Electroweak corrections to top-pair production at lepton colliders

Emi Kou (LAL-Orsay)

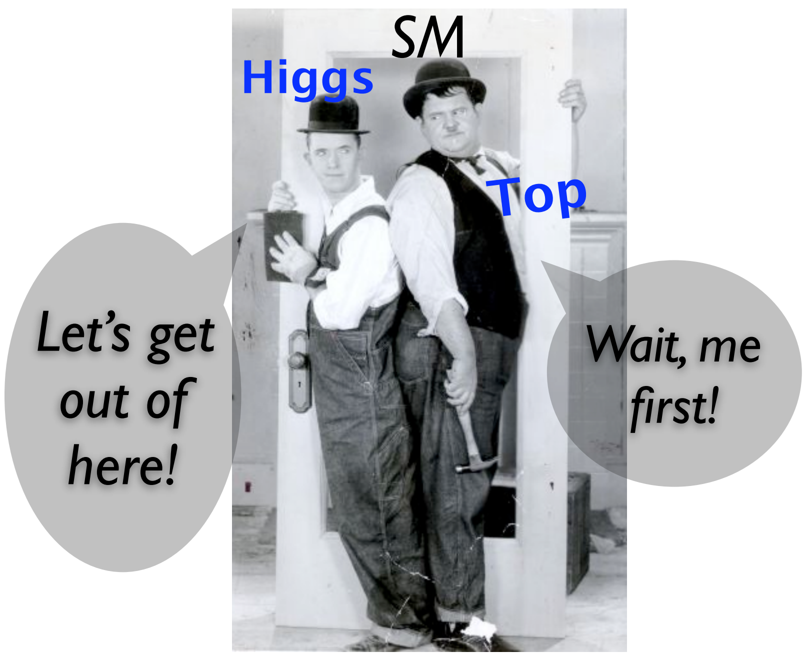
in collaboration with P.H. Khiem, Y. Kurihara and F. Le Diberder

based partially on arXive:1503.04247

LFC 15 @ ECT Trento, 7-11 September 2015*

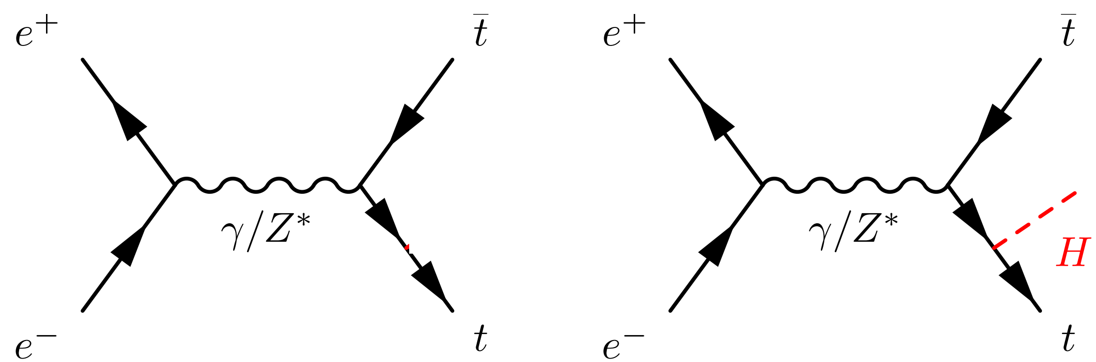
Top physics at ILC

Top being only fermion with mass close to electroweak scale make us think that it has a special role in the physics beyond the SM. Top physics at ILC may open an unique window for discovery of new physics.

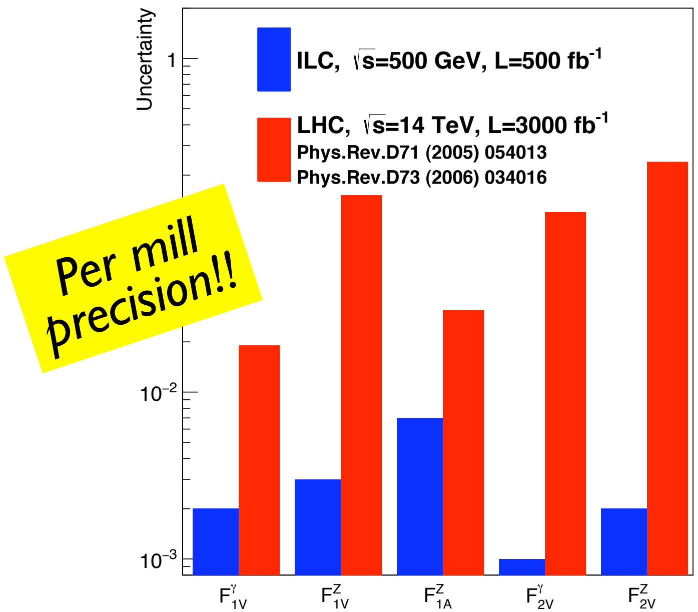


Top physics is one of the three pillars of ILC physics!

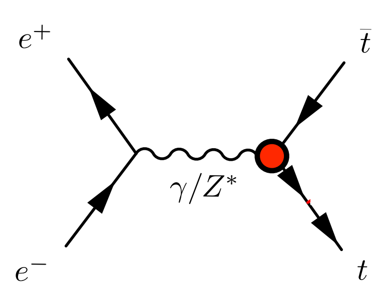
i) top mass measurement at the threshold
ii) $t\bar{t}$-Z/γ anomalous coupling measurement
iii) $t\bar{t}$-Higgs coupling measurement



