Testing the Standard Model with the lepton g-2

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Outline

- 1. μ: The muon g-2: recent theory progress
- 2. e: Testing the Standard Model with the electron g-2
- 3. τ: The tau g-2: opportunities or fantasies? Surprises?

Lepton magnetic moments: the basics

The beginning: g = 2

- **Uhlenbeck and Goudsmit in 1925 proposed:**

$$\vec{\mu} = g\frac{e}{2mc}\vec{s}$$

$$g = 2 \quad (\text{not } 1!)$$

- **Dirac 1928:**

$$(i\partial_\mu - eA_\mu)\,\gamma^\mu\psi = m\psi$$

- **A Pauli term in Dirac's eq would give a deviation…**

$$a\frac{e}{2m}\sigma_{\mu\nu}F^{\mu\nu}\psi \quad \rightarrow \quad g = 2(1+a)$$

…but there was no need for it! g=2 stood for ~20 yrs.

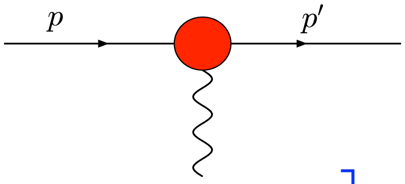
- **Kusch and Foley 1948:**

$$\mu_e^{\rm exp} = \frac{e\hbar}{2mc}\,(1.00119 \pm 0.00005)$$

- **Schwinger 1948 (triumph of QED!):**

$$\mu_e^{\rm th} = \frac{e\hbar}{2mc}\left(1+\frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- **Keep studying the lepton–γ vertex:**



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

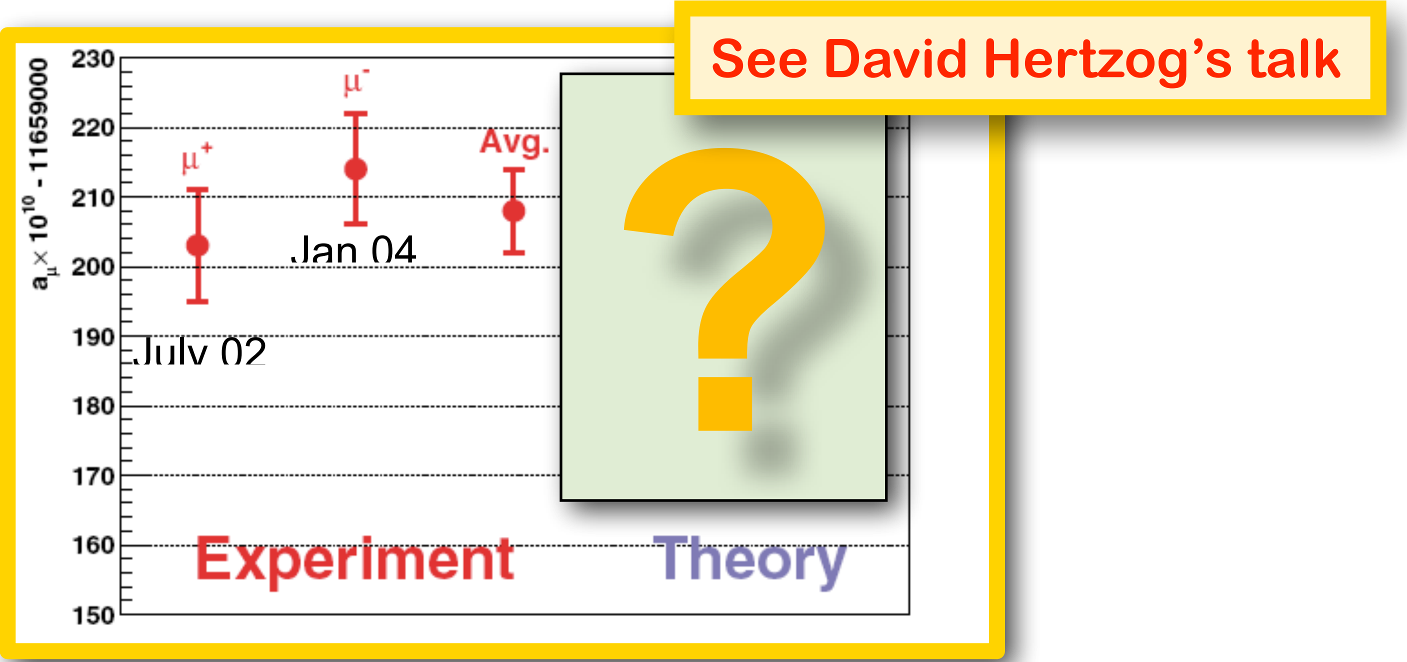
$$F_1(0) = 1 \qquad F_2(0) = a_l$$

A pure "quantum correction" effect!

μ

The muon g-2: recent theory progress

The muon g-2: the experimental result



- **Today:** $a_\mu^{EXP} = (116592089 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- **Future:** new muon g-2 experiments proposed at:
  - Fermilab E989, aiming at $\pm 16 \times 10^{-11}$, ie 0.14ppm
  - J-PARC proposal aiming at 0.1 ppm
- Are theorists ready for this (amazing) precision? No(t yet)

The muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857426\,(16)\,(\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$+ \; 24.05050988\,(28)\,(\alpha/\pi)^3$

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$+ \; 130.8773\,(61)\,(\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Lee, Marquard, Smirnov2, Steinhauser 2013 (electron loops, analytic),
Kurz, Liu, Marquard, Steinhauser 2013 (τ loops, analytic);
Steinhauser et al. 2015 (electron loops, analytic); work in progress

$+ \; 752.85\,(93)\,(\alpha/\pi)^5$ **COMPLETED!**

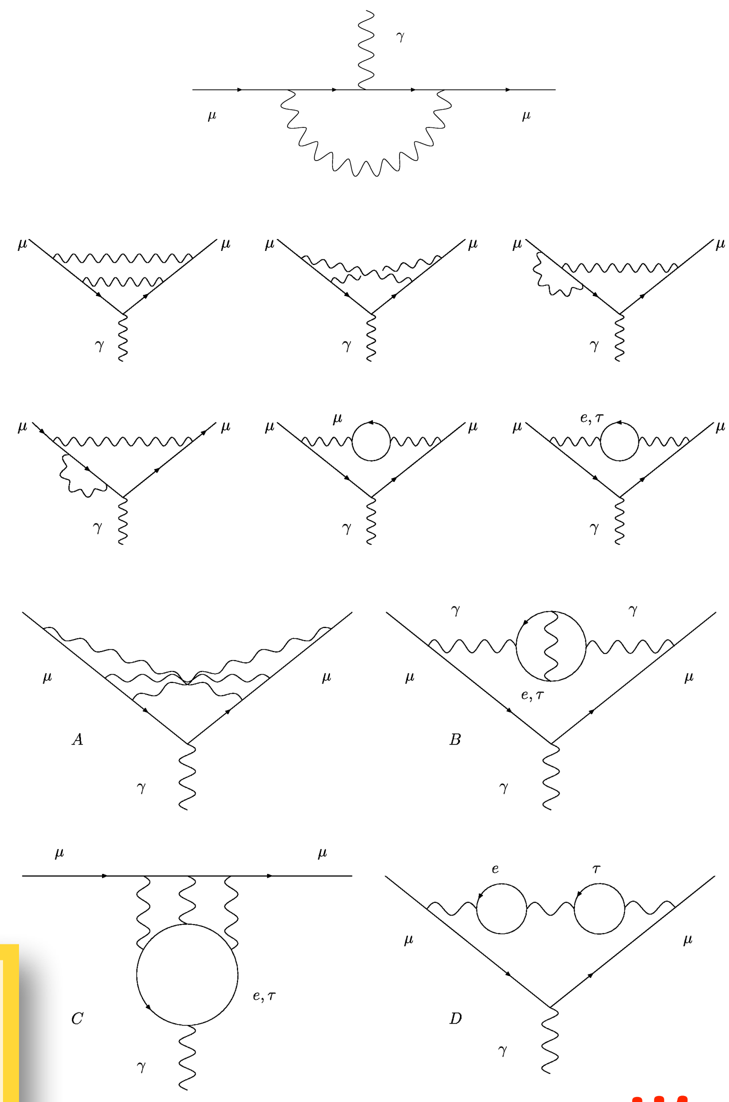
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim,…, Kataev, Kinoshita & Nio '06; Kinoshita et al. 2012 & 2015

Adding up, we get:

