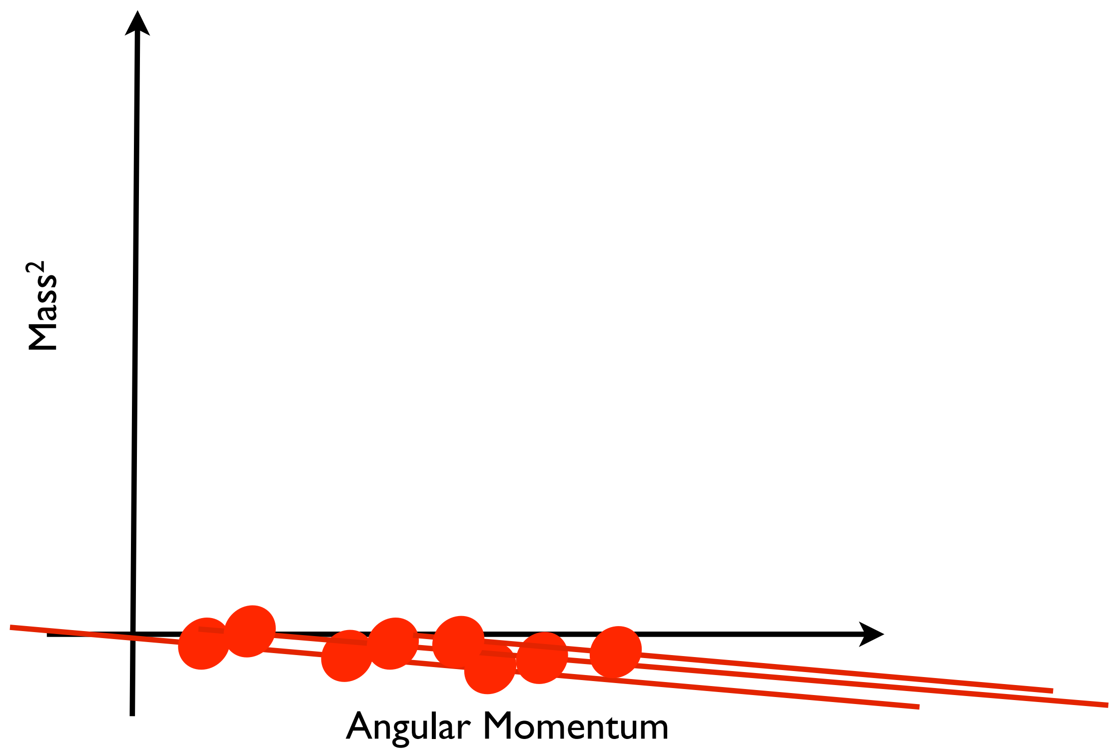

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Can one build higher spin gauge theories?

- Subject with a long history and, again, Italian pioneers…

TEORIA RELATIVISTICA DI PARTICELLE CON MOMENTO INTRINSECO ARBITRARIO

Nota di ETTORE MAJORANA

Sunto. - *L'autore stabilisce equazioni d'onda lineari nell'energia e relativisticamente invarianti per particelle aventi momento angolare intrinseco comunque prefissato.*

E. Majorana, Nuovo Cimento 9 (1932) 335

Free massless higher spins

- Example I: Maxwell
  - Field equations: $\partial^\lambda F_{\lambda\mu} = 0 \quad \Rightarrow \quad \Box A_\mu - \partial_\mu \partial \cdot A = 0$
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- Example II: linearised gravity (i.e. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$)
  - Field equations: $\Box h_{\mu\nu} - \partial_\mu \partial \cdot h_\nu - \partial_\nu \partial \cdot h_\mu + \partial_\mu \partial_\nu h_\lambda{}^\lambda = 0$
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Particle of arbitrary spin s:

Fronsdal (1978)

$$\mathcal{F}_{\mu_1 \cdots \mu_s} \equiv \Box \varphi_{\mu_1 \cdots \mu_s} - \partial_{(\mu_1} \partial \cdot \varphi_{\mu_2 \cdots \mu_s)} + \partial_{(\mu_1} \partial_{\mu_2} \varphi_{\mu_3 \cdots \mu_s)\lambda}{}^\lambda = 0$$

- Gauge symmetry: $\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \cdots \mu_s)}$
- Gauge invariance requires: $\xi_{\mu_1 \cdots \mu_{s-3} \lambda}{}^\lambda = 0$

Very nice, but…

Anonym (1930-????)

Come on! Higher-spin interactions are inconsistent!