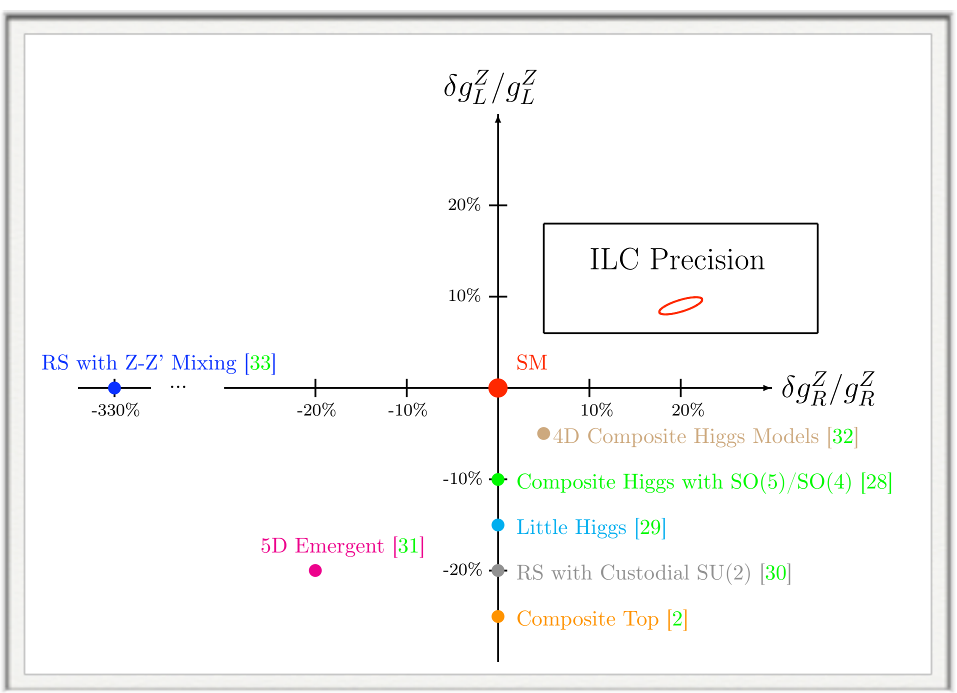
The $t\bar{t}$-Z/γ anomalous coupling

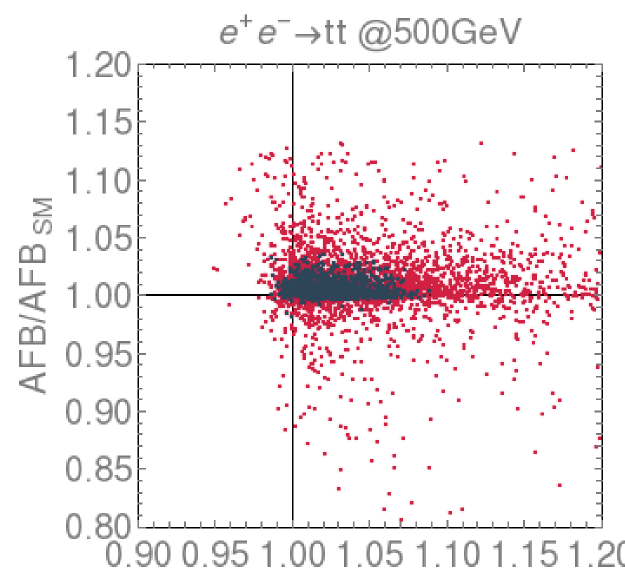


M.S. Amjad et al.
1505.06020
see talk by R. Poeschl



- The $t\bar{t}$-Z/γ anomalous coupling is one of the important probe of new physics. Many new physics models predict significant deviation from SM.
- The latest evaluation shows that the form factors can be measured at a per mill precision!

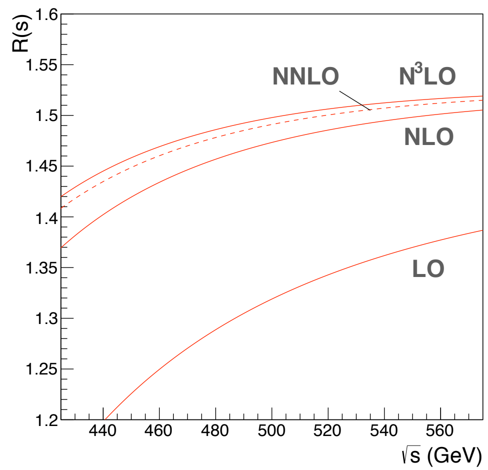




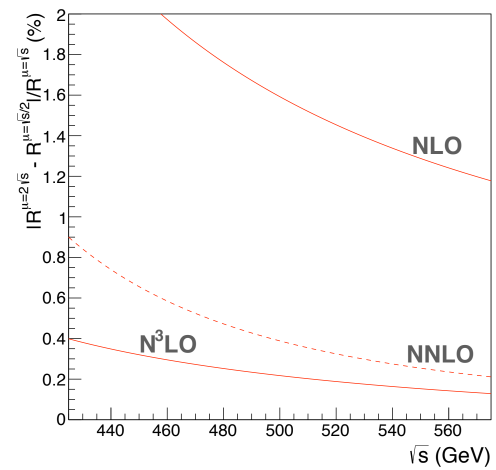
D. Barducci et al.
1504.05407
see talk by D. Barducci

