Indirect determinations of the top quark mass

Paride Paradisi

University of Padua

LC15
8 September 2015, ECT*, Trento, Italy

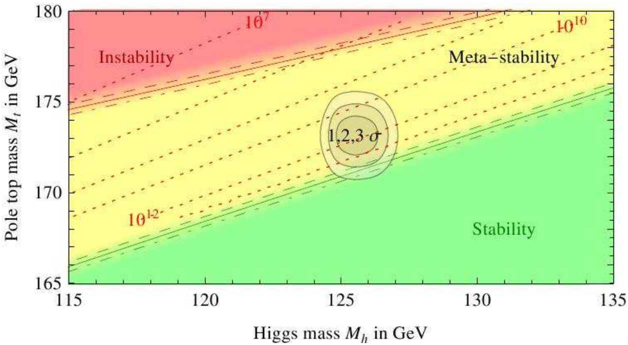
Plan of Talk

Based on arXiv:1508.05332 in collaboration with Giudice and Strumia

1. The role of M_t in particle physics and cosmology
2. M_t dependence of observables in the heavy-top limit
3. Extracting M_t from flavour data
4. Future determinations of M_t from flavour
5. Extracting M_t from electroweak precision data
6. Conclusions and future prospects

The role of M_t in particle physics and cosmology

- **A more precise determination of $M_t = vy_t$ will add important information to our knowledge of particle physics and cosmology. Since y_t is sizeable, it plays a crucial role in the predictions of the SM at the quantum level.**
  - **Stability of the EW vacuum:** if no NP modifies the short-distance behaviour of the SM, top-quark loops destabilise the Higgs potential even for $\delta M_t \sim 2$ GeV.
  - **Inflation:** small changes in M_t have importnt effects in the evolution of the universe at the inflationary epoch and determine the viability of scenarios of Higgs inflation.



Current determination of Mt @ collider

- **The most precise quoted value of the top-quark pole mass comes from the combination of LHC and Tevatron measurements** [LHC & Tevatron '14]

$$(M_t)_{\rm pole} = 173.34 \pm 0.76 \ {\rm GeV}$$

  - M_t **is not a physical observable**: Its extraction is done through final-state invariant masses, kinematic distributions, total rates especially sensitive to M_t.
  - **"Monte-Carlo mass"**: In the context of hadron colliders, the extraction of M_t suffers from a variety of effects linked to hadronization like bound-state effects of the $t\bar{t}$ pairs, parton showering, and other non-perturbative corrections. The extraction of M_t relies on modelling based on Monte-Carlo generators.

- **More robust determination of** M_t: through observables calculable in terms of $(M_t)_{\rm run}$ in perturbative QCD such as the total inclusive $t\bar{t}$ cross section. $(M_t)_{\rm run}$ is then translated into $(M_t)_{\rm pole}$ by a relation now known at four-loops in QCD.

$$(M_t)_{\sigma_{t\bar{t}}} = \begin{cases} 172.9 \pm 2.6 \ {\rm GeV} & \text{ATLAS} \\ 176.7 \pm 2.9 \ {\rm GeV} & \text{CMS} \end{cases}$$

- e^+e^- **collider operating at the** $t\bar{t}$ **threshold:** scans of the $t\bar{t}$ pair production would reach a statistical accuracy on the mass measurement of about 20–30 MeV [Seidel et al,'13]. Recent N^{3}LO calculations can relate such measurements to a well-defined M_t, with a theoretical uncertainty below about 50 MeV [Beneke et al.,'15].

Gauge-less theory

- **Sensitivity of observables to M_t:** the large y_t enable us to infer M_t from SM quantum effects. The observables that are more sensitive to M_t are identified working in the heavy-top limit $M_t \gg M_W, M_Z$ [Barbieri et al.,'92].

