Theory status of four-fermion production at e^+e^- colliders

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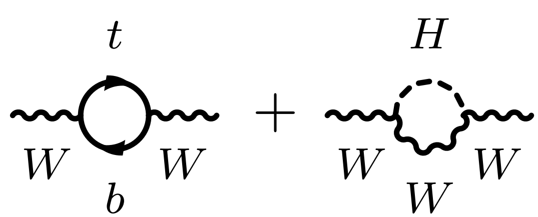
Searching for new physics after the Higgs discovery:

- Direct production of new particles

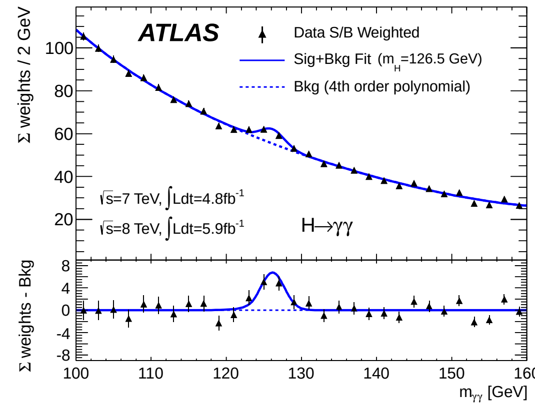
  $\Rightarrow$ collider energies above scale Λ_{NP} of new physics (LHC13/14, 100 TeV FCC)

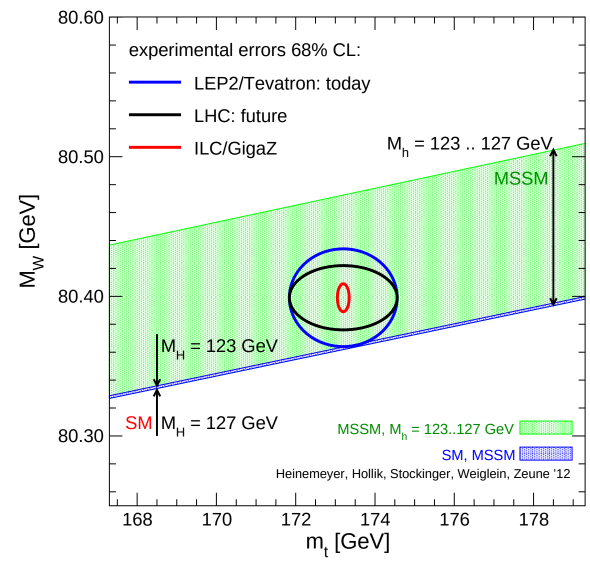
- New-physics effects below Λ_{NP}
  - virtual effects (EWPT)



  - anomalous gauge boson/Higgs couplings
  - precise measurements of input parameters: M_W, m_t, ...

  $\Rightarrow$ higher precision: future e^-e^+ colliders



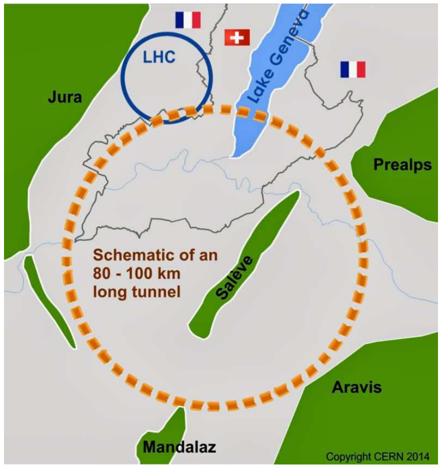


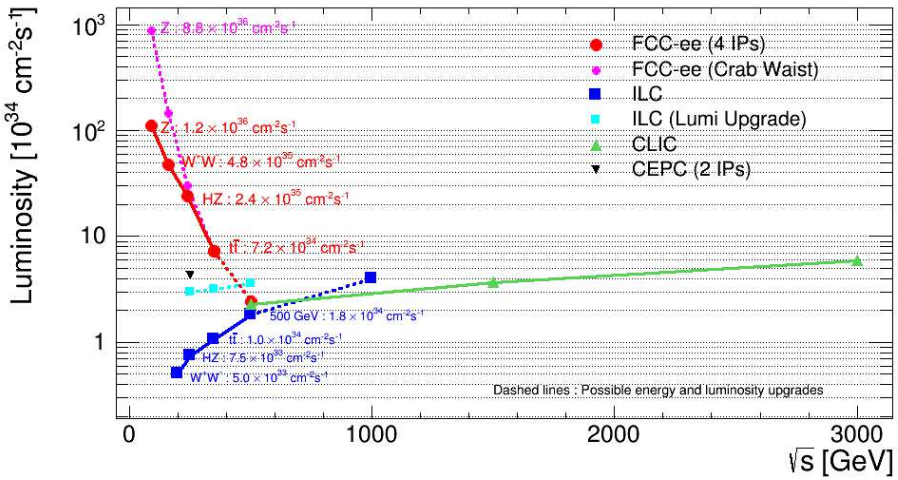
$\Rightarrow$ complementary approaches, both needed to identify new physics

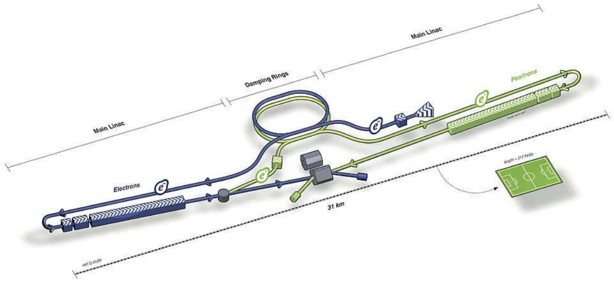
Future e^+e^- colliders

several concepts:(Talks by Yokoya, D'Enterria)

- FCC-ee, CEPC (90–$240/350$ GeV)
- ILC (90–$500/1000$ GeV),
- CLIC (≤ 3 TeV)



