Measuring a_{μ}^{HLO} in the spacelike region

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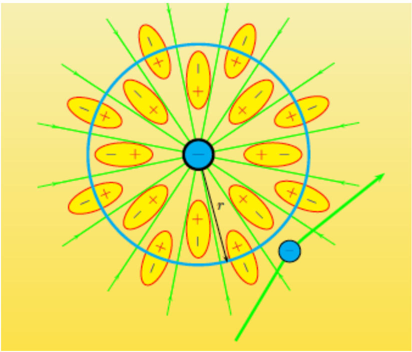
[*based on* Phys.Lett. B746 (2015) 325-32]

Castello di Trento ("Trint"), watercolour, 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495) British Museum, London

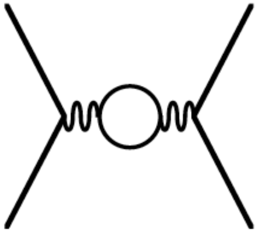
LFC15: Physics prospects for Linear and other Future Colliders after the discovery of the Higgs

Trento, September 7-11, 2015

α_{em} running and the Vacuum Polarization



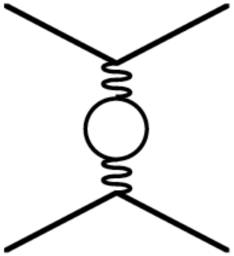
- Due to Vacuum Polarization effects $\alpha_{em}(q^2)$ is a running parameter from its value at vanishing momentum transfer to the effective q^2.



➢ The "Vacuum Polarization" function $\Pi(q^2)$ can be "absorbed" in a redefinition of an effective charge:

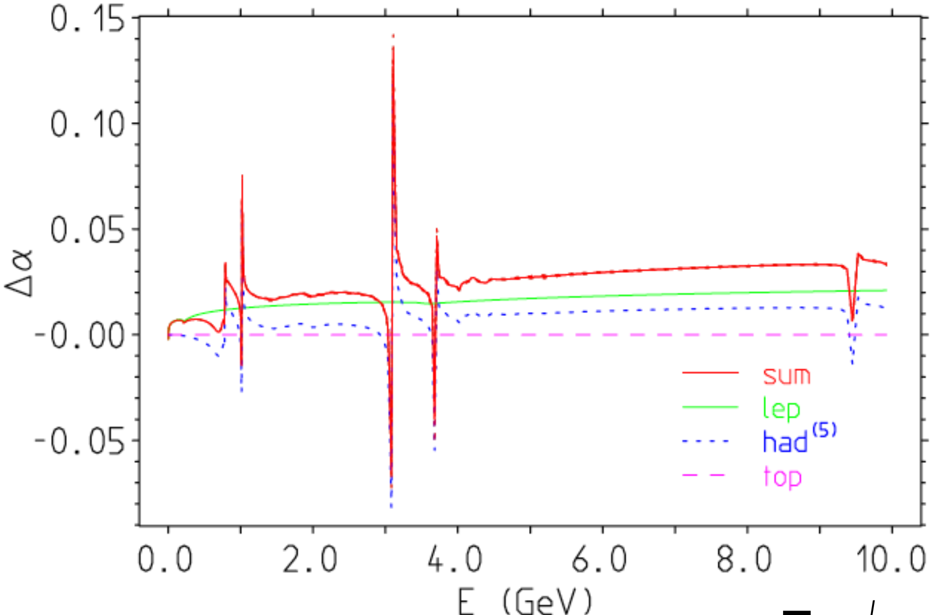
$$e^2 \rightarrow e^2(q^2) = \frac{e^2}{1+(\Pi(q^2)-\Pi(0))} \qquad \alpha(q^2) = \frac{\alpha(0)}{1-\Delta\alpha}; \quad \Delta\alpha = -\Re e\left(\Pi(q^2)-\Pi(0)\right)$$

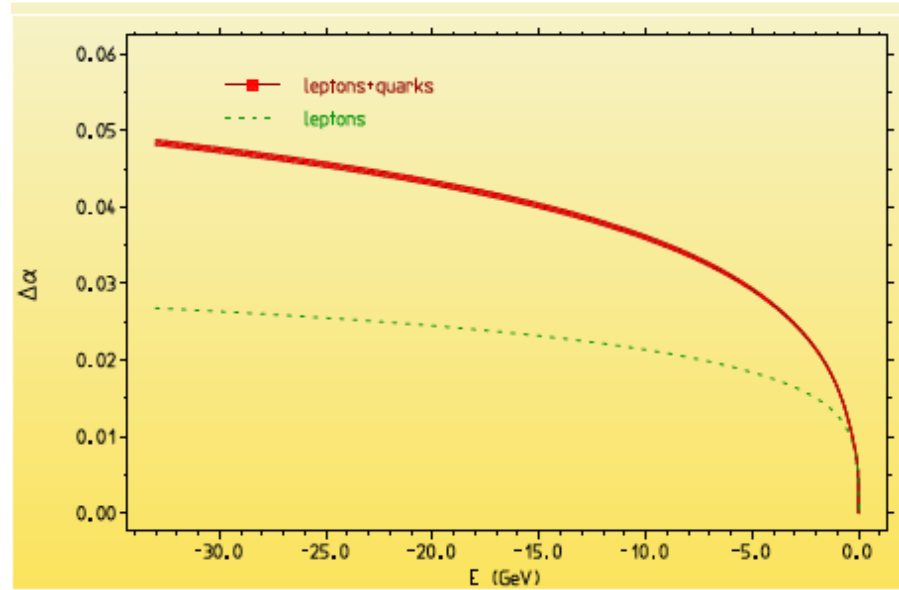
$$\Delta\alpha = \Delta\alpha_l + \Delta\alpha^{(5)}_{had} + \Delta\alpha_{top}$$



➢ $\Delta\alpha$ takes a contribution by non perturbative hadronic effects ($\Delta\alpha^{(5)}_{had}$) which exibits a different behaviour in time-like and spacelike region

Running of α_{em}



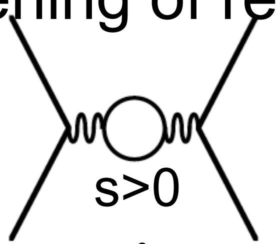


$E=\sqrt{s}$

$E=-\sqrt{-t}$

Time-like

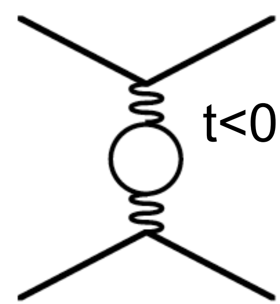
Behaviour characterized by the opening of resonances



$$\Delta\alpha^{(5)}_{had}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s-M_Z^2-i\varepsilon)}$$

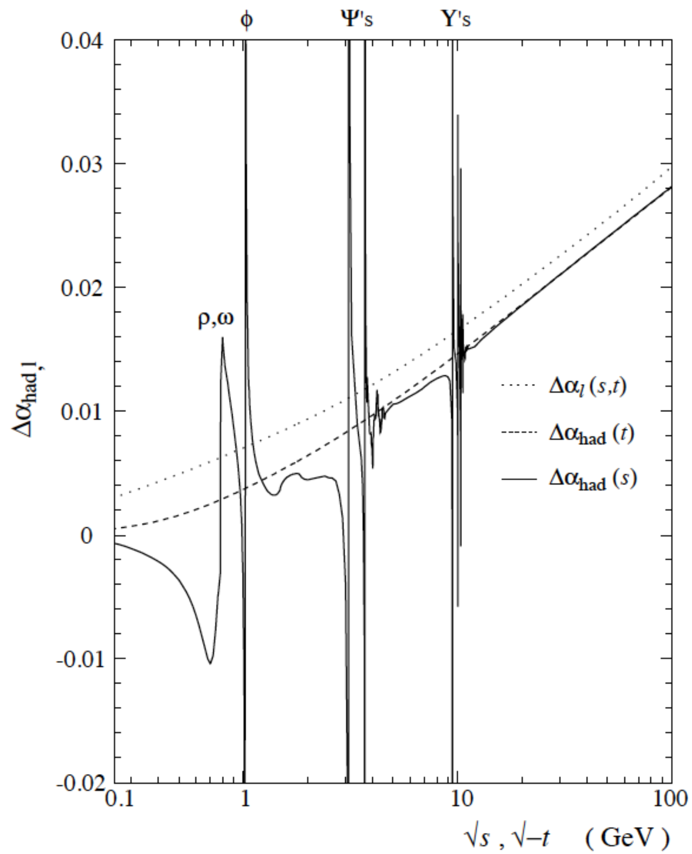
Space-like

Very smooth behaviour



Measurement of α_{em} running

- A direct measurement of $\alpha_{em}(q^2)$ in space/time like region can prove the running of α_{em}
- It can provide a test of "duality" (fare way from resonances)
- It has been done in past by few experiments at e^+e^- colliders by comparing a "well-known" QED process with some reference (obtained from data or MC)

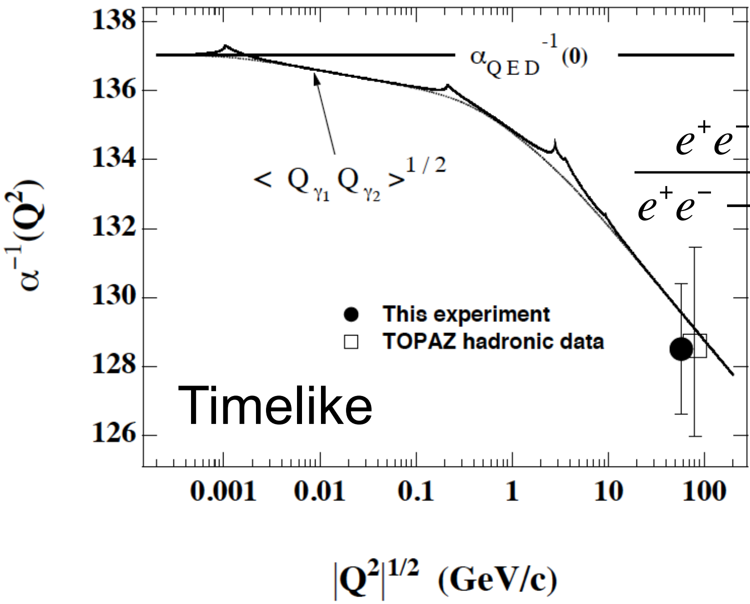


$$\left(\frac{\alpha(q^2)}{\alpha(q_0^2)}\right)^2 \sim \frac{N_{signal}(q^2)}{N_{norm}(q_0^2)}$$

