Higgs effects in $t\bar{t}$ production near threshold in e^+e^- annihilation

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LFC15: physics prospects for Linear and other Future Colliders after the discovery of the Higgs
ECT, Trento, Italy, September 7 - 11, 2015

MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864 [hep-ph]
MB, Maier, Piclum, Rauh, 1506.06865 [hep-ph]

including earlier results from

MB, Kiyo, Schuller, Part I 1312.4791 [hep-ph] and Part II in preparation;
Marquard, Piclum, Seidel, Steinhauser, 1401.3004 [hep-ph];
MB, Jantzen, Ruiz-Femenia 1004.2188 [hep-ph];
Jantzen, Ruiz-Femenia 1307.4337 [hep-ph];
MB, Piclum, Rauh, 1312.4792 [hep-ph]

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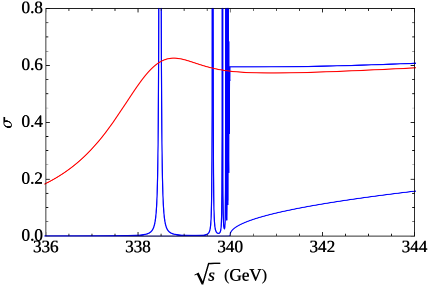
Pair production threshold – Strong Coulomb force and Weak Decay

Ultra-precise mass measurement
Unique QCD dyanmics

Unstable top quark, with strong Coulomb force



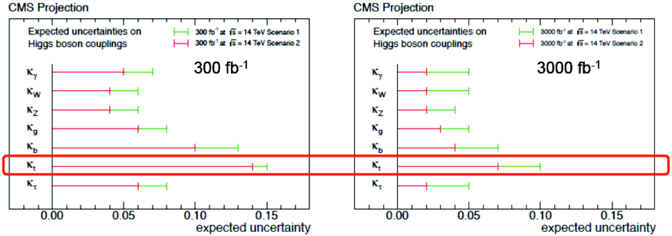
Smallest structure in particle physics known to exist (10^{-17}m).
Sensitivity to Higgs Yukawa force, too?

Yukawa coupling measurements

Indirect though loop effects in Higgs production and decay.
Directly, from associated production $t\bar{t}H$.

LHC prospects

- Scenario 1: all systematic uncertainties are left unchanged.
- Scenario 2: theoretical uncertainties are scaled by a factor of 1/2, while other systematic uncertainties are scaled by the square root of the integrated luminosity.

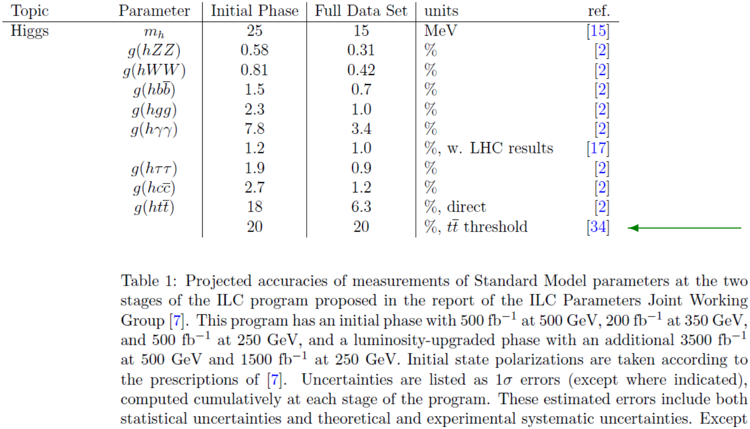


Tentative conclusion: a ~5% uncertainty on the top Yukawa coupling at the LHC should be achievable!