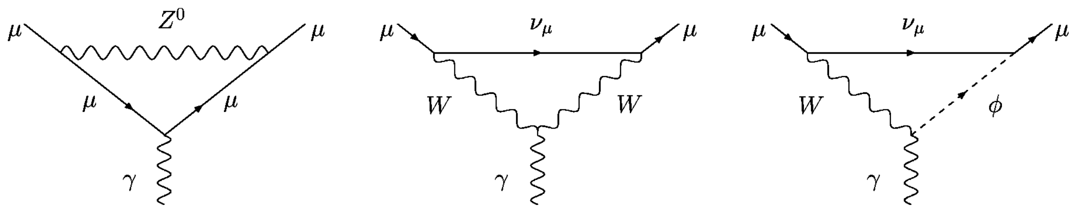
$a_\mu^{QED} = 116584718.941\,(21)(77) \times 10^{-11}$

from coeffs, mainly from 4-loop unc ↲ ↳ from δα(Rb)

with $\alpha = 1/137.035999049(90)$ [0.66 ppb]



- **One-loop term:**



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

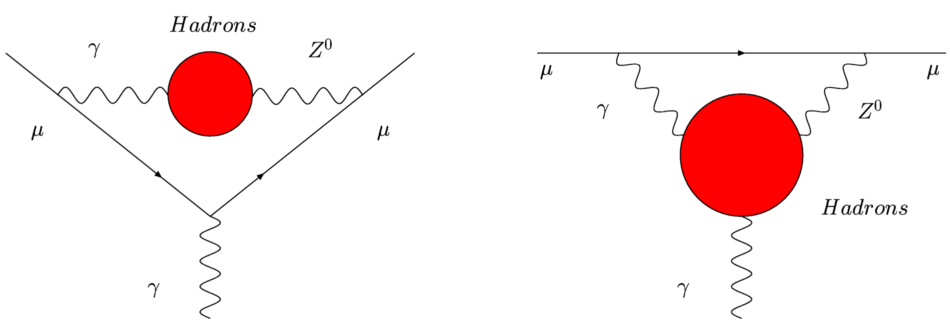
- **One-loop plus higher-order terms:**

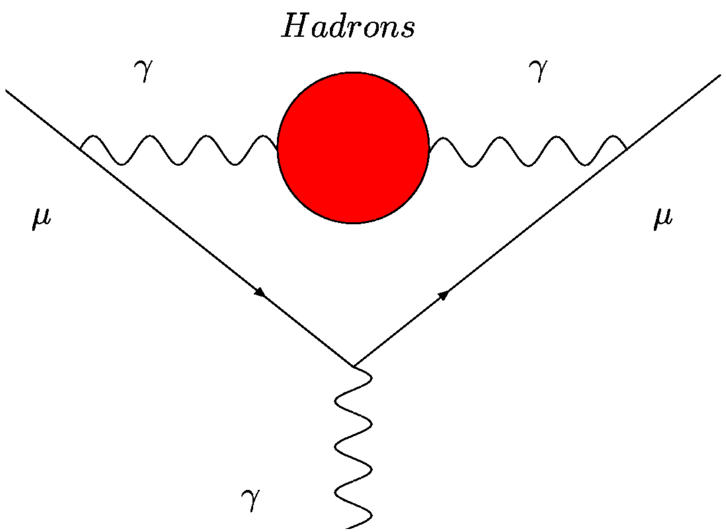
$a_\mu^{\rm EW} = 153.6\ (1) \times 10^{-11}$

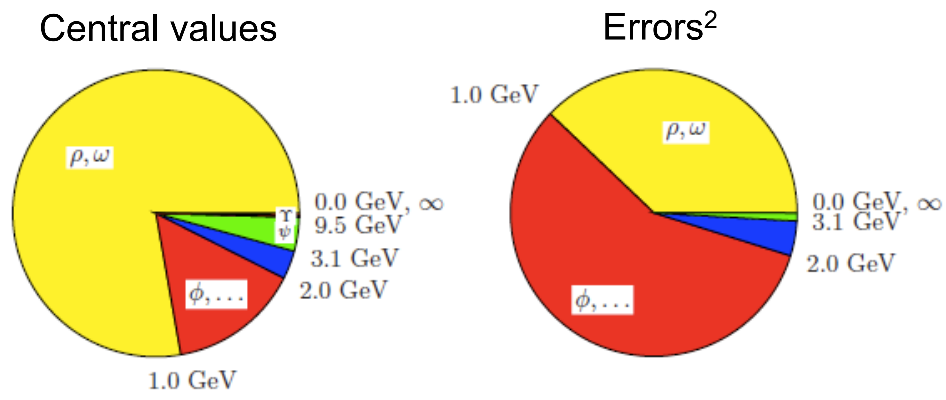
with $M_{\rm Higgs}$ = 125.6 (1.5) GeV

Hadronic loop uncertainties and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.







F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \qquad a_\mu^{\rm HLO} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds \, K(s) \, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

$a_\mu^{\rm HLO} = 6903\ (53)_{\rm tot} \times 10^{-11}$ — F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$= 6923\ (42)_{\rm tot} \times 10^{-11}$ — Davier et al, EPJ C71 (2011) 1515 (incl. BaBar & KLOE10 2π)

$= 6949\ (37)_{\rm exp}\ (21)_{\rm rad} \times 10^{-11}$ — Hagiwara et al, JPG 38 (2011) 085003

Radiative Corrections are crucial! S. Actis et al, Eur. Phys. J. C66 (2010) 585

Alternatively, exchanging the x and s integrations in a_μ^{HLO} one gets:

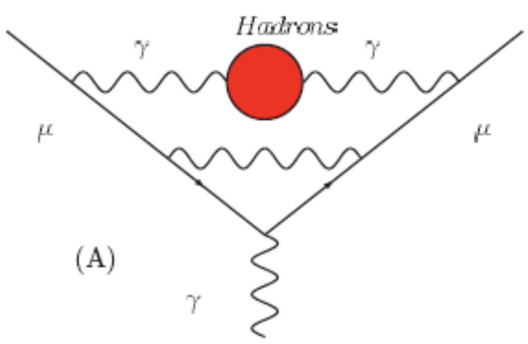
$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)] \qquad t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

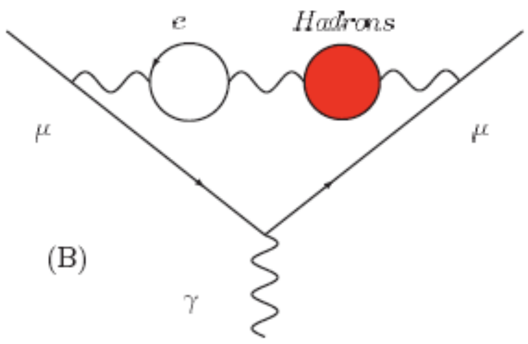
involving the hadronic contribution to the running of α in the space-like region, which can be extracted from Bhabha scattering data!

Requires measuring the Bhabha cross section at relatively small angles at better than 10^{-4} accuracy. Challenging, but with dedicated work it may become feasible at flavor factories.

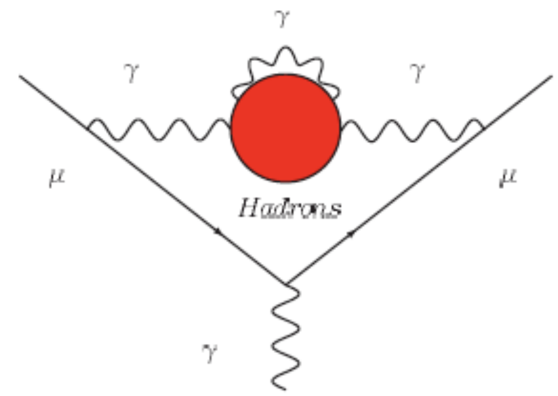
Carloni Calame, MP, Trentadue, Venanzoni, PLB 746 (2015)

- **HNLO: Vacuum Polarization**





Already included in a_μ^{HLO}

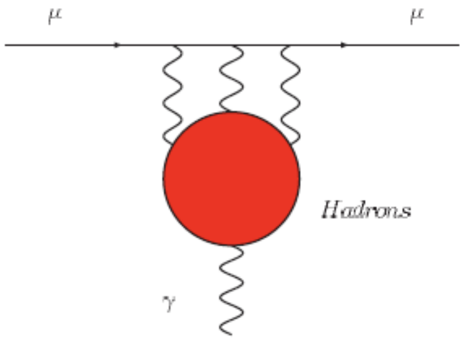


$O(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{HNLO}(vp) = -98\ (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

HNLO: Light-by-light contribution



- Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.