N_{signal} can be Bhabha process, muon pairs, etc…
N_{signal} can be Bhabha process, $\gamma\gamma$ pairs, Theory, etc…

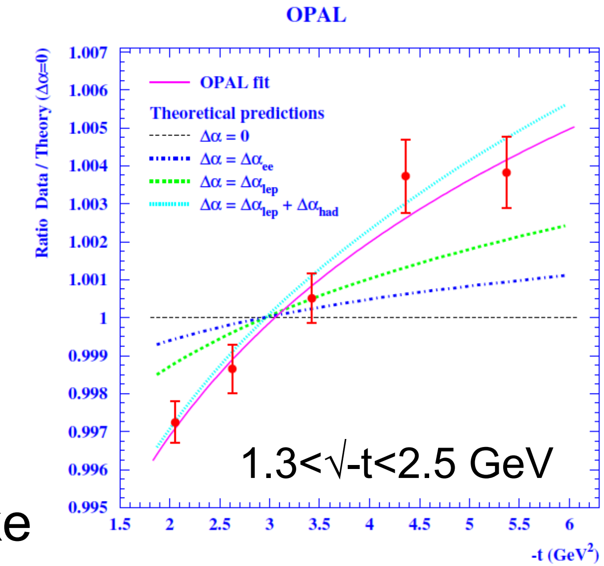
Measurement of α_{em} running

e+e- collider TRISTAN at √s=57.8 GeV,

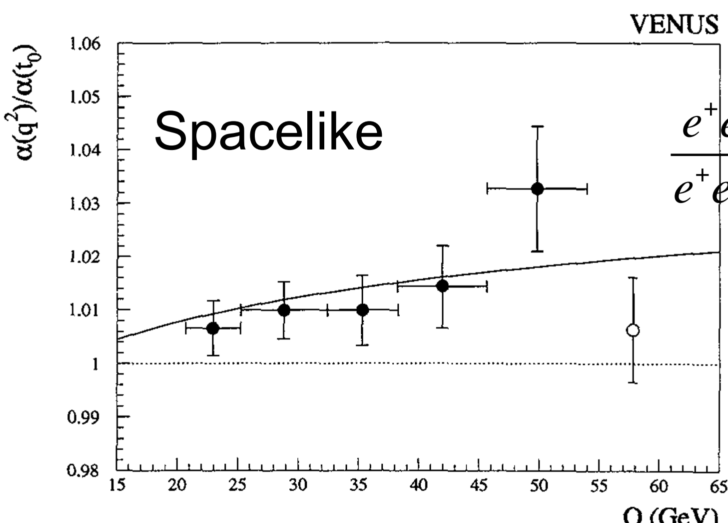


$$\frac{e^+e^- \to \mu^+\mu^-}{e^+e^- \to e^+e^-\mu^+\mu^-}$$

e+e- collider LEP at √s=189 GeV, using **Bhabha** events

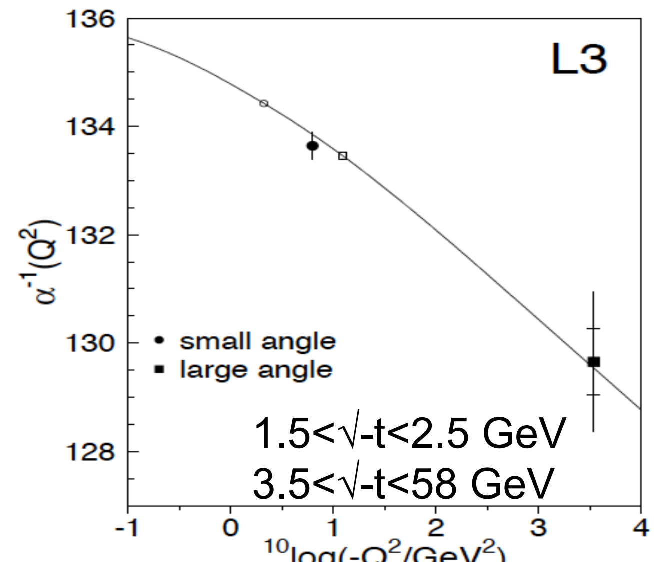


Spacelike



$$\frac{e^+e^- \to e^+e^-}{e^+e^- \to \mu^+\mu^-}$$

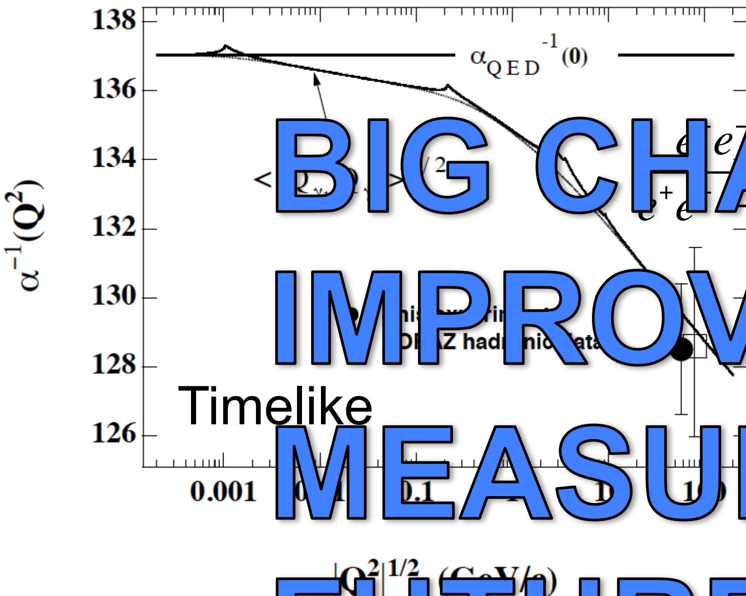
10<√-t<54 GeV

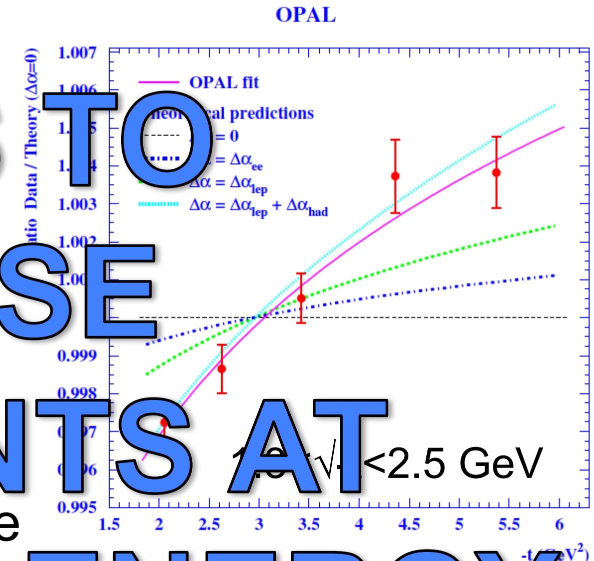


Measurement of α_{em} running

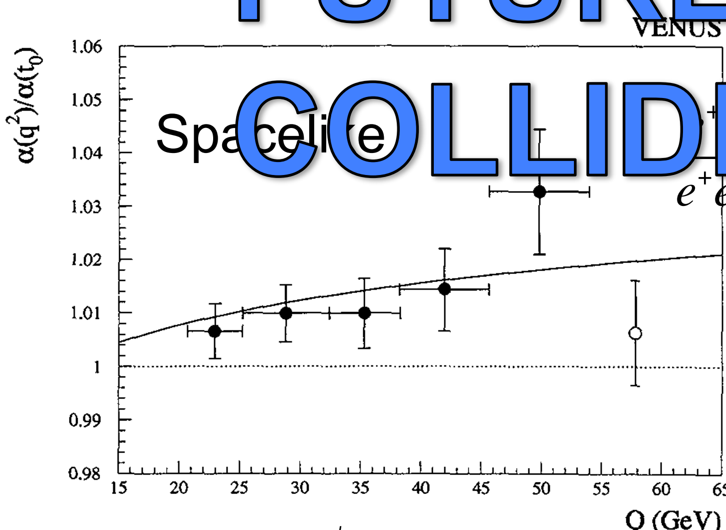
e+e- collider TRISTAN at √s=57.8 GeV,

e+e- collider LEP at √s=189 GeV, using **Bhabha** events





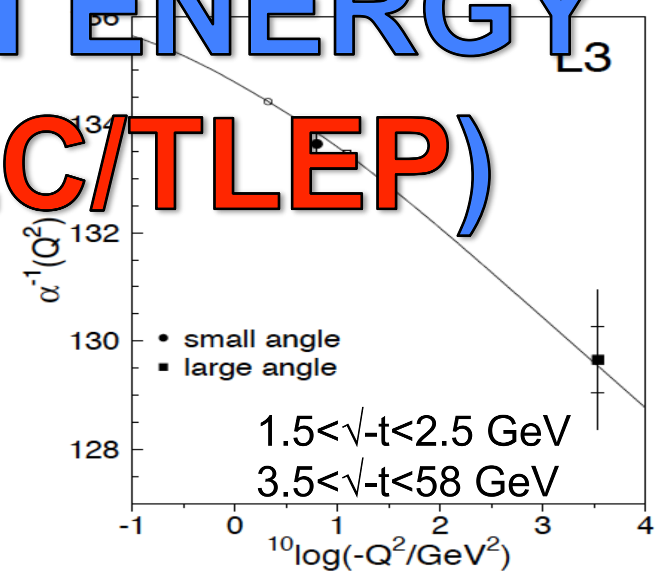
Spacelike



$e^+e^- \to e^+e^-$

$e^+e^- \to \mu^+\mu^-$

10<√-t<54 GeV

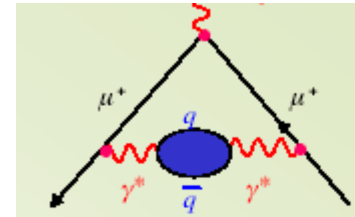


BIG CHANCES TO IMPROVE THESE MEASUREMENTS AT FUTURE HIGH ENERGY COLLIDER (ILC/TLEP)