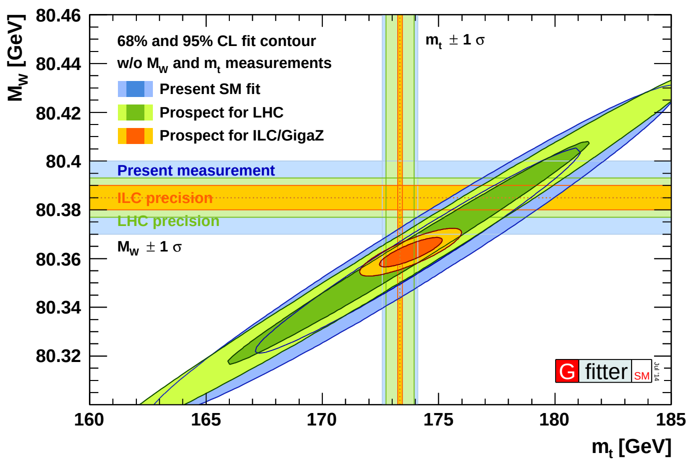




Key input parameters to SM fit:

M_W, m_t, $\sin\theta_w^{\ell,\mathrm{eff}}$

Prospects at future e^-e^+ colliders



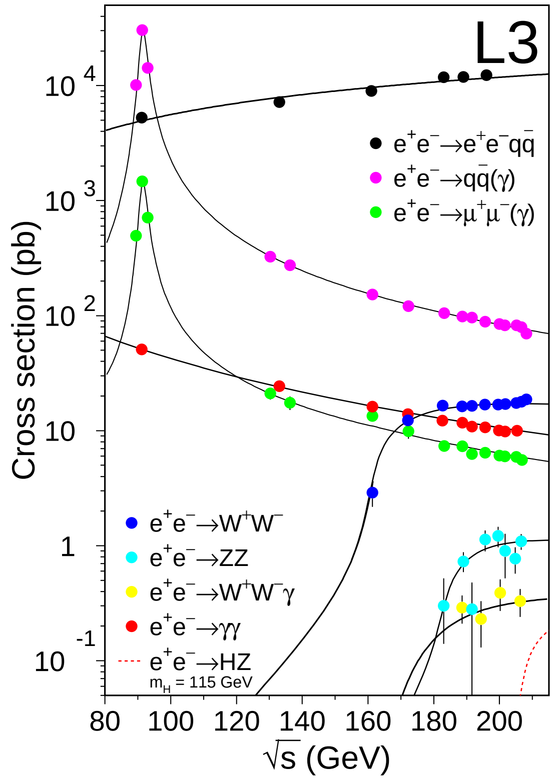
- $\Delta M_W \sim \begin{cases} 3-4\,\mathrm{MeV} & \text{(ILC)} \\ 1\,\mathrm{MeV} & \text{(FCCee)} \end{cases}$
- $\Delta \sin\theta^l_{\mathrm{eff}} \sim \begin{cases} 1.3\times10^{-5} & \text{(ILC)} \\ 0.3\times10^{-5} & \text{(FCCee)} \end{cases}$
- Triple gauge couplings: reduction of errors by one or two magnitudes compared to LEP (ILC)

	LHC	LHC	ILC/GigaZ	ILC	ILC	ILC	TLEP	SM prediction
$\sqrt{s}$ [TeV]	14	14	0.091	0.161	0.161	0.250	0.161	-
$\mathcal{L}[\mathrm{fb}^{-1}]$	300	3000		100	480	500	3000×4	-
ΔM_W [MeV]	8	5	-	4.1-4.5	2.3-2.9	3.6	1.2	4.2(3.0)
$\Delta\sin^2\theta^\ell_{\mathrm{eff}}$ $[10^{-5}]$	36	21	1.3	-	-	-	0.3	3.0(2.6)

(Snowmass EW report 13)

Four-fermion production

- Explored at LEP 2 with $\mathcal{L} = 3\,\text{fb}^{-1}$ from $\sqrt{s} = 161.3$-206.6 GeV
- Precision tests of standard model:
  - cross-section measurements
  - m_W, Γ_W
  - triple-vector boson couplings
- Important for precision physics program at any future e^-e^+ collider
- Pushes methods of perturbative QFT for complicated processes:
  - gauge invariant definition of signals
  - consistent treatment of finite-width effects
  - different scales
  - many Feynman diagrams



This talk

- Overview of LEP2 4-fermion signatures and theory status
- Theoretical issues
- Theory developments since LEP2
- Outlook

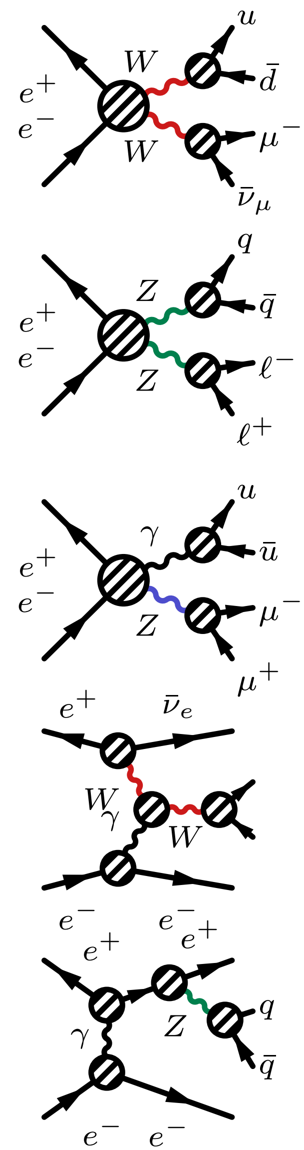
Resources

- LEP2 Theory: Grünewald et al. [hep-ph/0005309]
- LEP2 Experiment: Schael et al. [arXiv:1302.3415]
- ILC: [arXiv:1504.01726]
- FCCee WG2 `http://tlep.web.cern.ch/content/wg2-exp`

