Cosmological Higgstory of the vacuum instability

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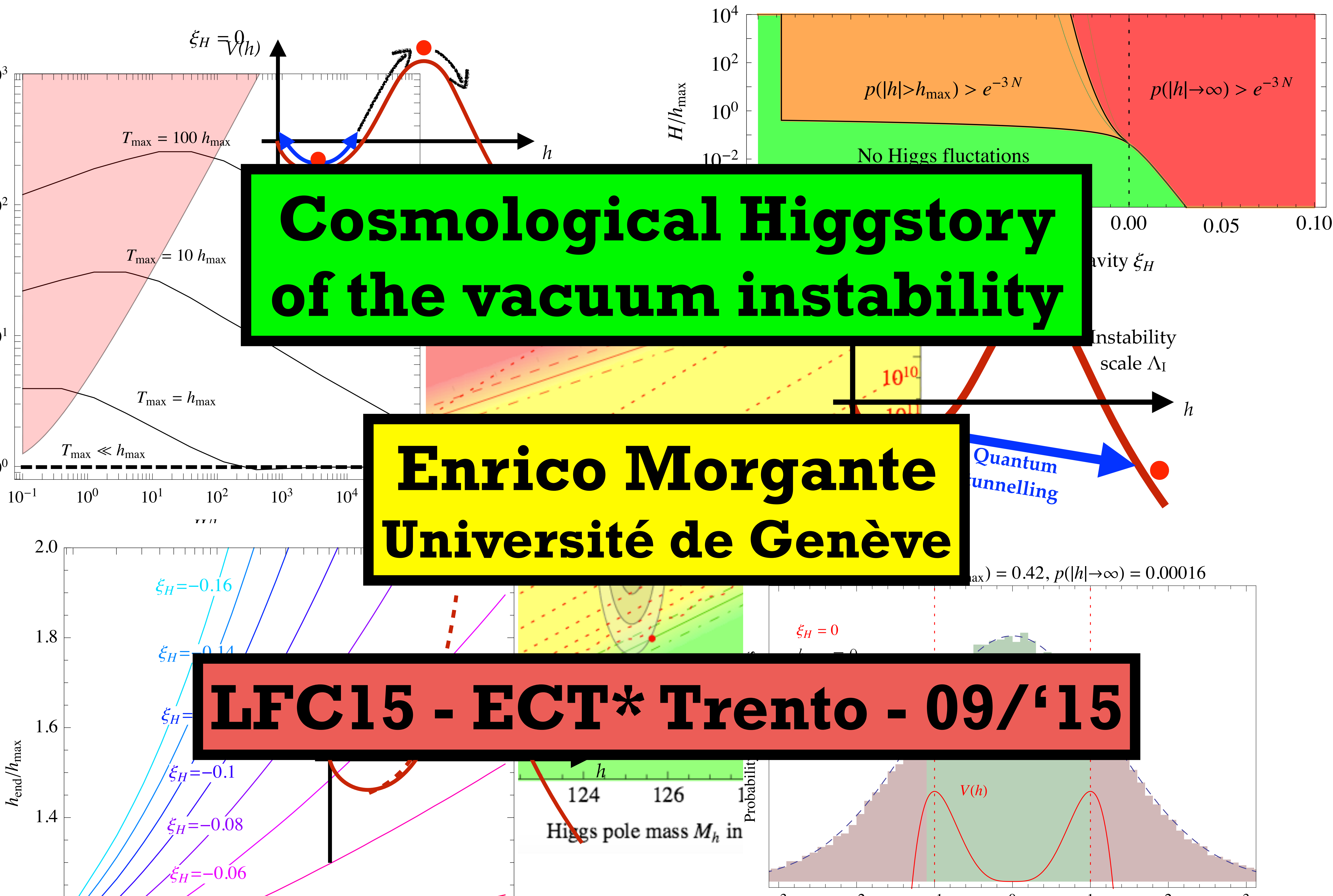


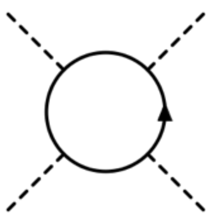
with Espinosa, Giudice, Riotto, Senatore, Strumia, Tetradis

1505.04825 (accepted by JHEP)

The Higgs vacuum instability

Extrapolate SM up to Planck scale. We assume Higgs is SM like, no BSM physics.