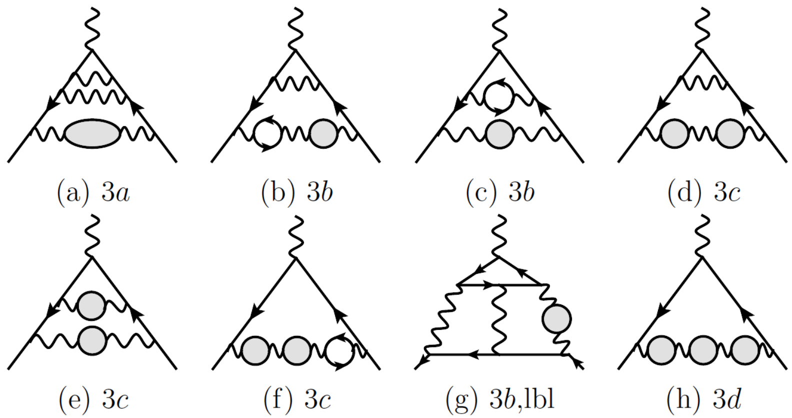
- This term had a troubled life! Latest values:

$a_\mu^{HNLO}(lbl) = + 80\ (40) \times 10^{-11}$	Knecht & Nyffeler ’02
$a_\mu^{HNLO}(lbl) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein ’03
$a_\mu^{HNLO}(lbl) = +105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein ’09
$a_\mu^{HHO}(lbl) = +116\ (39) \times 10^{-11}$	Jegerlehner & Nyffeler ’09

Results based also on Hayakawa, Kinoshita ’98 & ’02; Bijnens, Pallante, Prades ’96 & ’02

- “Bound” $a_\mu^{HNLO}(lbl) < \sim 160 \times 10^{-11}$ Erler, Sanchez ’06, Pivovarov ’02; also Boughezal, Melnikov ’11
- Pion exch. in holographic QCD agrees. D.K.Hong, D.Kim ’09; Cappiello, Catà, D’Ambrosio ’11
- Lattice? Very hard but promising Tom Blum et al. 2015
- Dispersive approach proposed Colangelo, Hoferichter, Procura, Stoffer 1402.7081, 1506.01386

HNNLO: Vacuum Polarization



$O(\alpha^4)$ contributions of diagrams containing hadronic vacuum polarization insertions:

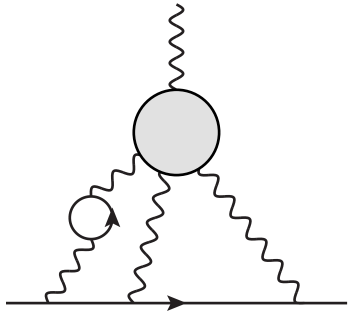
$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

HNNLO: Light-by-light

$$a_\mu^{HNNLO}(lbl) = 3\ (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



The muon g-2: SM vs. Experiment

Adding up all contributions, we get the following SM predictions and comparisons with the measured value:

$a_\mu^{EXP} = 116592091\ (63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda=\mu_\mu/\mu_p$ from CODATA'10

$a_\mu^{\rm SM} \times 10^{11}$	$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM}$	σ
116 591 809 (66)	$282\ (91) \times 10^{-11}$	3.1 [1]
116 591 829 (57)	$262\ (85) \times 10^{-11}$	3.1 [2]
116 591 855 (58)	$236\ (86) \times 10^{-11}$	2.8 [3]

with the "conservative" a_μ^{HNLO}(lbl) = 116 (39) x 10^{-11} and the LO hadronic from:

[1] Jegerlehner & Nyffeler, Phys. Rept. 477 (2009) 1
[2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10 2π)
[3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10 2π)

Note that the th. error is now about the same as the exp. one

- Δa_μ can be explained by errors in QED, EW, HNLO, g-2 EXP, HLO, or, we hope, by New Physics!
- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$

$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$

$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

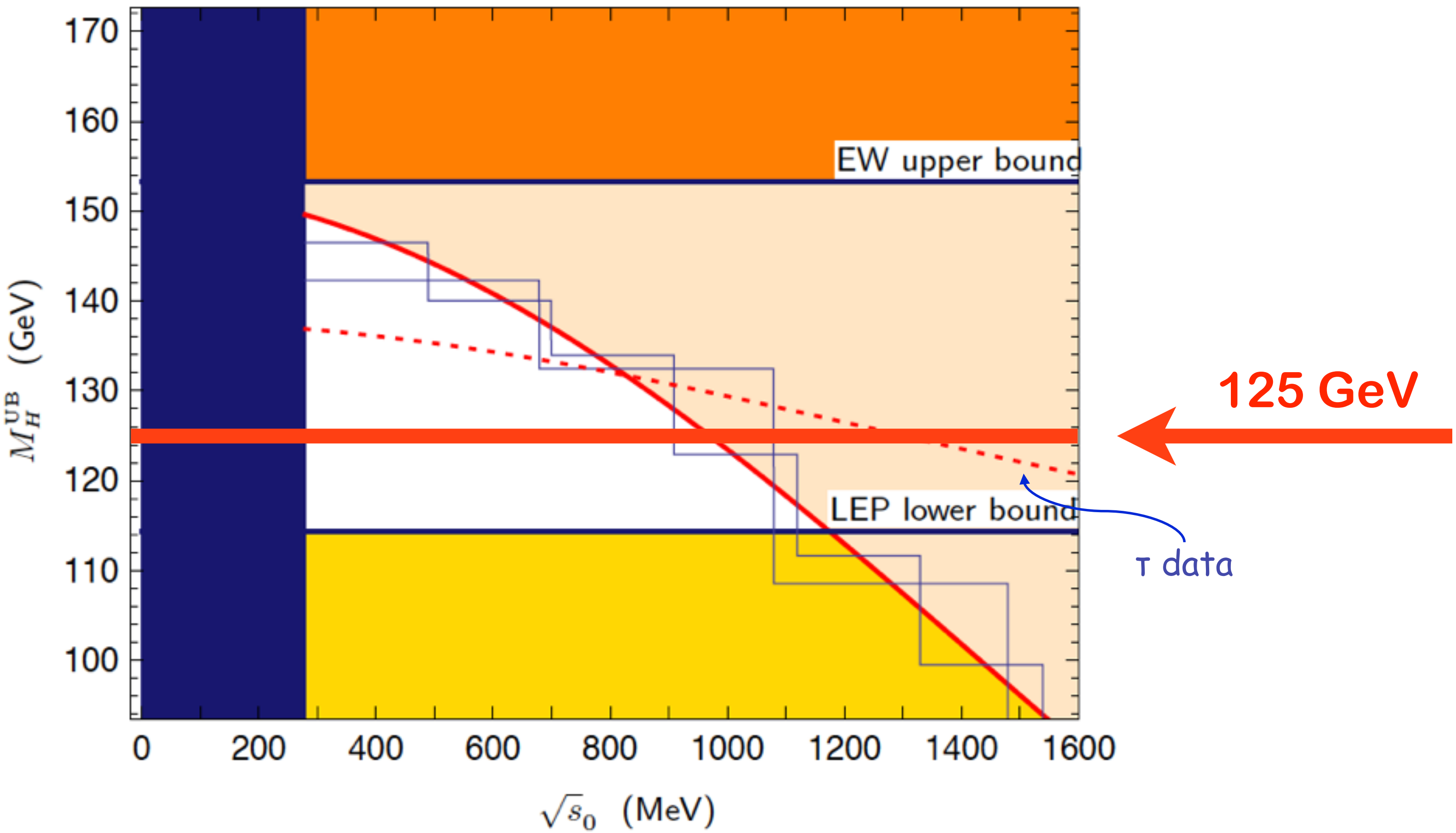
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

(ε>0), in the range:

$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2] \longrightarrow$$

- **How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?**



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The muon g-2: connection with the SM Higgs mass (3)

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

- Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV) where $\sigma(s)$ is precisely measured.

