# Theory overview of top mass and couplings

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# Outline

- Top mass
  - Highlight of a few key issues
  - Dwell on some recent beautiful results
  - Implications for experiment
- Top couplings
  - Rough status of what is known
  - and what we might eventually know

Reviews e.g.:
Snowmass (2013) write-up A. Juste et al, arXiv: 1310.0799
S. Moch et al (2014) MITP workshop, arXiv:1405.481


WEIGHT GAIN FORMULA
TOP
mass
GOLD
DƯỠNG GIÚP TĂNG CÂN

# The last of the mass problems?

I. Newton (1687)

- We thought we had solved it in the 17th century
    - (i) resistance force and (ii) gravitational coupling

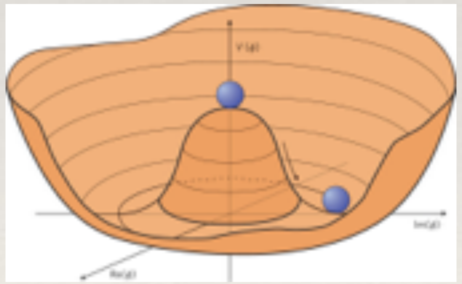
**Gravity holds universe together**

- New insight in 1905: condensed energy
    - Non-trivial for proton

A. Einstein (1905)

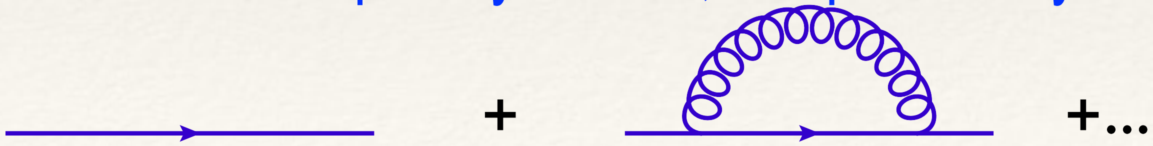
K. Wilson; Durr et al (2008)

- Yet newer insight: coupling to condensate



R, Brout, F. Englert, P. Higgs, Kibble, Hagen, Guralnik (1964 -2012)

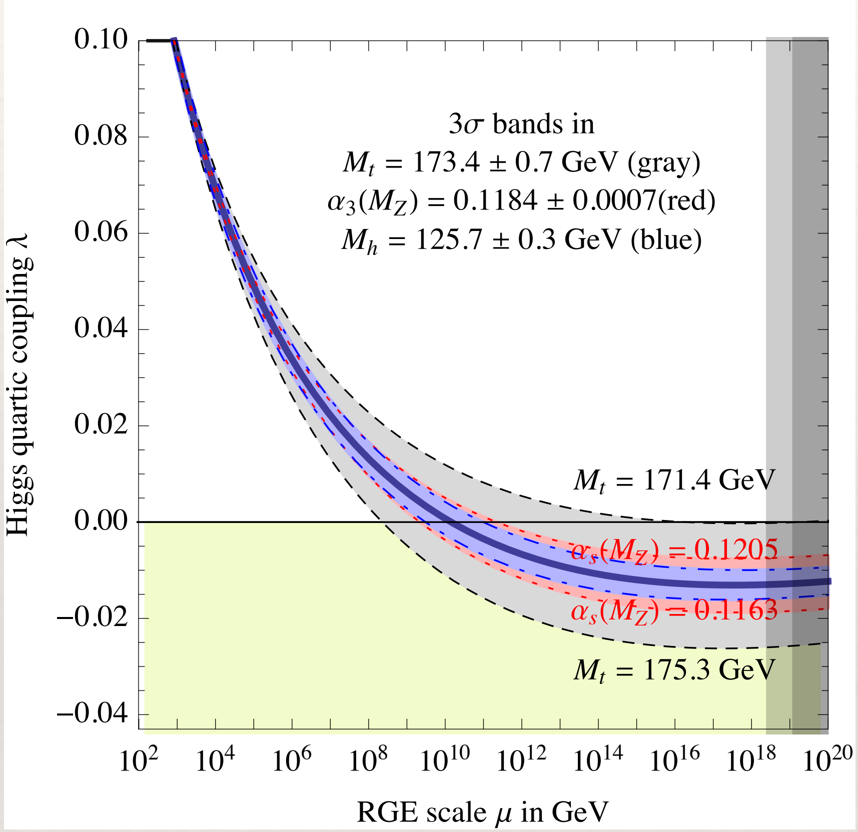
- Finally
    - Mass of confined particle? Conceptually solved, but practically subtle



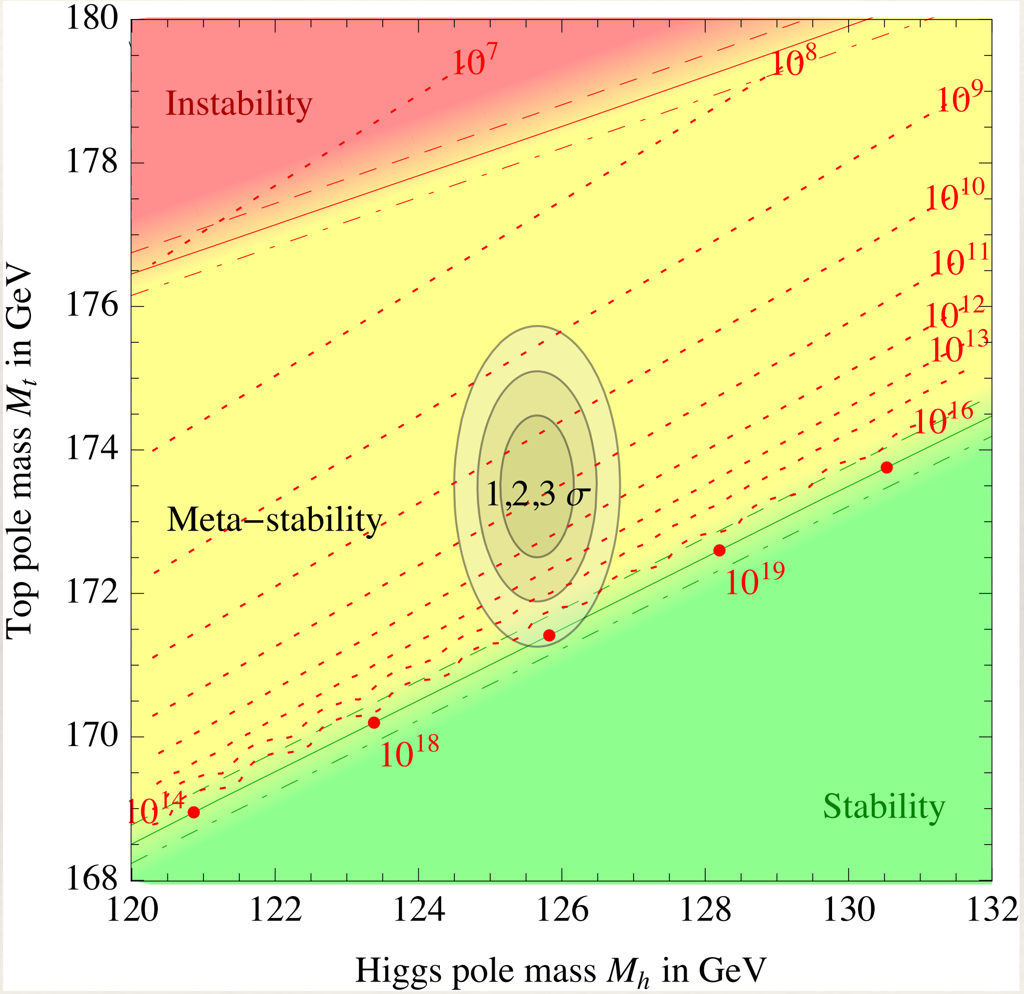
**Does top make the universe fall apart?**

# State of the Vacuum

- Top quark dominant in loop corrections that make the Higgs 4-pt coupling evolve. Full two-loop analysis:



Buttazzo et al (July 2013)



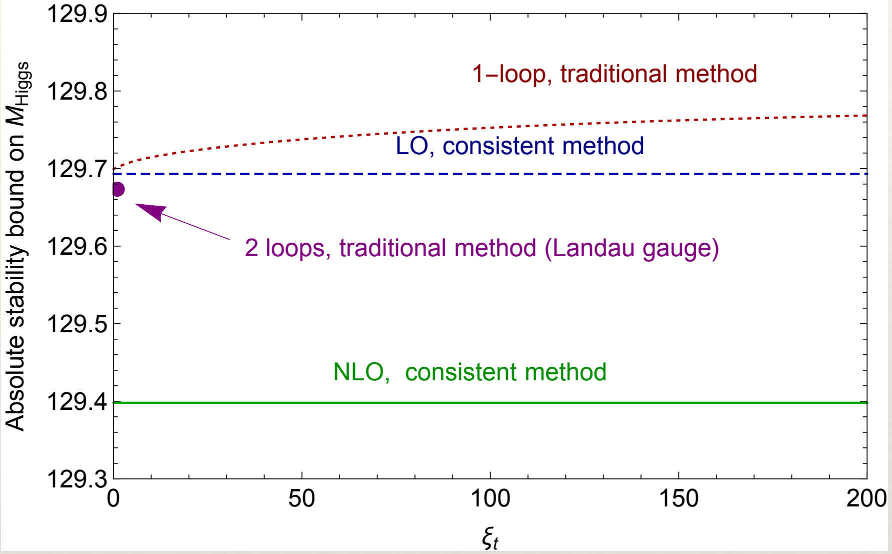
- Depends on precise top quark mass
  - within 300 MeV or so
    - But no practical worries about universe expiring

# Consistent effective potential and the top mass

- Effective potentials are not gauge invariant

Andreassen, Frost, Schwartz..