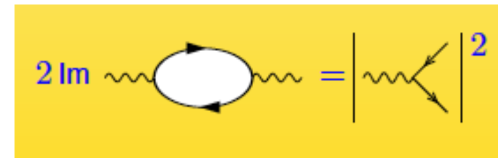
a_μ^{HLO} calculation, traditional way: time-like data

a_μ=(g-2)/2

$$\boxed{a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} \sigma_{e^+e^-\to hadr}(s)K(s)\,ds}$$



$$a_\mu^{HLO.} = \frac{\alpha}{\pi^2}\int_0^\infty \frac{ds}{s} K(s)\,\mathrm{Im}\,\Pi_{had}(s) \quad \sigma_{e^+e^-\to hadr}(s) = \frac{4\pi}{s}\mathrm{Im}\,\Pi_{had}(s)$$



$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \sim \frac{1}{s}$$

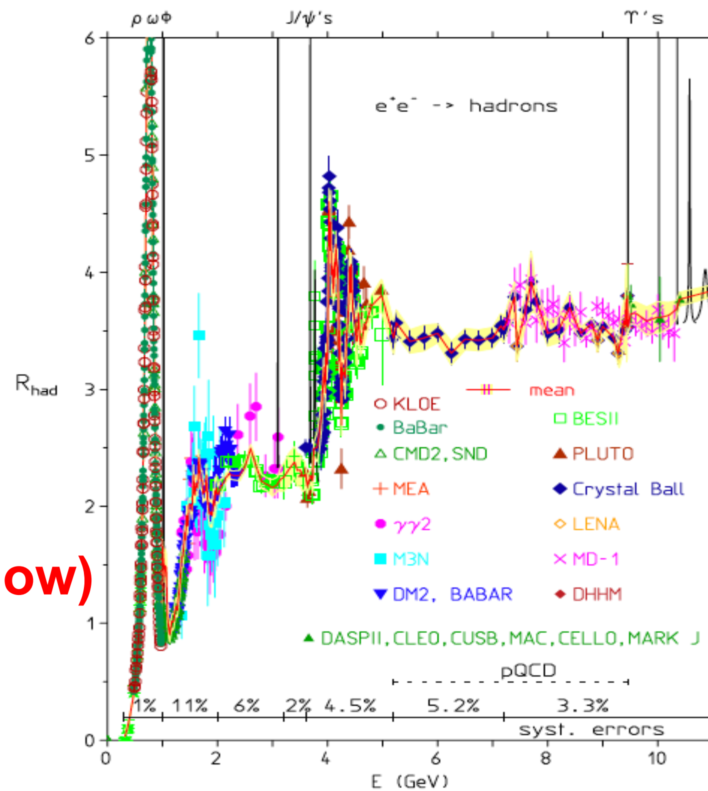
Traditional way: based on precise experimental (time-like) data:

$$a_\mu^{had} = (689.7 \pm 4.4)\cdot 10^{-10}$$

Main contribution in the low energy region

$\delta a_\mu^{exp} \rightarrow$ 1.5 10^{-10} = 0.2% on a_μ^{HLO} (from 0.7% now)

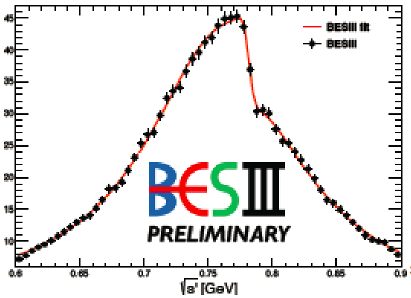
NEW G-2 at FNAL and JPARC

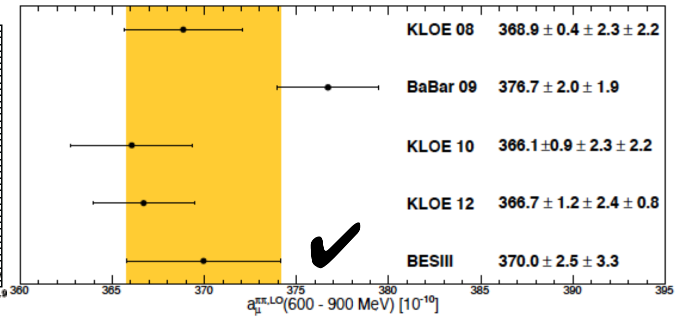


New results on a_μ^{HLO}

– BES

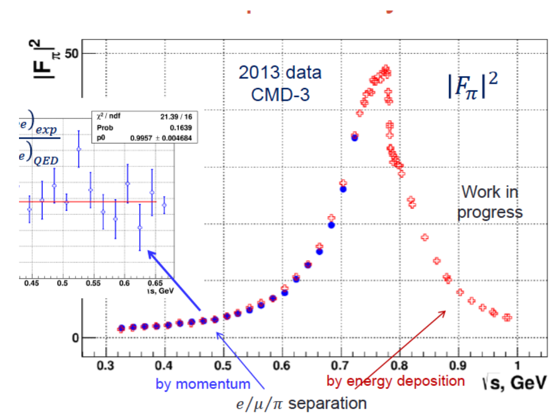
- New; 1/3[rd] data; $\pi\pi$ results consistent with others (arxiv:1507.08188)





– VEPP-2000

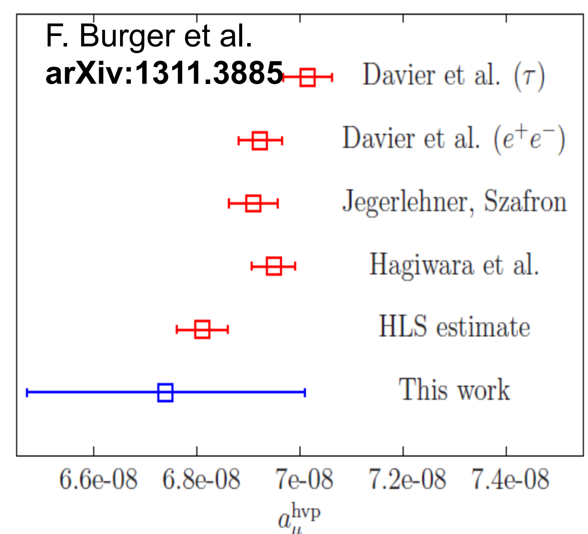
- New results this year at ~0.6% on $\pi\pi$
- Aim at ~0.3% by 2017 (the ultimate goal)



– Lattice

- Results on a_μ^{HLO} with 4% error; 4x worse than the dispersive approach (~1% uncertainty)

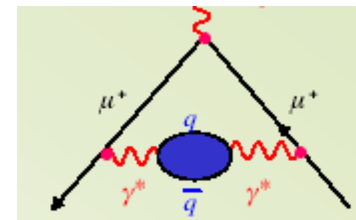
$$a_\mu^{\rm hvp} = 6.74(21)(18)\cdot 10^{-8} \quad (N_f = 2+1+1)$$
$$a_\mu^{\rm hvp} = 6.91(01)(05)\cdot 10^{-8} \quad (\text{dispersive analysis})\,.$$



a_μ^{HLO} evaluation in spacelike region: alternative approach

$a_\mu=(g-2)/2$

$$a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Pi_{had}\left(-\frac{x^2}{1-x}m_\mu^2\right)dx$$



x =Feynman parameter

See also G.Fedotovich, proceedings of PHIPSI08

$$t = \frac{x^2 m_\mu^2}{x-1} \qquad 0 \le -t < +\infty$$

t<0

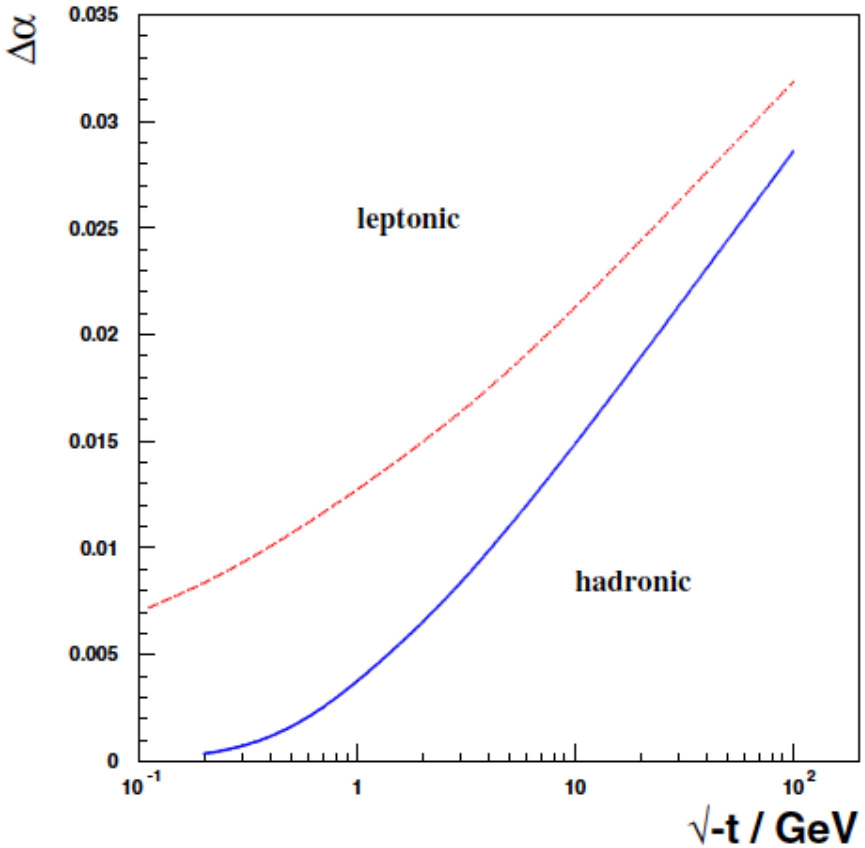
$$x = \frac{t}{2m_\mu^2}\left(1-\sqrt{1-\frac{4m_\mu^2}{t}}\right); \qquad 0 \le x < 1; \qquad t = -s\sin^2\left(\frac{\vartheta}{2}\right)$$