- Vice versa, assuming we now have a SM Higgs with M_H = 125 GeV, if we bridge the M_H discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

μ

Limiting two-Higgs-doublet models with the muon g-2

A. Broggio, E.J. Chun, MP, K. Patel, S. Vempati

arXiv:1409.3199 (JHEP 2014)

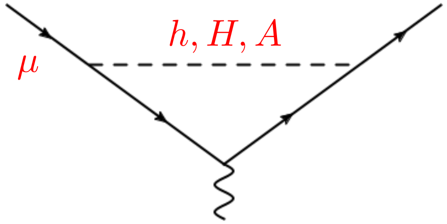
One-loop contribution:

$$\delta a_\mu^{\rm 2HDM}(1{\rm loop}) = \frac{G_F\, m_\mu^2}{4\pi^2\sqrt{2}} \sum_{j=h,H,A,H^\pm} \left(y_\mu^j\right)^2 r_\mu^j\, f_j(r_\mu^j)$$

For $r_\mu^j = m_\mu^2/M_j^2 \ll 1$:

roughly scales with m_μ^4!

$$f_{h,H}(r) \sim -\ln r - 7/6 + O(r) > 0$$
$$f_A(r) \sim +\ln r + 11/6 + O(r) < 0$$
$$f_{H^\pm}(r) \sim -1/6 + O(r) < 0$$



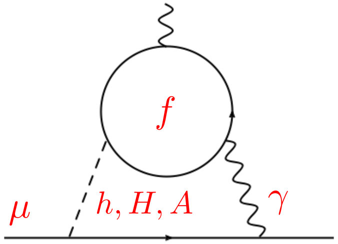
Two-loop Barr-Zee type diagrams:

m_f^2/m_μ^2 w.r.t. 1 loop

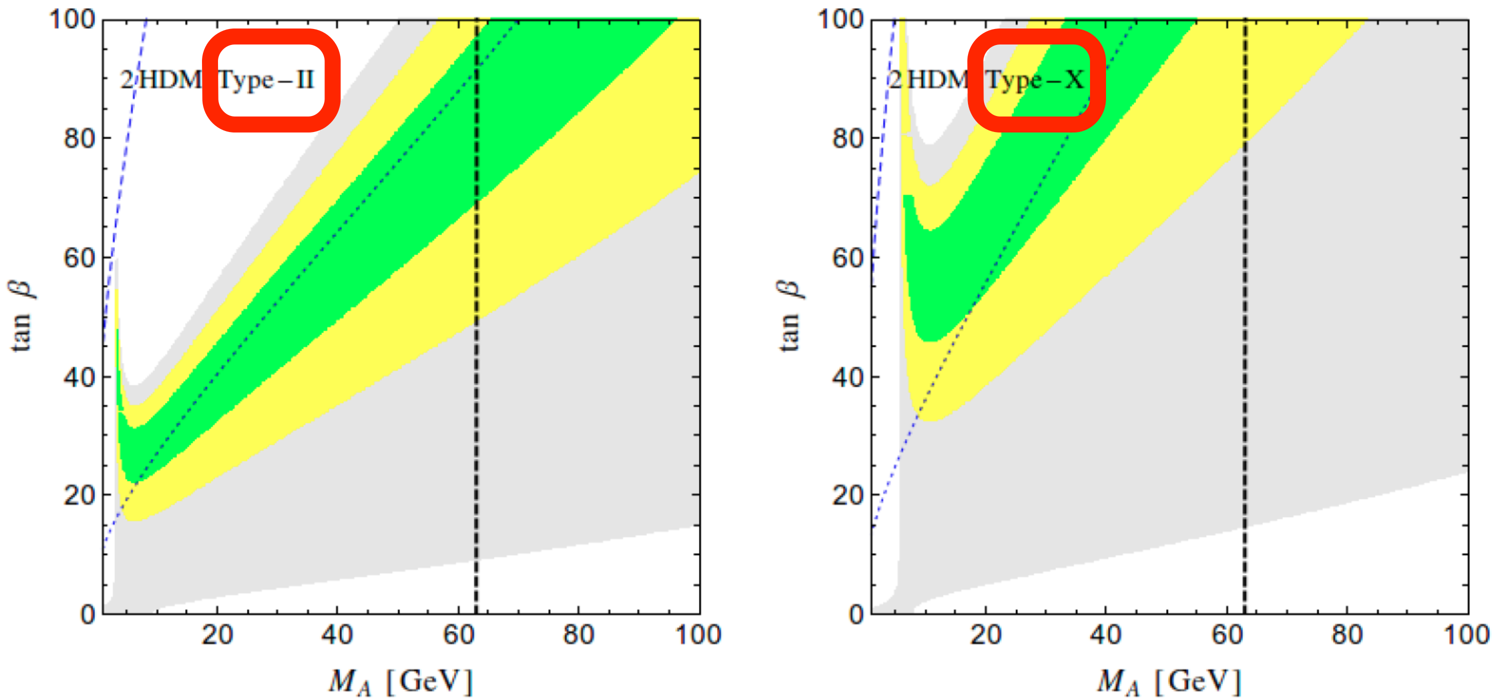
$$\delta a_\mu^{\rm 2HDM}(2{\rm loop-BZ}) = \frac{G_F\, m_\mu^2}{4\pi^2\sqrt{2}} \frac{\alpha_{\rm em}}{\pi} \sum_{f;\, i=h,H,A} N_f^c\, Q_f^2\, y_\mu^i\, y_f^i\, r_f^i\, g_i(r_f^i)$$

$$g_{h,H}(r) < 0$$
$$g_A(r) > 0$$

In type II models, for $M_A \gtrsim$ 3GeV: $(2{\rm loop})_A > (1{\rm loop})_A$. Similar for X.

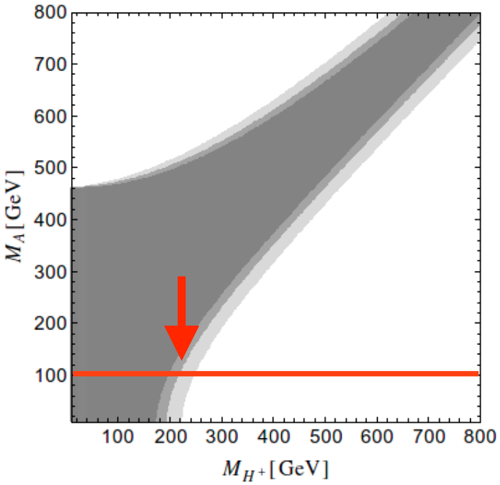


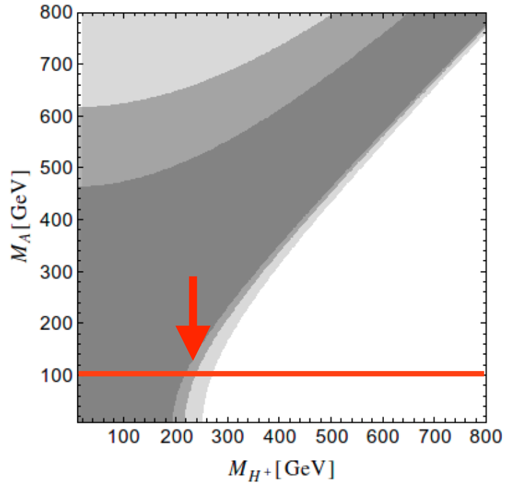
Limiting 2HDMs with a_μ



The 1σ, 2σ and 3σ regions allowed by Δa_μ in the M_A-$\tan\beta$ plane taking the limit of $\beta - \alpha = \pi/2$ and $M_{h(H)} = 126$ (200) GeV in type II (left panel) and type X (right panel) 2HDMs. The regions below the dashed (dotted) lines are allowed at 3σ (1.4σ) by Δa_e. The vertical dashed line corresponds to $M_A = M_h/2$.

The contribution of the τ loop is enhanced by a factor $\tan^2\beta$ both in type II and in X models; it is suppressed by $1/\tan^2\beta$ in models of type I and Y.



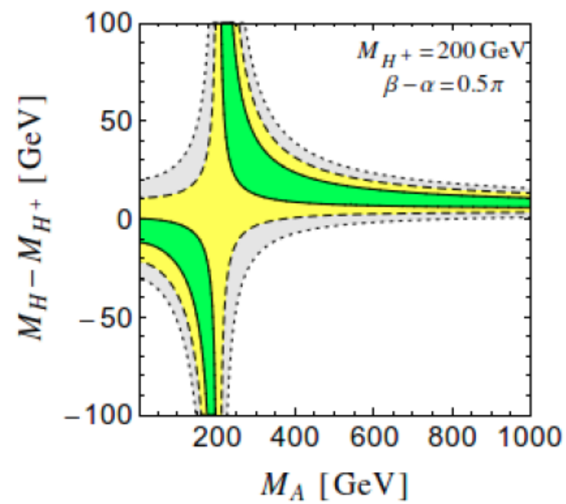


Vacuum stability and perturbativity constraints ($\beta - \alpha = \pi/2$ and $M_h = 126$ GeV). Left: allowed regions for $\Delta M \equiv M_H - M_{H^\pm} = \{20, 0, -30\}$ GeV (darker to lighter), $\lambda_{\max} = \sqrt{4\pi}$. Right: $\lambda_{\max} = \{\sqrt{4\pi}, 2\pi, 4\pi\}$, $\Delta M = 0$. $\tan\beta = 50$, but negligible change for $\tan\beta \in [5, 100]$.