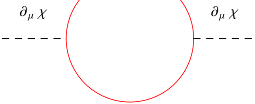
- **Gauge-less theory:** theory with massive quarks, the Higgs boson h, and 3 Goldstone bosons $\vec{\chi}$ related by the equivalence theorem to the longitudinal components of the W and Z

$$L = y_t \, \bar{t}_R \, H^T \begin{pmatrix} V_{ti} \, d_{iL} \\ -t_L \end{pmatrix} + \mathrm{h.c.}\,, \qquad H = \frac{1}{\sqrt{2}} e^{\frac{i\vec{\sigma}\cdot\vec{\chi}}{v}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

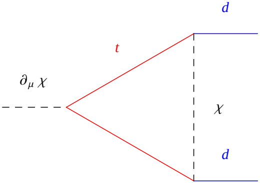
$$\begin{aligned} L = \quad & - \quad \frac{y_t}{\sqrt{2}} \left(\cos|\vec{\chi}|/v\right)(v+h)\,\bar{t}t \\ & + \quad y_t \left(\frac{\sin|\vec{\chi}|/v}{|\vec{\chi}|/v}\right)\left(1+\frac{h}{v}\right)\left[\frac{i}{\sqrt{2}}\chi^0 \bar{t}\gamma_5 t + \left(\chi^+ \bar{t}_R V_{ti}\, d_{iL} + \mathrm{h.c.}\right)\right], \end{aligned}$$

- **Top-less effective theory:** the next step is to integrate out the top quark and this will generate a set of effective operators whose coefficients describe the leading top-mass dependence in the large M_t limit.

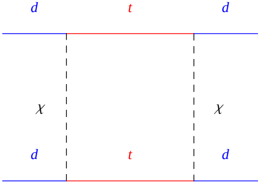
$\Delta\rho \propto y_t^2$



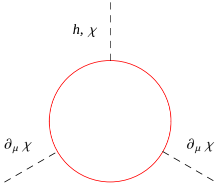
$A(Z\to b\bar{b},\, B_s\to\mu^+\mu^-,\, K\to\pi\nu\bar{\nu}) \propto \frac{y_t^3}{M_t}$



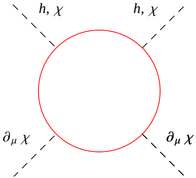
$\Delta m_B,\, \epsilon_K \propto \frac{y_t^4}{M_t^2}$



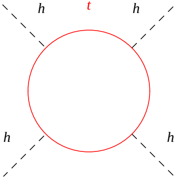
$g_{hZZ}, g_{hWW}, g_{ZWW} \propto \frac{y_t^3}{M_t}$



$A(WW\to WW) \propto \frac{y_t^4}{M_t^2}$



$A(hh\to hh) \propto y_t^4$



[Giudice, P.P. & Strumia, '15]

M_t dependence of observables in the heavy-top limit

- $\boldsymbol{\Delta\rho}$ is generated by the dim-4 operator $(\partial_\mu\chi)^2$ and therefore $\boldsymbol{\Delta\rho \propto y_t^2}$

$$\Delta\rho = \frac{3y_t^2}{32\pi^2} = \frac{3G_F M_t^2}{8\sqrt{2}\pi^2}$$

- $\boldsymbol{Z \to b\bar{b}}$, $\boldsymbol{K \to \pi\nu\bar{\nu}}$ and $\boldsymbol{B_s \to \ell^+\ell^-}$ arise from the dim-5 operator $(\bar{d}_L\gamma^\mu d_L)(\partial_\mu\chi^0)$ with coefficient of order $|V_{td}|^2 y_t^3/(16\pi^2 M_t)$.

$$\frac{g}{c_W}\,\bar{d}_i\left[(g_L + \Delta g_L)^{ij} P_L + g_R^{ij} P_R\right] \not{Z} d_j\,, \qquad \Delta g_L^{ij} = \frac{V_{ti}^* V_{tj}\, y_t^2}{32\pi^2}\,,$$

$$\Gamma(Z \to b\bar{b}) = \frac{\rho\, G_F M_Z^3}{\pi\sqrt{2}}\left[(g_L^{bb} + \Delta g_L^{bb})^2 + (g_R^{bb})^2\right]$$

$$\mathcal{H}^{\rm eff}_{K\to\pi\nu\bar{\nu}} = \frac{\Delta g_L^{sd}}{2v^2}\,(\bar{s}_L\gamma^\mu d_L)(\bar{\nu}_L^\ell\gamma_\mu\nu_L^\ell) + \text{h.c.}\,,$$

$$\mathcal{H}^{\rm eff}_{B_s\to\ell^+\ell^-} = -\frac{\Delta g_L^{bs}}{2v^2}\,(\bar{b}_L\gamma^\mu s_L)(\bar{\ell}_L\gamma_\mu\ell_L) + \text{h.c.}$$