Higher-spin theories in a nutshell

- 1930's: first problems
  - Difficulties with electromagnetic coupling — Fierz and Pauli (1939)
- 1960's: extra problems & no-go theorems
  - No interactions compatible with a non-trivial S-matrix — Weinberg (1964); Coleman and Mandula (1967)
  - Loss of causality in an external e.m. background — Velo and Zwanziger (1969)
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  - Consistent cubic vertices in flat & (A)dS spaces — Bengtsson^2 and Brink (1983); Berends, Burgers and van Dam (1984); Fradkin and Vasiliev (1986)
- 1990's: Vasiliev's theory
  - Full non-linear interactions with $\Lambda \neq 0$ — Vasiliev (1990)

Well, maybe they are only unconventional...

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Key: more than two derivatives (when D>3)

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- 2000's: AdS/CFT — Sezgin and Sundell (2002); Klebanov and Polyakov (2002); … Giombi and Yin (2009); … Maldacena and Zhiboedov (2011)
  - Higher-spin gauge theories as bulk duals of weakly coupled QFTs

Why such a long history?

The *unconventional* nature of HS interactions

- One needs <u>infinitely many particles</u>
  - Including a spin-s field calls for both lower and higher spins — Berends, Burgers, Van Dam (1985)
  - Good news: <u>higher spins require gravity</u>
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- Is a <u>cosmological constant</u> necessary?

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If you have a problem, simplify it!

- Higher-spin interactions are not easy to handle: it would be very useful to extract info from a *simplified setup*!

- Time-honoured trick: reduce the number of dimensions of spacetime

- Goal: look for a simpler, but *still non-trivial* theory

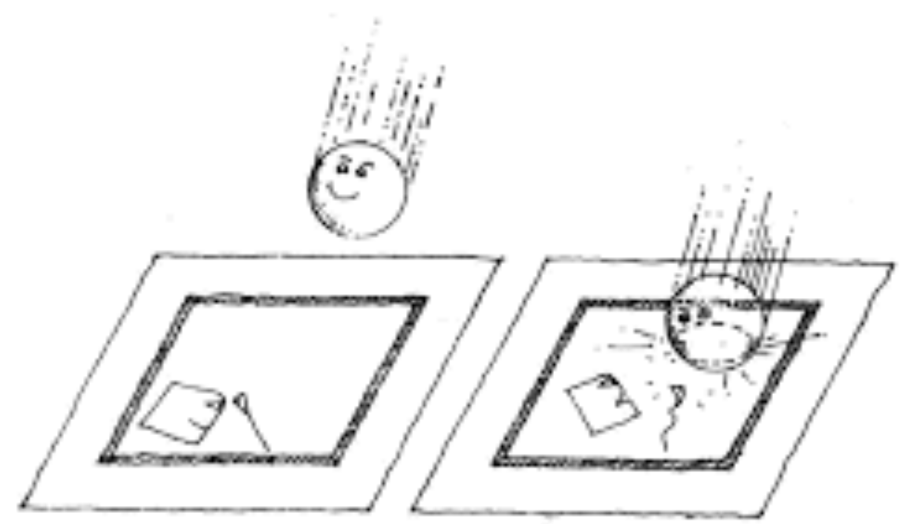

Fig. 14 Un Circolo appare nella stanza chiusa del Quadrato.

D=2+1: a "theoretical laboratory" for gravity

- D = 2+1: no irreps of arbitrary helicity for the little group of *massless* particles ⇒ no spin in the usual sense

- Still… look at Fronsdal equations:

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- We can consider Fronsdal equations in D=2+1
  - No wave solutions for s>1 (no local d.o.f.)
  - Nothing really new: no gravitons in D=2+1, yet black holes exist

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- D = 2+1: no irreps of arbitrary helicity for the little group of *massless* particles ⇒ <u>no spin in the usual sense</u>

- Still… look at Fronsdal equations:

$$\mathcal{F}_{\mu_1 \dots \mu_s} \equiv \Box \varphi_{\mu_1 \dots \mu_s} - \partial_{(\mu_1} \partial \cdot \varphi_{\mu_2 \dots \mu_s)} + \partial_{(\mu_1} \partial_{\mu_2} \varphi_{\mu_3 \dots \mu_s)\lambda}{}^{\lambda} = 0$$

- We can consider Fronsdal equations in D=2+1
  - No wave solutions for s>1 (no local d.o.f.)
  - Nothing really new: no gravitons in D=2+1, yet black holes exist

- What can we learn from this *apparently* too simple example? And how?