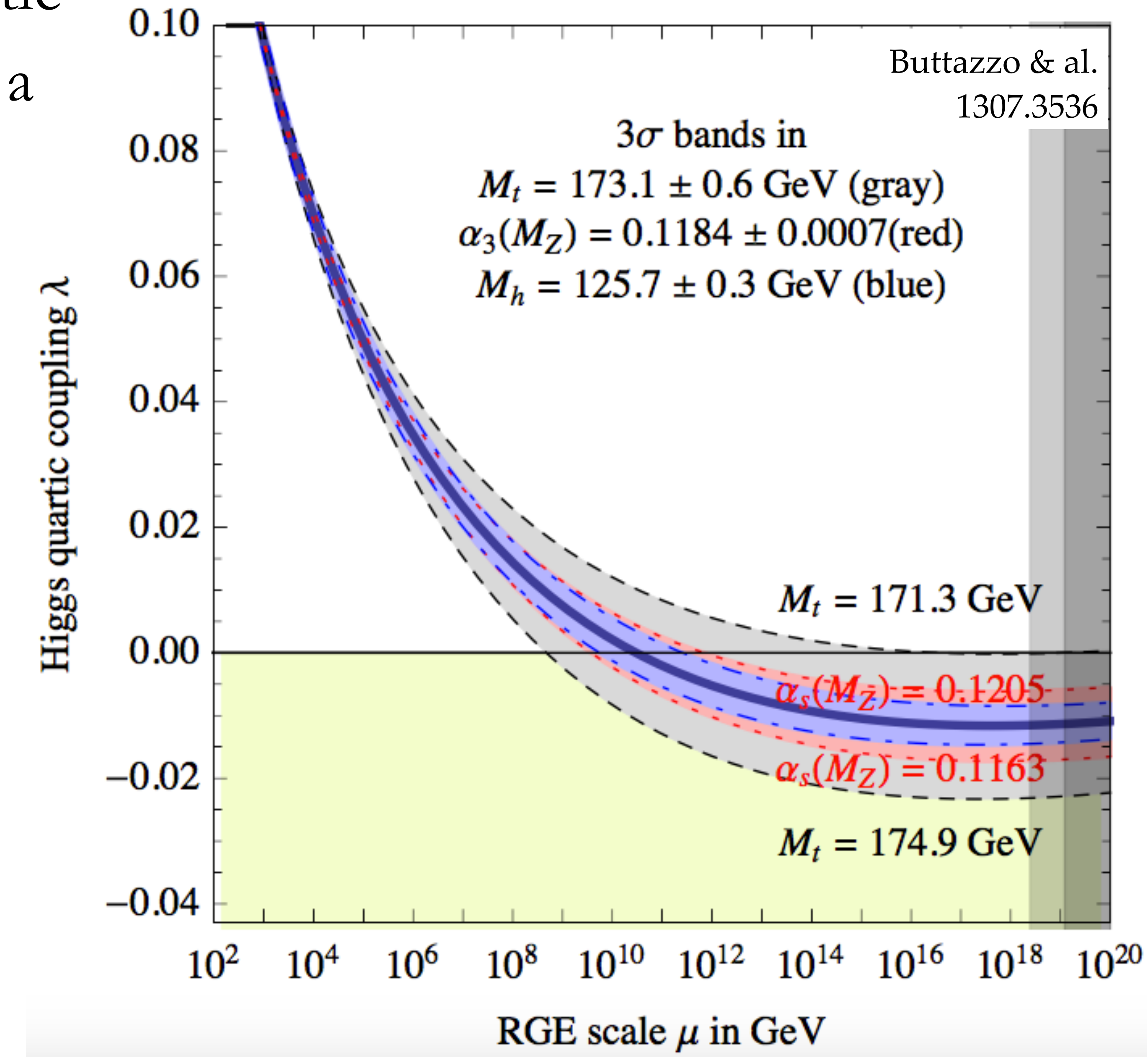
Following the SM RG equations, the quartic Higgs coupling becomes negative around a scale of $10^{10} \div 10^{11}$ GeV, due to top loop contribution

top loop:  $\longrightarrow \dfrac{d\lambda}{d\log\mu} \sim -\dfrac{y_t^4}{16\pi^2}$