  - but their extrema are gauge invariant, and scale invariant
    - find that stability bound is gauge-dependent in perturbation theory



  - Consistent treatment to order ħ combines, at LO, tree-level with one-loop
  - Find

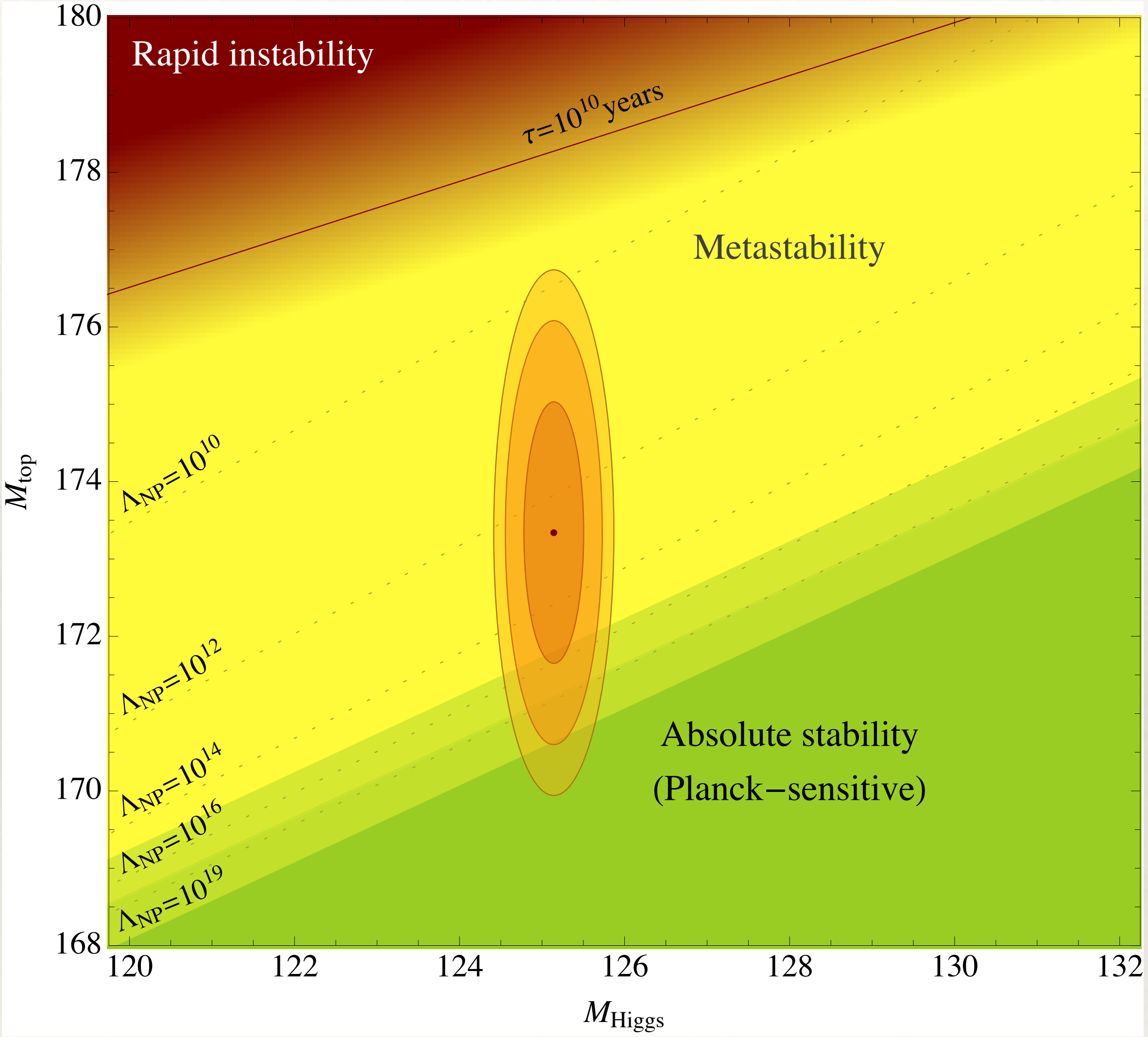
$$\frac{m_t^{\text{pole}}}{\text{GeV}} < (171.22 \pm 0.28) + 0.12\,(\frac{m_h^{\text{pole}} - 125.14\ \text{GeV}}{0.24\ \text{GeV}})$$

    - stability bound on top pole mass: 171.2

# Consistent effective potential and the top mass

- Together with testing for new physics in a consistent way they find

Andreassen, Frost, Schwartz..



# Implications of a large mass

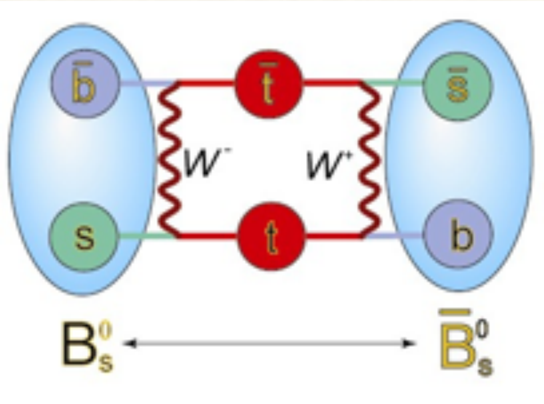
- Top decays before it hadronizes fully
  - the only “bare”(=undressed by QCD) quark
  - gives us access to its spin (i.e. LH and RH couplings)
- For QCD interactions of the top, the natural scale to put in the running QCD coupling is $m_t$.
  - good for perturbative approach

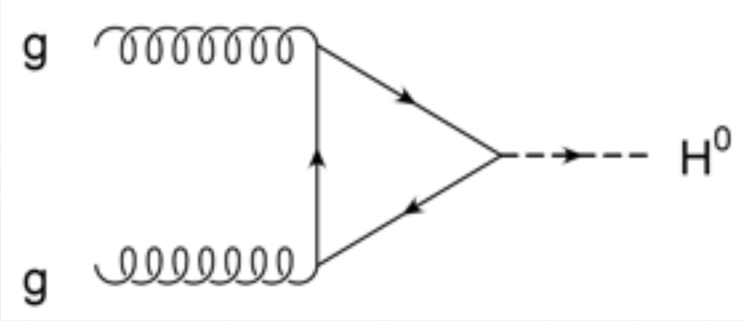
$$\alpha_s(m_t) \simeq 0.1$$

    - ✓ but not always good enough

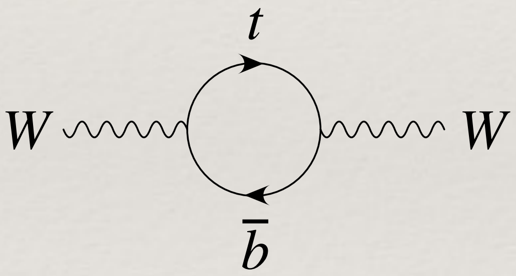
# Virtual top

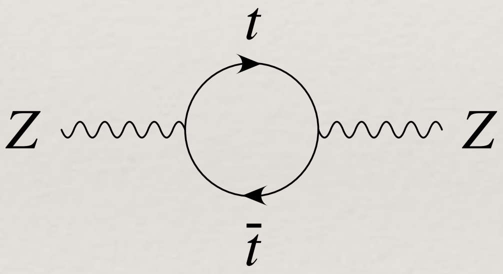
- Virtual top make other things really happen





  - in a loop integral a fixed mass scale always occurs in the result
  - even more if there is no particle with (roughly) equal mass to compensate





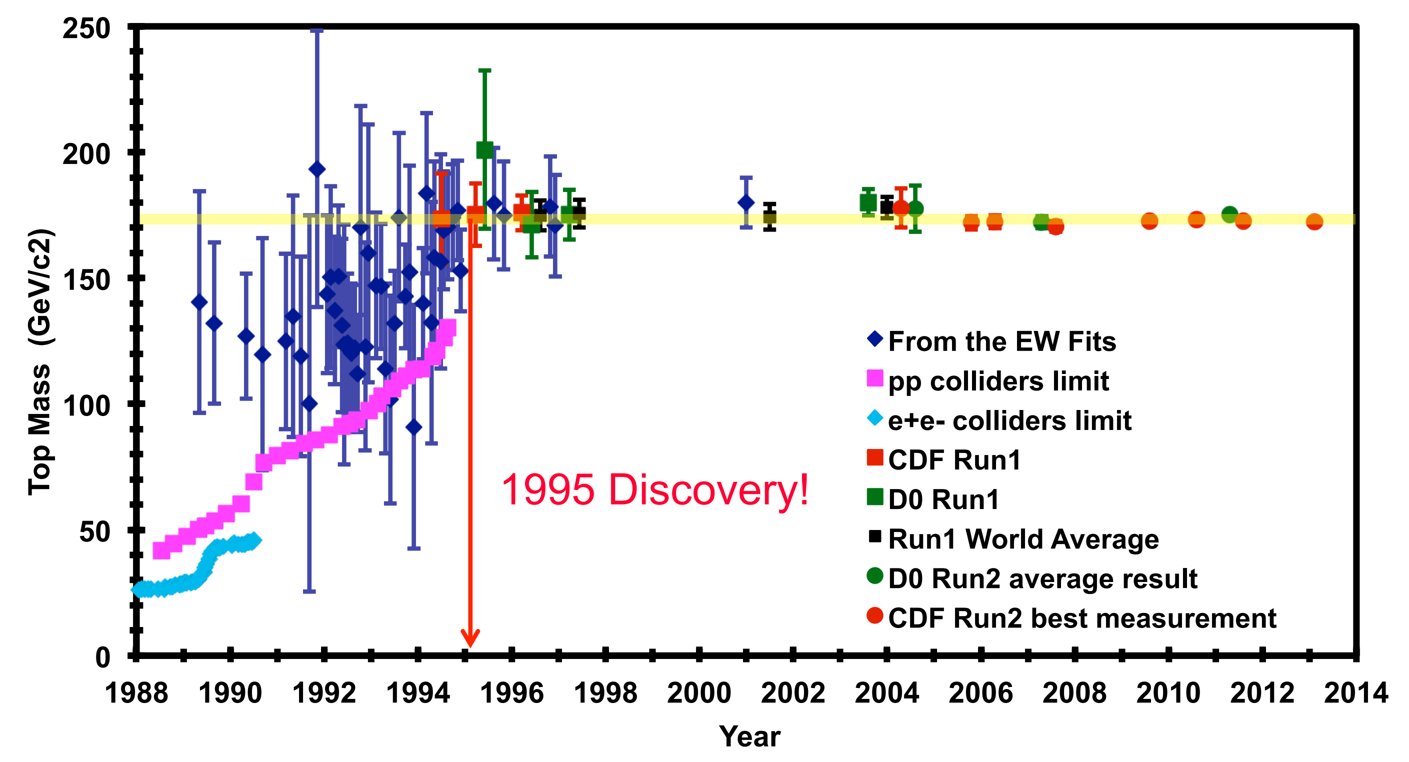
- Express the W mass in terms of 3 fundamental weak parameter, with loop corrections