$M_A \lesssim 100$ GeV → $M_{H\pm} \lesssim 200$ GeV

All values of M_A are allowed by EW precision tests if $M_H \sim M_{H\pm}$.

Regions allowed by EW precision constraints: green, yellow, gray for $\Delta\chi^2_{\rm EW} < 2.3, 6.2, 11.8$, i.e. 68.3, 95.4, 99.7% confidence intervals.



Limiting two-Higgs-doublet models

- Type I and Y models cannot account for the present value of Δa_μ due to their lack of $\tan^2\beta$ enhancements.

- In type II (and Y) models the $BR(b \to s\gamma)$ sets a strong lower bound on $M_{H\pm}$ of order 380 GeV [Hermann, Misiak, Steinhauser 2012] $\to$ hardly any space left for the light A required by Δa_μ.

- In type X models, no such strong bounds on $M_{H\pm}$ from $BR(b \to s\gamma)$, only model-indep. LEP bound $M_{H\pm} \gtrsim 80$ GeV.

- Therefore, out of type I, II, X, Y models, only type X is consistent with all the constraints we considered, provided that $M_A \lesssim 100$ GeV, $80 \lesssim M_{H\pm} \lesssim 200$ GeV , $M_H \sim M_{H\pm}$, large $\tan\beta$. Work in progress for additional constraints.

Testing the SM with the electron g-2

G.F. Giudice, P. Paradisi & MP

arXiv:1208.6583 (JHEP 2012)

The QED prediction of the electron g-2

e

$a_e^{QED} = + (1/2)(\alpha/\pi) - 0.328\,478\,444\,002\,55(33)\,(\alpha/\pi)^2$

Schwinger 1948 Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78...$ → $O(10^{-18})$ in a_e

$A_2^{(4)}(m_e/m_\mu) = 5.197\,386\,68\,(26) \times 10^{-7}$

$A_2^{(4)}(m_e/m_\tau) = 1.837\,98\,(33) \times 10^{-9}$

$+ 1.181\,234\,016\,816\,(11)\,(\alpha/\pi)^3$ → $O(10^{-19})$ in a_e

Kinoshita; Barbieri; Laporta, Remiddi; … , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$A_1^{(6)} = 1.181\,241\,456\,587...$

$A_2^{(6)}(m_e/m_\mu) = -7.373\,941\,62\,(27) \times 10^{-6}$

$A_2^{(6)}(m_e/m_\tau) = -6.5830\,(11) \times 10^{-8}$

$A_3^{(6)}(m_e/m_\mu, m_e/m_\tau) = 1.909\,82\,(34) \times 10^{-13}$

$- 1.91206\,(84)\,(\alpha/\pi)^4$ → $0.2\ 10^{-13}$ in a_e

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012 & 2015; Kurz, Liu, Marquard & Steinhauser 2014: analytic heavy virtual lepton part.

$+ 7.79\,(34)\,(\alpha/\pi)^5$ **Complete Result! (12672 mass indep. diagrams!)**

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807; PRD 91 (2015) 3, 033006

→ $0.2\ 10^{-13}$ in a_e **NB: $(\alpha/\pi)^6 \sim O(10^{-16})$**

The SM prediction of the electron g-2

- The SM prediction is:

$$a_e^{SM}(\alpha) = a_e^{QED}(\alpha) + a_e^{EW} + a_e^{HAD}$$

- The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [value from Codata10]

$$a_e^{EW} = 0.2973\,(52) \times 10^{-13}$$

- The Hadronic contribution, at LO+NLO+NNLO, is:

Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{HAD} = 17.10\,(17) \times 10^{-13}$$

$$a_e^{HLO} = +\,18.66\,(11) \times 10^{-13}$$

$$a_e^{HNLO} = [-2.234(14)_{vac} + 0.39(13)_{lbl}] \times 10^{-13}$$

$$a_e^{HNNLO} = +\,0.28\,(1) \times 10^{-13}$$

- Which value of α should we use to compute a_e^{SM}?

The electron g-2 gives the best determination of alpha

e

- **The 2008 measurement of the electron g-2 is:**

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- **Equate** $a_e^{SM}(\alpha) = a_e^{EXP}$ → **best determination of alpha (2015):**

$$\alpha^{-1} = 137.035\,999\,157\,(33) \qquad [0.24\ \text{ppb}]$$

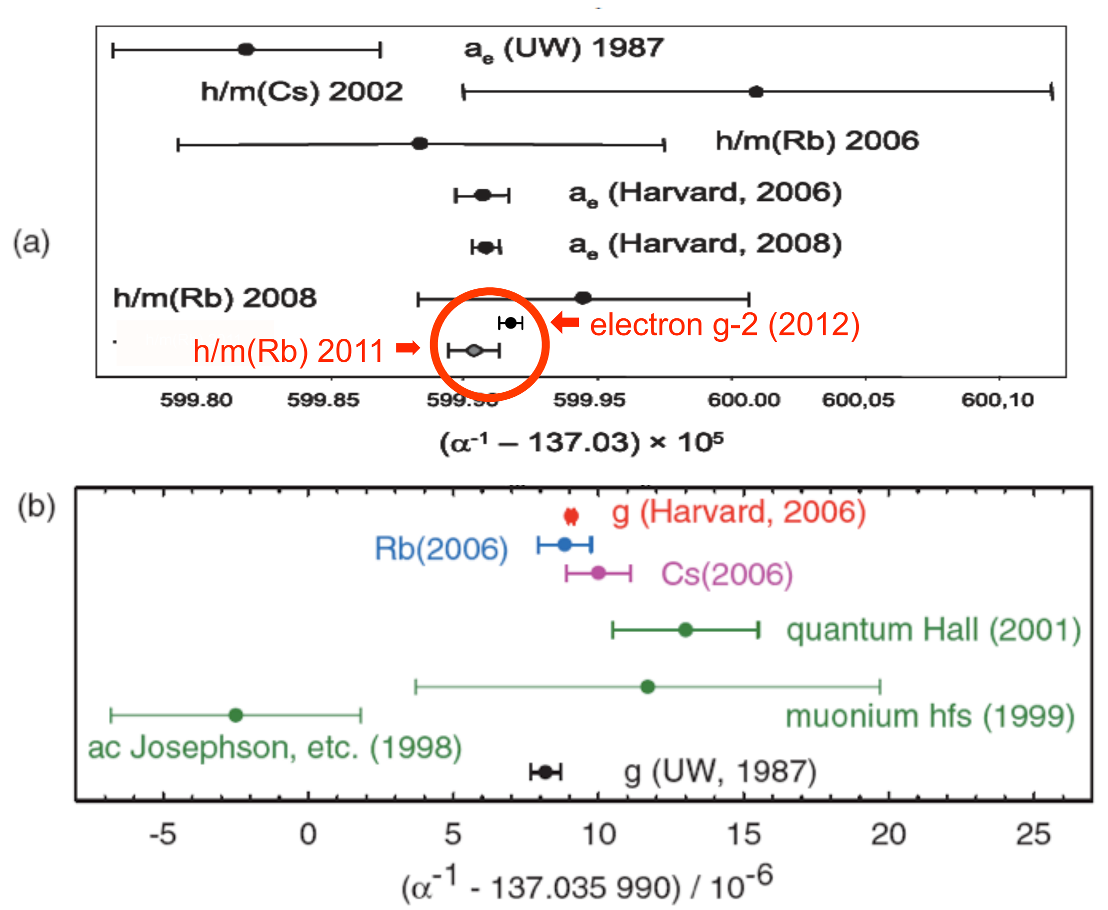
- **Compare it with other determinations (independent of a_e):**

$\alpha^{-1} = 137.036\,000\,0\,(11)$ [7.7 ppb] PRA73 (2006) 032504 (Cs)

$\alpha^{-1} = 137.035\,999\,049\,(90)$ [0.66 ppb] PRL106 (2011) 080801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!

Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902
Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801
Bouchendira et al, PRL106 (2011) 080801

The electron g-2: SM vs Experiment

e

- Using α = 1/137.035 999 049 (90) [^{87}Rb, 2011], the SM prediction for the electron g-2 is

$$a_e^{SM} = 115\,965\,218\,16.5\,(0.2)\,(0.2)\,(0.2)\,(7.6) \times 10^{-13}$$

(0.2) → δC_4^{qed}; (0.2) → δC_5^{qed}; (0.2) → δa_e^{had}; (7.6) → from $\delta\alpha$

- The EXP-SM difference is (note the negative sign):

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -9.2\,(8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1σ).
NB: The 4-loop contrib. to a_e^{QED} is $-556 \times 10^{-13} \sim 70\, \delta\Delta a_e$!
(the 5-loop one is 6.2×10^{-13})

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie ($10^{-13}$ units):

$$\underbrace{(0.2)_{\rm QED4}, \quad (0.2)_{\rm QED5}, \quad (0.2)_{\rm HAD}}_{(0.4)_{\rm TH}}, \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\rm EXP}}$$

← may drop to 0.2

- The $(g-2)_e$ exp. error may soon drop below 10^{-13} and work is in progress for a significant reduction of that induced by $\delta\alpha$.

→ sensitivity of 10^{-13} may be reached with ongoing exp. work

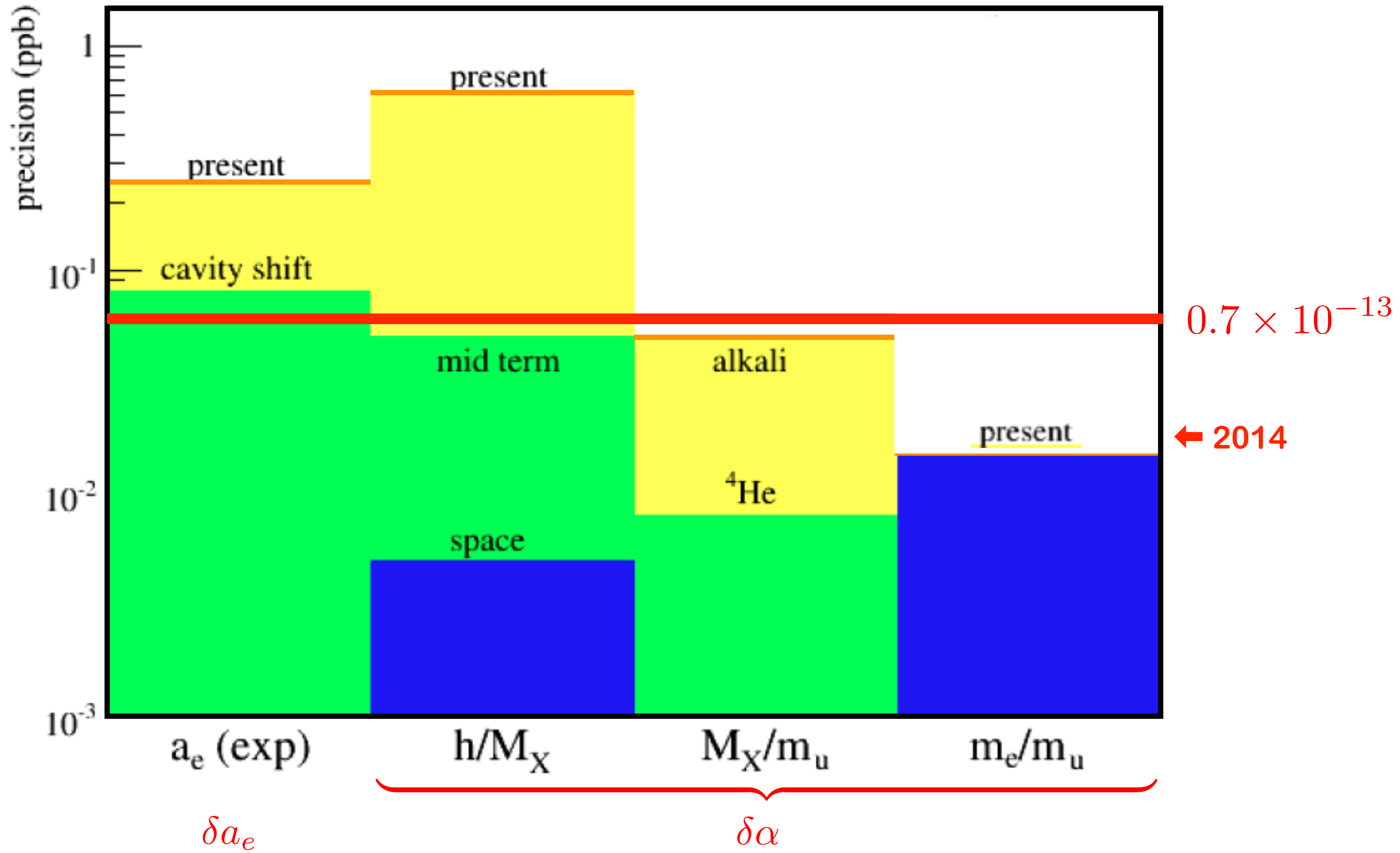
- In a broad class of BSM theories, contributions to a_l scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}$$

The electron g-2 sensitivity and NP tests (2)



Summary of the exp. contributions to the relative uncertainty of δα and δaₑ (in ppb).

F. Terranova & G.M. Tino, PRA89 (2014) 052118

The electron g-2 sensitivity and NP tests (3)

e

- The experimental sensitivity in Δa_e is not very far from what is needed to test if the discrepancy in $(g\text{-}2)_\mu$ also manifests itself in $(g\text{-}2)_e$ under the naive scaling hypothesis.

- NP scenarios exist which violate Naive Scaling. They can lead to larger effects in Δa_e and contributions to EDMs, LFV or lepton universality breaking observables.

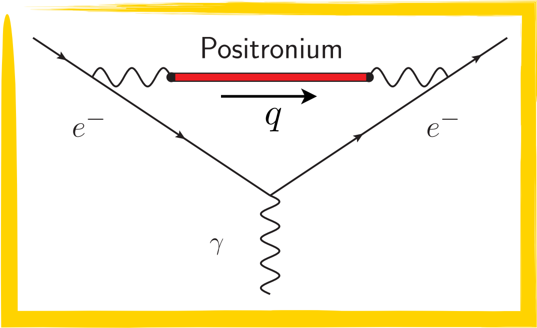
- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), Δa_e can reach 10^{-12} (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

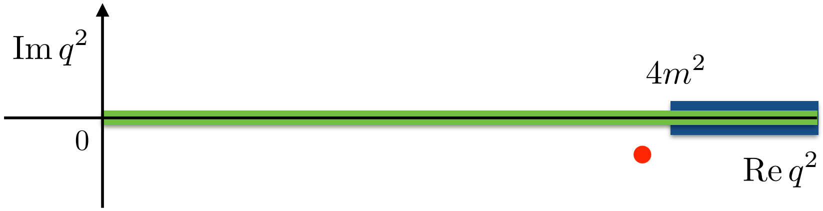
Positronium contribution to the electron g-2

M. Fael & MP,   arXiv:1402.1575 (PRD 2014)

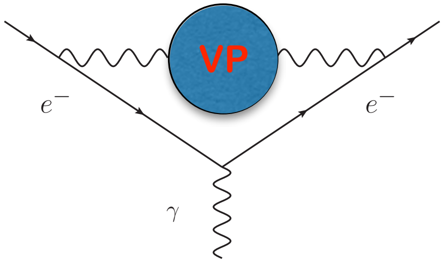
- **The leading contribution of positronium to a_e comes from:**

Mishima 1311.7109; Fael & MP 1402.1575; Melnikov et al. 1402.5690; Eides 1402.5860; Hayakawa 1403.0416





- **The e^+e^- bound states appear as poles in the vac. pol. $\Pi(q^2)$ right below the branch-point $q^2 = (2m)^2$. Their contribution is:**



$$a_e(\mathrm{vp}) = \frac{\alpha}{\pi^2}\int_0^\infty \frac{ds}{s}\, \mathrm{Im}\,\Pi(s+i\epsilon)\, K(s)$$

$$a_e^{\mathrm{P}} = \frac{\alpha^5}{4\pi}\zeta(3)\underbrace{\left(8\ln 2 - \frac{11}{2}\right)}_{K(4m^2)} = 0.9\times 10^{-13} = 1.3\left(\frac{\alpha}{\pi}\right)^5$$