Four-fermion processes

Classification of signatures at LEP

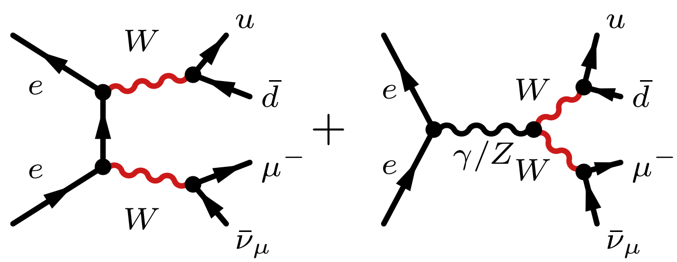
- WW: Cross section,
  M_W, Γ_W,
  branching ratios,
  anomalous couplings
- ZZ: Cross section
- $Z\gamma$: Cross section,
  anomalous couplings
- $We\nu$: Cross section,
  anomalous couplings
- Zee: Cross section



Theory issues

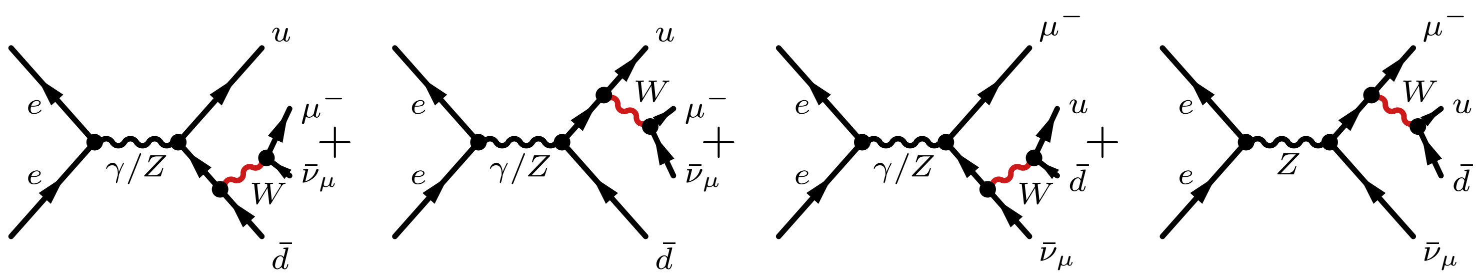
4-fermion production at tree level, e.g. $e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}$

Double resonant ('signal') diagrams (CC3):



But 10 diagrams in total:

CC3 +



Only sum gauge invariant

Need consistent scheme for finite width effects: (Beenakker et al. 96)

$\sqrt{s}$	200 GeV	500 GeV	1 TeV	5 TeV
Running width	672.96(3)	225.45(3)	62.17(1)	123.76(1)
Constant width	673.08(4)	224.05(3)	56.90(1)	2.212(6)

Extensive study of several schemes for LEP2

(no attempt at completeness, see e.g. Grünewald et.al. [hep-ph/0005309])

- Minimal modifications to standard perturbation theory
  - "Constant width": use propagator

$$\frac{\mathrm{i}}{p^2 - M^2 + \mathrm{i}M\Gamma}$$

(violates gauge invariance but mostly harmless at tree level)

  - running width with modified vertices, "overall factors",...
  - complex mass scheme: replace $M^2 \to M^2 - \mathrm{i}M\Gamma$, everywhere, also in Feynman rules e.g.

$$\cos\theta_w = \frac{M_W}{M_Z} \Rightarrow \sqrt{\frac{M_W^2 - \mathrm{i}M_W\Gamma_W}{M_Z^2 - \mathrm{i}M_Z\Gamma_Z}}$$

Extensive study of several schemes for LEP2

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- Minimal modifications to standard perturbation theory
  - "Constant width": use propagator

$$\frac{\mathrm{i}}{p^2 - M^2 + \mathrm{i}M\Gamma}$$

- Rearrange perturbation theory using hierarchy Γ/M:
  - Pole scheme: expand around complex pole $\mu^2 = M^2 - iM\Gamma$ of propagator

$$\mathcal{A}(s)|_{p^2 \sim M^2} = \frac{\mathcal{R}(\mu)}{p^2 - \mu^2} + \mathcal{N}(p^2)$$

(Stuart 91; Aeppli,v. Oldenbourgh, Wyler 93)

Schemes for finite width effects

Extensive study of several schemes for LEP2

(no attempt at completeness, see e.g. Grünewald et.al. [hep-ph/0005309])

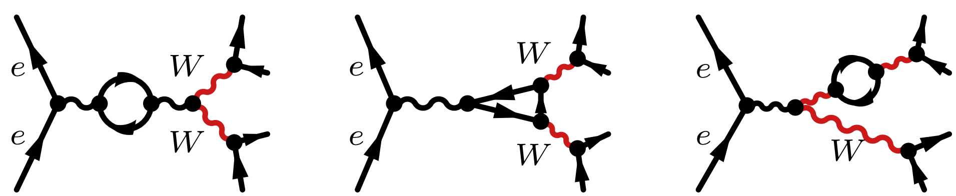
- Minimal modifications to standard perturbation theory
  - "Constant width": use propagator

$$\frac{\mathrm{i}}{p^2 - M^2 + \mathrm{i}M\Gamma}$$

- Rearrange perturbation theory using hierarchy Γ/M:
  Pole scheme: expand around complex pole of propagator