$$\Delta r_{top} = -\frac{3}{8\pi^2}\frac{G_F}{\sqrt{2}\tan^2\theta_w}m_t^2$$

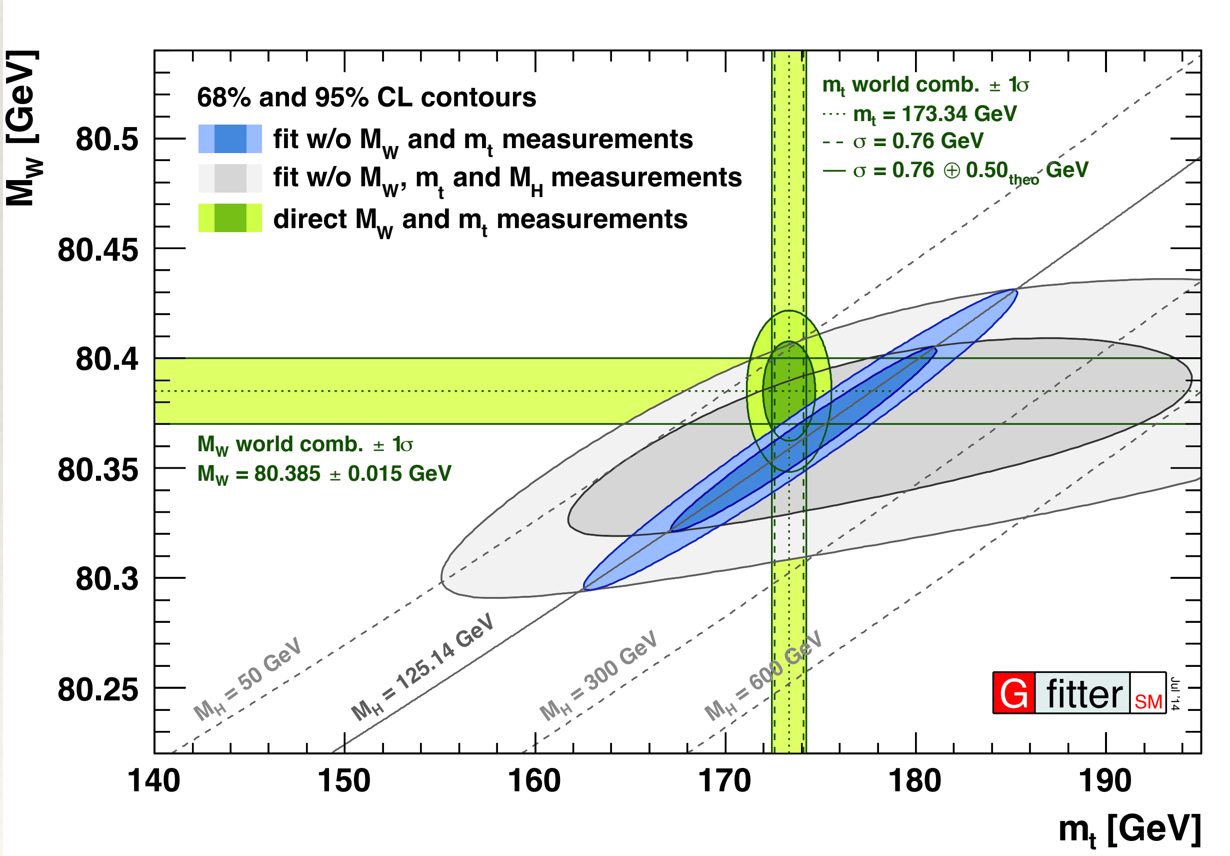
$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F\sin^2\theta_w}\frac{1}{1-\Delta r(m_t, m_H)}$$

$$\Delta r_{Higgs} = \frac{3}{8\pi^2}\frac{G_F}{\sqrt{2}\tan^2\theta_w}m_W^2\Big(2\ln(m_H/m_Z) - 5/6\Big)$$

# Virtual top mass

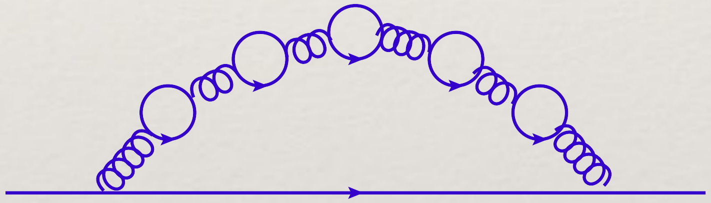


Now impressive consistency between top, Higgs, W mass



# Top mass

- Electron mass definition is“easy”: defined by pole in full propagator
    - ✓ If particle momentum satisfies pole condition (p2=m2),  can propagate to ∞
        - ⇒ there is no real ambiguity what electron “pole” mass is
- But: quarks are confined, so physical on-shell quarks cannot exist
    - ✓ Leads to non-perturbative ambiguity of few hundred MeV
        - (revealed by all-order pQCD!)



# Heavy quark mass, definition(s)

$$= \frac{1}{\not p - m_0 - \Sigma(p, m_0)}$$

$$m_0 \frac{\alpha_s}{\pi}\left[\frac{1}{\epsilon} + \text{finite stuff}\right]$$

To make finite, substitute $m_0 = m_R\left(1 + \frac{\alpha_s}{\pi}\left[\frac{1}{\epsilon} + z_{\text{finite}}\right]\right)$

Mass definitions differ in the choice of

Pole mass: pretend quarks are free and long-

$$\frac{1}{\not p - m_0 - \Sigma(p, m_0)} = \frac{c}{\not p - M}$$

MSbar mass: treat mass as a coupling $z_{\text{finite}} = 0$

# Pole mass issues

- Most natural definition for a free (stable) particle (electron, Z-boson)

Kronfeld

  - gauge invariant and IR safe to all orders
- But quarks are confined, so pole mass has intrinsic uncertainty of order $\Lambda_{\mathrm{QCD}}$
  - Full QCD has no pole at the top quark mass
    - ✓ Finite width of top does not “screen” this

Smith, Willenbrock

  - Reproduced in perturbation theory

Bigi, Shifman, Uraltsev, Vainshtein,
Beneke, Braun, Smith, Willenbrock

$$\Sigma(m,m) \approx \sum_n \alpha_s^{n+1}\beta_0^n n!$$

Renormalon behaviour
order $\Lambda_{\mathrm{QCD}}$ uncertaint

# Heavy quark mass schemes

- Various definitions other than the pole and MSbar schemes have been made
- PS (potential subtracted) mass
  - Substract from the pole mass the IR part of the ttbar Coulomb potential
    - ✓ The two parts have the same IR sensitivity

$$m^{\mathrm{PS}} = M - \tfrac{1}{2}\int_{|q|<\mu_f} \frac{d^3q}{(2\pi)^3} V(q)$$

Beneke

    - ✓ V known to 3-loop

Beneke, Kiyo, Schuller;
Smirnov$^2$, Steinhauser; Anzai,
Kiyo, Zumino

- 1S mass
  - Half the perturbative mass of (fictitious) $1^3S_1$ state

$$m^{1\mathrm{S}} = M + \tfrac{1}{2}E_1^{pt}$$

Hoang, Teubner

# Some $\mathrm{m}_{\mathrm{pole}}$ observations

- Perturbative ("asymptotic") expansion of pole mass

$$m_{\mathrm{pole}} = m_{\overline{\mathrm{MS}}} \times (1 + 0.047 + 0.01 + 0.003 + \ldots)$$

Melnikov, van Ritbergen

  - -> uncertainty about 500 MeV (or less)
  - Uncertainty in pole mass about 300 MeV
  - resultant uncertainty in MSbar mass smaller than

$$m_{\overline{\mathrm{MS}}}(3-\mathrm{loop}) - m_{\overline{\mathrm{MS}}}(2-\mathrm{loop})$$

    - ✓ → NNNNLO?

# MSbar vs pole mass at 4 loop

Marquard, Smirnov, Smirnov, Steinhauser

- Important progress: 4-loop relations between top quark masses

$$M = c_m(\mu) m(\mu)$$

$$c_m^{(4)}\Big|_{n_l=5} = 827.37 \pm 21.5 + 408.88\, l_{\overline{\rm MS}} + 86.574\, l_{\overline{\rm MS}}^2 + 22.023\, l_{\overline{\rm MS}}^3 + 3.2227\, l_{\overline{\rm MS}}^4\,, \qquad (12)$$

$$l_{\overline{\rm MS}} = \ln(\mu^2/m^2)$$

  - Use of various specialized codes (FORM, FIRE, FIESTA,..), many of the (master) loop integrals done numerically.
  - This is also sufficient, together with N3LO Coulomb potential, for 4-loop relations to PS and 1S masses

$$M = m\Big(1 + 0.4244\,\alpha_s + 0.8345\,\alpha_s^2 + 2.375\,\alpha_s^3 + (8.49 \pm 0.25)\,\alpha_s^4) + \ldots\Big)$$

- Result

$$M = 163.643 + 7.557 + 1.617 + 0.501 + 0.195 \pm 0.005 \text{ GeV}$$

- Numerically: nice progression!! No sign of an impending renormalon

# Impact on MSbar mass

Marquard, Smirnov, Smirnov, Steinhauser

- Study how a different threshold mass measurement leads to MSbar mass

| input<br>#loops | $m^{\mathrm{PS}} =$<br>171.792 | $m^{\mathrm{1S}} =$<br>172.227 | $m^{\mathrm{RS}} =$<br>171.215 |
|---|---|---|---|
| 1 | 165.097 | 165.045 | 164.847 |
| 2 | 163.943 | 163.861 | 163.853 |
| 3 | 163.687 | 163.651 | 163.663 |
| 4 | 163.643 | 163.643 | 163.643 |
| 4 ($\times$1.03) | 163.637 | 163.637 | 163.637 |