Gravity in D=2+1

- Einstein-Hilbert action

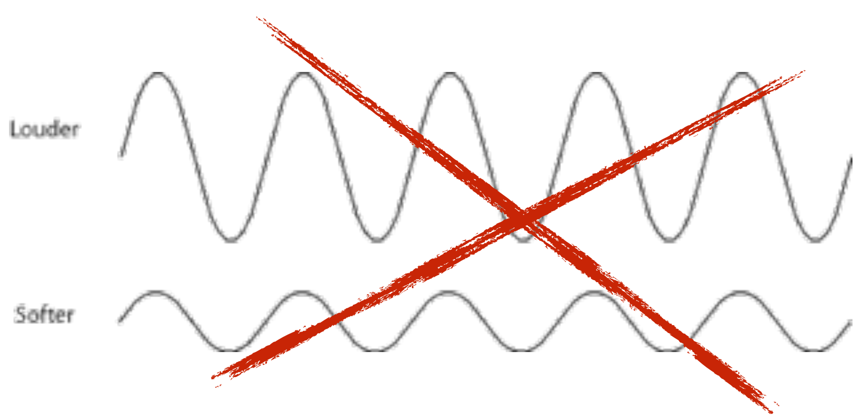
$$I = \frac{1}{16\pi G}\int \epsilon_{abc}\left(e^a \wedge R^{bc} + \frac{1}{3l^2}\, e^a \wedge e^b \wedge e^c\right)$$

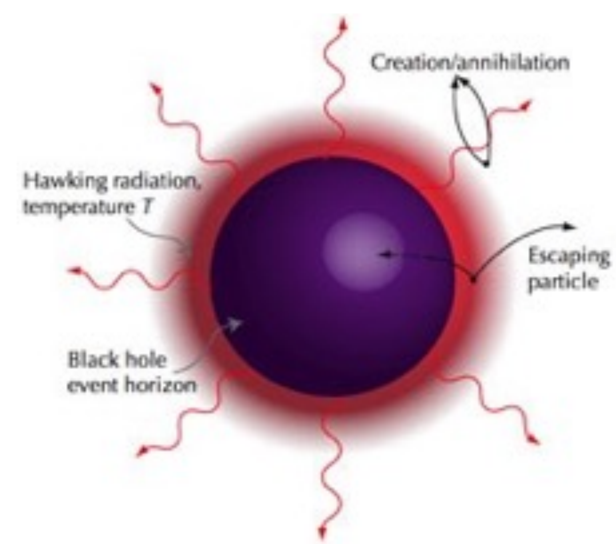
- Field equations

$$R_l^{ab} \;\equiv\; d\omega^{ab} + \omega^{ac}\wedge\omega_c{}^b + \frac{1}{l^2}\, e^a \wedge e^b \;=\; 0 \qquad \leftarrow \text{constant curvature!}$$

$$T^a \;\equiv\; de^a + \omega^a{}_b \wedge e^b \;=\; 0$$

- No dynamics, but thermodynamics!