- $\boldsymbol{\Delta m_{B_q}}$ and $\boldsymbol{\epsilon_K}$ arise from dim-6 operator involving four d_L fields.

$$\mathcal{H}^{\rm eff}_{\Delta F=2} = \frac{y_t^2 (V_{ti}^* V_{tj})^2}{256\pi^2\, v^2}(\bar{d}_{iL}\gamma^\mu d_{jL})(\bar{d}_{iL}\gamma_\mu d_{jL}) + \text{ h.c.}$$

M_t dependence of observables in the heavy-top limit

- **Triple gauge boson vertices and *WW* scattering:** dim-5 or dim-6 operators such as $h(\partial_\mu\chi)^2$, $\chi(\partial_\mu\chi)^2$, $h^2(\partial_\mu\chi)^2$, and $\chi^2(\partial_\mu\chi)^2$ sensitive on M_t^2. Experimental sensitivity too poor to allow for any significant determination of M_t.

- ***hh* → *hh* scattering:** it receives a correction $\mathcal{O}(y_t^4/16\pi^2)$. This explains the importance of the top-mass measurement for vacuum stability considerations.

- $\boldsymbol{B \to X_s\gamma}$: the coefficient of the dim-6 operator $m_b\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}$ is estimated to be $(eV_{tb}V_{ts}^*/16\pi^2)\times(y_t^2/M_t^2)$ with no power sensitivity on M_t.

$$BR(B \to X_s\gamma) \propto M_t^{0.38}$$

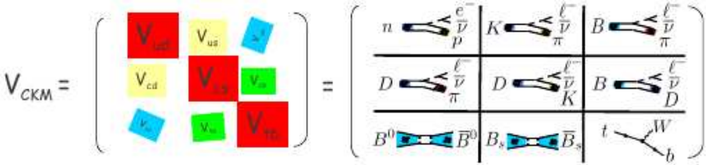
- $\boldsymbol{h \to \gamma\gamma, \gamma Z}$: is induced by the operators $hF_{\mu\nu}^2$, $hF_{\mu\nu}Z_{\mu\nu}$, $h(\partial_\mu\chi^0)\partial_\nu F_{\mu\nu}$, with coefficients of order $(e^2/16\pi^2)\times(y_t/M_t)$ for the first two dim-5 operators and $(e/16\pi^2)\times(y_t^2/M_t^2)$ for the third dim-6 operator.

$$\Gamma(h \to \gamma\gamma) \propto M_t^{0.037}, \qquad \Gamma(h \to Z\gamma) \propto M_t^{0.014}$$

- $\boldsymbol{h \to WW^*, ZZ^*}$: comes from the dim-5 operator $h(\partial_\mu\chi)^2$ with coefficient $\mathcal{O}(y_t^3/16\pi^2 M_t)$. Even a futuristic measurement of the branching ratio at 1% could not determine M_t with an error better than 50 GeV.

$$\frac{\Delta\Gamma(h \to WW^*, ZZ^*)}{\Gamma(h \to WW^*, ZZ^*)} = -\frac{5\,y_t^2}{32\pi^2}$$

Extracting M_t from flavour data



- **New Physics Unitary Triangle fit:** when searching for NP, the four CKM parameters are determined by tree-level observables which are expected to be not affected by NP effects.
- **Standard Model Unitary Triangle fit:** if we assume the SM to be the true theory, the best determination of the CKM comes from the loop-induced processes ΔM_d, $\Delta M_d/\Delta M_s$, ϵ_K and $\sin 2\beta$
- **Hybrid Unitary Triangle fit:** assuming the SM to be exactly valid the extraction of M_t from flavour processes is done fixing the four CKM parameters from the most precise measurements independent on M_t, even if they arise at loop level.