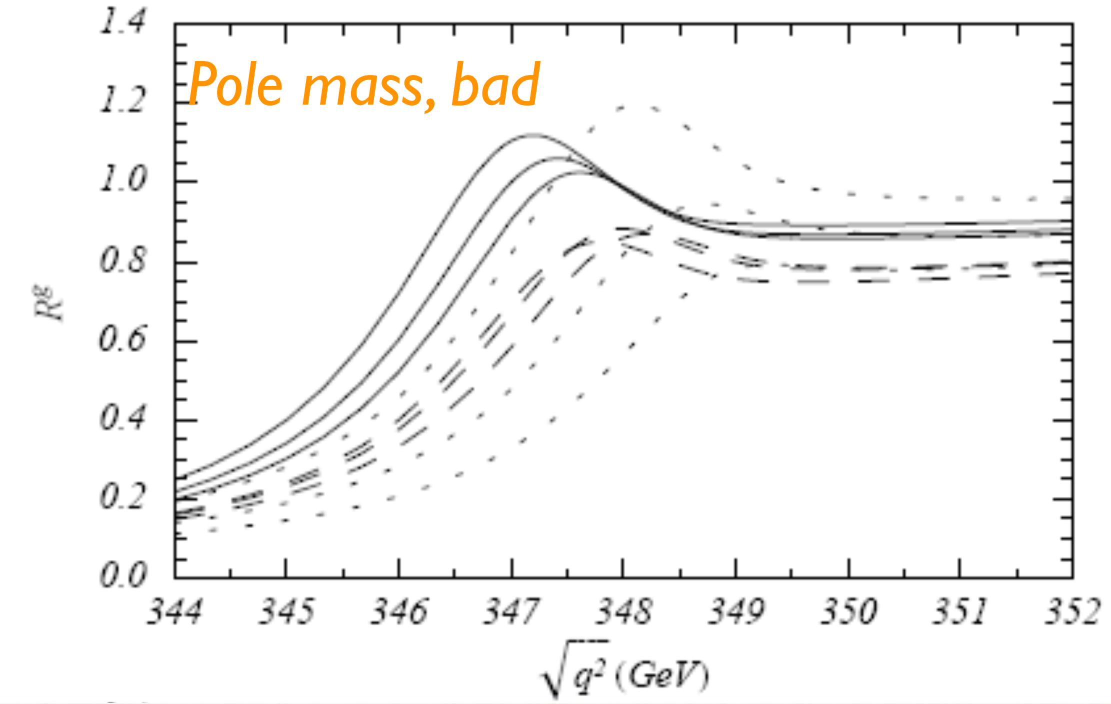
- 3-loop still gives 200-250 MeV shifts
- 4-loop only gives further {44,8,20} MeV shifts
  - final remaining uncertainty estimate {23,7,11} MeV

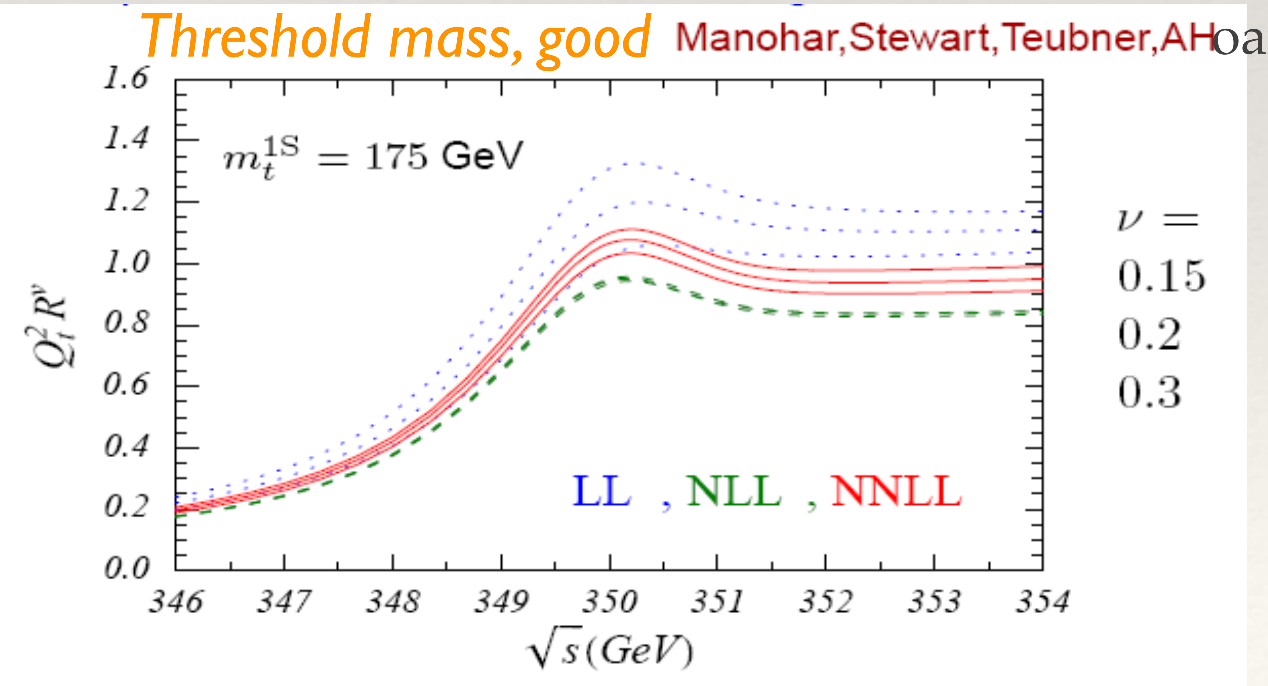
# Top threshold mass

- Scan the ttbar threshold at linear collider by varying beam energy. The opening of the top channel leads to “smooth” theta-function
- Distribution can be measured very precisely. with calculation using Schrodinger equation and appropiate short-distance mass
- Also sensitive to top quark width, allows good measurement
- Calculation non-relativistic effective field theory. Two small parameters: $\alpha_s$ and v.

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times \left\{1\,(\mathrm{LL}); \alpha_s, v\,(\mathrm{NLL}); \alpha_s^2, \alpha_s v, v^2\,(\mathrm{NNLL})\right\}$$

- Choice of top quark mass scheme matters..

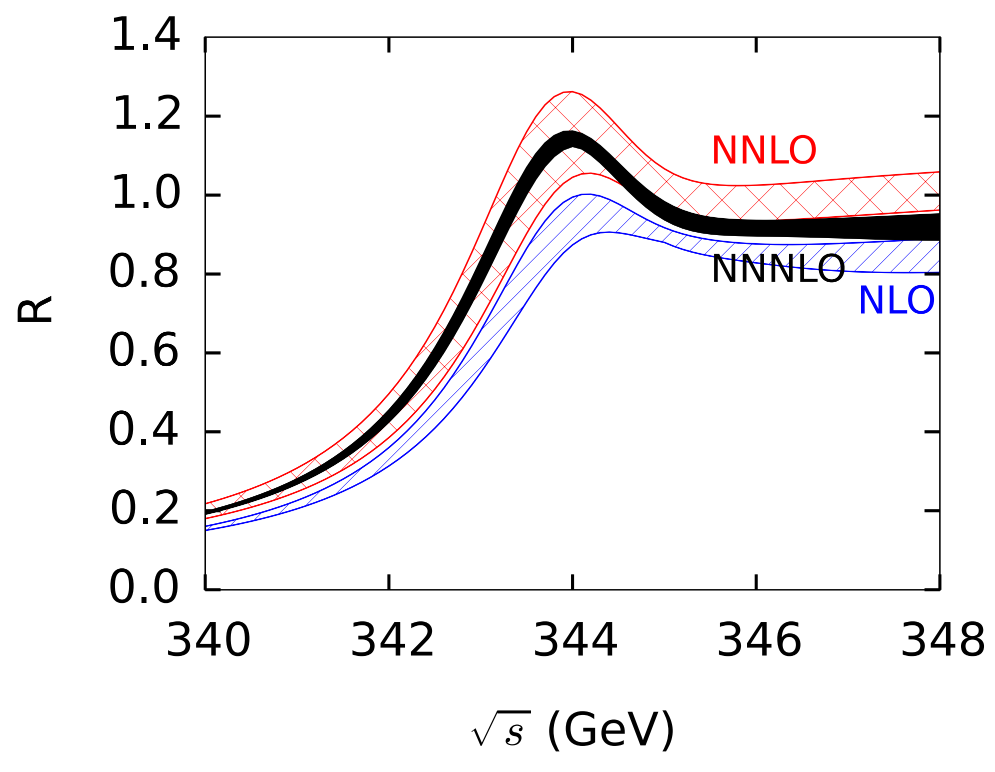

Pole mass, bad


Threshold mass, good Manohar,Stewart,Teubner,AHoang

# N3LO for ttbar S-wave threshold production at e+e- collider

- Now finally the full N3LO cross section, including the last non-logarithmic terms, is known
  - Heroic effort, and it was worth it! QCD calculation under control

Beneke, Kiyo, Marquard,
Penin, Piclum, Steinhauser

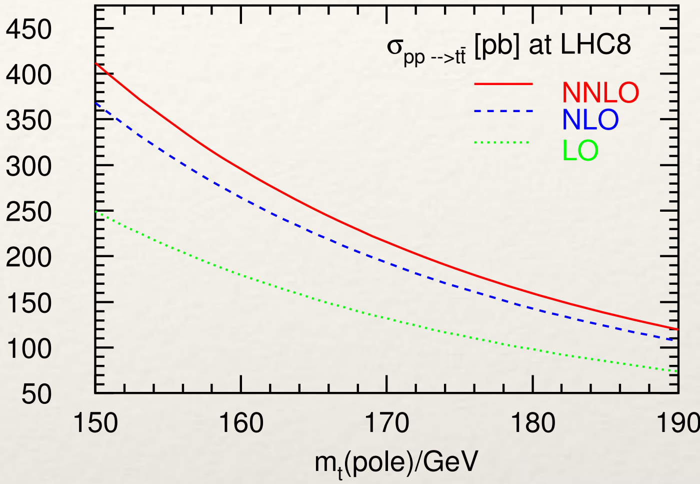


  - Dramatic scale reduction N2LO ⟶ N3LO. Negative correction beyond the peak.
  - QCD uncertainty on top quark mass can go below 50 MeV.
    - ✓ But are also non-QCD effects to study: EW, Higgs, Beamstrahlung, non-resonant terms..