Mishima 1311.7109

- **Of the same magnitude of the exp. unc. of a_e & the "naively rescaled" muon Δa_μ. Of the same order of α as the 5-loop term!**

- Melnikov, Vainshtein & Voloshin (MVV) 1402.5690 determined a nonpert. contrib. of the e^+e^- continuum right above threshold that cancels one-half of a_e^P:

$$a_e(\text{vp})^{\text{cont,np}} = -\frac{|\alpha|^5}{8\pi}\zeta(3)\left(8\ln 2 - \frac{11}{2}\right)$$

- Check: the total positronium poles + continuum nonperturbative contribution to a_e arising from the threshold region at LO in α is:

$$a_e^{\text{thr}}(\text{vp}) = -\frac{\alpha}{\pi}K(4m^2)\,\text{Re}\,A(1)$$

with

$$A(\beta) = -\frac{\alpha^2}{2}\left[\gamma + \psi\left(1 - \frac{i\alpha}{2\beta}\right)\right] = \frac{\alpha^2}{2}\sum_{k=1}^{\infty}\zeta(k+1)\left(\frac{i\alpha}{2\beta}\right)^k$$

so that

$$a_e^{\text{thr}}(\text{vp}) = \frac{\alpha^5}{8\pi}\zeta(3)\,K(4m^2) = \frac{a_e^{\text{P}}}{2}$$

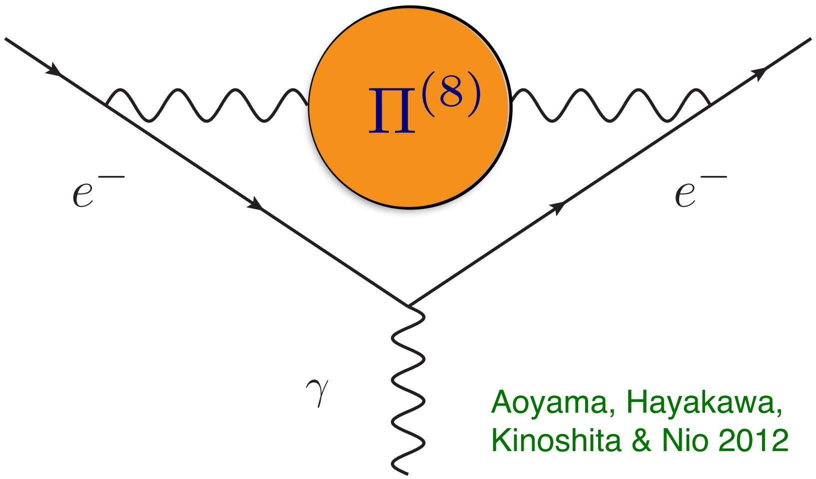
Positronium contribution to the electron g-2 (III)

- So, should we add this total threshold contribution $a_e^P/2$ to the perturbative QED 5-loop result of Kinoshita and collaborators?

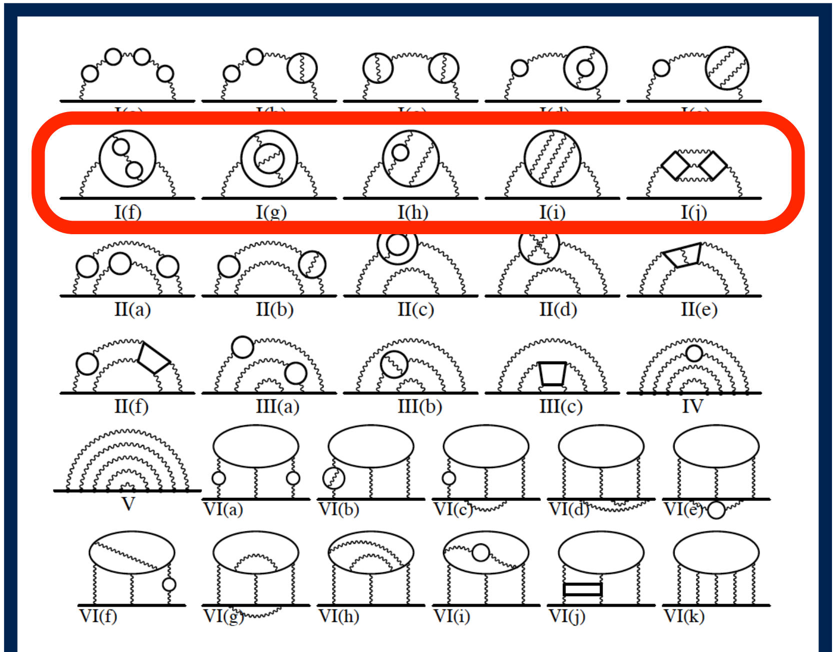
- Using the Coulomb Green's function, MVV 1402.5690 argued that it is already contained in the contribution of $O(\alpha^5)$.

- Hayakawa 1403.0416 claimed that positronium contributes to a_e only through a specific class of diagrams of $O(\alpha^7)$.

- To address this question we studied the 5-loop QED contribution to a_e arising from the insertion of the 4-loop VP in the photon line. This has been computed via:



Aoyama, Hayakawa, Kinoshita & Nio 2012



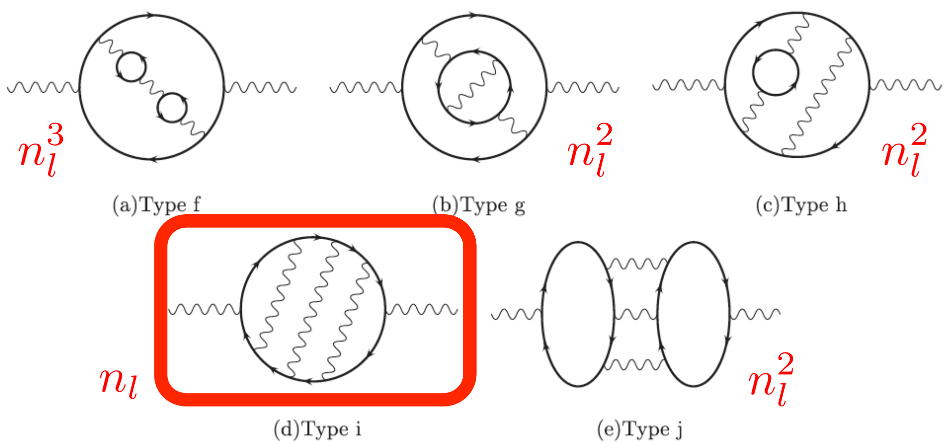
Positronium contribution to the electron g-2 (IV)

- **Using explicit expressions for $\Pi^{(8)}(q^2)$ (Baikov, Maier, Marquard '13) in**

$$a_e^{(10)}(\mathrm{vp}) = -\frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Pi^{(8)}\left(-\frac{m^2x^2}{1-x}\right)$$

we obtain:

$$a_e^{(10)}(\mathrm{vp}) \;=\; n_e\,\frac{\alpha^5}{8\pi}\zeta(3)\,K(4m^2) + \cdots \;=\; \frac{a_e^{\mathrm{P}}}{2} + \cdots$$



- **$a_e^P/2$ is already included in the 5-loop contribution of class I(i).**
- **There is no additional contrib of QED bound states beyond PT!**

M.A. Braun 1968; Barbieri, Christillin, Remiddi 1973

The tau g-2: opportunities or fantasies?

Work in progress in collaboration with
S. Eidelman, D. Epifanov, M. Fael, L. Mercolli

arXiv:1301.5302
arXiv:1310.1081
arXiv:1506.03416

The SM prediction of the tau g-2

τ

- The Standard Model prediction of the tau g-2 is:

a_τ^{SM}	=	117324	(2)	x 10^{-8}	QED
	+	47.4	(0.5)	x 10^{-8}	EW
	+	337.5	(3.7)	x 10^{-8}	HLO
	+	7.6	(0.2)	x 10^{-8}	HHO (vac)
	+	5	(3)	x 10^{-8}	HHO (lbl)

$$a_\tau^{SM} = 117721\ (5) \times 10^{-8}$$

Eidelman & MP 2007

- $(m_\tau/m_\mu)^2 \sim 280$: great opportunity to look for New Physics, and a "clean" NP test too…

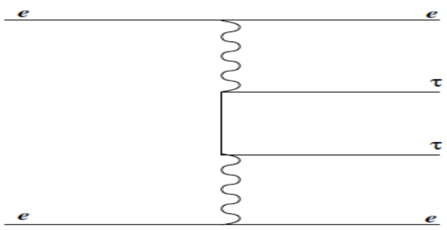
	Muon	Tau
aEW/aH	1/45	1/7
aEW/ δaH	3	10

… if only we could measure it!!