SM computation of e+e- → $t\bar{t}$

- ✳QCD corrections are known up to N^3LO



(a) Perturbation series

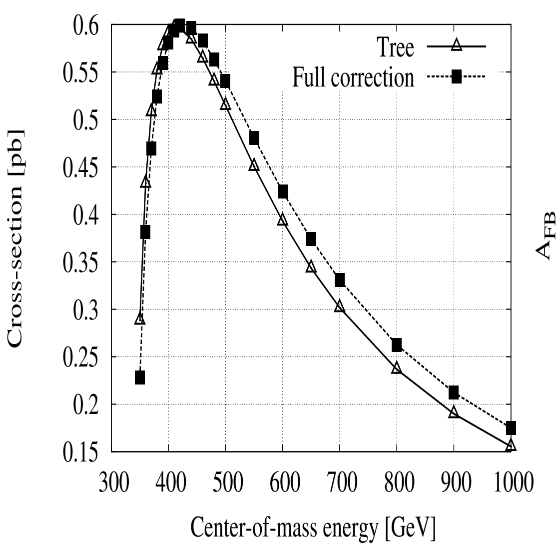


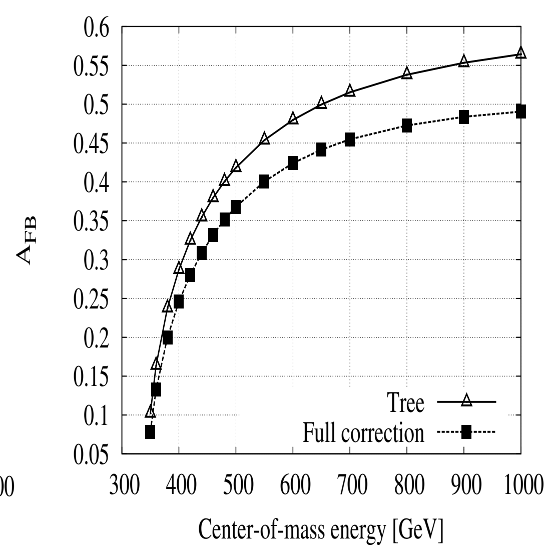
(b) Scale variations

QCD correction (N^3LO) is at the per mil level

Kiyo, Maier, Maierhofer, Marquard, NCP B823 ('09)
Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber. NPB750 ('06)
Hoang, Mateu, Zebarjad, NPB813 ('09)

- ✳Electroweak corrections are known at one-loop level





EW correction at one-loop is ~5% for cross section ~10% for A_{FB}

Fleischer, Leike, Riemann, Werthenbach, EJPC31 ('03)
Kheim, Fujimoto, Ishikawa, Kaneko, Kato, arXive:1211.1112

SM computation of e+e- → $t\bar{t}$

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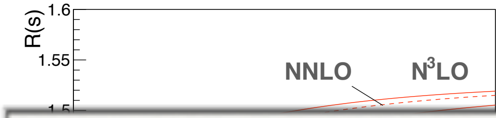
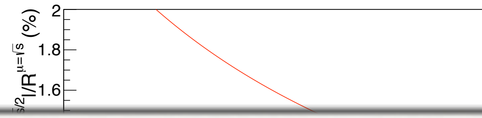
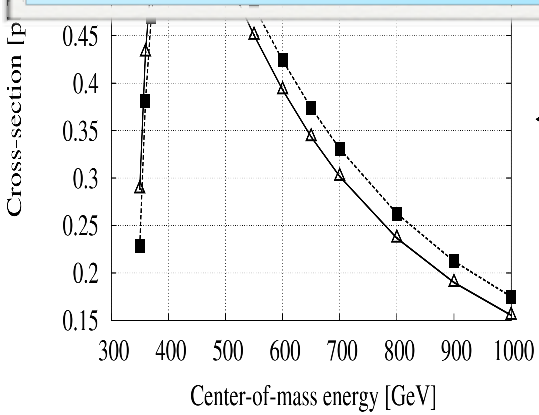


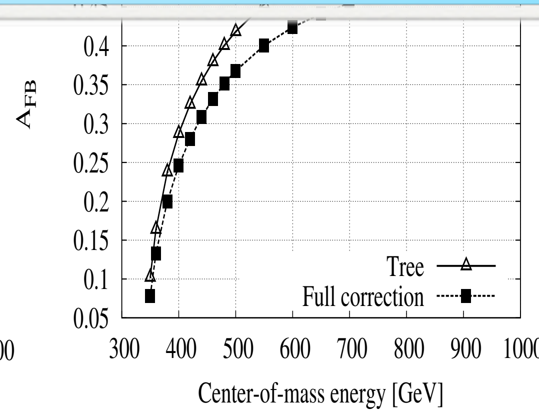


QCD correction (N³LO) is

23 ('09)
sch,
9)

- ✳ *It has been known since long that the NLO EW correction to the $t\bar{t}$ production is quite sizable.*
- ✳ *ILC can (easily) access to these EW corrections and further perform a precision measurement of the SM couplings.*
- ✳ *To search for NP effect, we have to make the EW correction* ***under control*** *(for now, nobody knows how to compute the NNLO EW correction!)*





is

~10% for A_{FB}

Fleischer, Leike, Riemann, Werthenbach, EJPC31 ('03)
Kheim, Fujimoto, Ishikawa, Kaneko, Kato,
arXive:1211.1112

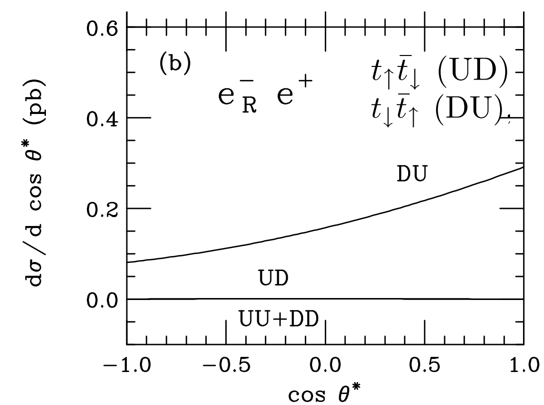
Spin correlation as a tool for precision

Parke and Shadmi PLB387 ('96)

Top production and decays are very different form the other fermions. Many (interesting) angular correlations emerge, which can be used to extract various information.

Polarized beam useful!

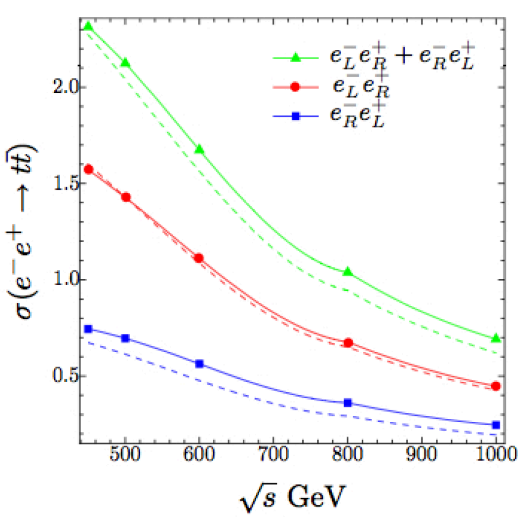
ILC pol. beam option: $(e^-, e^+) = (\pm 0.8, \mp 0.3)$

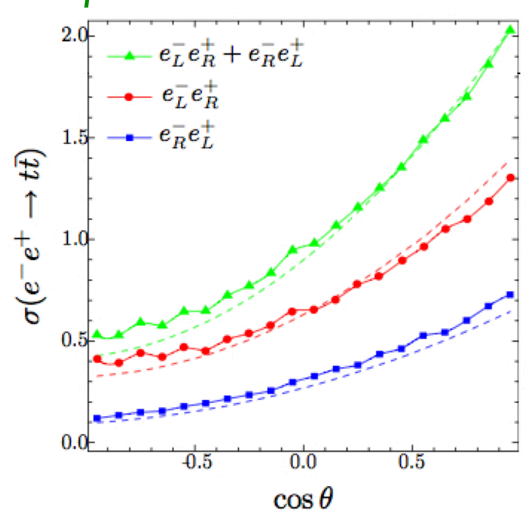


Motivation

Can we use the angular correlation to control NLO EW contributions?