[From A. Juste, and 1307.7135]

Yukawa coupling measurements

LC prospects



Topic	Parameter	Initial Phase	Full Data Set	units	ref.
Higgs	m_h	25	15	MeV	[15]
	$g(hZZ)$	0.58	0.31	%	[2]
	$g(hWW)$	0.81	0.42	%	[2]
	$g(hb\bar{b})$	1.5	0.7	%	[2]
	$g(hgg)$	2.3	1.0	%	[2]
	$g(h\gamma\gamma)$	7.8	3.4	%	[2]
		1.2	1.0	%, w. LHC results	[17]
	$g(h\tau\tau)$	1.9	0.9	%	[2]
	$g(hc\bar{c})$	2.7	1.2	%	[2]
	$g(ht\bar{t})$	18	6.3	%, direct	[2]
		20	20	%, $t\bar{t}$ threshold	[34]

Table 1: Projected accuracies of measurements of Standard Model parameters at the two stages of the ILC program proposed in the report of the ILC Parameters Joint Working Group [7]. This program has an initial phase with 500 fb^{-1} at 500 GeV, 200 fb^{-1} at 350 GeV, and 500 fb^{-1} at 250 GeV, and a luminosity-upgraded phase with an additional 3500 fb^{-1} at 500 GeV and 1500 fb^{-1} at 250 GeV. Initial state polarizations are taken according to the prescriptions of [7]. Uncertainties are listed as 1σ errors (except where indicated), computed cumulatively at each stage of the program. These estimated errors include both statistical uncertainties and theoretical and experimental systematic uncertainties. Except

[From 1506.05992, LCC Physics Working Group, Fujii et al.]

Theory

Non-perturbative but weak coupling. Expansion in α_s and $v = \sqrt{\frac{E}{m}} = \sqrt{\frac{\sqrt{q^2} - 2m_t}{m_t}}$, while $\alpha_s/v = O(1)$

$$R \sim v \sum_k \left(\frac{\alpha_s}{v}\right)^k \cdot \Big\{ 1\,\text{(LO)};\, \alpha_s, v\,\text{(NLO)};\, \alpha_s^2, \alpha_s v, v^2\,\text{(NNLO)};\, \ldots \Big\}$$

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi(q^2) = i \int d^4x\, e^{iq\cdot x}\,\langle 0|T(j_\mu(x) j_\nu(0))|0\rangle, \qquad j^\mu(x) = [\bar{Q}\gamma^\mu(\gamma_5)Q](x)$$

Summation through Schrödinger equation.

$$\text{Im}\,\Pi(E) = \frac{N_c}{2m^2} \underbrace{\sum_{n=1}^{\infty} Z_n \times \pi\delta(E_n - E)}_{\text{bound states}} + \Theta(E)\, \underbrace{\text{Im}\,\Pi(E)_{\text{cont}}}_{\text{continuum}}$$

$$R \equiv \frac{\sigma_{e^+e^- \to WWb\bar{b}X}}{\sigma_0} = 12\pi e_t^2 K\ \text{Im}\,\Pi(E + i\Gamma_t) + [\text{EWC + non-resonant}]$$

Tools

Non-relativistic effective field theory and threshold expansion (defines the matching procedure!)
See [MB, Kiyo, Schuller, arXiv:1312.4791 [hep-ph]]

Relevant scales: $m_t \approx 175\,\mathrm{GeV}$ (hard), $m_t\alpha_s \approx 30\,\mathrm{GeV}$ (soft, potential) and the ultrasoft scale (us) $m_t\alpha_s^2 \approx 2\,\mathrm{GeV}$.

$$\begin{tikzcd}
\mathcal{L}_{\mathrm{QCD}}\,[Q(h,s,p),\ g(h,s,p,us)] \arrow[d] & \mu > m_t \\
\mathcal{L}_{\mathrm{PNRQCD}}\,[Q(p),\ g(us)] & \mu < m_t v
\end{tikzcd}$$

See mulit-loop fixed-order calculations to match PNRQCD, then perturbation theory in Coulomb background in PNRQCD. Can be extended systematically to any order. 3rd order is current technological limit.

$$\Pi^{(v)}(q^2) = \frac{N_c}{2m^2}\, c_v \left[c_v - \frac{E}{m} \left(c_v + \frac{d_v}{3} \right) \right] G(E) + \ldots$$

$$G(E) = \frac{i}{2N_c(d-1)} \int d^d x\, e^{iEx^0}\, \langle 0|\, T(\, [\chi^\dagger \sigma^i \psi](x)\, [\psi^\dagger \sigma^i \chi](0))|0\rangle_{|\mathrm{PNRQCD}}\,,$$