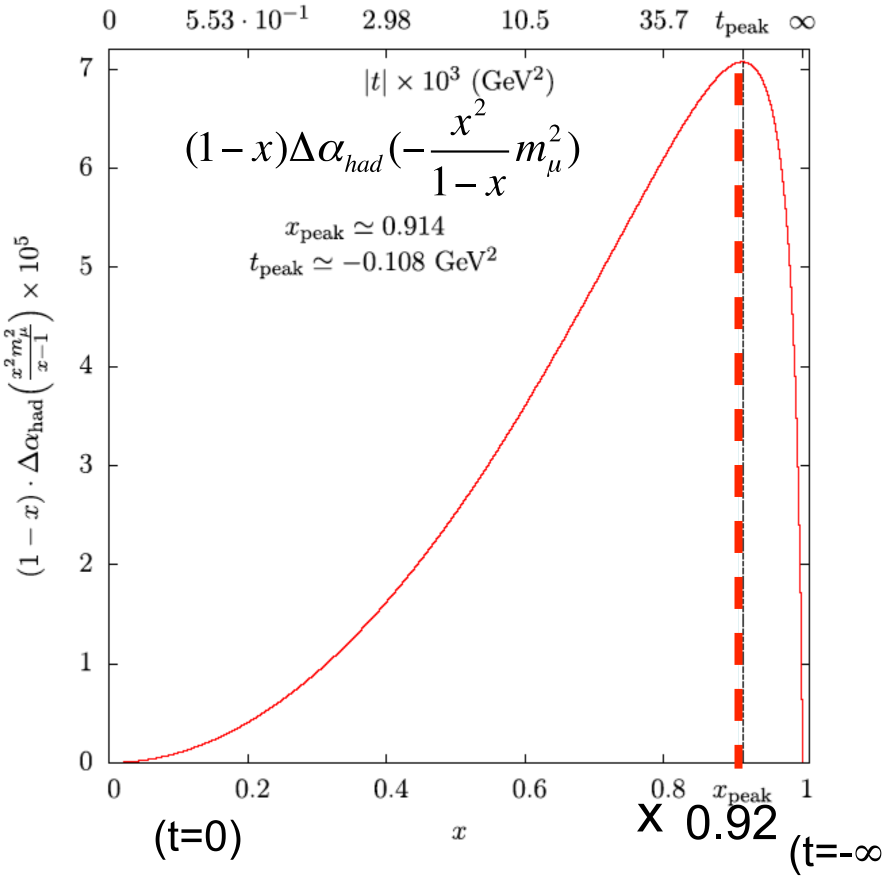
$$\Delta\alpha_{had}(t) = -\Pi_{had}(t) \quad \mathit{for}\ t<0$$

$$a_\mu^{HLO} = -\frac{\alpha}{\pi}\int_0^1 (1-x)\Delta\alpha_{had}\left(-\frac{x^2}{1-x}m_\mu^2\right)dx$$

For t<0

Behaviors





$\Delta\alpha$~log(-t)
Dominated at low |t| by leptonic contribution

A. Arbuzov et al., Eur. Phys. J. C 34 (2004) 267

High |t|-values are depressed by 1-x (a kind of analogy with time-like region)
The integrand is peaked at ~x=0.92
→t=-0.11 GeV^2 (~330 MeV) for which
$\Delta\alpha_{had}$(0.92)~ 10^{-3}

Experimental considerations

Using Bhabha at small angle (to emphasize t-channel contribution) to extract $\Delta\alpha$:

$$\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 \sim \frac{d\sigma_{ee\to ee}(t)}{d\sigma^0_{MC}(t)}$$

Where $d\sigma^0_{MC}$ is the MC prediction for Bhabha process with α(t)=α(0), and there are corrections due to RC…

$$\Delta\alpha_{had}(t) = 1 - \left(\frac{\alpha(t)}{\alpha(0)}\right)^{-1} - \Delta\alpha_{lept}(t)$$

$\Delta\alpha_{lep}(t)$ theoretically well known!

Which experimental accuracy we are aiming at?
$\delta\Delta\alpha_{had}$~1/2 fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t)$.

If we assume to measure $\delta\Delta\alpha_{had}$ at 5% at the peak of the integrand ($\Delta\alpha_{had}$ ~10^{-3} at x=0.92) → fractional accuracy on $d\sigma(t)/d\sigma^0_{MC}(t)$ ~ 10^{-4} !

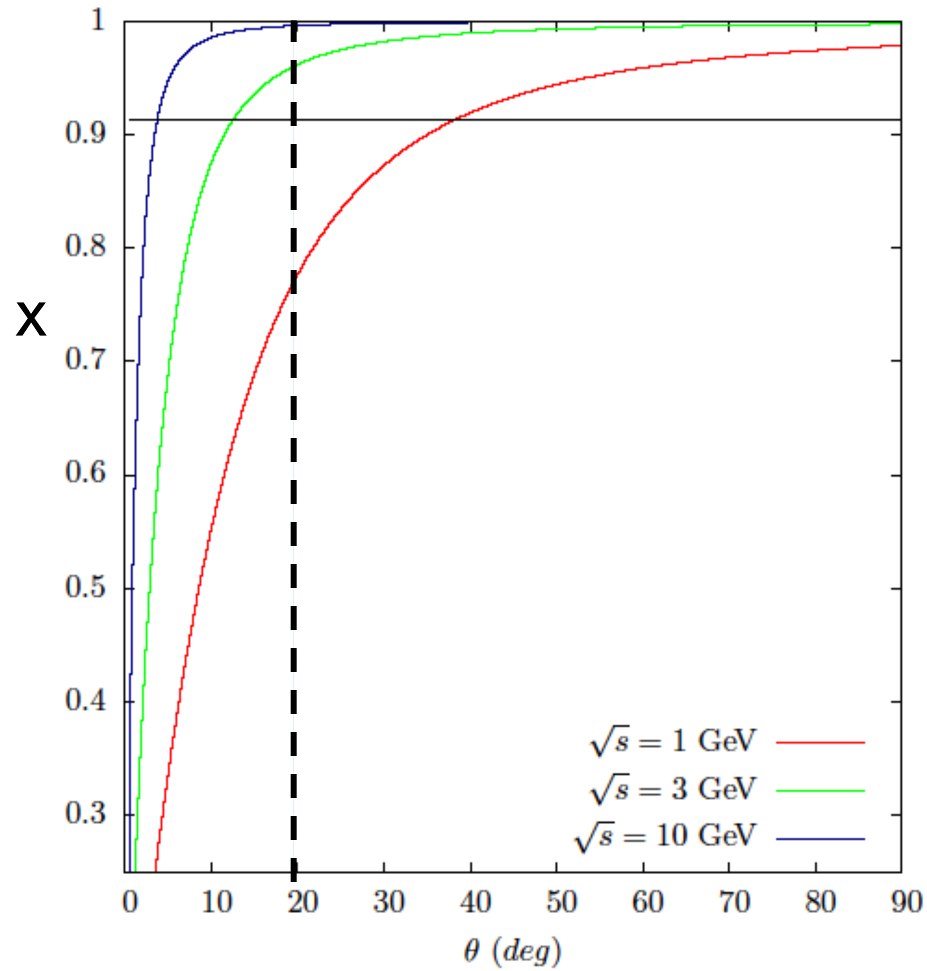
Very challenging measurement (one order of magnitude improvement respect to date) for systematic error

Experimental considerations - II

Most of the region (up to x~0.98) can be covered with a low energy machine (like Dafne/VEPP-2000 or tau/charm-B-factories)

Example:
Covering up to 60^0 at $\sqrt{s}$=1 GeV can arrive at x= 0.95(!)

A different situation can be obtained at tau/charm/ B-factories (and at future ILC/TLEP machines) where smaller angles (below 20^o) are needed



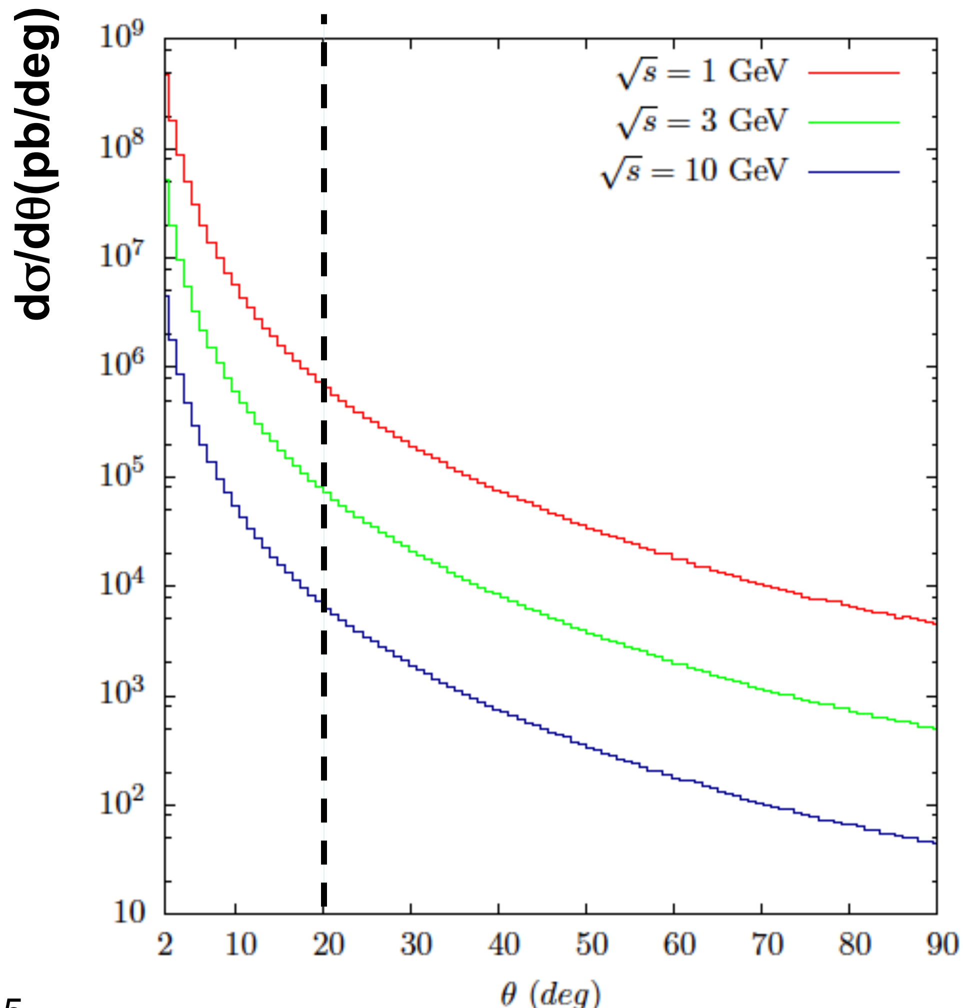
$$t = -s\sin^2(\frac{\vartheta}{2})$$

Statistical consideration

10^{-4} accuracy on Bhabha cross section requires at least 10^8 events which at 20° mean at least:

- O(1) fb^{-1} @ 1 GeV
- O(10) fb^{-1} @ 3 GeV
- O(100) fb^{-1} @ 10 GeV

These luminosities are within reach at flavour factories!