GRACE e+e- ---> ttbar with initial polarization: now available





For example, we have confirmed that the size of the NLO correction depend strongly on the initial state polarization.

Khiem, E.K. Kurihara, Le Diberder arXiv:1503:04247

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e and Shadmi
B387 ('96)

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ILC pol. beam option:
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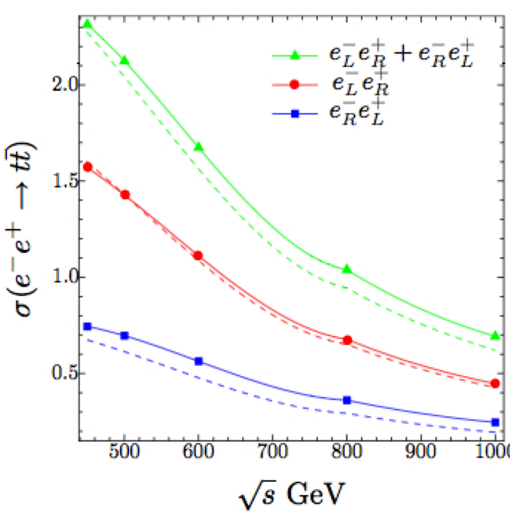
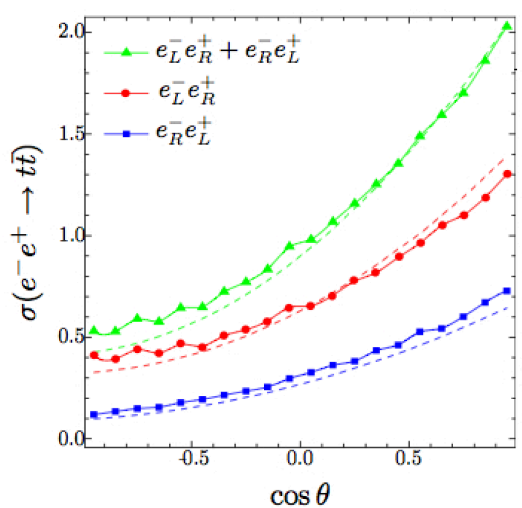
- *In this talk, we introduce a method we developed to take a full use of the angular correlation to study the $t\bar{t}$ production at ILC.*
- *We show our study at the LO, including a few systematic effects (6 fermion background, top and W width effect etc).*
- *Then, we show our preliminary results on the NLO study.*





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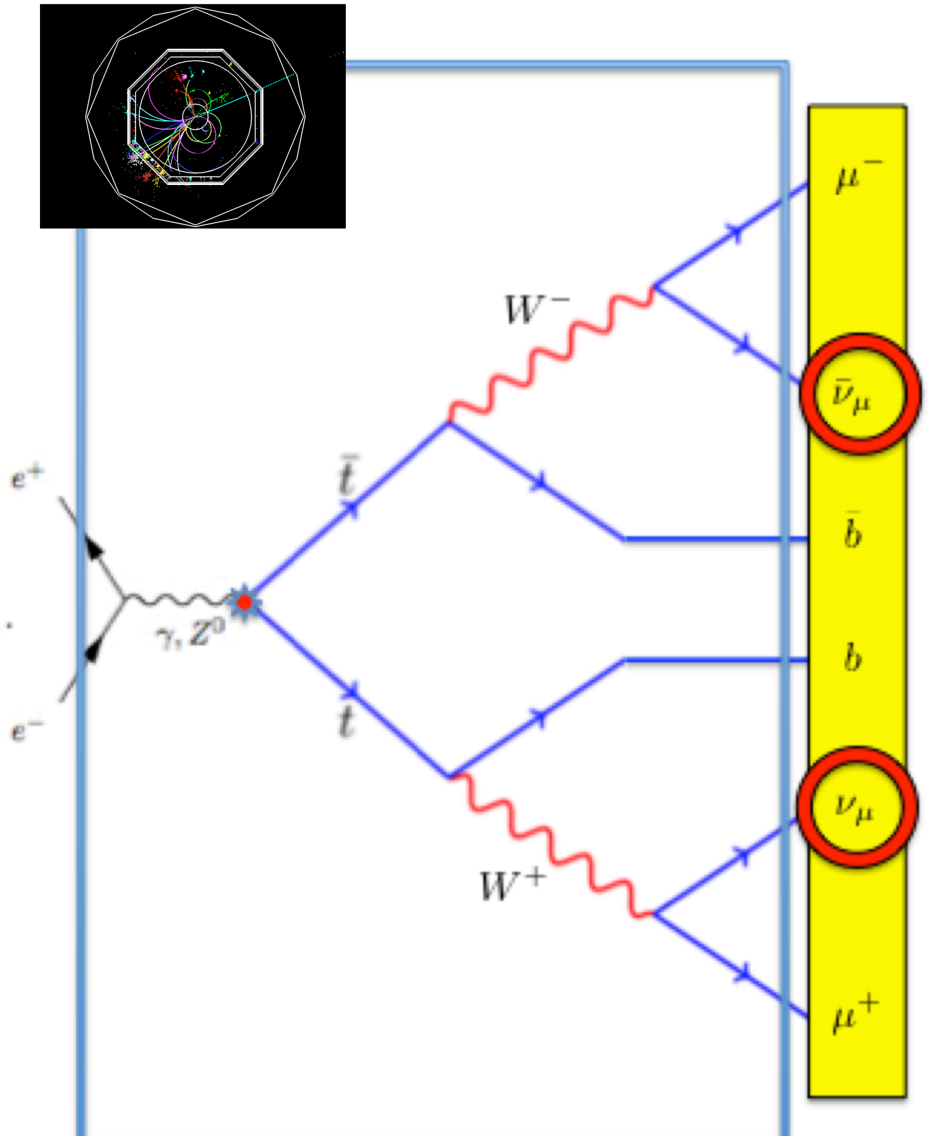
Khiem, E.K. Kurihara, Le Diberder
arXiv:1503:04247