$$|V_{us}|, \qquad |V_{cb}|, \qquad \gamma, \qquad \beta$$

CKM parameters in the Hybrid scheme

- For our analysis we will need the following combinations (where $\lambda_i = V_{id} V_{is}^*$)

$$
\begin{aligned}
|V_{td} V_{tb}^*| &= |V_{us}||V_{cb}| \frac{\sin\gamma}{\sin(\gamma+\beta)} \left[1 + \mathcal{O}(\lambda^4)\right] \\
|V_{ts} V_{tb}^*| &= |V_{cb}| \left[1 - \frac{\lambda^2 \sin(\gamma-\beta)}{2\sin(\gamma+\beta)} + \mathcal{O}(\lambda^4)\right] \\
\mathrm{Re}\lambda_t &= -|V_{cb}|^2 |V_{us}| \frac{\sin\gamma\cos\beta}{\sin(\gamma+\beta)} \left[1 + \lambda^2 \left(\frac{1}{2} - \frac{\sin\gamma}{\cos\beta\sin(\gamma+\beta)}\right) + \mathcal{O}(\lambda^4)\right] \\
\mathrm{Re}\lambda_c &= -|V_{us}| \left[1 - \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4)\right], \\
\mathrm{Im}\lambda_t &= -\mathrm{Im}\lambda_c = |V_{cb}|^2 |V_{us}| \frac{\sin\gamma\sin\beta}{\sin(\gamma+\beta)} \left[1 + \frac{\lambda^2}{2} + \mathcal{O}(\lambda^4)\right]
\end{aligned}
$$

- SM predictions for flavour observables are often expressed in terms of the running $\overline{\mathrm{MS}}$ top quark mass $\overline{m}_t(m_t)$, related to the pole top mass M_t by

$$
\frac{M_t}{\overline{m}_t} = 1 + 0.4244\,\alpha_s + 0.8345\,\alpha_s^2 + 2.375\,\alpha_s^3 + (8.49 \pm 0.25)\,\alpha_s^4 = 1.060302(35)
$$

Present values and future uncertainties

Observable	Now (2015)	Error 2020	Error 2025
$M_W(\mathrm{GeV})$	80.385(15)	8	5
$\sin^2\theta_{\mathrm{W}}$	0.23116(13)	13	1.3
$\alpha_s(M_Z)$	0.1184(7)	?	?
$\lvert V_{cb}\rvert \times 10^3$	40.9(11)	4	3
$\lvert V_{ub}\rvert \times 10^3$	3.81(40)	10	8
$\sin 2\beta$	0.679(20)	16	8
γ	$(73.2^{+6.3}_{-7.0})^\circ$	3°	1°
$\Delta m_{B_d}(\mathrm{ps}^{-1})$	0.510(3)	–	–
$\Delta m_{B_s}(\mathrm{ps}^{-1})$	17.757(21)	–	–
$B(B_s \to \mu^+\mu^-) \times 10^9$	2.8(7)	3	1.3
$B(K^+ \to \pi^+\nu\bar{\nu}) \times 10^{11}$	$17.3^{+11.5}_{-10.5}$	0.8	0.4
$B(K_L \to \pi^0\nu\bar{\nu}) \times 10^{11}$	–	2	0.3
$\lvert\epsilon_K\rvert \times 10^{-3}$	2.228(11)	–	–
η_B	0.55(1)	0.5	0.2
$f_{B_s}(\mathrm{MeV})$	226(5)	2	1
$\hat{B}_{B_s}$	1.33(6)	2	0.7
f_{B_s}/f_{B_d}	1.204(16)	10	5
$\hat{B}_{B_s}/\hat{B}_{B_d}$	1.03(8)	2	0.5

[Giudice, P.P. & Strumia, '15]