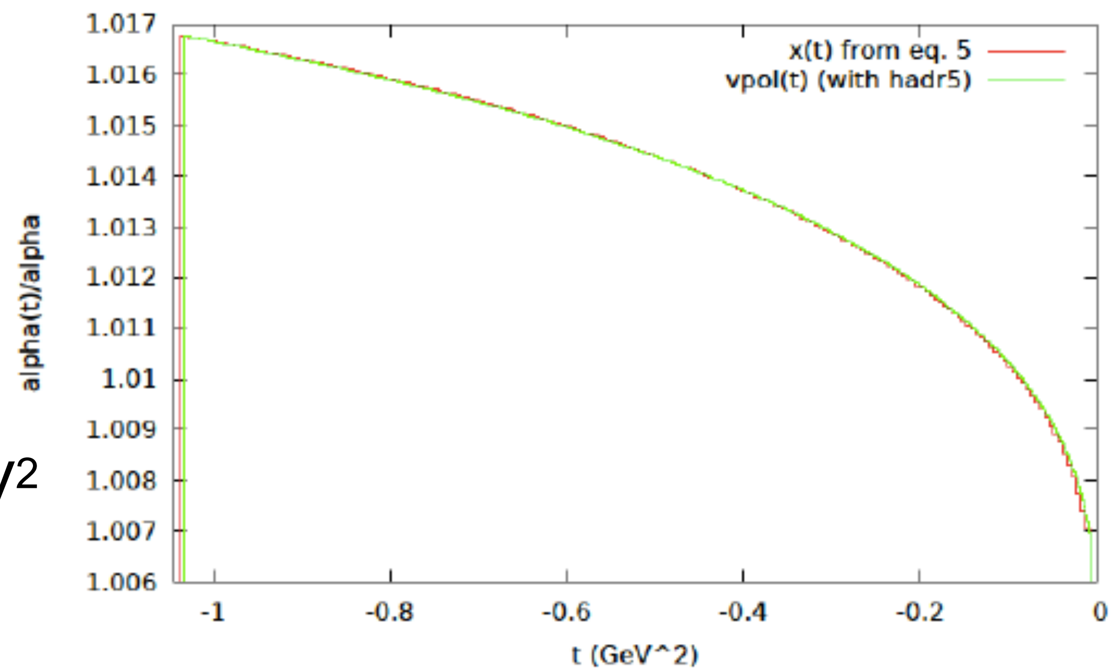
Additional considerations: s-channel

At low energy (<10 GeV) above 10^0 there is still a sizeable contribution from s-channel.
At LO no difficulty to deconvolute the cross section for the s-channel

Test with Babayaga:

s=1 GeV
$10^o<\theta<170^o$

$d\sigma_{born}/dt=1.52$ mb/GeV^2



However this picture changes with Rad. Corr.

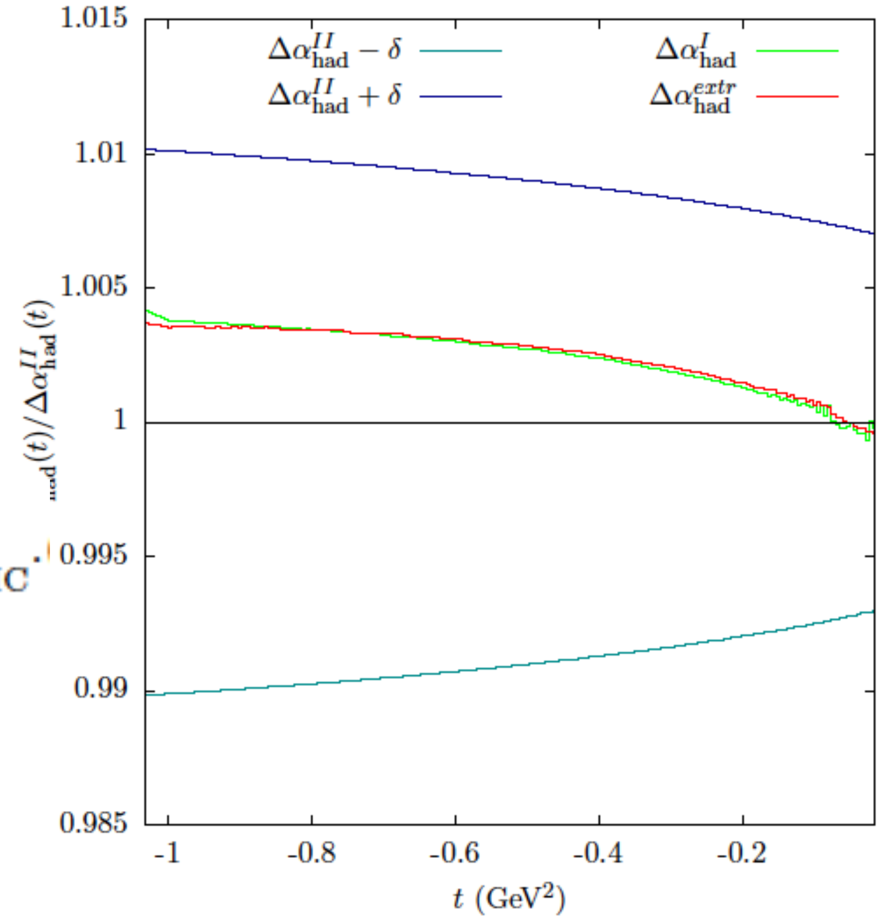
Additional considerations: Rad. Corr.

A Monte Carlo procedure has been developed to check if $\Delta\alpha_{had}(t)$ can be obtained by a minimization procedure with a different $\Delta\alpha_{had}(t)'$ inside

$$\frac{d\sigma}{dt}\bigg|_{\mathrm{data}} = \frac{d\sigma}{dt}\Big(\alpha(t), \alpha(s)\Big)\Big|_{\mathrm{MC}},$$

→

$$\frac{d\sigma}{dt}\bigg|_{j,\mathrm{data}} = \frac{d\sigma}{dt}\Big(\bar{\alpha}(t) + \frac{i_j}{N}\delta(t), \alpha(s)\Big)\Big|_{j,\mathrm{MC}}.$$



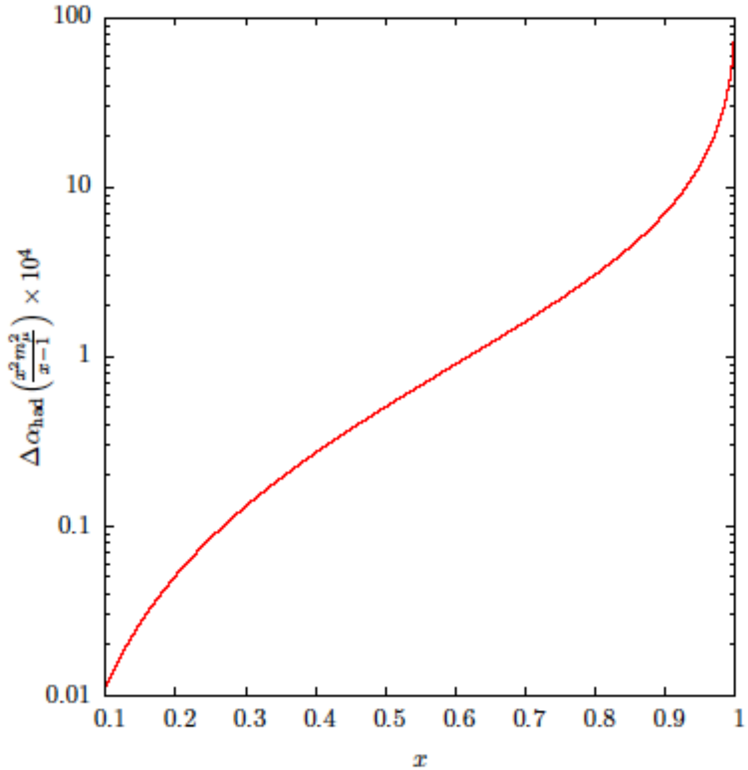
$\Delta\alpha_{had}(t)$ is obtained with$<10^{-4}$ error !

Additional consideration: Normalization

To compare Bhabha absolute cross section from data with MC we need Luminosity of the machine.

Two possibilities:

1) Use Bhabha at very small angle where the uncertainty on $\Delta\alpha_{had}$ can be neglected (for example at E_{beam}=1 GeV and θ=5°, $\Delta\alpha_{had}$ ~10^{-5}).
2) Use a process with $\Delta\alpha_{had}$=0, like e+e- → $\gamma\gamma$. However very difficult to determine it at 10^{-4} accuracy.



Option 1) looks better to us as some of the common systematics cancel in the measurement !

Measurement of DAFNE Luminosity with KLOE/KLOE-2 at 10^{-4}?

F. Ambrosino et al [KLOE] Eur. Phys. J. C 47, 589–596 (2006)

Table 2. Summary of the corrections and systematic errors in the measurement of the luminosity