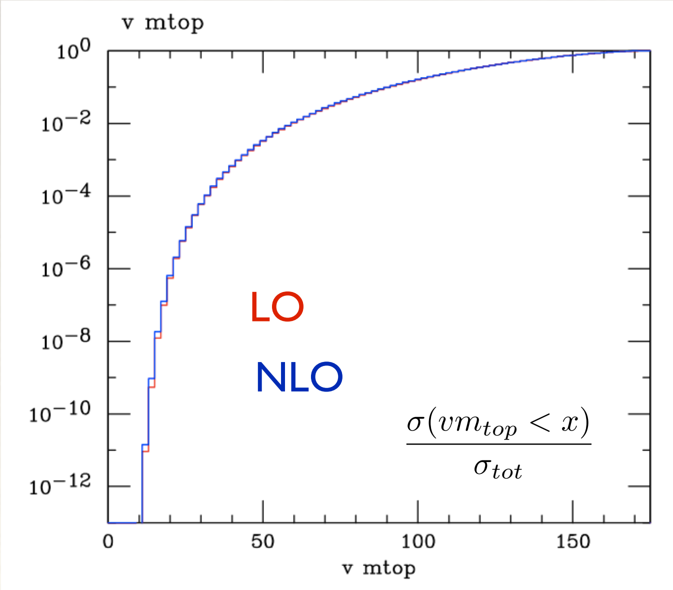
# Mass by proxy

- Of course, one does not need to reconstruct the top quark from its decays. Needs to solve implicit equation

$$\sigma^{\text{exp}}(\{Q\}) = \sigma^{\text{th}}(m_t, \{Q\})$$



- using an observable σ that is optimally sensitive to $m_t$.
  - Adjust $m_t$ to fit data best.
- When extracting ttbar cross section, IR sensitive region is minute fraction of total result.
  - Pole mass should be fine here; can interpret “$m_{top}$” in MC as pole mass, with small error (unlike $e^+e^-$)



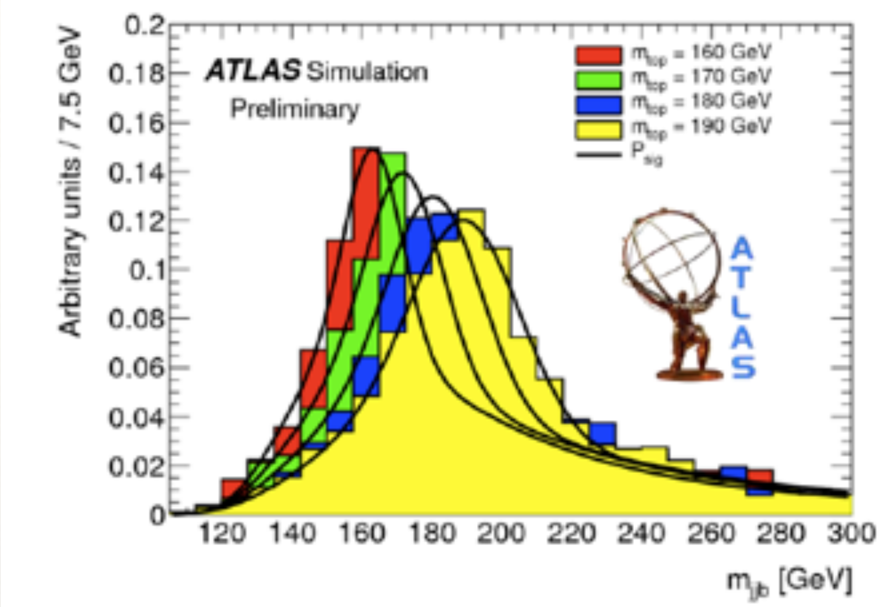
[Mangano at TOP2013]

# Experimental mass determinations

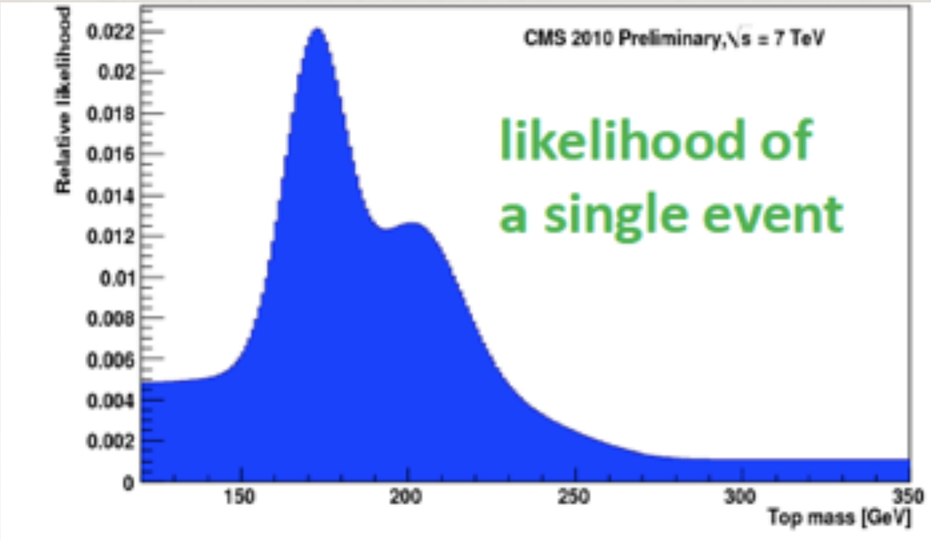
In theory there is no difference between theory and practice; in practice there is

P. Uwer, talk at SM@LHC, quoting Yogi Berra

- What do experiments do?
  - Template method: compare observables in data with MC templates generated with different masses



  - Matrix element method: build event likelihood for full (LO) top quark matrix element, with full kinematics