- Take gauge invariance as guideline: Fermion loop scheme
  Resummation of fermionic part of self-energy (Beenakker et al. 96)

$$\frac{\mathrm{i}}{p^2 - M^2 + \Sigma^{(\mathrm{f})}(p^2)}$$

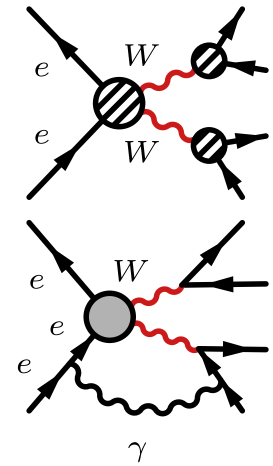


closed fermion loops form gauge invariant subset of diagrams

NLO calculations in double pole approximation

- Factorizable EW corrections to production, decay of on-shell Ws
- Nonfactorizable soft photon corrections

  (Berends et al. 98; Denner et al. 99)
- Implemented in Monte-Carlo programs used at LEP2: RacoonWW (Denner et al. 99), YFSWW (Jadach et al. 99)



- Estimate of DPA accuracy at NLO

$$\Delta\sigma_{\rm DPA} \sim \frac{\Gamma_W}{M_W} \times \frac{\alpha}{\pi} \sim \mathcal{O}(0.1\%)$$

- Loss of accuracy at production threshold

$$\Delta\sigma_{\rm DPA} \sim \frac{\Gamma_W}{\sqrt{s}-2M_W} \times \frac{\alpha}{\pi} \sim \mathcal{O}(1\%)$$

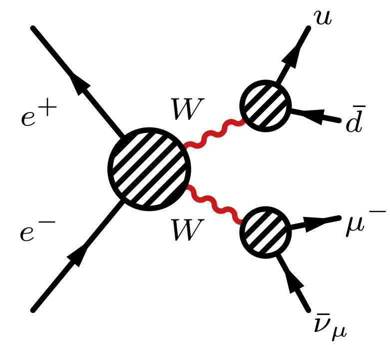
Status at LEP2

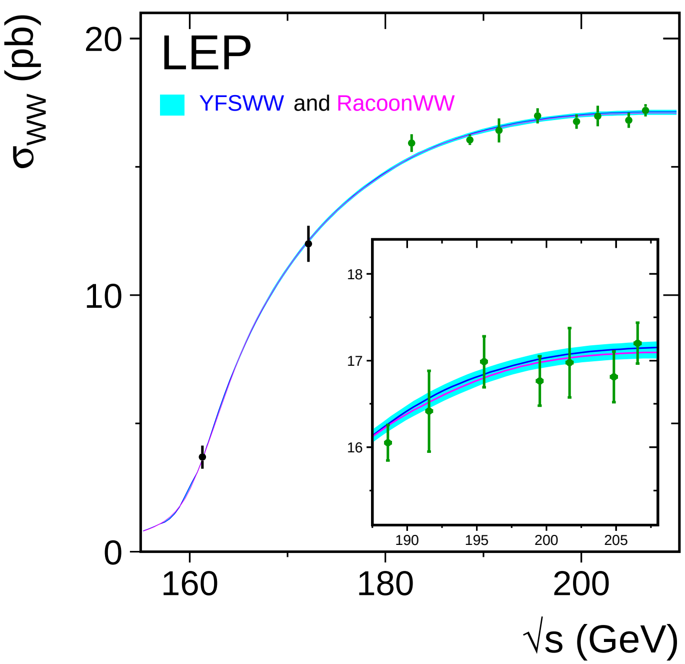
- σ_{WW}: 1%-level agreement with NLO theory

  RacoonWW (Denner et al.), YFSWW (Jadach et al.)

- Residual theory uncertainty $\Delta\sigma_{WW} \sim 0.5\%$

- ILC (FCCee): Luminosity increase $\times 100$ (10^4)
  Reduction of theory error to $< 0.1\%$ realistic?

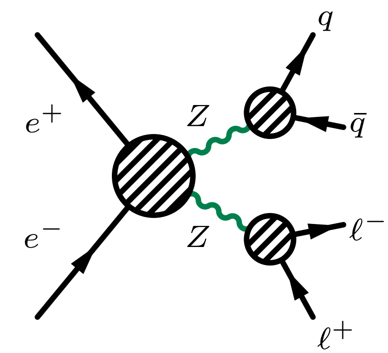


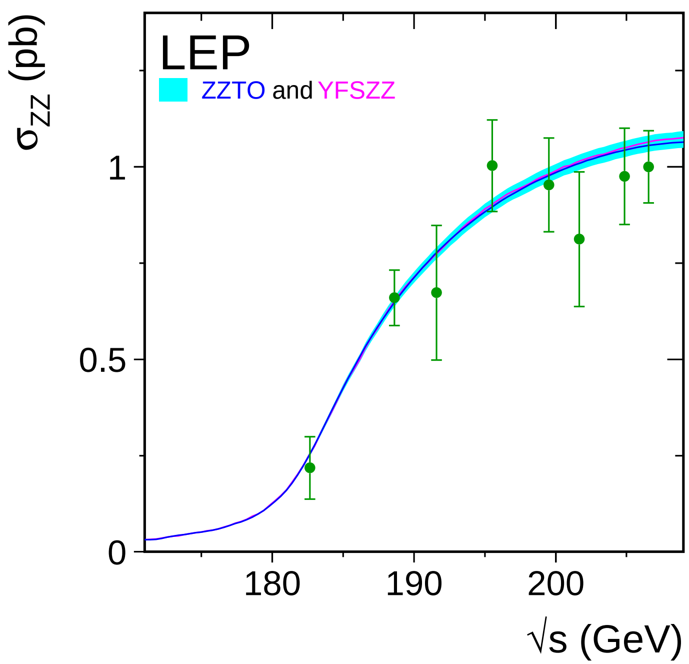


- σ_{ZZ}: Agreement with NLO QED theory in pole approximation/fermion loop scheme

  `YFSZZ` (Jadach et al.), `ZZTO` (Passarino), `Gentle` (Bardin et al.)

- Residual theory uncertainty $\Delta\sigma_{ZZ} \sim 2\%$
- FCCee: Accuracy $\Delta\sigma_{ZZ} < 1\%$.