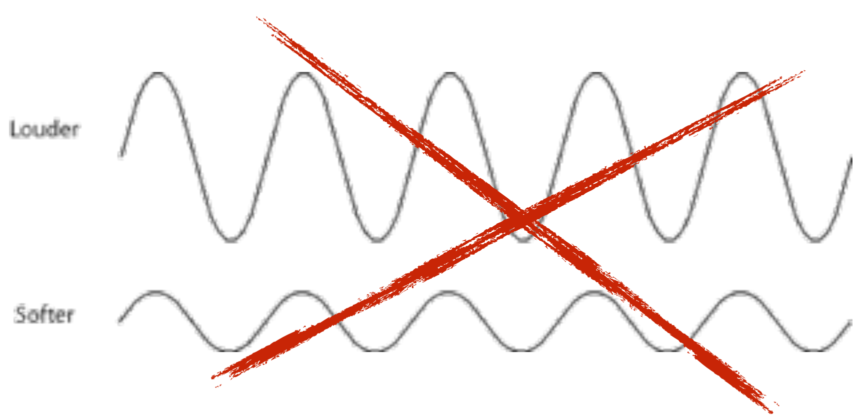
Gravity in D=2+1

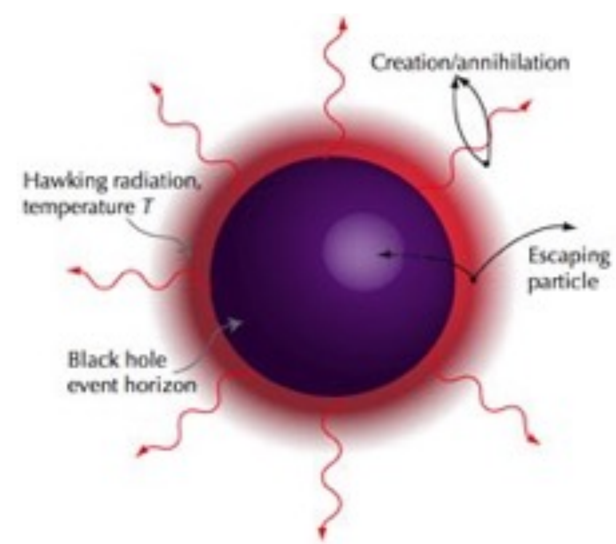
- Einstein-Hilbert action

$$I = \frac{1}{16\pi G}\int \epsilon_{abc}\left(e^a \wedge R^{bc} + \frac{1}{3l^2}\, e^a \wedge e^b \wedge e^c\right)$$

- A couple of useful tricks…
  - $\omega_\mu{}^a = \frac{1}{2}\,\epsilon^a{}_{bc}\,\omega_\mu{}^{b,c}$
  - so(2,2) ≃ so(1,2) ⊕ so(1,2) ≃ sl(2,R) ⊕ sl(2,R)

- No dynamics, but thermodynamics!





Gravity in D=2+1

- Einstein-Hilbert action

$$I = \frac{1}{8\pi G}\int \left(e^a \wedge R_a + \frac{1}{6l^2}\,\epsilon_{abc}\, e^a \wedge e^b \wedge e^c \right)$$

- A couple of useful tricks…
  - $\omega_\mu{}^a = \frac{1}{2}\,\epsilon^a{}_{bc}\,\omega_\mu{}^{b,c}$
  - so(2,2) ≃ so(1,2) ⊕ so(1,2) ≃ sl(2,R) ⊕ sl(2,R)
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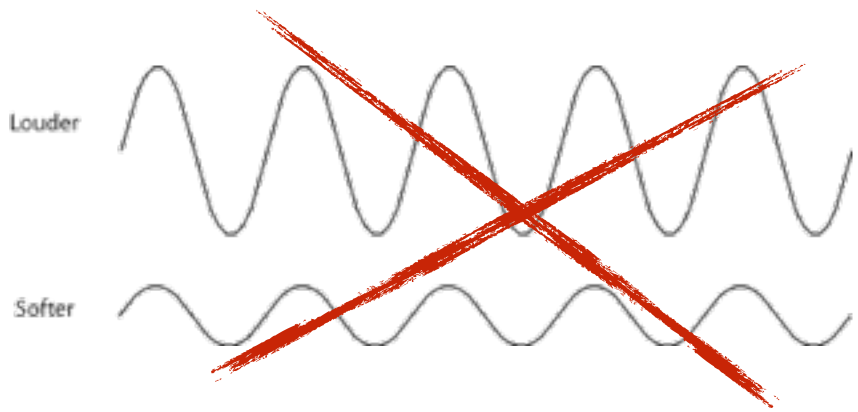

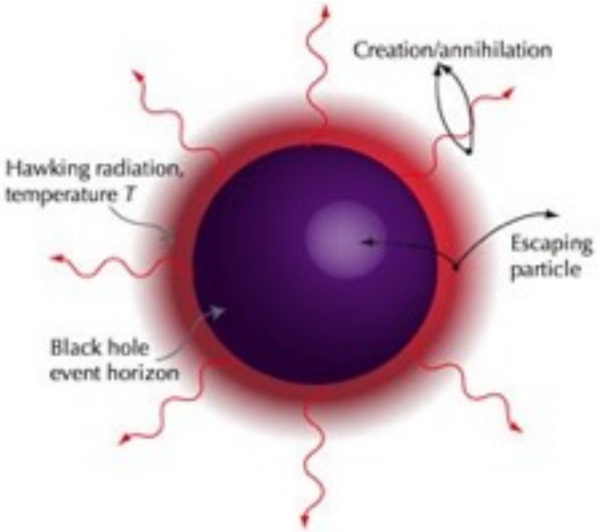

Gravity in D=2+1

- Einstein-Hilbert action

Achúcarro, Townsend (1986); Witten (1988)

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3l^2}\, e\wedge e\wedge e\right) \quad \text{with} \quad \begin{cases} e = e^a J_a \\ \omega = \omega^a J_a \end{cases}$$

- A couple of useful tricks…
  - $\omega_\mu{}^a = \frac{1}{2}\epsilon^a{}_{bc}\,\omega_\mu{}^{b,c}$
  - so(2,2) ≃ so(1,2) ⊕ so(1,2) ≃ sl(2,R) ⊕ sl(2,R)