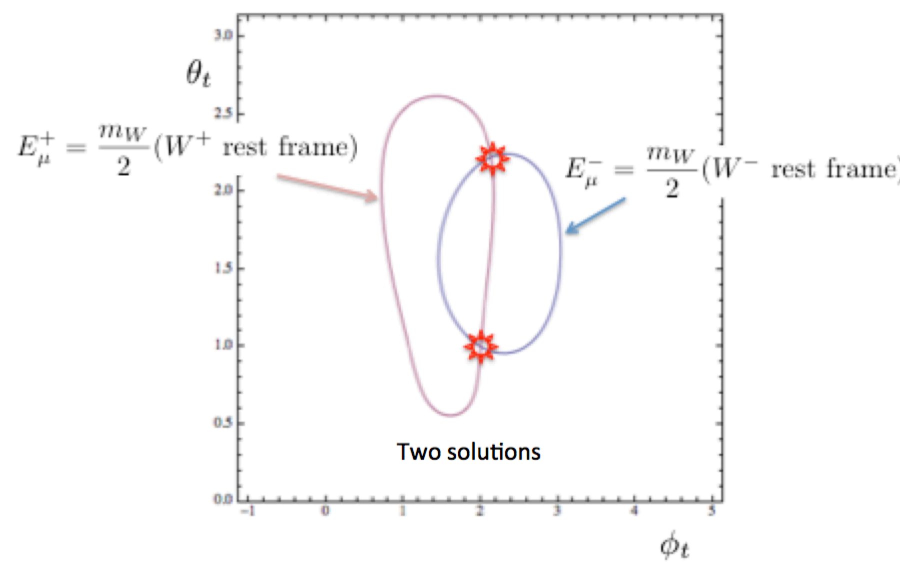
Full kinematical reconstruction using pure leptonic final states



To have a full kinematical information, we try to use the fully leptonic final state:

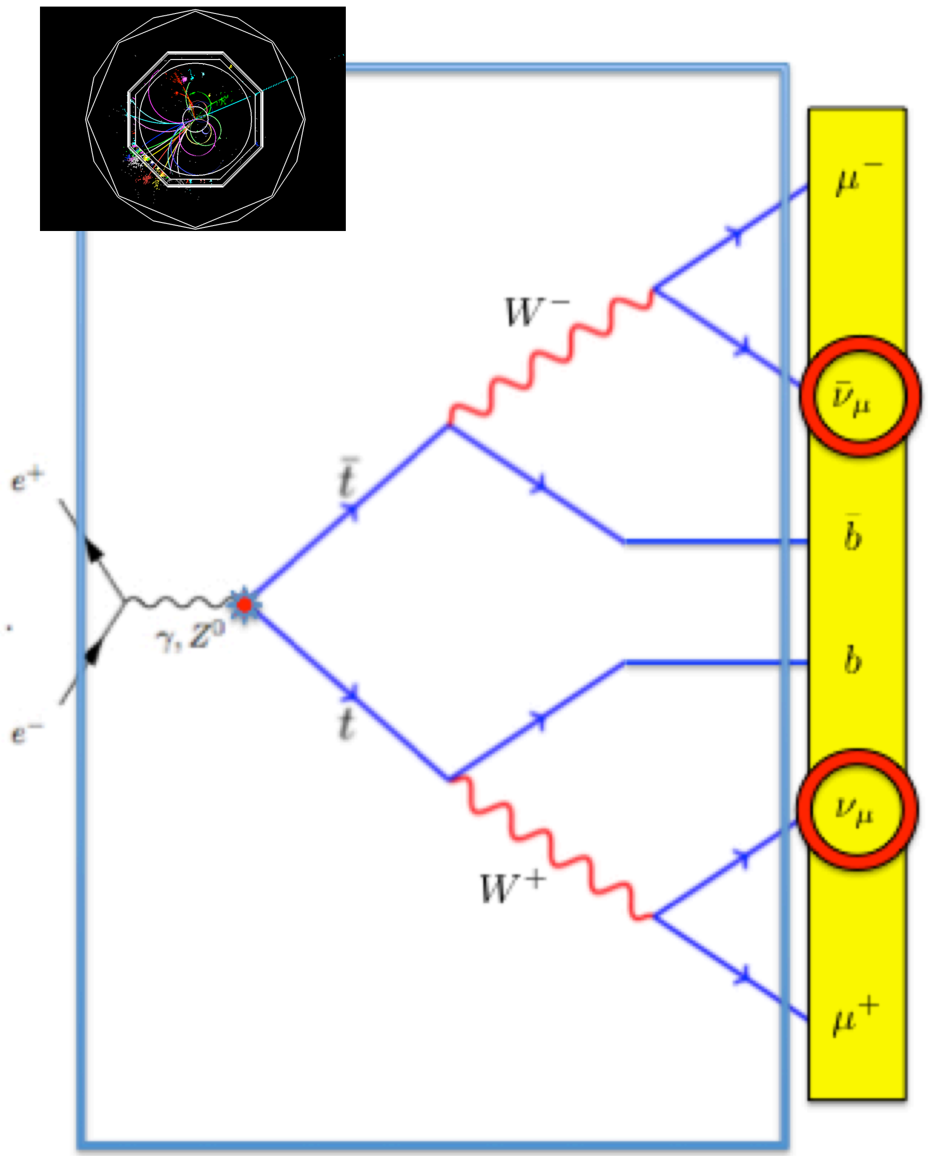
Angles for mu+, mu- and b, anti-b (7 angles)
Energies of mu+ and mu-
Energies of b and anti-b

Reconstruction of top polar/azimuthal angles



Without bottom energy information, we are left with ambiguities of two solutions.