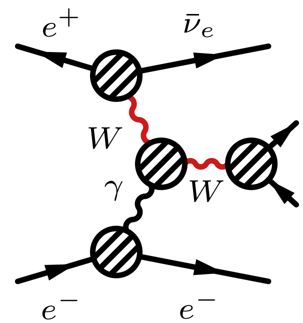


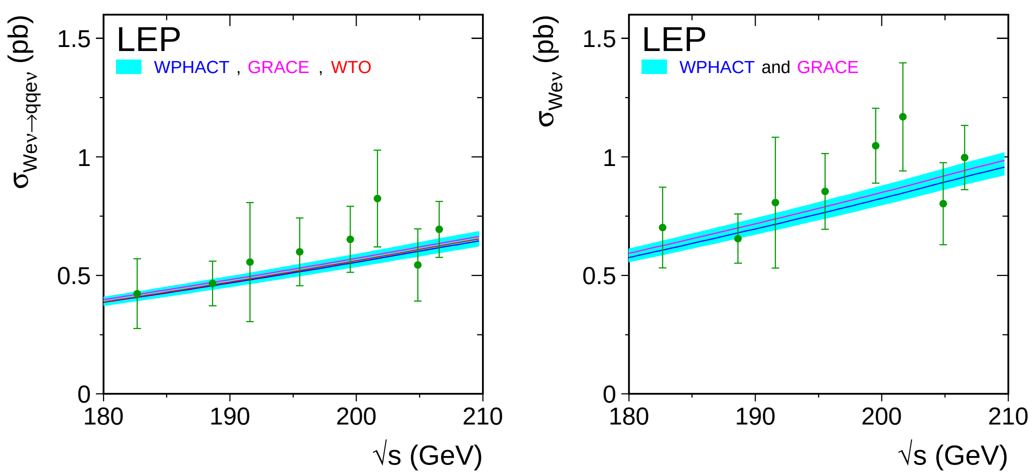
- $\sigma_{We\nu}$: agreement with theory predictions in fermion-loop scheme/Born+ISR

  WPHACT (Accomando/Ballestrero), WTO (Passarino),

  grc4f (Fujimoto et al.)

- Forward e^--scattering: finite m_e required, respecting gauge invariance essential
- Residual theory uncertainty $\Delta\sigma_{We\mu} \sim 5\%$
- FCCee: Accuracy $\Delta\sigma_{We\nu} < 1\%$.





Theory developments after LEP2:

W-pair production:

- Complete NLO calculation for charged current $e^+e^- \to 4f$
  (Denner et al. 05)
- Log-enhanced NNLO corrections for $\hat{s} \gg M_W$
  $\Rightarrow$ CLIC (Kühn et al. 07)
- NLO and leading NNLO correction in threshold expansion
  $\Rightarrow$ ILC/FCCee (Beneke et al. 07, Actis et al. 08)
- Sensitivity to anomalous couplings in EFT approach
  (Buchalla et al. 13; Wells/Zhang 15)

Other results for vector-boson production in e^-e^+:

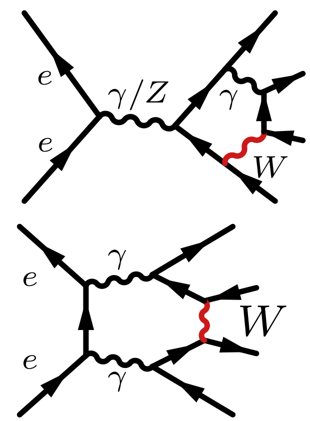
- NLO EW corrections for triple gauge-boson production
  - WWZ, ZZZ (Su et al. 08; Wei et al. 09; Boudjema et al. 09)
  - $\gamma\gamma Z$ (Yu et al. 13)
  - $WW\gamma$ (Chen et al. 14)

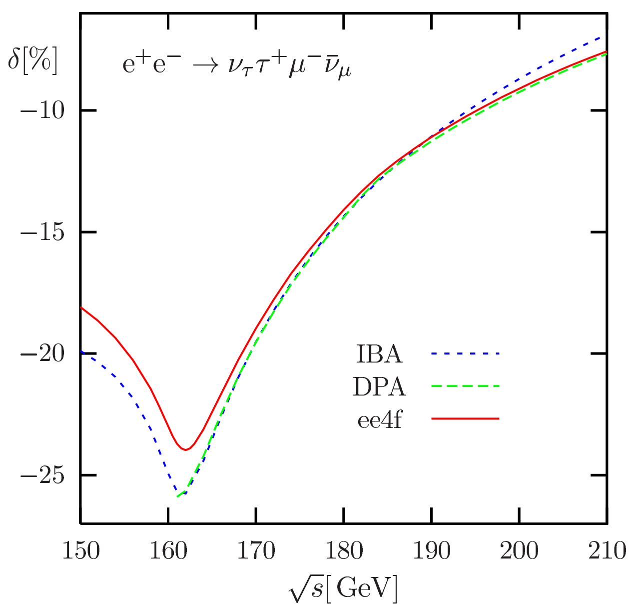
Full NLO calculation for $e^+e^- \to 4f$ (Denner, Dittmaier, Roth, Wieders 05)

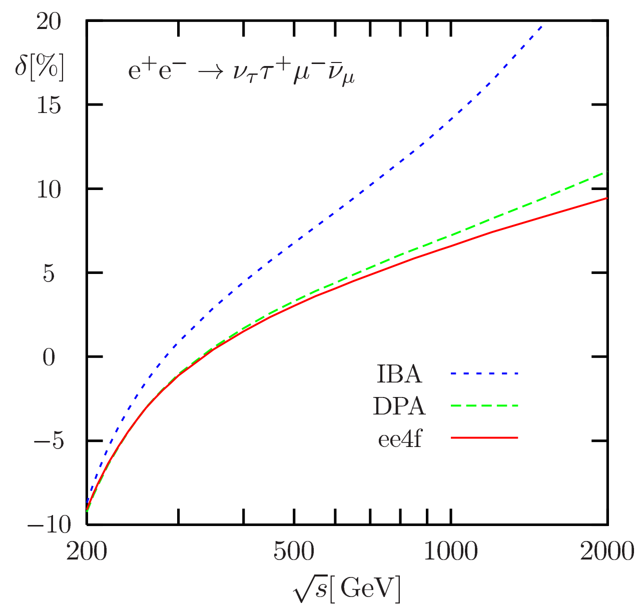
- More than 1000 1-loop diagrams
  5-point, 6-point loop integrals