- **Δm_{B_s}: SM prediction 2015**

$$\Delta m_{B_s} = \frac{16.9 \pm 1.4}{\text{ps}} \left(\frac{\sqrt{\hat{B}_{B_s}} f_{B_s}}{261\ \text{MeV}} \right)^2 \left(\frac{M_t}{173.34\ \text{GeV}} \right)^{1.52} \left(\frac{|V_{ts} V_{tb}^*|}{0.0400} \right)^2 \left(\frac{\eta_B}{0.55} \right)$$

- **Error budget 2015**

$$\delta(\Delta m_{B_s}) = \left(\pm 1.04_{\hat{B}_{B_s}^{1/2} f_{B_s}} \pm 0.91_{|V_{cb}|} \pm 0.31_{\eta_B} \right) \text{ps}^{-1}$$

- **Error budget 2025**

$$\delta(\Delta m_{B_s}) = \left(\pm 0.17_{\hat{B}_{B_s}^{1/2} f_{B_s}} \pm 0.25_{|V_{cb}|} \pm 0.06_{\eta_B} \right) \text{ps}^{-1}$$

- **M_t prediction: 2015**

$$(M_t)_{\Delta m_{B_s}} = (179 \pm 10)\ \text{GeV}$$

- **M_t prediction: 2020 & 2025**

$$\delta(M_t)_{\Delta m_{B_s}} \approx \begin{cases} \pm 3.6\ \text{GeV} & (2020) \\ \pm 2.1\ \text{GeV} & (2025) \end{cases}$$

- **Δm_{B_d}: SM prediction 2015**

$$\Delta m_{B_d} = \frac{0.54 \pm 0.08}{\mathrm{ps}} \left(\frac{\sqrt{\hat{B}_{B_d}} f_{B_d}}{214\ \mathrm{MeV}} \right)^2 \left(\frac{M_t}{173.34\ \mathrm{GeV}} \right)^{1.52} \left(\frac{|V_{td} V_{tb}^*|}{0.0088} \right)^2 \left(\frac{\eta_B}{0.55} \right),$$

- **Error budget 2015**

$$\delta(\Delta m_{B_d}) = \left(\pm 0.056_{\hat{B}_{B_d}^{1/2} f_{B_d}} \pm 0.029_{|V_{cb}|} \pm 0.001_{\beta} \pm 0.047_{\gamma} \pm 0.009_{\eta_B} \right) \mathrm{ps}^{-1}$$

- **Error budget 2015**

$$\delta(\Delta m_{B_d}) = \left(\pm 0.007_{\hat{B}_{B_d}^{1/2} f_{B_d}} \pm 0.008_{|V_{cb}|} \pm 0.001_{\beta} \pm 0.007_{\gamma} \pm 0.002_{\eta_B} \right) \mathrm{ps}^{-1}$$

- **M_t prediction: 2015**

$$(M_t)_{\Delta m_{B_d}} = (167 \pm 16)\ \mathrm{GeV}$$

M_t prediction: 2020 & 2025

$$\delta(M_t)_{\Delta m_{B_d}} \approx \begin{cases} \pm 6.5\ \mathrm{GeV} & (2020) \\ \pm 2.7\ \mathrm{GeV} & (2025) \end{cases}$$

- $B_s \to \mu^+\mu^-$: **SM prediction 2015**

$$\mathcal{B}(B_s \to \mu^+\mu^-) = (3.35 \pm 0.06) \times 10^{-9}\, R_{t\alpha}\, R_s$$

$$R_{t\alpha} = \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.18} \left(\frac{M_t}{173.34\,\mathrm{GeV}}\right)^{3.06}$$

$$R_s = \left(\frac{f_{B_s}}{226\,\mathrm{MeV}}\right)^2 \left(\frac{|V_{cb}|}{0.0409}\right)^2 \left(\frac{|V_{ts}V_{tb}^*/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s}{1.607\,\mathrm{ps}}$$

$$\mathcal{B}(B_s \to \mu^+\mu^-) = (3.35 \pm 0.23) \times 10^{-9} \left(\frac{M_t}{173.34\,\mathrm{GeV}}\right)^{3.06}$$