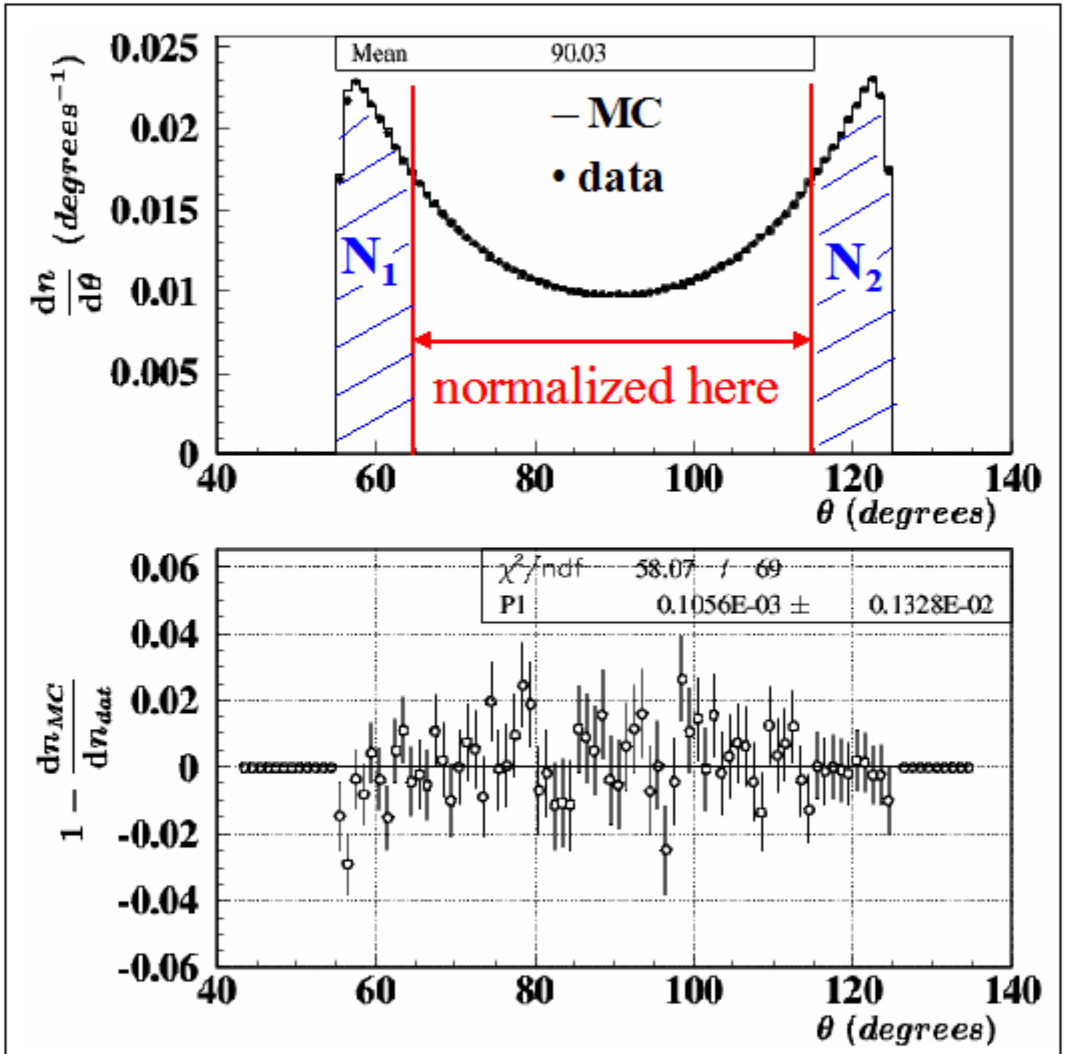
	correction (%)	systematic error (%)
angular acceptance	+0.25	0.25
tracking	–	0.06
clustering	+0.14	0.11
background	−0.62	0.13
cosmic veto	+0.40	–
energy calibration	–	0.10
center of mass energy	+0.10	0.10
	+0.34	0.32

Adding in quadrature: 0.3 %

(can be improved by a factor 10?)

From F. Nguyen 2006

Polar angle systematics



✓ global agreement is very good

but the cut occurs in a steep region of the distributions

⇒ estimate of border mismatches

✓ after normalizing MC to make it coincide with data in the region $65^o < \theta < 115^o$, we estimate as a systematic error:

$$\frac{N^{dat}_{[55:65]+[115:125]} - N^{MC}_{[55:65]+[115:125]}}{N^{dat}_{TOT}} \sim 0.25\%$$

Can be improved at 10^{-4}?

A measurement of the Luminosity at 10^{-4} at LEP

Giovanni Abbiendi

INFN - Bologna

Eur. Phys. J. C 45, 1–21 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02389-3

THE EUROPEAN PHYSICAL JOURNAL C

Measurement of the running of the QED coupling in small-angle Bhabha scattering at LEP

The OPAL Collaboration

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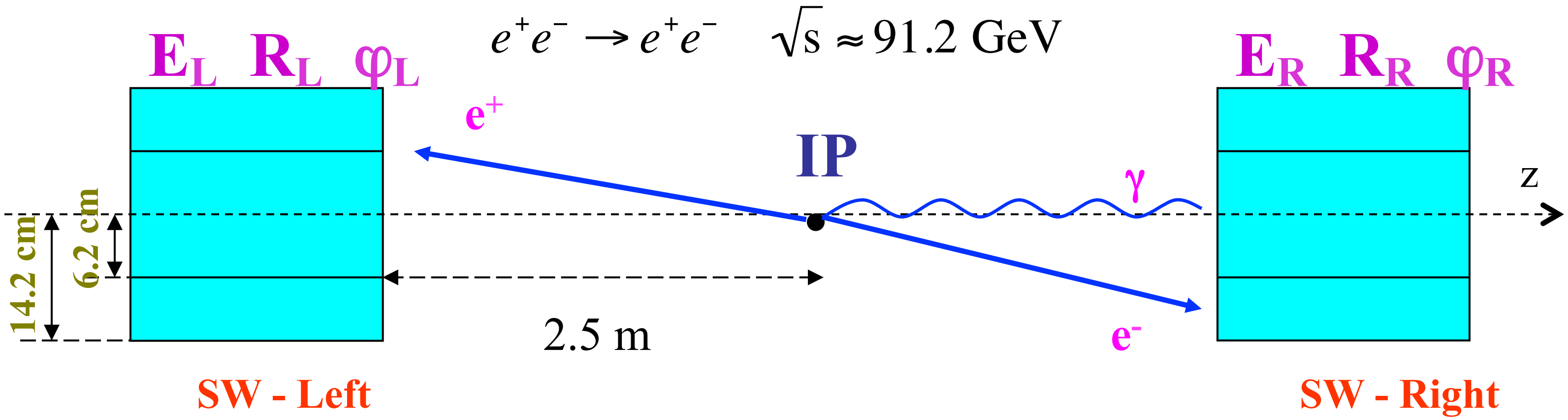
Small-angle Bhabha scattering in OPAL



2 cylindrical calorimeters encircling the beam pipe at ± 2.5 m from the Interaction Point

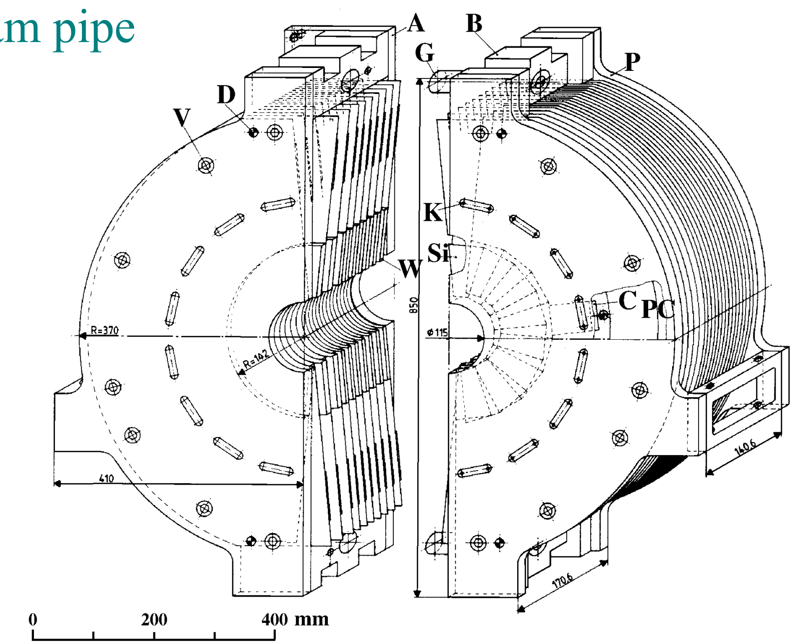
19 Silicon layers

18 Tungsten layers

Total Depth 22 X_0
(14 cm)

Each detector layer divided into 16 overlapping wedges

Sensitive radius: **6.2 – 14.2 cm**, corresponding to **scattering angle** of **25 – 58 mrad** from the beam line



Final Error on Luminosity

After all the effort on Radial reconstruction the dominant systematic error is related to Energy (mostly tail in the E response and nonlinearity)

Quantitatively: (OPAL Collaboration, Eur.Phys.J. C14 (2000) 373)

	Systematic Error ($\times 10^{-4}$)
Energy	1.8
Inner Anchor	1.4
Radial Metrology	1.4