$\Rightarrow$ new methods for six-point diagrams

- fully differential calculation
- complex mass scheme for W decay width
- DPA not sufficient at threshold and for $\sqrt{s} > 500$ GeV

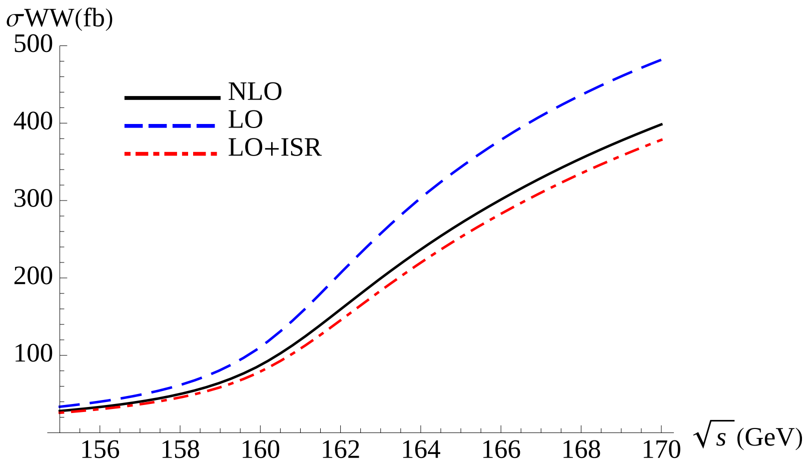


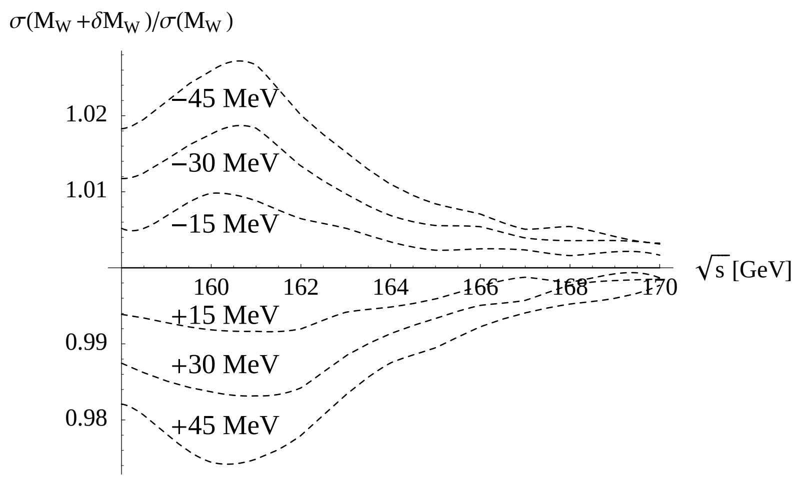




- **ILC** $\Delta M_W \lesssim 4$ MeV from threshold scan
  $\Leftrightarrow \Delta\sigma_{WW} \ll 1\%$ prediction for $\sqrt{s} \sim 160 - 170$ GeV

- **FCCee** goal $\Delta M_W < 1$ MeV
  theory uncertainty dominant!





Enhanced corrections in threshold limit $\beta = \sqrt{1 - \frac{4M_W^2}{s}} \to 0$:

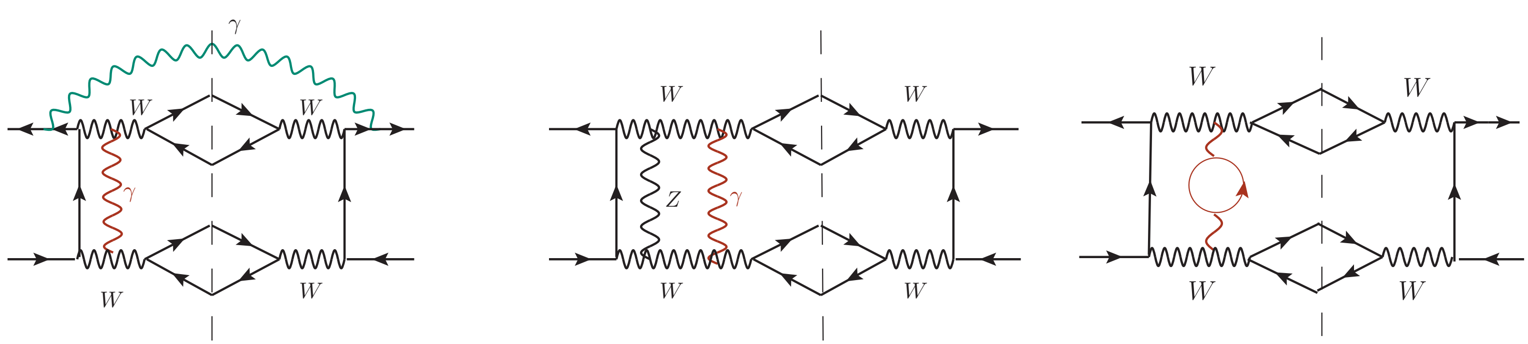
soft threshold logarithms $\sim (\alpha \log^2 \beta)^n$, Coulomb correction $\sim (\alpha/\beta)^n$