3rd order ingredients

- Bound state quantities (S-wave)
  - E_n – Kniehl, Penin, Smirnov, Steinhauser (2002); MB, Kiyo, Schuller (2005); Penin, Sminrov, Steinhauser (2005)
  - $|\psi_n(0)|^2$ – MB, Kiyo, Schuller (2007); MB, Kiyo, Penin (2007)
- Matching coefficients
  - a_3 – Anzai, Kiyo, Sumino (2009); Smirnov, Sminrov, Steinhauser (2009)
  - c_3 – Marquard, Piclum, Seidel, Steinhauser (2014) [2009]
- Continuum (PNRQCD correlation function)
  - ultrasoft – MB, Kiyo (2008)
  - potential – MB, Kiyo, Schuller, in preparation (2015) [2007]
  - P-wave – MB, Piclum, Rauh (2013)

Note: logarithmically enhanced 3rd order terms known before or resummed [Hoang et al. 2001-2013; Pineda et al. 2002-2007]. But non-log terms are as large in individual terms.
2nd order available since end of 1990s.

Mass issues

- Pole mass cannot be determined with an accuracy better than $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ [MB, Braun, 1994; Bigi et al., 1994].
  Leads to spurious shifts in the peak position of the $t\bar{t}$ cross section [MB, 1998]
- Solution: intermediate mass definition, which can be related precisely to the $\overline{\mathrm{MS}}$ mass (→ top Yukawa coupling) AND avoids spurious shifts.

  Potential-subtracted mass [MB, 1998]

$$m_{\mathrm{PS}}(\mu_f) \equiv m_{\mathrm{pole}} + \frac{1}{2}\int_{|\vec{q}\,|<\mu_f} \frac{d^3\vec{q}}{(2\pi)^3}\, \tilde{V}_{\mathrm{Coulomb}}(\vec{q}\,)$$

Cancellation of large perturbative contributions from the IR. In the following use $m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$.

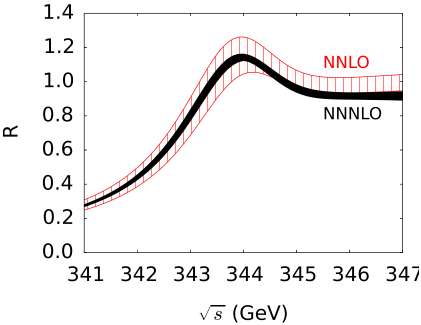
- Mass relation

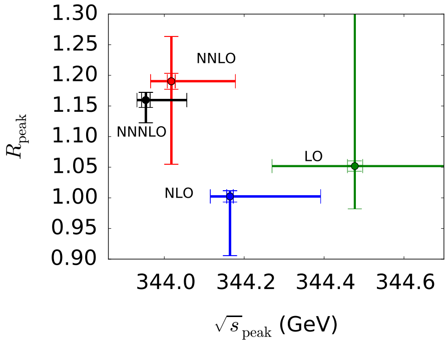
$$m_{\mathrm{PS}}(\mu_f) - \overline{m}(\overline{m}) = \underbrace{[m_{\mathrm{PS}}(\mu_f) - m_{\mathrm{pole}}]}_{\text{known to } \mathcal{O}(\mu_f\alpha_s^4) \text{ [hep-ph/0501289]}} + \underbrace{[m_{\mathrm{pole}} - \overline{m}(\overline{m})]}_{\text{known to } \mathcal{O}(m_t\alpha_s^4) \text{ [1502.01030]}}$$

Conversion precision ≈ 20 MeV [Marquard et al., 2015]

NNNLO

[MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864]
Photon exchange and Z-vector coupling only.
$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$, $\alpha_s(m_Z) = 0.1185 \pm 0.006$, $\sin^2\theta_W = 0.23$,
$\mu = (50\ldots80\ldots350)\,\mathrm{GeV}$, $\mu_w = 350\,\mathrm{GeV}$.





Position shift: 310 MeV (LO to NLO) 150 MeV (to NNLO) 64 MeV (to NNNLO)
Improvement of factor 3 in uncertainty in peak height.