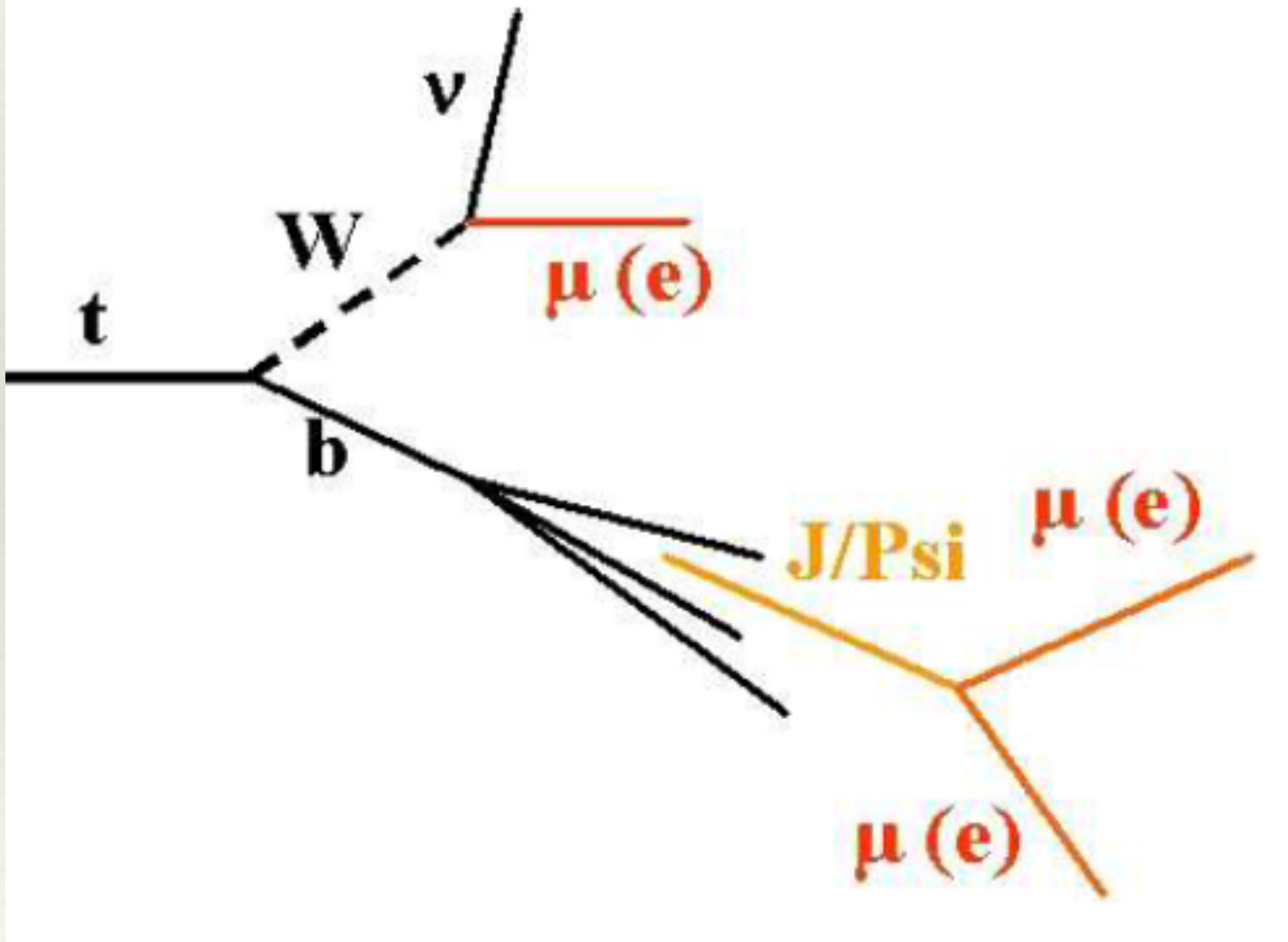
likelihood of a single event

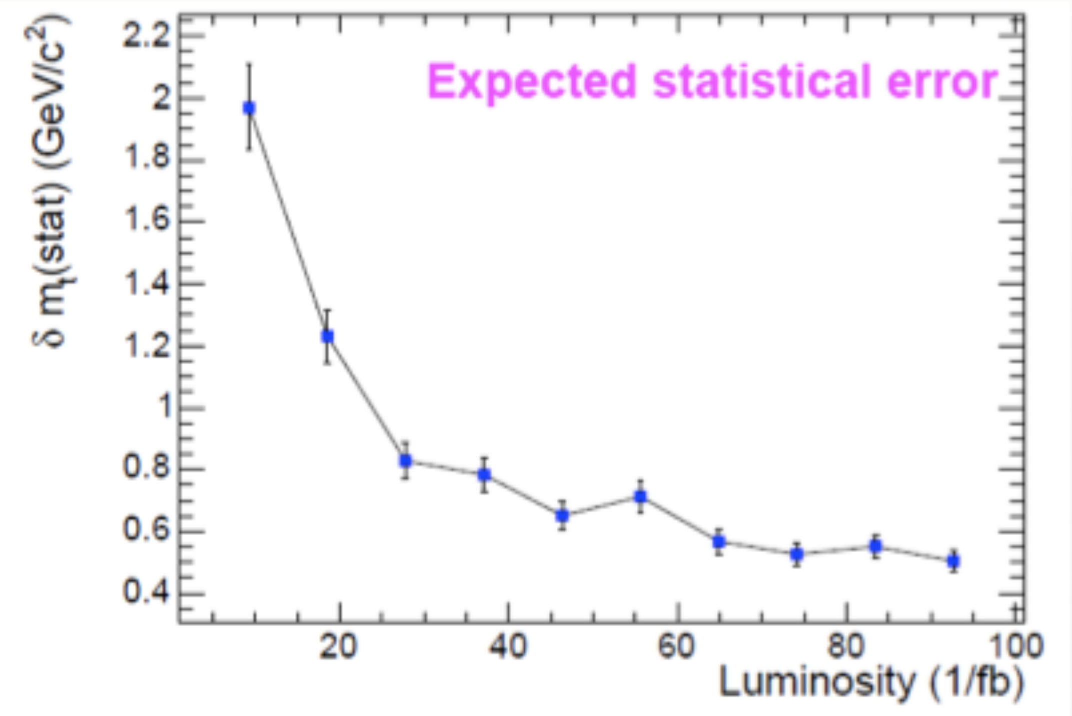
  - “Ideogram”, and other methods

# $M_{lb}$ in leptonic top-quark decays

[R. Chierici, A. Dierlamm
CMS Note 2006/058], Karchilava

Interesting idea : infer top mass from correlation with e/mu and J/Psi invariant mass





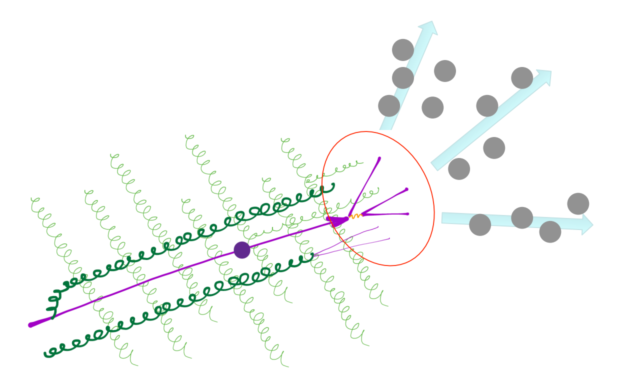
# What top mass is measured?

- Most involve MC's that are LO, so they could never tell the difference between different mass definitions.
- So what mass do hadron colliders determine?
    - Pole mass? "Pythia" mass?
        - ✓ Typically the path from data to a value for m involves a Monte Carlo, itself driven by a mass parameter.
        - ✓ Path goes via (shower) cuts, efficiencies, hadronization models etc

# Monte Carlo top mass

Hoang, Stewart
Moch et al

- MC mass does not depend on observables. Related to soft radiation for that MC.
- Hadronization affects the MC mass value
  - Has aspects of top (or B)-meson mass!



$$m_t^{\mathrm{MC}} = m_t^{\mathrm{MSR}} + \Delta^{\mathrm{MSR}}$$

  - Use methods from B-meson physics to extract field theory mass. But uncertainty order 1 GeV.
- To relate to field theory mass, would need mass-sensitive observable, that can also be computed in MC
  - Calculation beyond LO (LL)
    - ✓ controlled errors. Factorize observable to control top mass in each factor.

Hoang, Stewart

  - Hadron level observables
- E.g. massive thrust, DIS for massive quarks, ttbar at high pT

Hoang et al

# Proxy mass: determing the MSbar mass

- How to determine the MSbar mass?
  - Problem: on-shell condition of final state top leads tot the pole mass

$$\mathrm{Im}\left[\frac{1}{p^2 - M^2 + i\varepsilon}\right] = \pi\,\delta(p^2 - M^2)$$

- By proxy
  - compute cross section using pole mass $\sigma_{tt}(M, \alpha_s)$
  - replace pole mass by MSbar mass
  - Now fit to data, extract MSbar mass

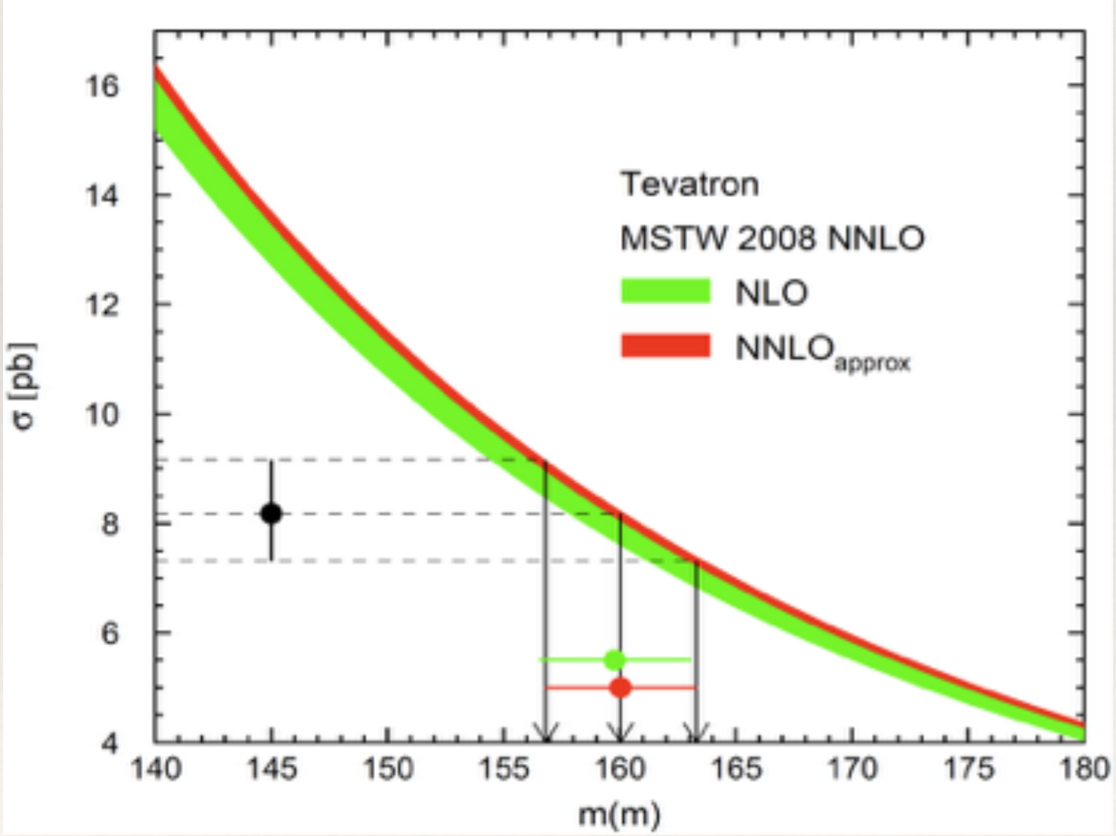
Langenfeld, Moch, Uwer

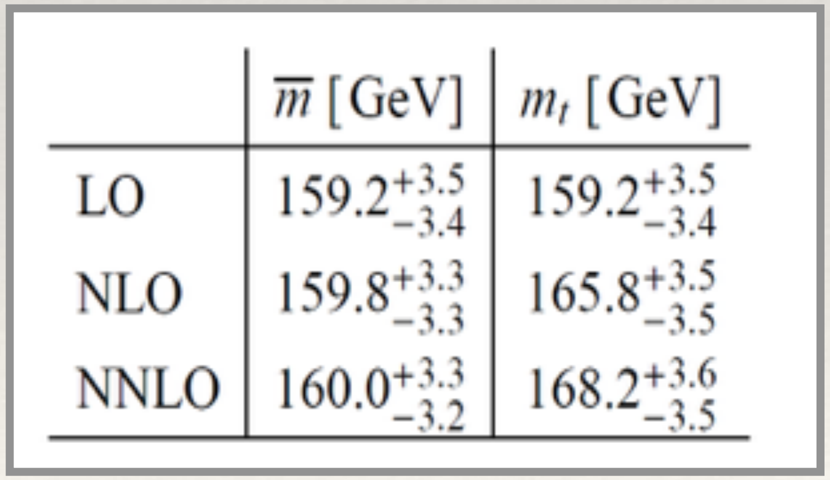
# MSbar mass extraction

Langenfeld, Moch, Uwer

- Accuracy limited by mt sensitivity and PDF uncertainties
- Other proposals for mass-sensitive observables:
  - (moments of) the invariant mass distribution  Frederix, Maltoni
  - tt+1 jet rate  Alioli, Fernandez, Fuster, Irles, Moch, Uwer