Full kinematical reconstruction using pure leptonic final states



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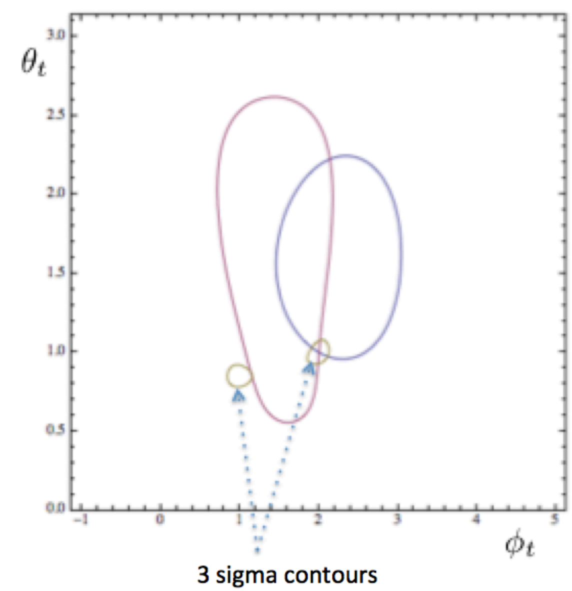
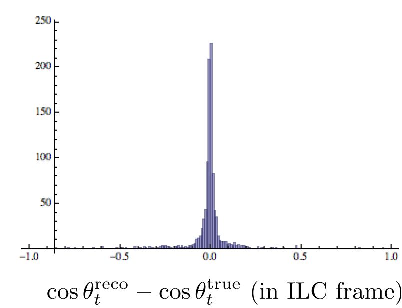


Yes, we can!



Rough idea of bottom energy is enough to distinguish two solutions!

Helicity amplitudes

$$e^-(\lambda_{e^-})e^+(\lambda_{e^+}) \longrightarrow t(\lambda_t)\bar{t}(\lambda_{\bar{t}})$$

$$t \to W^+ b \qquad |s_t, \lambda_t\rangle \to |s_{W^+}, \lambda_{W^+}; s_b, \lambda_b\rangle$$

$$\hookrightarrow l^+\nu \qquad |s_{W^+}, \lambda_{W^+}\rangle \to |s_{l^+}, \lambda_{l^+}; s_\nu, \lambda_\nu\rangle$$

$$\bar{t} \to W^- \bar{b} \qquad |s_{\bar{t}}, \lambda_{\bar{t}}\rangle \to |s_{W^-}, \lambda_{W^-}; s_{\bar{b}}, \lambda_{\bar{b}}\rangle$$

$$\hookrightarrow l^-\bar{\nu} \qquad |s_{W^-}, \lambda_{W^-}\rangle \to |s_{l^-}, \lambda_{l^-}; s_{\bar{\nu}}, \lambda_{\bar{\nu}}\rangle$$

$$\mathcal{M}^{\lambda_{e^-}\lambda_{e^+}} = \sum_{\lambda_t\lambda_{\bar{t}}\lambda_{W^+}\lambda_{W^-}} \mathcal{M}^{\lambda_{e^-}\lambda_{e^+}}_{\lambda_t\lambda_{\bar{t}}} \mathcal{M}^{\lambda_t}_{\lambda_{W^+}\lambda_b} \mathcal{M}^{\lambda_{\bar{t}}}_{\lambda_{W^-}\lambda_{\bar{b}}} \mathcal{M}^{\lambda_{W^+}}_{\lambda_{l^+}\lambda_\nu} \mathcal{M}^{\lambda_{W^-}}_{\lambda_{l^-}\lambda_{\bar{\nu}}}$$

Let us assume J=1

ttbar production

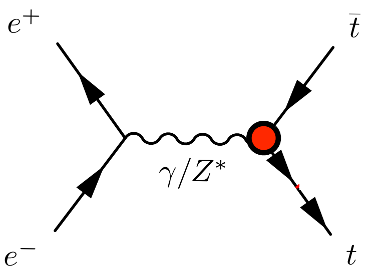
$$\mathcal{M}^{\lambda_{e^-}\lambda_{e^+}}_{\lambda_t\lambda_{\bar{t}}} = 4\pi\sqrt{\frac{s}{p_i p_f}} \sum_J (2J+1) e^{i(\lambda_i - \lambda_f)\phi_t} d^J_{\lambda_i\lambda_f}(\theta_t) T^{\lambda_{e^-}\lambda_{e^+}}_{\lambda_t\lambda_{\bar{t}}}$$

top decay

$$\mathcal{M}^{\lambda_t}_{\lambda_{W^+}\lambda_b} = \sum_{J,M} \left(\frac{2J+1}{4\pi}\right)^{1/2} e^{i(M-\lambda)\phi^*_{W^+}} d^J_{M\lambda}(\theta^*_{W^+}) A^M_{\lambda_{W^+}\lambda_b}$$

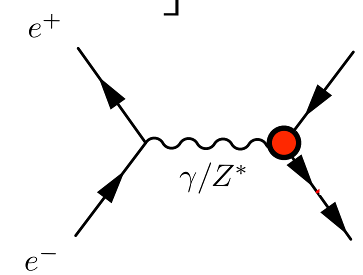
W decay

$$\mathcal{M}^{\lambda_{W^+}}_{\lambda_{l^+}\lambda_\nu} = \sum_{J,M} \left(\frac{2J+1}{4\pi}\right)^{1/2} e^{i(M-\lambda)\phi^{**}_{e^+}} d^J_{M\lambda}(\theta^{**}_{l^+}) A^M_{\lambda_{l^+}\lambda_\nu}$$