The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine a_τ measuring its spin precession in a magnetic field.

- DELPHI's result, from $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):



$$a_\tau = -0.018\ (17)$$

PDG 2014

- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

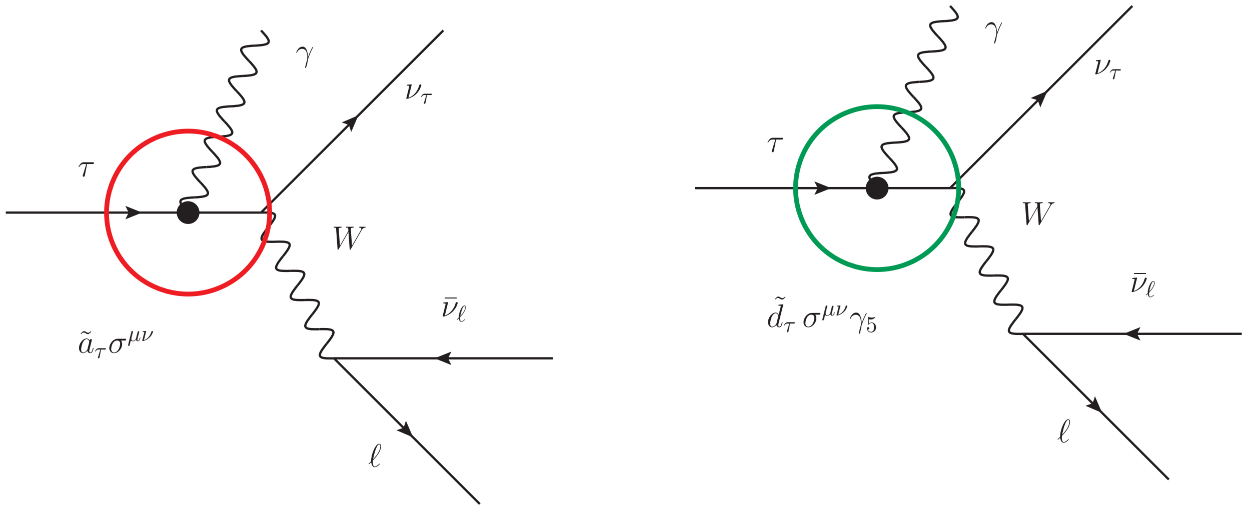
$$-0.007 < a_\tau^{NP} < 0.005 \quad (95\%\ \text{CL})$$

Gonzáles-Sprinberg et al 2000

- Bernabéu et al, propose the measurement of $F_2(q^2=M_\Upsilon^2)$ from $e^+e^- \rightarrow \tau^+\tau^-$ production at B factories. NPB 790 (2008) 160

- Must employ the polarized differential decay rate.
- Study the full phase space.

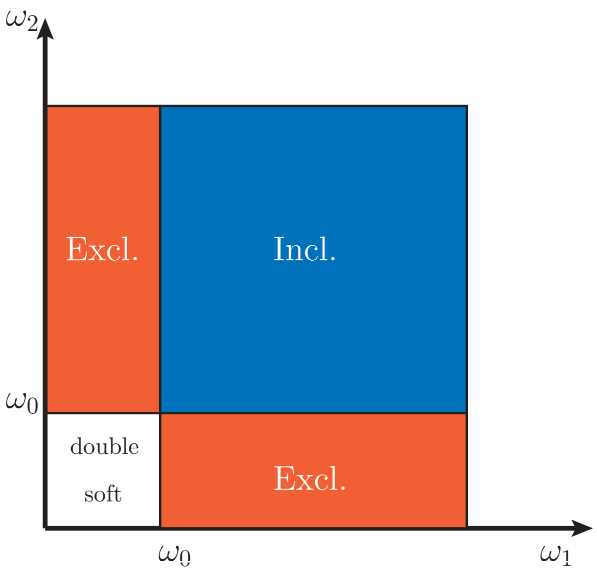
$$d\Gamma = d\Gamma_0 + \left(\frac{m_\tau}{M_W}\right)^2 d\Gamma_W + \frac{\alpha}{\pi} d\Gamma_{\rm NLO} + \tilde{a}_\tau \, d\Gamma_{\rm a} + \tilde{d}_\tau \, d\Gamma_{\rm d}$$



- Under investigation for Belle-II analysis. Should at least improve the Delphi bound. Work in progress.

The branching ratio of radiative μ and τ leptonic decays for a minimum photon energy ω_0:

$$\mathcal{B}(\omega_0) \propto \int d\Phi_4 \left(d\Gamma_{\rm LO} + d\Gamma_{\rm virt}\right) + \int d\Phi_5 \, d\Gamma_{\gamma\gamma}$$



- $\mathcal{B}^{\rm Exc}(\omega_0)$: only one γ of energy $\omega > \omega_0$, additional second soft photon $\omega' < \omega_0$.

$$\mathcal{B}^{\rm Exc}(\omega_0) = \blacksquare$$

- $\mathcal{B}^{\rm Inc}(\omega_0)$: at least one γ of energy $\omega > \omega_0$.

$$\mathcal{B}^{\rm Inc}(\omega_0) = \blacksquare + \blacksquare$$

Radiative leptonic tau decays: branching ratios

τ

B.R. of radiative τ leptonic decays ($\omega_0 = 10$ MeV)	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm LO}$	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}^{\rm Inc}_{\rm NLO}$	$-1.06\,(1)_n(10)_N \times 10^{-3}$	$-5.8\,(1)_n(2)_N \times 10^{-5}$
$\mathcal{B}^{\rm Exc}_{\rm NLO}$	$-1.89\,(1)_n(19)_N \times 10^{-3}$	$-9.1\,(1)_n(3)_N \times 10^{-5}$
$\mathcal{B}^{\rm Inc}$	$1.728\,(10)_{\rm th}(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\rm Exc}$	$1.645\,(19)_{\rm th}(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\dagger}_{\rm EXP}$	$1.847\,(15)_{\rm st}(52)_{\rm sy} \times 10^{-2}$	$3.69\,(3)_{\rm st}(10)_{\rm sy} \times 10^{-3}$

(n): numerical errors

(N): uncomputed NNLO corr.

$\sim (\alpha/\pi) \ln r \ln(\omega_0/M) \times \mathcal{B}^{\rm Exc/Inc}_{\rm NLO}$

†BABAR - PRD 91 (2015) 051103

(th): combined $(n) \oplus (N)$

(τ): experimental error of τ lifetime: $\tau_\tau = 2.903(5) \times 10^{-13}$ s

	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\Delta^{\rm Exc}$	$2.02\,(57) \times 10^{-3} \to 3.5\sigma$	$1.2\,(1.0) \times 10^{-4} \to 1.1\sigma$

Fael, Mercolli, MP, arXiv:1506.03416 (JHEP 2015)

Conclusions

- The muon discrepancy is $\Delta a_\mu \sim 3\div3.5\ \sigma$. Is it NP? New upcoming g-2 experiment: QED & EW terms ready for the challenge; how about the hadronic one? Future of hadronic LBL??
- Could Δa_μ be due to mistakes in the hadronic $\sigma(s)$? Very unlikely. Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energies (below ~ 1GeV).
- Can Δa_μ be solved by 2HDMs? Not by type I, II, and Y. Type X is still allowed by all the constraints we considered if $M_A \lesssim 100$GeV, $80 \lesssim M_{H\pm} \lesssim 200$GeV , $M_H \sim M_{H\pm}$ & large $\tan\beta$.
- The sensitivity of the electron g-2 has improved. It may soon be possible to test if Δa_μ manifests itself also in the electron g-2! A robust and ambitious experimental program is under way to improve α & a_e. The positronium contribution shouldn't be added!
- The tau g-2 is essentially unknown: we propose to measure it at Belle II via its radiative leptonic decays. BaBar's recent precise measurement of $\mathcal{B}(\tau \to e\bar{\nu}\nu\gamma)$ differs from our prediction by 3.5σ!

The End