$8.18^{+0.98}_{-0.87}$ pb



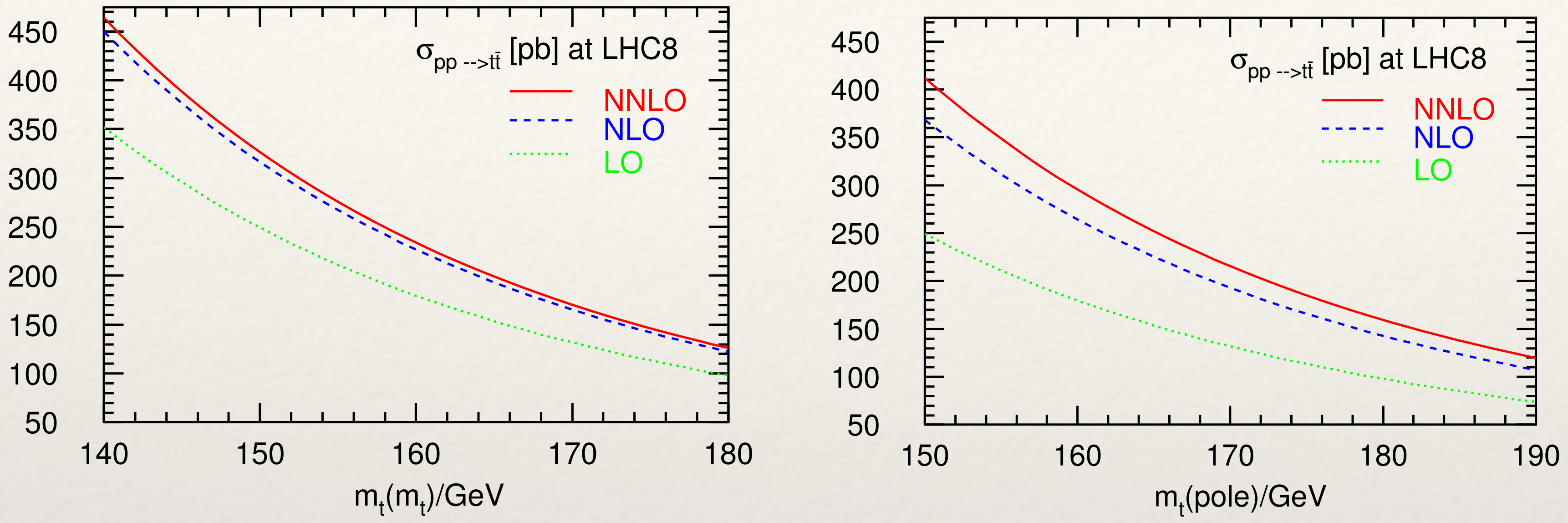


| | $\overline{m}$ [GeV] | $m_t$ [GeV] |
|---|---|---|
| LO | $159.2^{+3.5}_{-3.4}$ | $159.2^{+3.5}_{-3.4}$ |
| NLO | $159.8^{+3.3}_{-3.3}$ | $165.8^{+3.5}_{-3.5}$ |
| NNLO | $160.0^{+3.3}_{-3.2}$ | $168.2^{+3.6}_{-3.5}$ |

# MSbar mass extraction

- In spite of MLM's argument MSbar has better progression

Moch



  - and better scale dependence
- Same holds for the distribution invariant mass $m_{tt}$.
- From a correlated fit including the LHC and Tevatron ttbar cross section, to also gluon PDF and $\alpha_s$.

Alekhin, Bluemlein, Moch

$$m_t(m_t) = 162.3 \pm 2.3 \text{ GeV}$$

  - leading to the pole mass value

$$M = 171.2 \pm 2.4 \pm 0.7 \text{ GeV}$$

# Some other LHC mass proxies

Frixione, Mitov

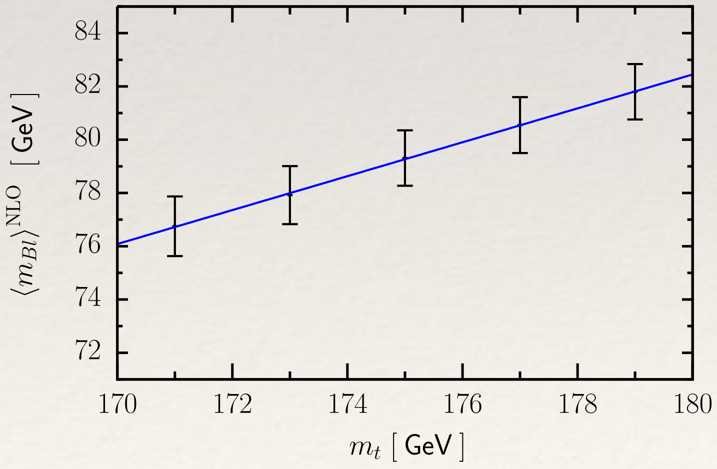
- In dilepton channel, can use shapes of various observables sensitive to top mass
  - study with NLO+PS+MadSpin
  - single-inclusive or mildly correlated (1,4,5) stable under above effects
    - ✓ 2,3 not -> be careful with using NNLO with stable tops
  - about 0.8 GeV theory error in studied scenario, with aMC@NLO

| Label | Observable |
|---|---|
| 1 | $p_T(\ell^+)$ |
| 2 | $p_T(\ell^+\ell^-)$ |
| 3 | $M(\ell^+\ell^-)$ |
| 4 | $E(\ell^+) + E(\ell^-)$ |
| 5 | $p_T(\ell^+) + p_T(\ell^-)$ |

Biswas, Melnikov, Schulze

- NLO study for $<m_{Bl}>$, possibly via J/psi, and other parton shower independent proxies
  - 1.5 GeV uncertainty. Partons showers do in general quite well in estimating uncertainties

Corcella, Mescia



# Top couplings

# Top SM couplings

Exp. tested?

- to W boson: flavor mixing, lefthanded
  - $g_W$ ~ 0.45

$$\frac{g}{\sqrt{2}} V_{tq} \, (\bar{t}_L \gamma^\mu q_L) W^+_\mu$$

√

- to Z boson: parity violating
  - $g_Z$ ~ 0.14

$$\frac{g}{4\cos\theta_w} \, \bar{t} \left( (1 - \frac{8}{3} \sin^2\theta_w)\gamma^\mu - \gamma^\mu\gamma^5 \right) t \, Z_\mu$$

?

- to photon: vectorlike, has charge 2/3
  - $e_t$ ~ 2/3

$$e_t \, \bar{t} \gamma^\mu t A_\mu$$

√?

- to gluon: vectorlike, non-trivial in color
  - $g_s$ ~ 1.12

$$g_s \left[ T_a^{\mathrm{SU}(3)} \right]^{ji} \bar{t}_j \gamma_\mu t_i A^a_\mu$$

√

- to Higgs: Yukawa type
  - $y_t$ ~ 1

$$y_t \, h \bar{t} t$$

√?

Top physics: check structure and strength of all these couplings

# tt+W,Z,γ

- Photon

Kardos, Trocsanyi

  - NLO + PS calculation
  - dominated by gluon fusion
  - Control sample/background for ttH, H⟶γγ
- Z

Garzelli, Kardos,
Papadopoulos,
Trocsanyi

  - NLO + PS calculation
  - not yet “seen”
- W
  - NLO + PS calculation
  - ttW at LHC has little sensitivity to tWb coupling
    - ✓ Use single top production here

Garzelli, Kardos,
Papadopoulos,
Trocsanyi

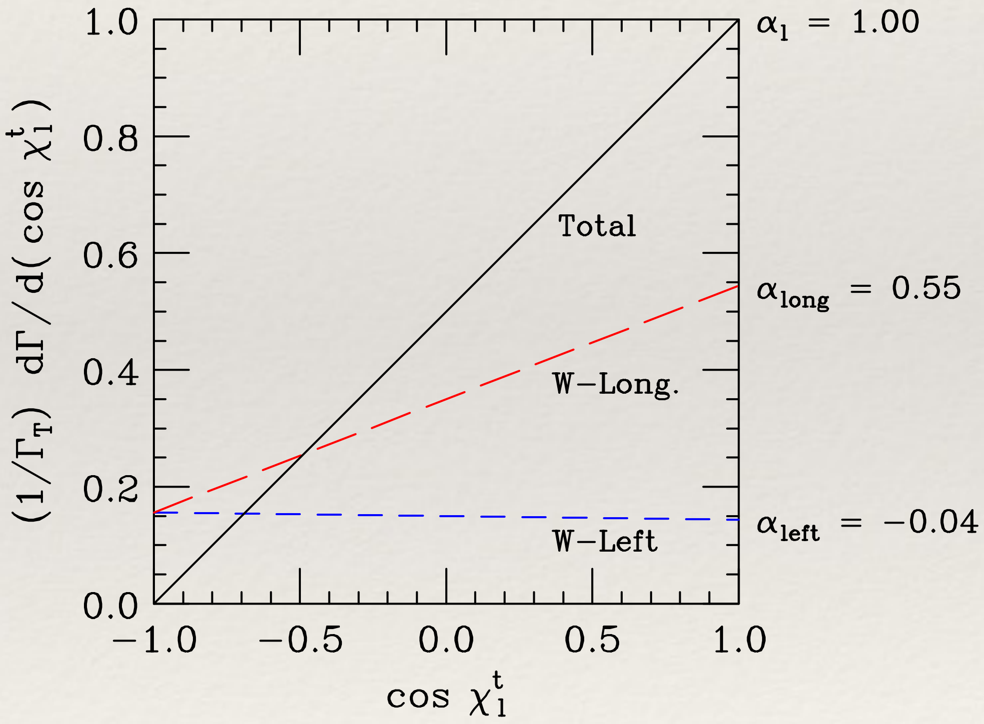
# Top self-analyzes its spin

- 100% correlation of charged lepton with top spin
    - Top self-analyzes its spin
    - Charged leptons easy to measure
- For spin-up top the polar angle distribution is

$$\frac{1}{\Gamma_T}\frac{d\Gamma_{(\uparrow)}}{d(\cos\theta_{e^+})} = \frac{1}{2}(1+\cos\theta_{e^+})$$