EFT approach: (Beneke/Falgari/CS/Signer/Zanderighi 07)

expansion in $\alpha \sim \frac{\Gamma_W}{M_W} \sim \frac{k_W^2 - M_W^2}{M_W^2} \sim \beta^2$

Leading NNLO corrections

- 2nd Coulomb correction $\sim \alpha^2/\beta^2 \sim \alpha$ (Fadin et al. 95)
- Coulomb-enhanced corrections $\sim \alpha^2/\beta \sim \alpha^{3/2}$ (Actis et al. 08)
- corrections at threshold $\Delta\sigma_{WW} \sim 0.5\%$
  $\Rightarrow$sufficient accuracy for ILC, more work for FCCee



Complete NLO EW corrections for all $e^-e^+ \to 4f$ processes

- Feasible with current methods $\Rightarrow$ accuracy $\Delta\sigma_{4f} < 1\%$ possible

  (Automation of NLO EW: Recola: Actis et al. 12; OpenLoops: Kallweit et al. 14)

NNLO?

Final words of R. Chierici at 8th FCC-ee workshop:

> *"Need to get to NNLO precision in α_{EW} for most processes, I am afraid"*

Complete NLO EW corrections for all $e^-e^+ \to 4f$ processes

- Feasible with current methods $\Rightarrow$ accuracy $\Delta\sigma_{4f} < 1\%$ possible

  (Automation of NLO EW: Recola: Actis et al. 12; OpenLoops: Kallweit et al. 14)

NNLO status

- NNLO QCD corrections for $pp \to VV$ at LHC (Grazzini et al. 11-14)
- Two-loop EW for $H \to \gamma\gamma$ (Passarino et al. 07)
- NNLO EW for $e^-e^+ \to VV$ more complicated;
  $e^-e^+ \to 4f$ out of reach today

Pole-approximation/EFT approach

- Pole approximation at NNLO:
  - required: NNLO for on-shell $e^-e^+ \to VV$, $V \to ff$;
  - Definition of factorizable/nonfactorizable real corrections not worked out yet.
- EFT approach for inclusive observables or near threshold
  - defined for single resonances at NNLO (Beneke et al. 04)
  - generalization to pair production at threshold appears feasible

ISR: resum leading logs

$$\beta_e = \frac{2\alpha}{\pi}\left(2\log\left(\frac{2M_W}{m_e}\right)-1\right)$$

in electron structure functions: (Skrzypek 92)

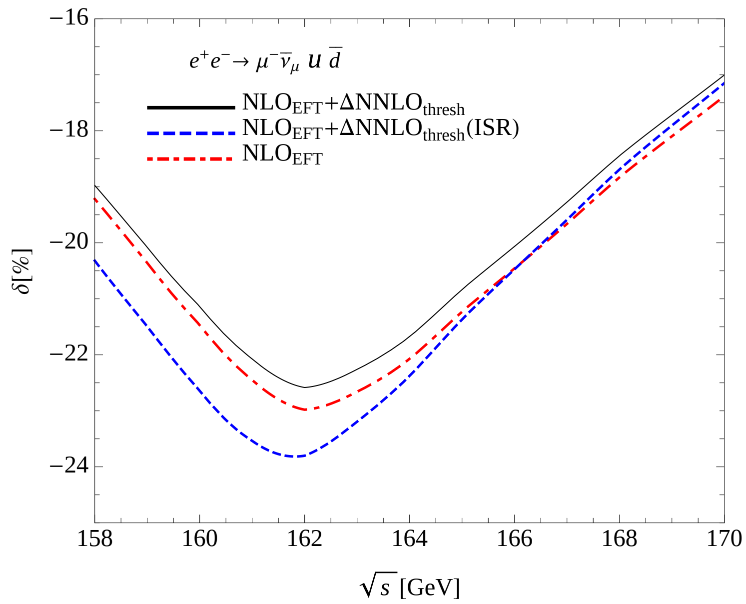
$$\sigma_{\text{NLO}}(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{ee}^{\text{LL}}(x_1)\Gamma_{ee}^{\text{LL}}(x_2)\left(\sigma_{\text{tree}} + \Delta\hat{\sigma}_{\text{NLO}}\right)$$

Estimate missing NLL $\mathcal{O}(\alpha\beta_e)$:

ISR for tree only $\Leftrightarrow$ also for NLO

Uncertainty $\sim 2\%$ at threshold

$\Rightarrow$ NLL resummation important



Four-fermion production

crucial process at any future e^+e^- collider

- M_W measurement from threshold or direct reconstruction
- anomalous couplings
- cross-section measurements

Full NLO EW corrections to $e^+e^- \to 4f$ (Denner et al. 05)

- accuracy of "a few 0.1%" from threshold to $\sim 500\,\mathrm{GeV}$.

NNLO

- Leading corrections at threshold
  and for large energies available (Actis et al. 08; Kühn et al. 07)
- NNLO EW for $e^-e^+ \to 4f$ beyond current state-of the art
- Pole approximation/EFT method
  need to be extended to NNLO

Beyond NLO—high energy

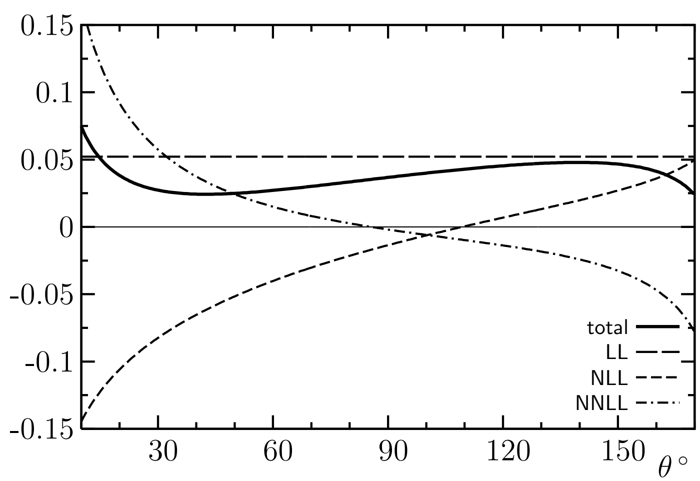
Enhanced Sudakov logarithms for high energies $s \gg M_W^2$