Total Experimental Systematic Error : 3.4×10^{-4}

Theoretical Error on Bhabha cross section: 5.4×10^{-4}

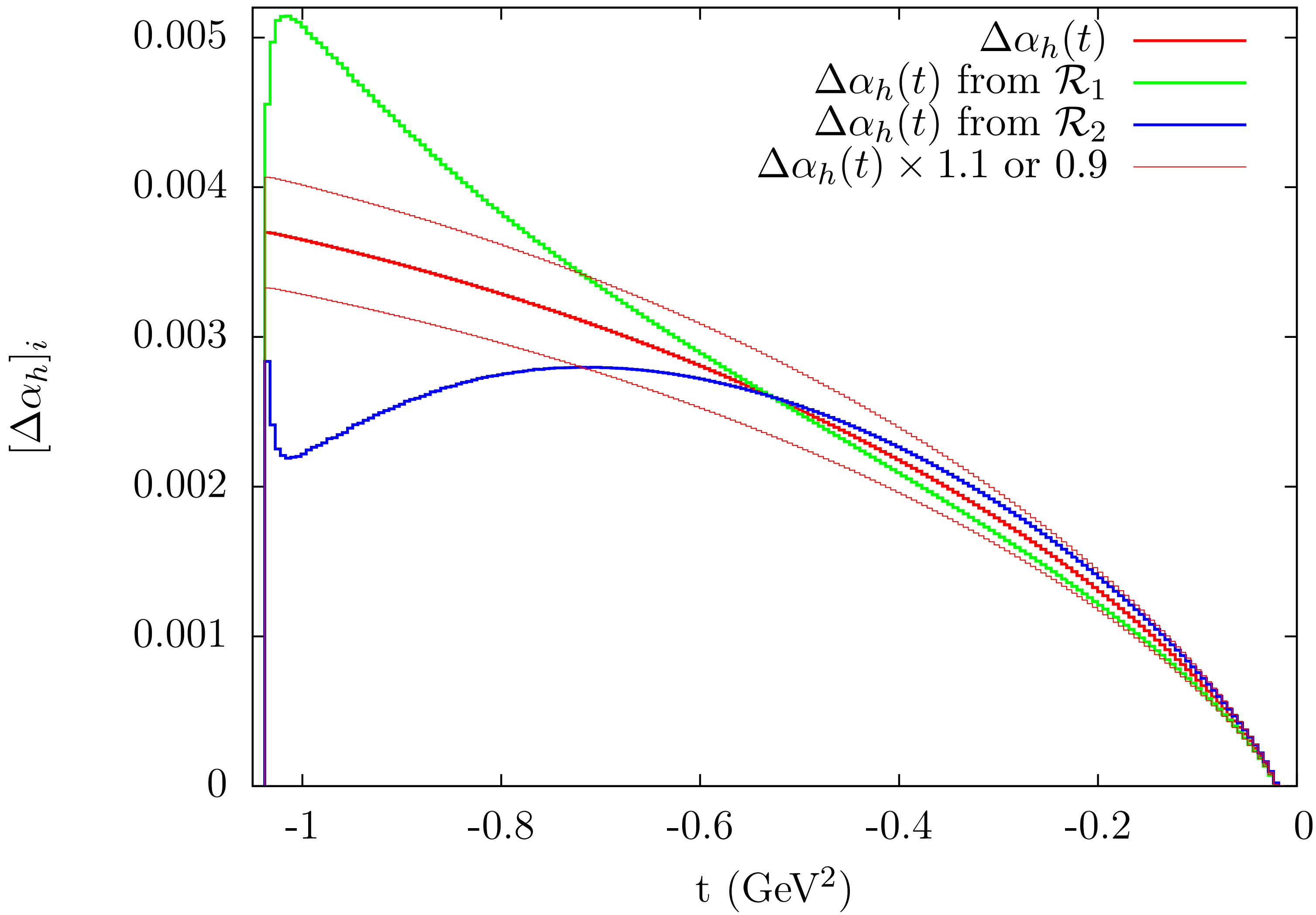
Conclusions

- Measuring α_{em} running in time-like and space like region appears to be very interesting. (Relatively) high q^2-values can be explored at ILC/TLEP
- An alternative formula for a_μ^{HLO} in spacelike region has been studied in details. It emphasizes low values of t (<1 GeV^2) and can be explored at low energy e+e- machines (VEPP2000/ DAFNE, τ/charm, B-factories)
- It requires to measure the Bhabha cross section at relatively small angles at (better than) 10^{-4} accuracy!
- Reaching such an accuracy demands a dedicated experimental and theoretical work for the next few years
- Can this method apply also at other (e^-e^-; fixed target) machines?

Thanks!

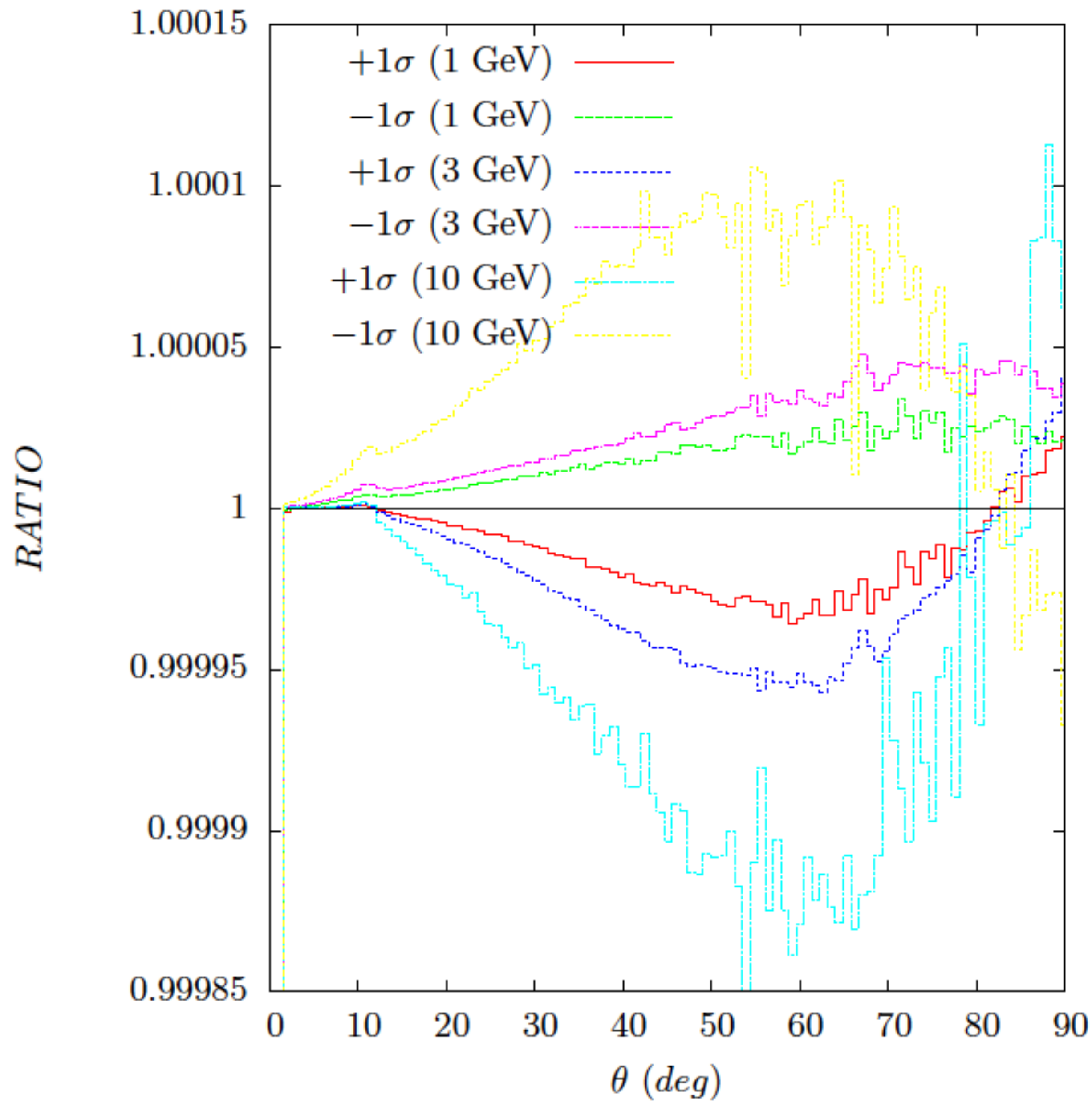
END

test



$\Delta\alpha_h(t)$
$\Delta\alpha_h(t)$ from $\mathcal{R}_1$
$\Delta\alpha_h(t)$ from $\mathcal{R}_2$
$\Delta\alpha_h(t) \times 1.1$ or 0.9
$[\Delta\alpha_h]_i$
t (GeV^2)
0.005
0.004
0.003
0.002
0.001
0
-1
-0.8
-0.6
-0.4
-0.2
0

$\Delta\alpha_{em}^{HAD}(s)$ dependence



Which is the best energy/angle configuration?