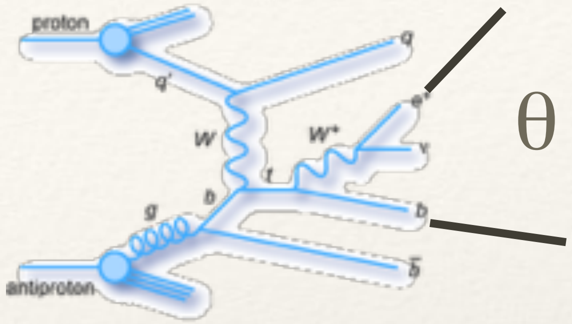
- Due to chiral structure of tWb coupling

$$\frac{d\ln\Gamma_f}{d\cos\chi_f} = \frac{1}{2}(1+\alpha_f\cos\chi_f)$$

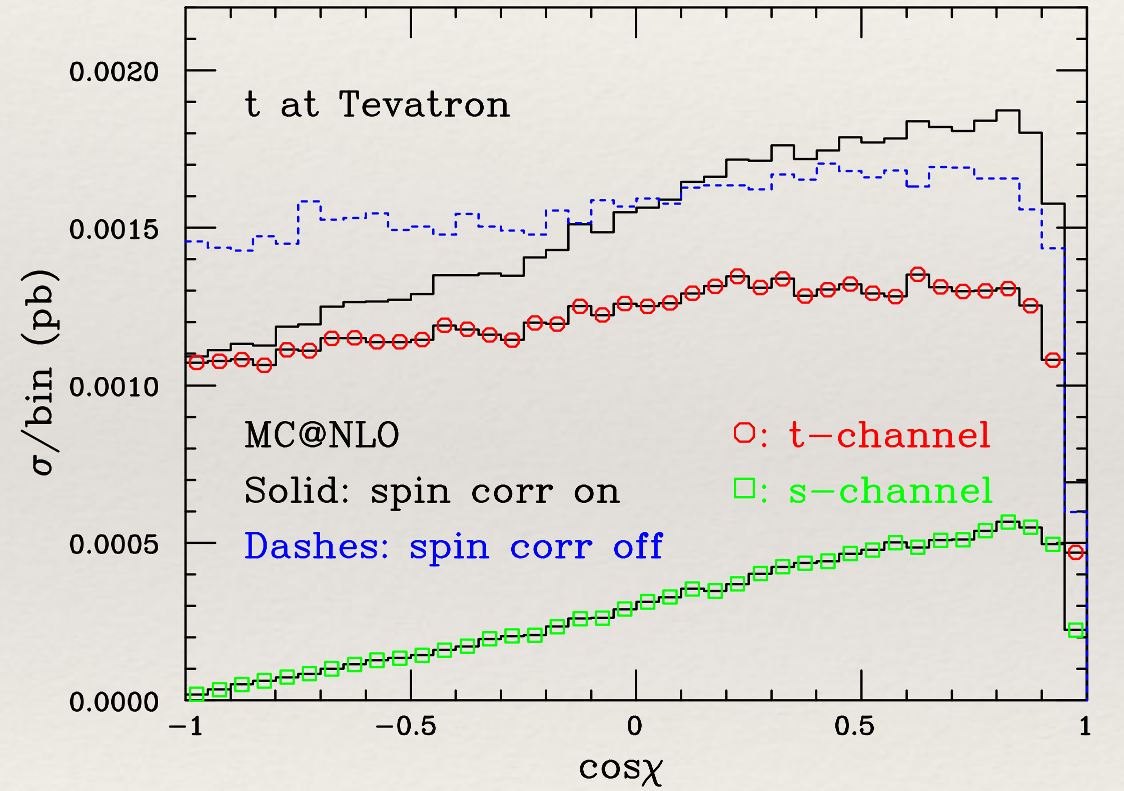


# Spin correlations for single top in MC@NLO

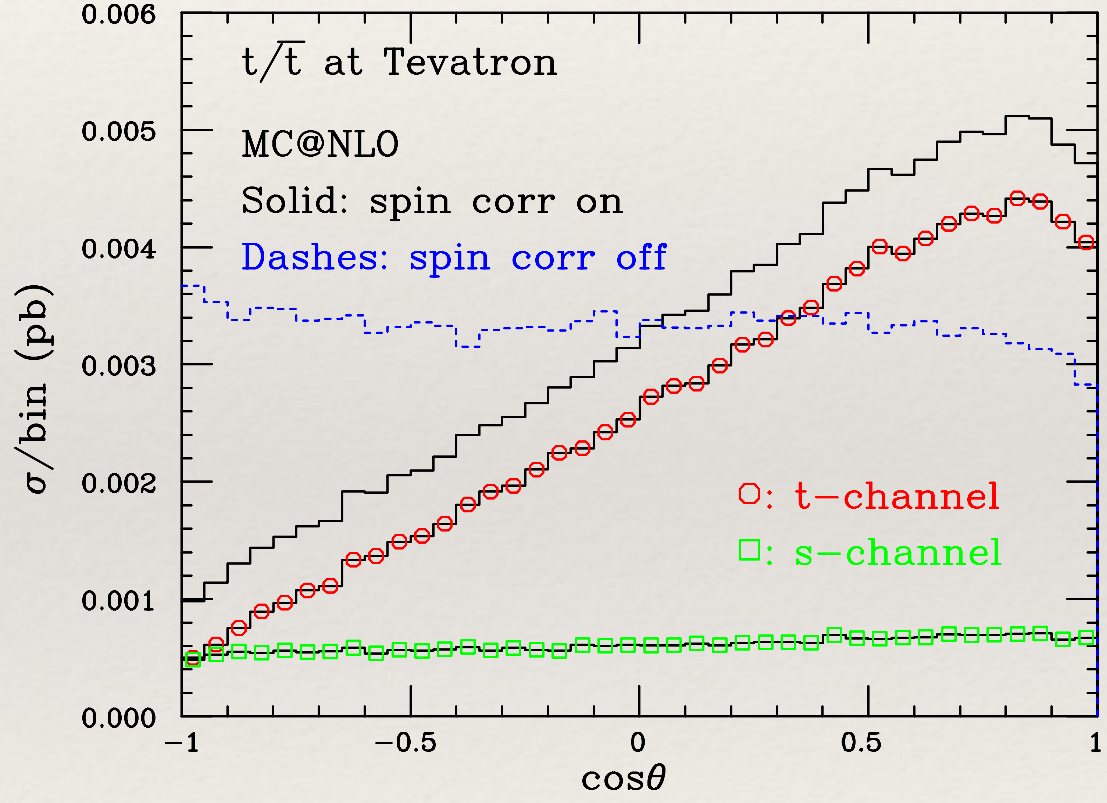
, Motylinski, Webber



- Top is produced polarized by EW interaction
  - 100% correlation between top spin and charged lepton direction
- Angle of lepton with appropriate axis is different per channel
- Method included “a posteriori”. Also used in POWHEG Aioli, Nason, Oleari, Re



Beam direction



Hardest, non-b jet

Robust correlation in NLO event generation
(Method to infer inclusive cross section?)

# ttH

- Should become very interesting for the new run
- $\sigma_{tth}\,(14) \simeq 4.6 \times \sigma_{tth}\,(8)$
- NLO calculations for signal

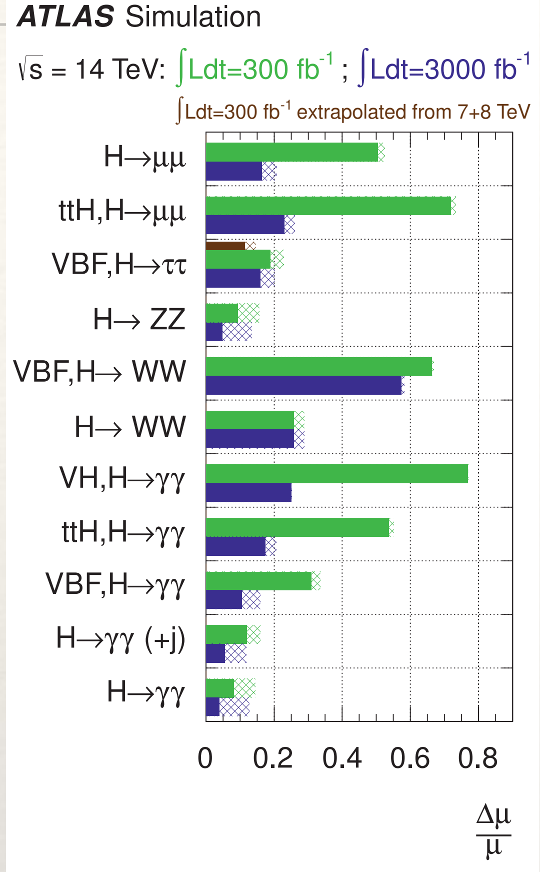
  Beenakker, Dittmaier, Kraemer, Plumper, Spira, Zerwas
  Dawson, Orr, Reina, Wackeroth

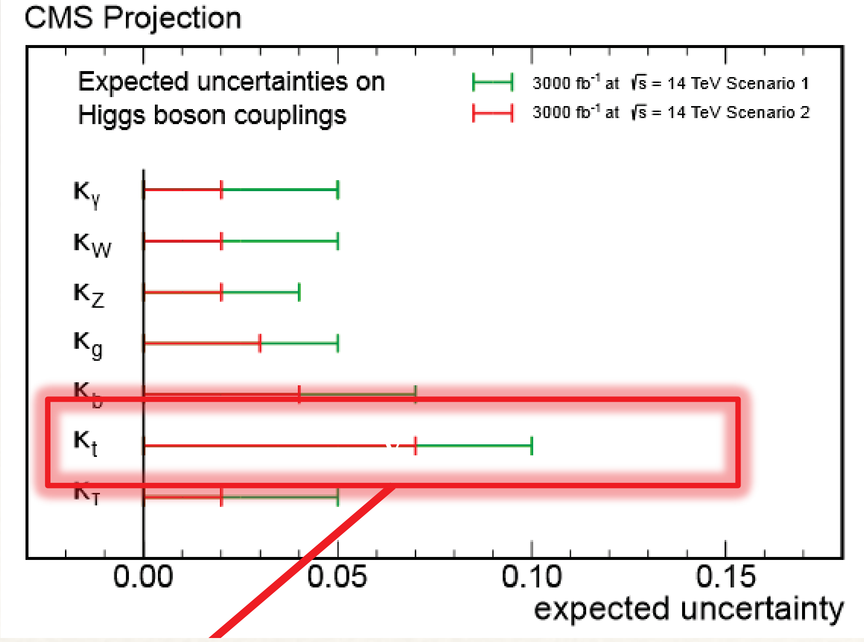
  - plus PS

    Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrieli
    Garzelli, Kardos, Papadopoulos, Trocsanyi
    Hartanto, Jaeger, Reina, Wackeroth

    - ✓ and spin correlations
  - plus EW

    Frixione, Hirschi, Pagani, Shao, Zaro
- and e.g. ttbb backgrounds to NLO(+PS)





Backgrounds difficult, but expect
10% accuracy in Yukawa coupling by 2030

# Conclusions

- Major progress at N3LO and N4LO for mass definition and threshold scan.
- Steady progress on proxy masses, and understanding MC masses.
- Top mass may be the last, but not the easiest theoretical problem to solve
  - Goes to the heart of data-theory comparison
- With new LHC run, EW couplings of top will be tested
- Theory seems ready for Yukawa coupling tests