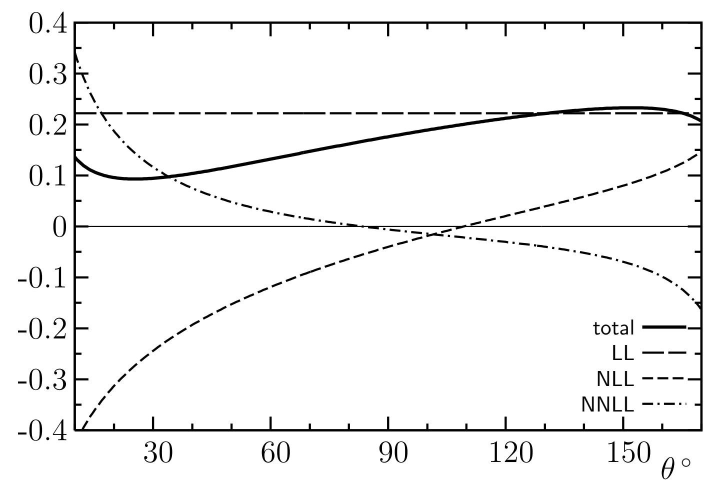
(Fadin et al. 00; Melles 01; Denner et al. 03; Beccaria et al. 03)

$$\underbrace{(\alpha \log^2(s/M_W^2))^n}_{\text{LL}}\,, \quad \underbrace{\alpha^n \log^{2n-1}(s/M_W^2))}_{\text{NLL}}\,, \quad \underbrace{\alpha^n \log^{2n-2}(s/M_W^2))}_{\text{NNLL}}\ldots$$

- NNLO-NNLL corrections for on-shell W-pair production:
  $\sim 5\%(s = 1$ TeV) – $15\%(s = 3$ TeV) (Kühn/Penin/Metzler 07)

$\Rightarrow$ need to be taken into account at CLIC and 2nd phase of ILC





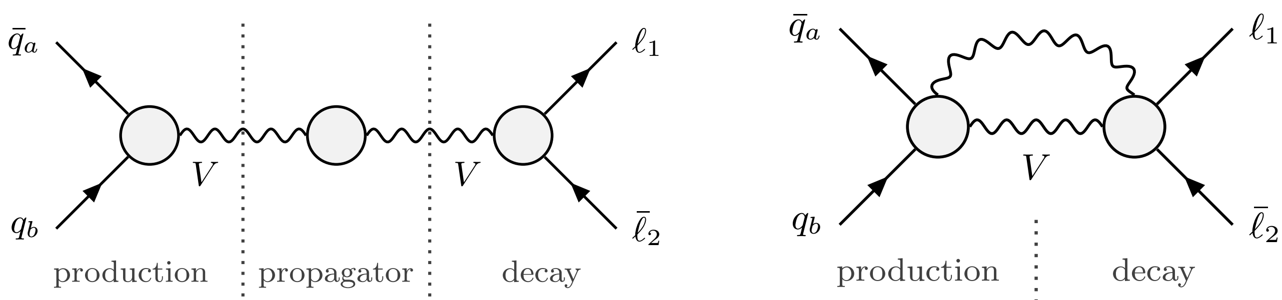
(transverse polarization)¡ 1 TeV 3 TeV

Pole approximation

Pole scheme: (Stuart 91; Aeppli/v.Oldenbourgh/Wyler 93) expand around complex pole of propagator $\mu^2 = M^2 + iM\Gamma$

$$\mathcal{A}(s) = \frac{R(s)}{s - M^2 + \Sigma(s)} + N(s)$$

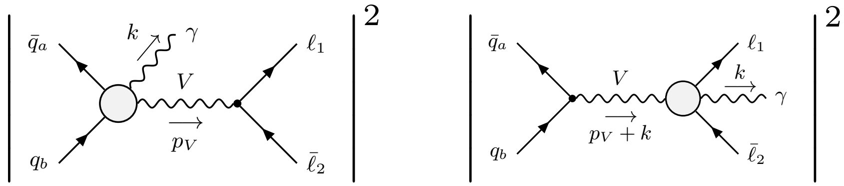
$$= \underbrace{\frac{R(\mu^2)}{s-\mu^2}\,\frac{1}{1+\Sigma'(\mu^2)}}_{\text{factorizable corrections}} + \underbrace{\left[\frac{R(s)}{s-M^2+\Sigma(s)} - \frac{R(\mu^2)}{s-\mu^2}\,\frac{1}{1+\Sigma'(\mu^2)}\right]_{s\to\mu^2}}_{\text{non-factorizable corrections}} + \text{non-res.}$$



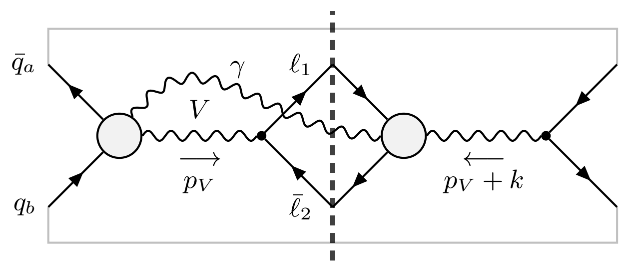
factorizable corrections to on-shell prod. and decay   non-fact. soft-photon corrections

Real corrections:

Factorizable corrections to on-shell production and decay:



Non-fact. corrections: resonance enhancement from soft photons



Applications of pole scheme

- Drell-Yan processes (Wackeroth/Hollik 96)
- Generalization to pair production:
  - $e^+e^- \to W^+W^- \to 4\mathrm{f}$ (Berends et. al. 98; Denner et.al. 99)