Lagrangian for $t\bar{t}$ production

$$\mathcal{L}_{\rm int} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t}\gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t} \sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$



$$\begin{aligned}
\mathcal{M}(e_L\bar{e}_R \to t_L\bar{t}_R)^{\gamma/Z} &= c_L^{\gamma/Z}[F_{1V}^{\gamma/Z} - \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}](1+\cos\theta)e^{-i\phi} \\
\mathcal{M}(e_L\bar{e}_R \to t_R\bar{t}_L)^{\gamma/Z} &= c_L^{\gamma/Z}[F_{1V}^{\gamma/Z} + \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}](1-\cos\theta)e^{-i\phi} \\
\mathcal{M}(e_L\bar{e}_R \to t_L\bar{t}_L)^{\gamma/Z} &= c_L^{\gamma/Z}\gamma^{-1}[F_{1V}^{\gamma/Z} + \gamma^2(F_{2V}^{\gamma/Z} + \beta F_{2A}^{\gamma/Z})]\sin\theta e^{-i\phi} \\
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\mathcal{M}(e_R\bar{e}_L \to t_L\bar{t}_R)^{\gamma/Z} &= -c_R^{\gamma/Z}[F_{1V}^{\gamma/Z} - \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}](1-\cos\theta)e^{i\phi} \\
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\end{aligned}$$

where $\beta^2 = 1 - 4m_t^2/s, \gamma = \sqrt{s}/(2m_t)$ and the overall factors $c_{L/R}^{\gamma/Z}$ are:

$$c_L^\gamma = -1, \quad c_R^\gamma = -1, \quad c_L^Z = \left(\frac{-1/2 + s_w^2}{s_w c_s}\right)\left(\frac{s}{s - m_Z^2}\right), \quad c_R^Z = \left(\frac{s_w^2}{s_w c_s}\right)\left(\frac{s}{s - m_Z^2}\right)$$

where $s_w = \sin\theta_w$ and $c_w = \cos\theta_W$, with θ_W being the weak mixing angle.

$$F_{1V}^{\gamma,SM} = -\frac{2}{3},\ F_{1A}^{\gamma,SM} = 0,\ F_{1V}^{Z,SM} = -\frac{1}{4s_w c_w}\left(1 - \frac{8}{3}s_w^2\right),\ F_{1A}^{Z,SM} = \frac{1}{4s_w c_w}, \quad F_{2V}^\gamma = Q_t(g-2)/2$$

Lagrangian for $t\bar{t}$ production

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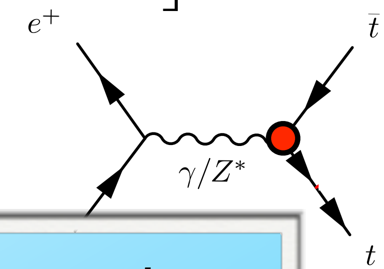

$$\mathcal{M}(e_L \bar{e}_R \to t_L \bar{t}_R)^{\gamma/Z} = c_L^{\gamma/Z}[F_{1V}^{\gamma/Z} - \beta F_{1A}^{\gamma/Z} + F_{2V}^{\gamma/Z}](1+\cos\theta)e^{-i\phi}$$

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- ✳ *Using distribution of the events in the 9 dimensional phase space, we fit 10 form factors (Re[$F^V{}_{1V}$, $F^V{}_{1A}$, $F^V{}_{2A}$, $F^V{}_{2A}$] and Im[$F^V{}_{2A}$] where V=γ,Z) !*
- ✳ *First, we test it with SM at LO.*

$$\mathcal{M}(e_R \bar{e}_L \to t_R \bar{t}_R)^{\gamma/Z} = c_R^{\gamma/Z}\gamma^{-1}[F_{1V}^{\gamma/Z} + \gamma^2(F_{2V}^{\gamma/Z} - \beta F_{2A}^{\gamma/Z})]\sin\theta e^{i\phi}$$

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Numerical analysis: matrix element method

We apply likelihood method to obtain our numerical result:

full matrix elements

$$\chi^2(\alpha_i) = -2\left(\sum_{e=1}^{N_{exp}} \ln|\mathcal{M}(\alpha_i)|^2 - N_{th}(\alpha_i)\right)$$

$\mathcal{M}(\alpha_i)$ is the full matrix element and α_i is the parameter (i.e. form factors) and α_0 is the normalization point, e.g. the SM LO value.

Covariance matrix

$$V_{ij}^{(-1)} = N\ \langle\omega_i\omega_j\rangle_0$$

$$N = N(\alpha_0)$$

$$\omega_i = \left.\frac{\partial \mid \mathcal{M} \mid^2 (\alpha)}{\partial\alpha_i}\right|_{\alpha^0} \frac{1}{\mid \mathcal{M} \mid^2 (\alpha^0)}$$

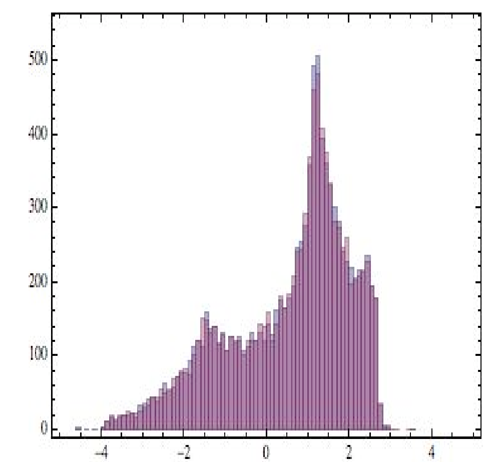


Figure 1: Distributions of the true and reconstructed values of ω_r, superimposed. The two distributions are barely distinguishable.