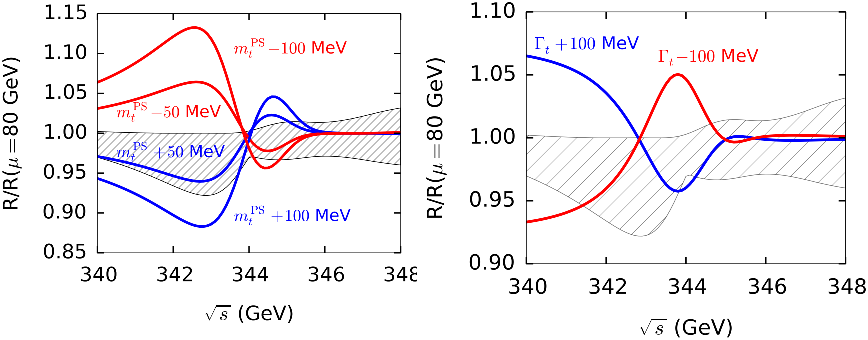
Sensitivity to (m_t, Γ_t) vs. theoretical uncertainty

Shaded band: Relative scale uncertainty

NNNLO $$\frac{\delta\sigma}{\sigma} = \pm(2\ldots3.5)\%$$

Superimposed: Variation with shifted top mass or width input normalized to reference.

[MB, Kiyo, Marquard, Penin, Piclum, Steinhauser, 1506.06864]



From QCD $t\bar{t}$ threshold to a realistic prediction

With 3rd order QCD effects known to a few percent, focus on non-QCD effects, potentially of the same order

- Axial-vector Z-coupling (not a non-QCD effect) [MB, Piclum, Rauh, 2013]
  NNLO+
- QED effects [Pineda, Signer, 2006; MB, Jantzen, Ruiz-Femenia, 2010]
  NLO+
- Initial state radiation (also QED) (formalism in MB, Falgari, Schwinn, Signer, Zanderighi, 2007)
  Formally NNLO, but large logs. Effectively LO.
- Electroweak matching coefficients absorptive parts [Hoang, Reisser, 2004] and electroweak corrections in general [Guth, Kühn, 1992]
  NNLO+ $[\alpha_{\mathrm{EW}} \sim \alpha_s^2]$
- Higgs contributions [Eiras, Steinhauser, 2006; MB, Maier, Piclum, Rauh, 2015]
  NNLO+

Non-resonant cross section

The pure-QCD calculation in the (P)NRQCD framework is technically inconsistent from NNLO. Uncancelled $1/\epsilon$ poles.

Electroweak effect. Must consider $e^+e^- \to W^+W^-b\bar{b}$.

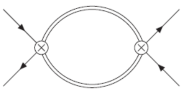
$$\sigma_{e^+e^-\to W^+W^-b\bar{b}} = \underbrace{\sigma_{e^+e^-\to [t\bar{t}]_{\text{res}}}(\mu_w)}_{\text{pure (PNR)QCD}} + \sigma_{e^+e^-\to W^+W^-b\bar{b}_{\text{nonres}}}(\mu_w)$$

- Non-resonant contributions: $\sigma_{e^+e^-\to W^+W^-b\bar{b}_{\text{nonres}}}(\mu_w)$
  Mostly inclusive, possibly invariant mass cuts.
  NLO+

  NLO [MB, Jantzen, Ruiz-Femenia, 2010; Penin, Piclum, 2011]
  Partial results only at NNLO [Hoang, Reisser, Ruiz-Femenia, 2010; Jantzen, Ruiz-Femenia, 2013; Ruiz-Femenia, 2014]

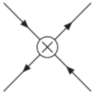
Unstable particle effective theory

Unstable particle EFT provides a systematic expansion of the amplitude in powers of Γ/m. [MB, Chapovsky, Signer, Zanderighi, 2003]



Resonant contributions

Production of an on-shell, non-relativistic $t\bar{t}$ pair and subsequent decay $t \to W^+ b$. Effective non-relativistic propagator contains on-shell width.