- **Error budget**

$$\delta\,\mathcal{B}(B_s \to \mu^+\mu^-) = (\pm 0.05_{\mathrm{th}} \pm 0.14_{f_{B_s}} \pm 0.17_{|V_{cb}|}) \times 10^{-9} \qquad (2015)$$

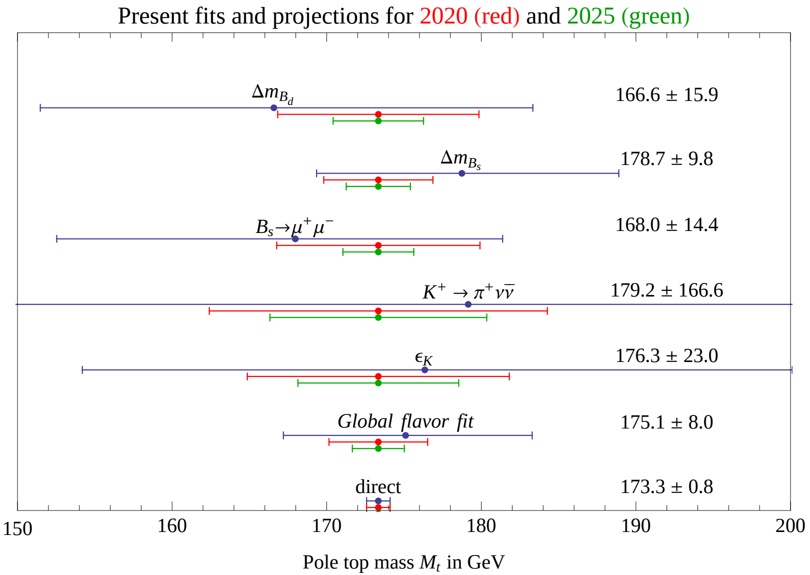
$$\delta\mathcal{B}(B_s \to \mu^+\mu^-) = (\pm 0.01_{\mathrm{th}} \pm 0.03_{f_{B_s}} \pm 0.05_{|V_{cb}|}) \times 10^{-9} \qquad (2025)$$

- M_t **predictions**

$$(M_t)_{B_s\mu\mu} = (167 \pm 14)\ \mathrm{GeV} \qquad (2015)$$

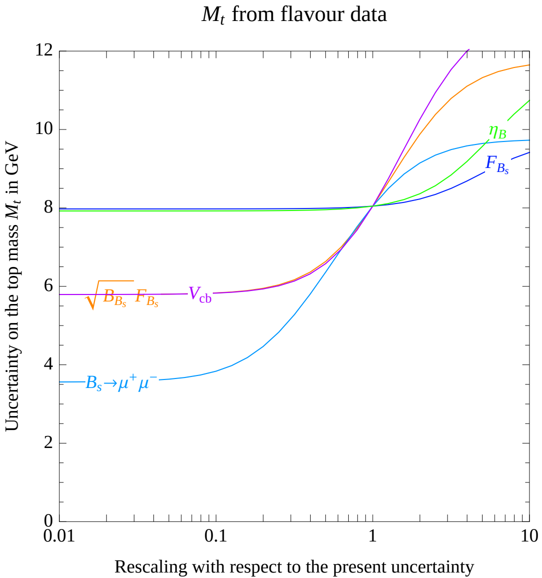
$$\delta(M_t)_{B_s\mu\mu} \approx \begin{cases} \pm 5.9\ \mathrm{GeV} & (2020) \\ \pm 2.5\ \mathrm{GeV} & (2025) \end{cases}$$