Numerical analysis: SM LO result

Statistical uncertainties and correlation with the SM LO as normalization

Kheim, E.K. Kurihara, Le Diberder: arXiv:1503:04247

$\mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma}$	$\mathcal{R}e\ \delta\tilde{F}_{1V}^{Z}$	$\mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma}$	$\mathcal{R}e\ \delta\tilde{F}_{1A}^{Z}$	$\mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma}$	$\mathcal{R}e\ \delta\tilde{F}_{2V}^{Z}$	$\mathcal{R}e\ \delta\tilde{F}_{2A}^{\gamma}$	$\mathcal{R}e\ \delta\tilde{F}_{2A}^{Z}$	$\mathcal{I}m\ \delta\tilde{F}_{2A}^{\gamma}$	$\mathcal{I}m\ \delta\tilde{F}_{2A}^{Z}$
0.0037	−0.18	−0.09	+0.14	+0.62	−0.15	0	0	0	0
	0.0063	+.14	−0.06	−0.13	+0.61	0	0	0	0
		0.0053	−0.15	−0.05	+0.09	0	0	0	0
			0.0083	+0.06	−0.04	0	0	0	0
				0.0105	−0.19	0	0	0	0
					0.0169	0	0	0	0
						0.0068	−0.15	0	0
							0.0118	0	0
								0.0069	−0.17
									0.0100

500 GeV&500 fb^{-1} Polarization 50/50 between ±80% and ±30%

- *Our result (at tree level) shows that the 10 form factors can be fitted simultaneously at less than a percent precision !*
- *Indeed, the matrix element method (with full kinematical reconstruction) is a powerful tool!*

Estimating the errors from various sources

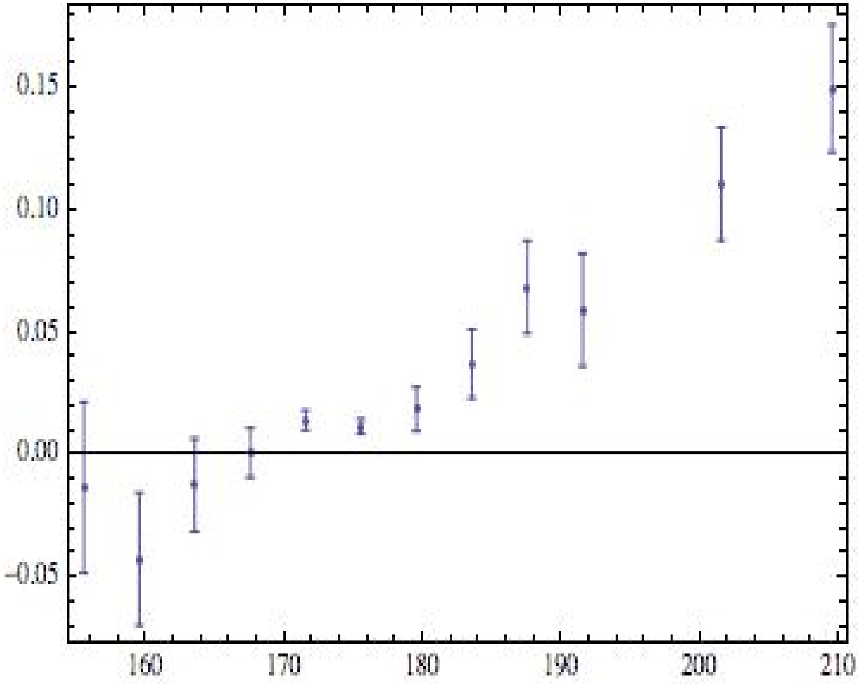
E.K. Kurihara, Le Diberder in preparation

- In real experimental analysis, we have extra errors coming from various sources, such as
  - ✓ The top and W widths effect
  - ✓ The wrong B assignment effect
  - ✓ The irreducible 6 fermion background effect
- By using the GRACE event generator, we estimate these systematic errors as bias by using the same numerical analysis method.
- These bias can be subtracted from the LO analysis of experimental data to correct this *naive* modeling.
- The bias from the top and W widths as well as from the wrong B assignment are found to be at per mill level.
- In the following, we focus on the irreducible background which comes from the 6 fermion final states ($\mu^+\mu^-\nu\bar{\nu}$ *$b\bar{b}$ with all possible intermediated states*).

Estimating the errors from 6 fermions

E.K. Kurihara, Le Diberder in preparation

- We analyze the events generated by GRACE which include all possible intermediate state (not only $t\bar{t}$) going to 6 fermions as before, i.e. as LO events. Then, compute the bias.
- For illustration, we first perform one dimensional fit for Re[F^Z_{2V}].
- The bias is obtained as 0.013±0.002 with 10^5 events. Thus, we observe one percent level 6 fermion background contributions at 6 sigma. For 10 parameter fit, we found that bias is slightly washed out.



The plot shows the bias on Re[F^Z_{2V}] caused by the non $t\bar{t}$ 6 fermion final states. This is computed with 100 % efficiency for kinematical reconstruction. We see that the 6 fermion contributions increases as the invariant mass of the top-to-be (μνb) increase.

First NLO results

E.K. Kurihara, Le Diberder in preparation

- First of all, we use GRACE event at NLO in EW which includes (both are narrow width approximation)
  - NLO $t\bar{t}$ production x LO top decay
  - LO $t\bar{t}$ production x NLO top decay
- We analyze GRACE $t\bar{t}$ events including NLO contribution as LO $t\bar{t}$ events. Then, compute the bias. These bias can be subtracted from the LO analysis to correct this *naive* modeling.
- For illustration, we first perform one dimensional fit for Re[F^Z_{1A}].
- The bias is obtained as 0.10±0.01 with 10^4 events, which is 10 sigma of shift (preliminary). This confirms that indeed, the NLO contributions can reach to ten percent level.
- We are now working on the 10 parameter fit where we find large deviations from LO in many of the form factors. This show that it is impossible to neglect NLO contribution in our analysis.

Conclusions

- In this talk, we have shown the sensitivity study of the ILC measurement of the $t\bar{t}$-Z/γ anomalous coupling, which is a powerful observable to search for physics beyond the SM.
- We propose to use the full angular distribution information to study the $t\bar{t}$-Z/γ coupling more in detail, which can be done with the fully leptonic final states.
- Using the matrix element method, we have shown that, at first, using the SM LO event distribution in the 9 dimensional phase space, we can fit 10 form factors simultaneously at less than a percent level.
- We have estimated the various errors and found that the 6 fermion background leads to one percent level errors (without any optimization).
- We have shown our first analysis result with NLO GRACE events (production and decay). Our preliminary results show that the impact of the NLO contributions to the form factors might be too large to persue the LO analysis.