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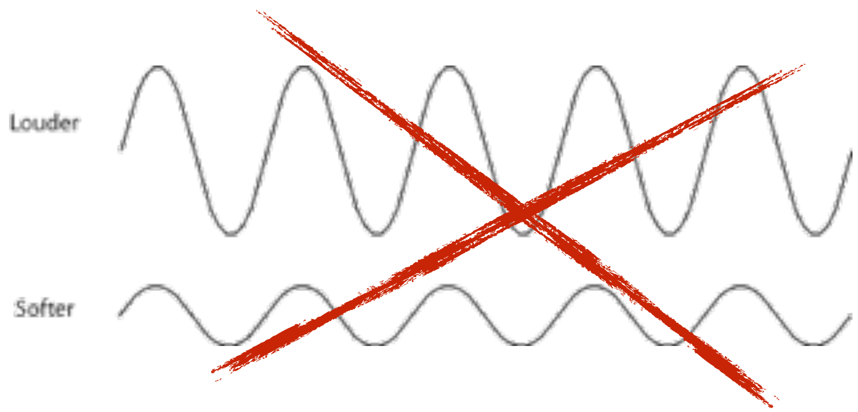

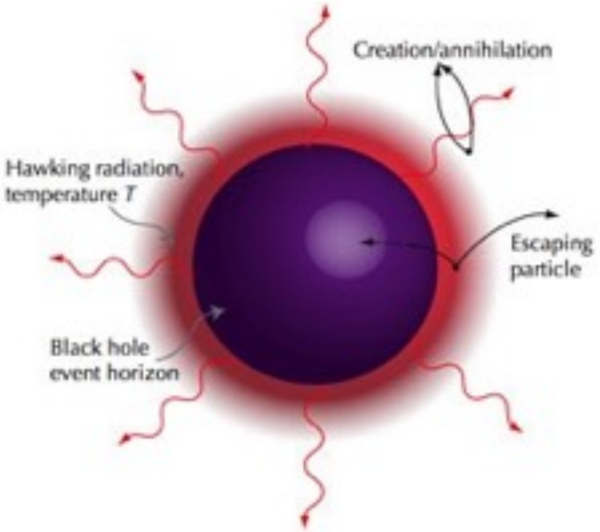

“Higher-spin” Chern-Simons action

Blencowe (1989); A.C., Pfenninger, Fredenhagen, Theisen (2010)

- Chern-Simons action

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e \wedge R + \frac{1}{3\ell^2}\, e \wedge e \wedge e \right)$$

$$\begin{cases} e = \left(e_\mu{}^a J_a + e_\mu{}^{ab}\, T_{ab} \right) dx^\mu \\ \omega = \left(\omega_\mu{}^a J_a + \omega_\mu{}^{ab}\, T_{ab} \right) dx^\mu \end{cases}$$

- Back to the metric (and Fronsdal)

$$g = \frac{1}{2}\,\mathrm{tr}\,(\, e_\mu e_\nu \,)\, dx^\mu dx^\nu\,, \qquad \phi = \frac{1}{6}\,\mathrm{tr}\,(\, e_\mu e_\nu e_\rho \,)\, dx^\mu dx^\nu dx^\rho$$

- All higher-spin fields have been packed up into two objects, e and ω

- Change their expansion and obtain a plethora of higher-spin theories!

What one gains with the restriction to D=2+1?

- No more need for infinitely many fields
  - Only “spins” s=2 and s=3 in the previous example
- Very compact formulation of the dynamics
  - One can use all tools that have been developed to study Chern-Simons theories in the last 40 years
- Main advantage: higher spins keep most of their peculiarities (that follow from gauge symmetry), but they are much closer to gravity in D=2+1
- One can try to extend what is known for gravity and *look for surprises*

Amazing surprises at hand!

- Asymptotic symmetries: infinitely many conserved charges for each spin related by a gargantuan symmetry (W-algebras)
  - Strong constraint for holography: boundary theories are highly constrained; reconstruct the quantum theory from 2D conformal field theories?

- Black holes can be built
  - Solutions with an horizon? OK!
  - The metric now “changes under higher-spin gauge transformations
  - They can “destroy” the horizon!

- Higher spin geometry?

The higher-spin way to quantum gravity

- If one scrutinises String Theory through the lens of QFT, one can question whether quantum consistency necessarily requires packing up infinitely many particles into a string

- Can one build models of quantum gravity adding *higher-spin particles* besides the graviton, but without resorting to the beautiful, but perhaps redundant structures of String Theory?

- Guiding principle: <u>gauge symmetry</u>

- We have a wonderful "theoretical laboratory" to test these ideas: work in <u>D=2+1</u> and enjoy the simplifications