Global flavor fit



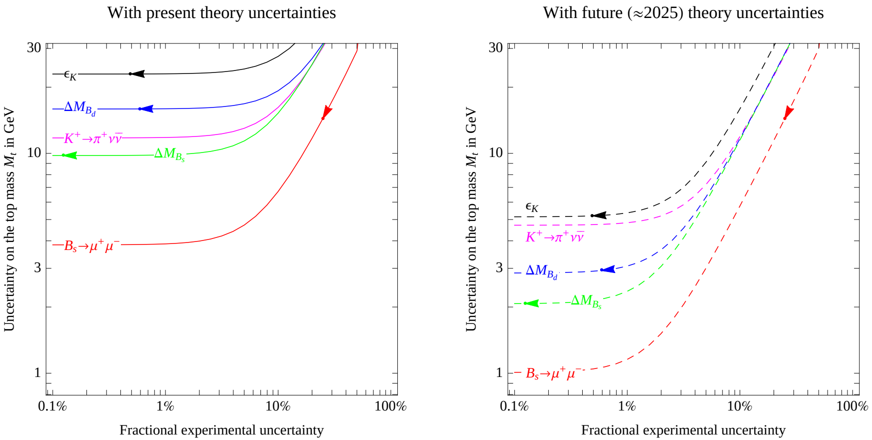
[Giudice, P.P. & Strumia, '15]

Impact of key observables



[Giudice, P.P. & Strumia, '15]

Impact of experimental improvements on the flavor fit



[Giudice, P.P. & Strumia, '15]

EW data and the ϵ_i parameters

- **EW fit**: EW observables depend on M_t only through the $\epsilon_1, \epsilon_2, \epsilon_3$ parameters describing corrections to the tree-level propagators of $W^\pm$, Z, and through the ϵ_b parameter that describes corrections to the $Zb\bar{b}$ vertex [Altarelli & Barbieri, '92].

- **SM prediction of ϵ_i:**

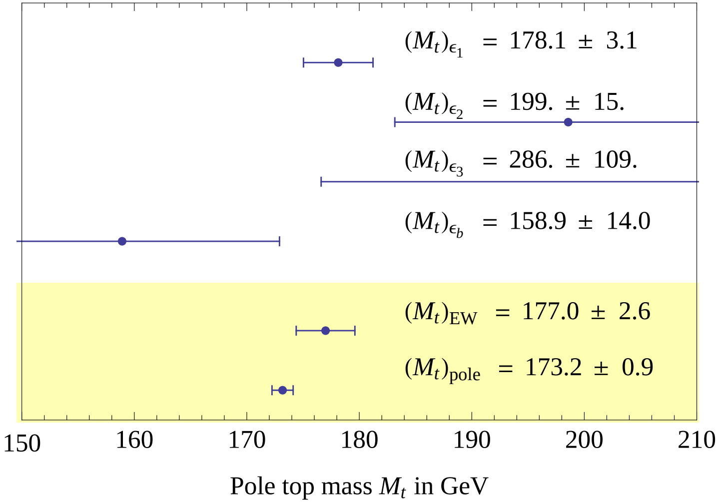
$$\begin{cases} \varepsilon_1 = +5.22 \times 10^{-3}\,(M_t/173.34\,\mathrm{GeV})^{3.15}\,(M_h/125.15\,\mathrm{GeV})^{-0.15} \\ \varepsilon_2 = -7.32 \times 10^{-3}\,(M_t/173.34\,\mathrm{GeV})^{-0.69}\,(M_h/125.15\,\mathrm{GeV})^{-0.03} \\ \varepsilon_3 = +5.28 \times 10^{-3}\,(M_t/173.34\,\mathrm{GeV})^{-0.01}\,(M_h/125.15\,\mathrm{GeV})^{0.11} \\ \varepsilon_b = -6.95 \times 10^{-3}\,(M_t/173.34\,\mathrm{GeV})^{2.18} \end{cases}$$

- In the large M_t limit the one-loop corrections to $\varepsilon_1 = \Delta\rho$ and $\varepsilon_b = -2\Delta g_L^{bb}$ grow as M_t^2, while ε_2 and ε_3 only have a milder $\ln M_t$ dependence.