Non-resonant contributions

All-hard region. Off-shell lines. Full theory diagrams expanded around $s = 4m_t^2$. No width in propagators.

$$i\mathcal{A} = \sum_{k,l} C_p^{(k)} C_p^{(l)} \int d^4x \, \langle e^- e^+ | \mathrm{T}[i\mathcal{O}_p^{(k)\dagger}(0)\, i\mathcal{O}_p^{(l)}(x)] | e^- e^+ \rangle + \sum_k C_{4e}^{(k)} \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

$$\mathcal{O}_p^{(v,a)} = \bar{e}_{c_2} \gamma_i (\gamma_5) e_{c_1} \, \psi_t^\dagger \sigma^i \chi_t \qquad \mathcal{O}_{4e}^{(k)} = \bar{e}_{c_1} \Gamma_1 e_{c_2} \, \bar{e}_{c_2} \Gamma_2 e_{c_1} \, ,$$

$$\sigma_{\text{non-res}} = \frac{1}{s} \sum_k \mathrm{Im} \left[C_{4e}^{(k)} \right] \langle e^- e^+ | i\mathcal{O}_{4e}^{(k)}(0) | e^- e^+ \rangle$$

Separately divergent and factorization (“finite-width”) scale-dependent.

Systematics of Yukawa coupling effects

Power counting of new parameters m_H, λ_t:

- $m_H \sim m_t$ or $m_H \sim m_t v$
- $\lambda_t \sim \alpha_s$ or $\lambda_t \sim \alpha_{\text{EW}} \sim \alpha_s^2$

Adopt $m_H \sim m_t$ and $\lambda_t = y_t^2/(4\pi)$ as electroweak coupling.

Higgs-exchange Yukawa potential is effectively local

$$\frac{y_t^2}{\vec{q}^{\,2} + m_H^2} \to \frac{y_t^2}{m_H^2} \qquad \Rightarrow \qquad \delta\sigma \propto -\frac{y_t^2}{m_H^2}\,\text{Im}[G_0(E)^2]_{\overline{\text{MS}}}$$

N3LO effect relative to $-g_s^2/\vec{q}^{\,2}$.

Need only single insertion of Higgs-exchange tree-level potential into Coulomb Green function.
Formally not the dominant Yukawa coupling effect.

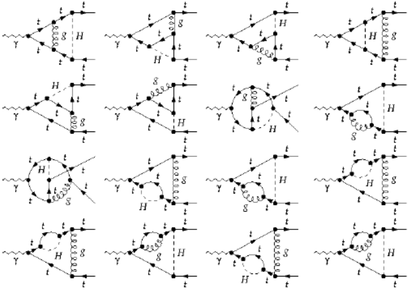
Short-distance Yukawa coupling effects

Modification of the $t\bar{t}$ production vertex

NNLO – 1-loop correction to vector-current matching [Guth, Kühn, 1991]
N3LO – Mixed 2-loop Higgs-QCD correction to vector-current matching [Eiras, Steinhauser, 2006]

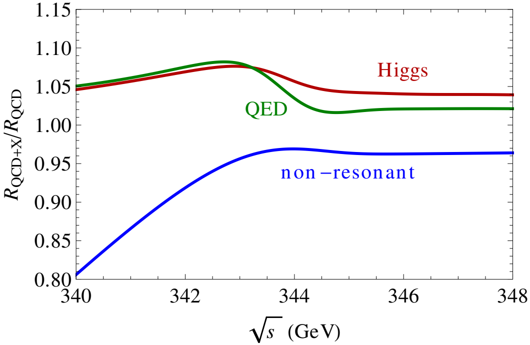
$$c_v = 1 - 0.103|_{\alpha_s} \underbrace{-0.022|_{\alpha_s^2} + 0.031|_{y_t^2}}_{\text{NNLO}} \underbrace{-0.070|_{\alpha_s^3} - 0.019|_{y_t^2\alpha_s}}_{\text{NNNLO}} + \ldots,$$

Local approximation of the Higgs potential required for consistent cancellation of factorization scale dependence.