$$x = \frac{t}{2m_\mu^2}\left(1-\sqrt{1-\frac{4m^2}{t}}\right)$$

$$-t = 9(1-\cos\theta)/2$$

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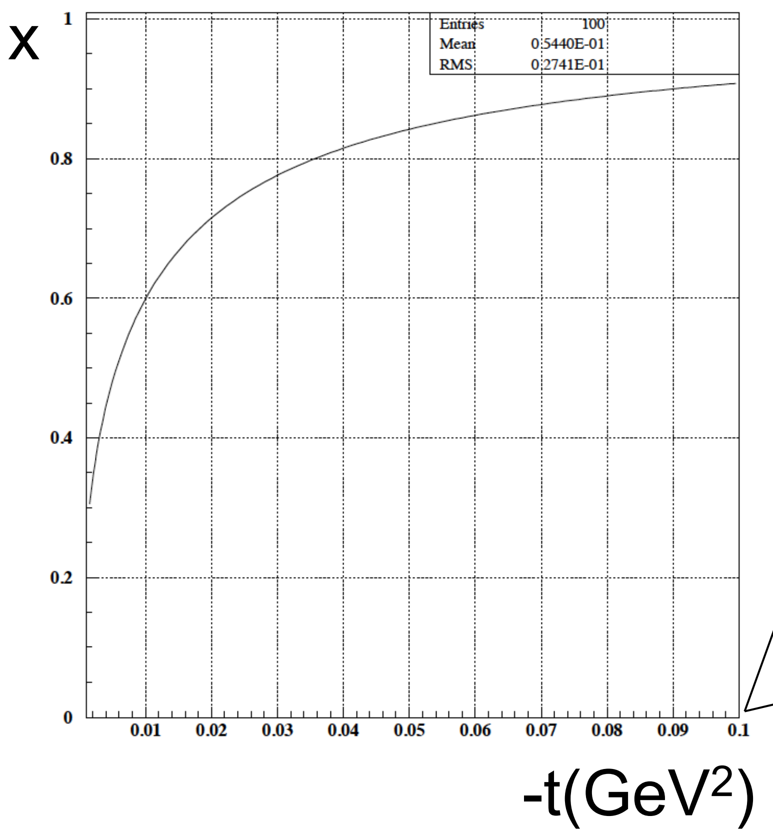
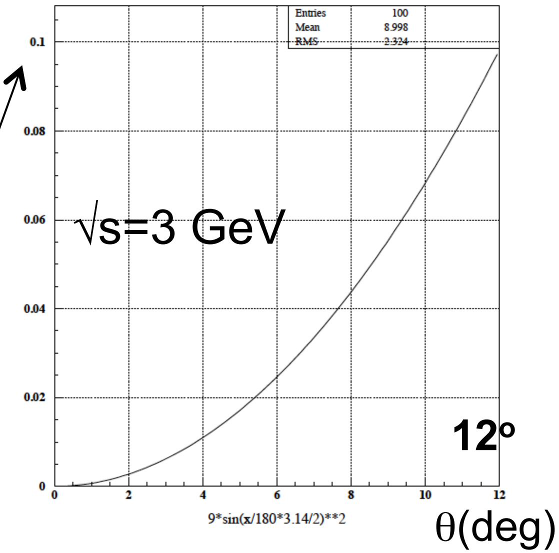
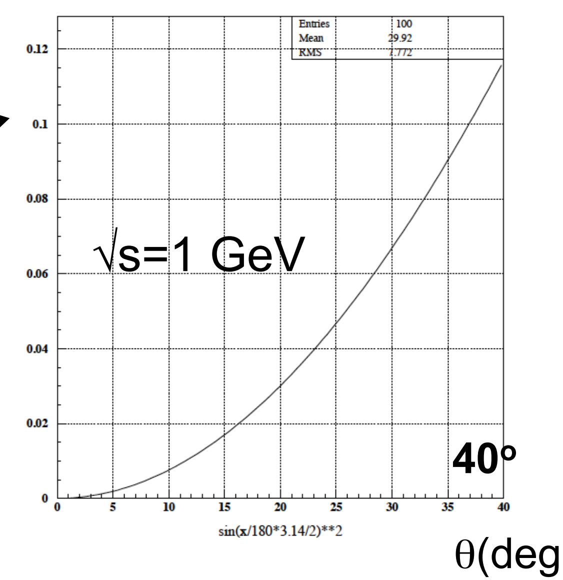
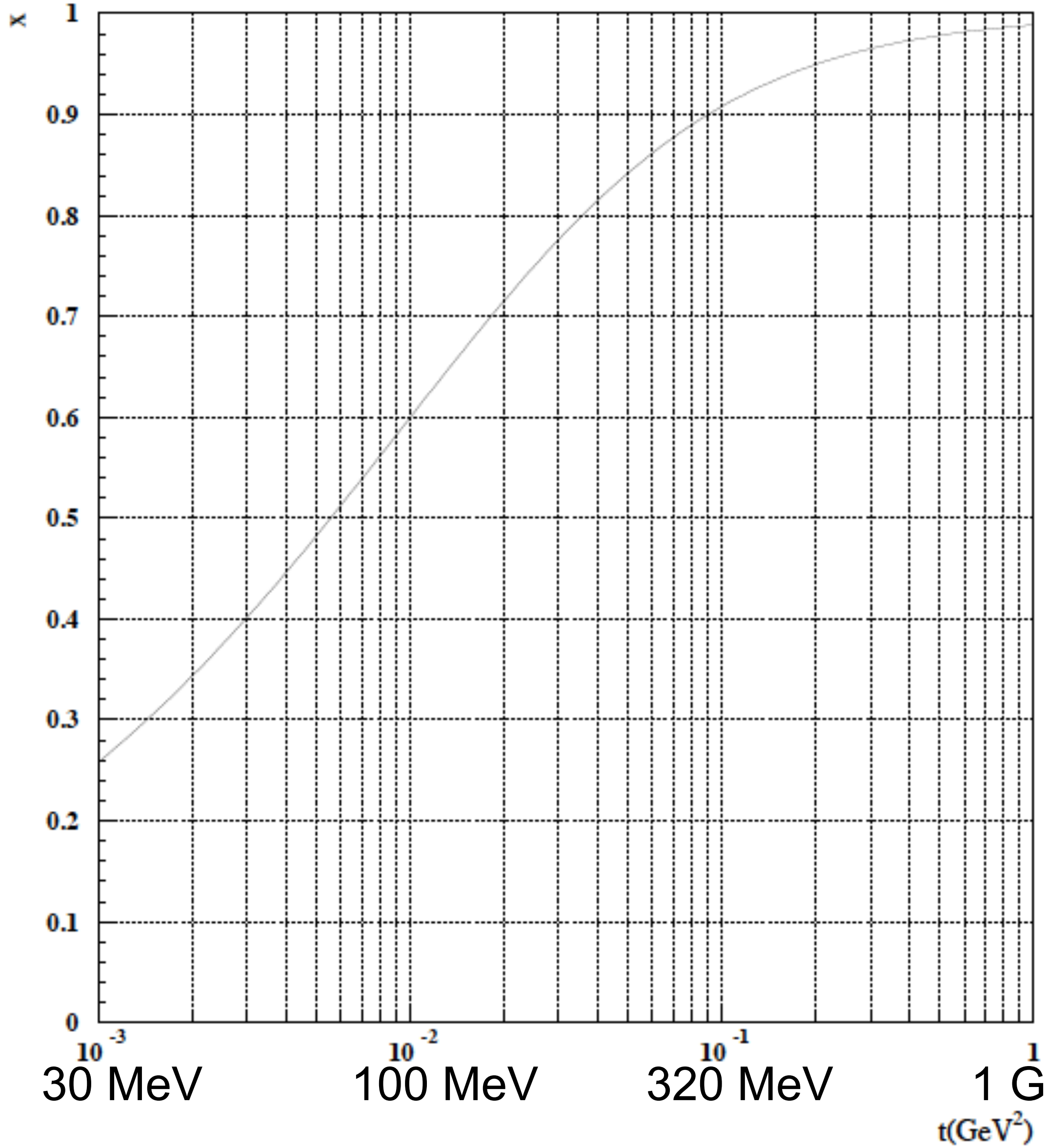






x vs t behaviour



x→1
t → ∞

x→0
t→0

30 MeV 100 MeV 320 MeV 1 GeV

Measuring a_μ^{HLO} in the spacelike region

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[arXiv:1504.02228, Phys.Lett. B746 (2015) 325-32]

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LFC15: Physics prospects for Linear and other Future Colliders after the discovery of the Higgs
Trento, September 7-11, 2015

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TRENTO, ITALY
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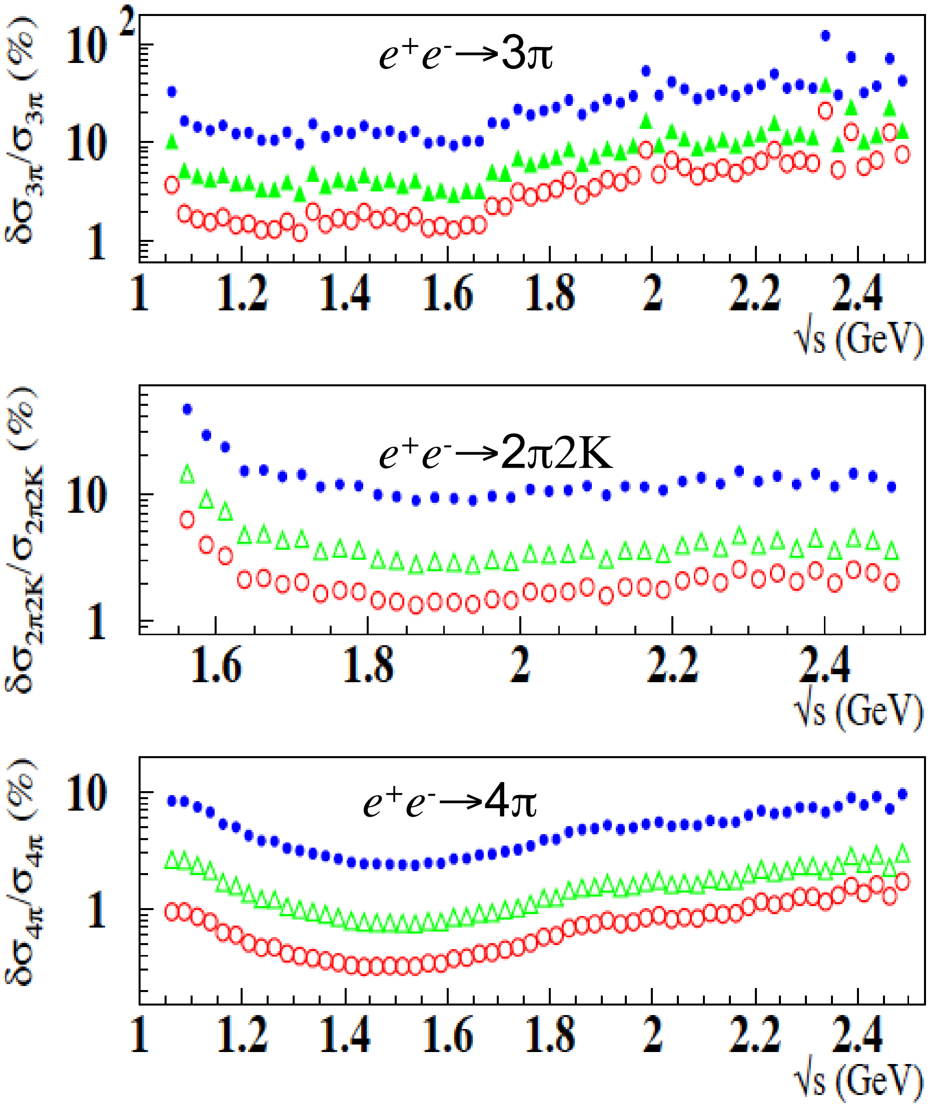
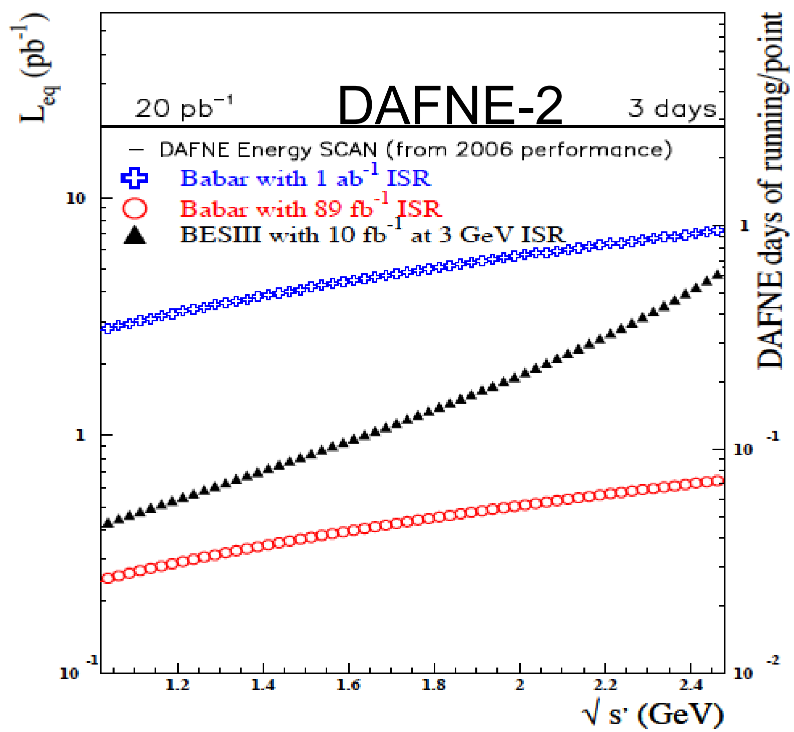
Muon g-2 Physics Workshop, Seattle, 14 July 2015

Impact of DAFNE-2 on exclusive channels in the range [1-2.5] GeV with a scan (Statistics only)

arXiv:1007.521

- **Published BaBar results: 89 fb^{-1} (ISR)**
- **▲ "BaBar" × 10 (890 fb^{-1})**
- **o KLOE-2 energy scan: 20 pb^{-1}/point @ L= 10^{32} cm^{-2} s^{-1}, 25 MeV bin ⇒ 1 year data-taking**





DAFNE-2 is statistically equivalent to 5÷10 ab^{-1} (Super)B-factory