- **Experimental measurement of ϵ_i:**

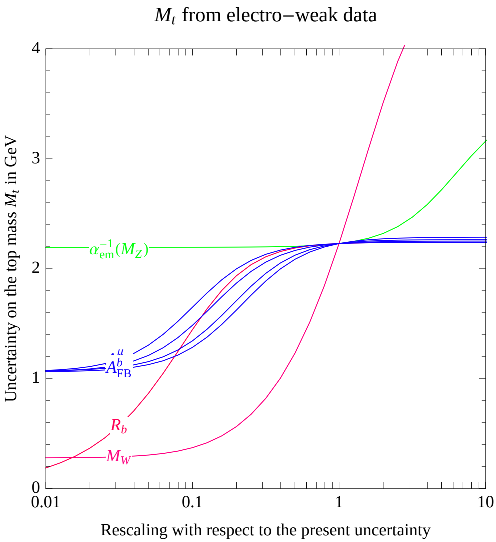
$$\begin{cases} \varepsilon_1 = +(5.6 \pm 1.0) \times 10^{-3} \\ \varepsilon_2 = -(7.8 \pm 0.9) \times 10^{-3} \\ \varepsilon_3 = +(5.6 \pm 0.9) \times 10^{-3} \\ \varepsilon_b = -(5.8 \pm 1.3) \times 10^{-3} \end{cases}$$

Determination of M_t from EW data



[Giudice, P.P. & Strumia, '15]

Impact of experimental improvements on the EW fit



[Giudice, P.P. & Strumia, '15]

Discussion of the EW fit

- M_W plays the key role, since $\delta M_t/M_t = 69\ \delta M_W/M_W$. Measuring M_W with a precision of 5 MeV can lead to $\delta M_t \sim 0.7\ \mathrm{GeV}$.
- The measurements of the various asymmetries have to be improved by more than a factor of 3 to improve the uncertainty on M_t.
- The uncertainty on M_t would be affected by $\alpha_{\rm em}(M_Z)$ only if its error were underestimated by more than a factor of 2.
- We can extract M_t using M_W as the only input quantity by means of Δr_W defined as the ratio of two determinations of the weak angle

$$\Delta r_W \equiv 1 - \frac{\pi\alpha_{\rm em}(M_Z)/\sqrt{2}G_{\rm F}M_Z^2}{M_W^2/M_Z^2(1-M_W^2/M_Z^2)} = (-25.4 \pm 0.95_{M_W} \pm 0.10_{\alpha_{\rm em}}) \times 10^{-3}$$

$$\Delta r_W = -\tan^{-2}\theta_{\rm W}\,\varepsilon_1 + (\tan^{-2}\theta_{\rm W} - 1)\,\varepsilon_2 + 2\,\varepsilon_3$$

$$= -24.0 \times 10^{-3}\left(\frac{M_t}{173.34\,\mathrm{GeV}}\right)^{2.50}\left(\frac{M_h}{125.15\,\mathrm{GeV}}\right)^{-0.14}$$

- M_t prediction from Δr_W

$$(M_t)_{M_W} = (177.7 \pm 2.8)\,\mathrm{GeV}$$

Conclusions

- Indirect determinations of M_t are very important especially in view of the theoretical ambiguities in the extraction of M_t from collider experiments.
- The Gauge-less limit of the SM allows to identify flavour and EW observables with power sensitivity on M_t at the quantum level.
- Flavour data determine $M_t = (175.1 \pm 8.0)$ GeV with Δm_{B_s} and $B_s \to \mu^+\mu^-$ being the best probe of M_t. Since they require only V_{cb} as CKM input, a complete joint fit with all CKM parameters is not necessary.
- Better measurements of $B_s \to \mu^+\mu^-$, better lattice computations of the hadronic parameters relevant for Δm_{B_s} and $B_s \to \mu^+\mu^-$ and improved calculations of short-distance effects will bring δM_t down to 3 GeV (1.7 GeV) by 2020 (2025).
- EW data determine $M_t = (177.0 \pm 2.6)\,\mathrm{GeV}$ and M_W is the best toppometer. The present uncertainty on M_W should be reduced by a factor of 3 by LHC experiments bringing M_t to a precision of about 0.7 GeV.

A global fit of all indirect determinations of M_t, from both EW and flavour data, will provide very significant information.