Putting together [MB, Maier, Piclum, Rauh, 2015]

- NNNLO QCD including P-wave
- NNNLO Higgs (top-Yukawa)
- NLO electroweak (QED)
- NLO non-resonant



NNNLO (QCD+Higgs) + NLO (QED+non-resonant)

[MB, Maier, Piclum, Rauh 1506.06865]
Inclusive $e^+e^- \to W^+W^- b\bar{b}$ cross section
$m_{t,\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$, $\alpha_s(m_Z) = 0.1185 \pm 0.006$, $\sin^2\theta_W = 0.2229$,
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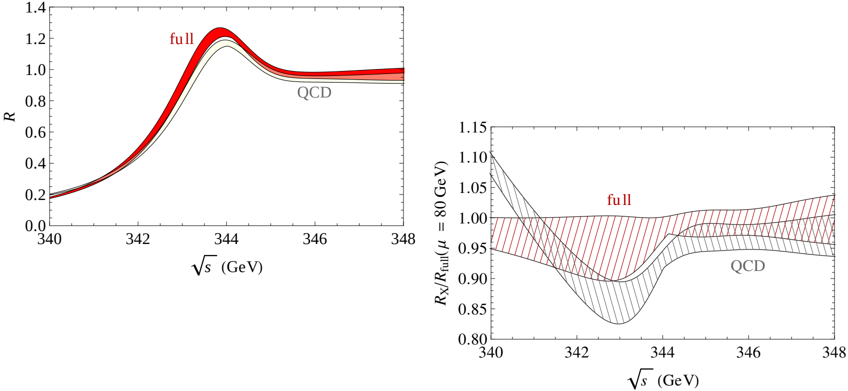
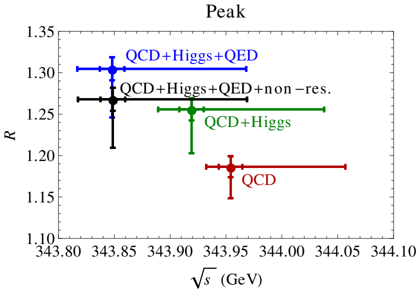
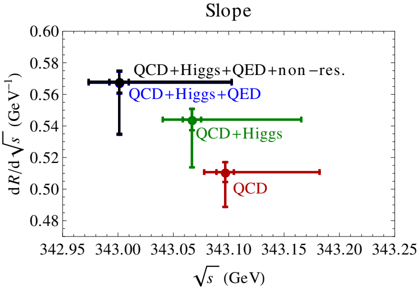


Peak and maximal slope position





Sensitivity to y_t

- Add

$$\Delta\mathcal{L} = -\frac{c_{\rm NP}}{\Lambda^2}(\phi^\dagger\phi)(\bar{Q}_3\widetilde{\phi}t_R) + \text{h.c.}$$

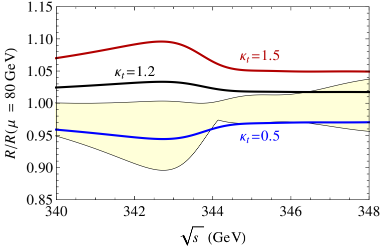
to the SM Lagrangian

$$\kappa_t \equiv \frac{y_t}{\sqrt{2}m_t/v} = 1 + \frac{c_{\rm NP}}{\Lambda^2}\frac{v^3}{\sqrt{2}m_t}$$

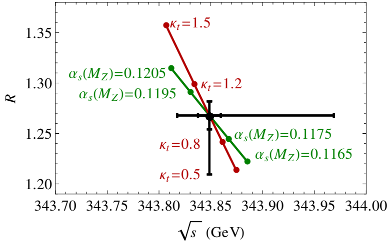
Treat top mass and Yukawa coupling as independent parameters.

- In the framework of the SM effective Lagrangian (SM + dim-6) there are many more and possibly more important anomalous coupling effects.

[MB, Maier, Piclum, Rauh, 2015]



Peak position of (κ_t, α_s) vs. th. uncertainty



Summary

I $e^+e^- \to t\bar{t}X$ cross section near threshold now computed at NNNLO in (PNR)QCD + top-Yukawa effects

- Sizeable 3rd order corrections and reduction of theoretical uncertainty to about $\pm 3\%$.

II Realistic predictions for $e^+e^- \to W^+W^-b\bar{b}$ near top-pair threshold

- NLO available, including cuts invariant mass cuts.
  NNLO needed. Residual uncertainty should then be small.

III Parameter dependences $(m_t, \Gamma_t, y_t, \alpha_s)$ can be studied.

- (m_t, Γ_t) with unrivaled accuracy.
- y_t with 20% accuracy from threshold already challenging.

In many cases, theoretical uncertainties now of the same order as the expected statistical + systematic.