The effective potential, parametrised as

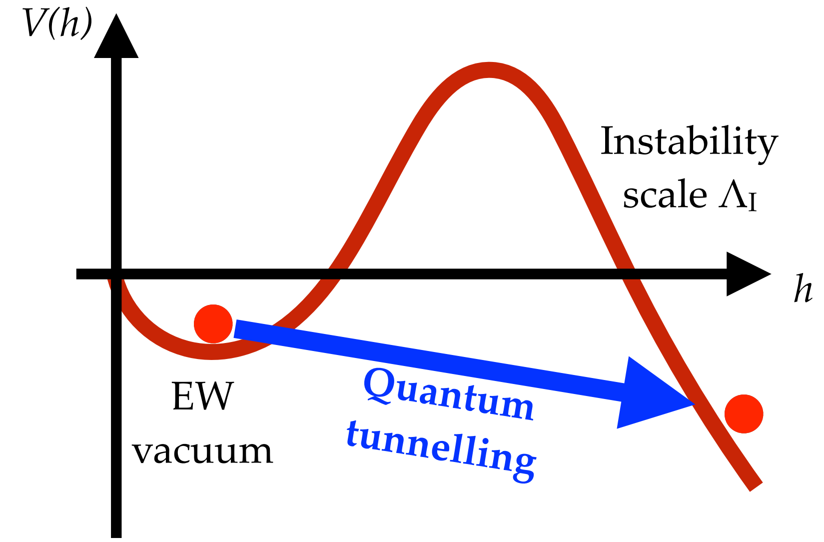
$$V(h) = \lambda(h)\frac{h^4}{4}$$

becomes negative at a scale Λ_I



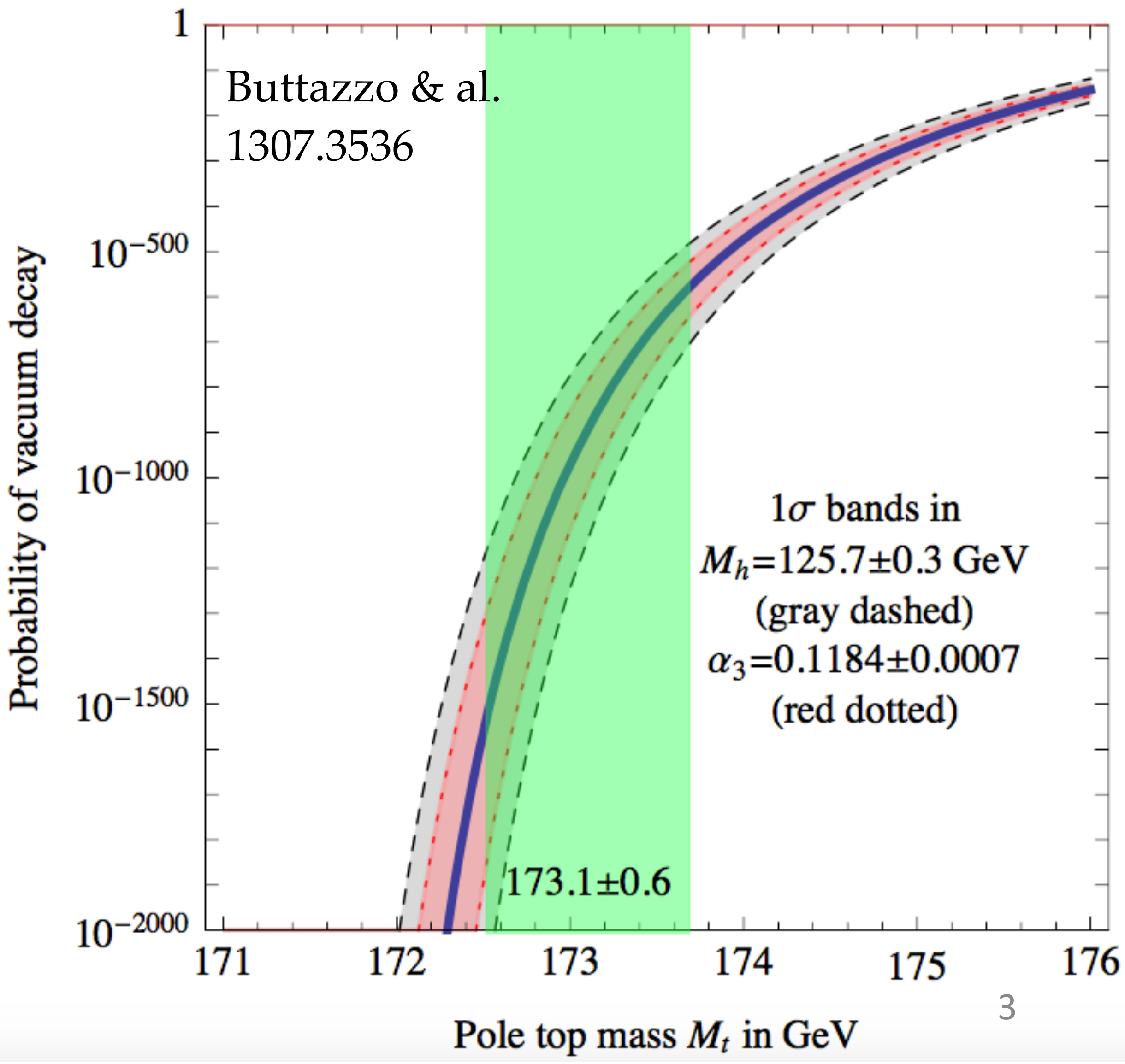
The Higgs vacuum instability

We live in an unstable false vacuum,
which can be destabilised by quantum tunnelling.



"True vacuum" energy large and negative.

$\Rightarrow$ a region of Anti - de Sitter space forms



The tunnelling probability can be computed as

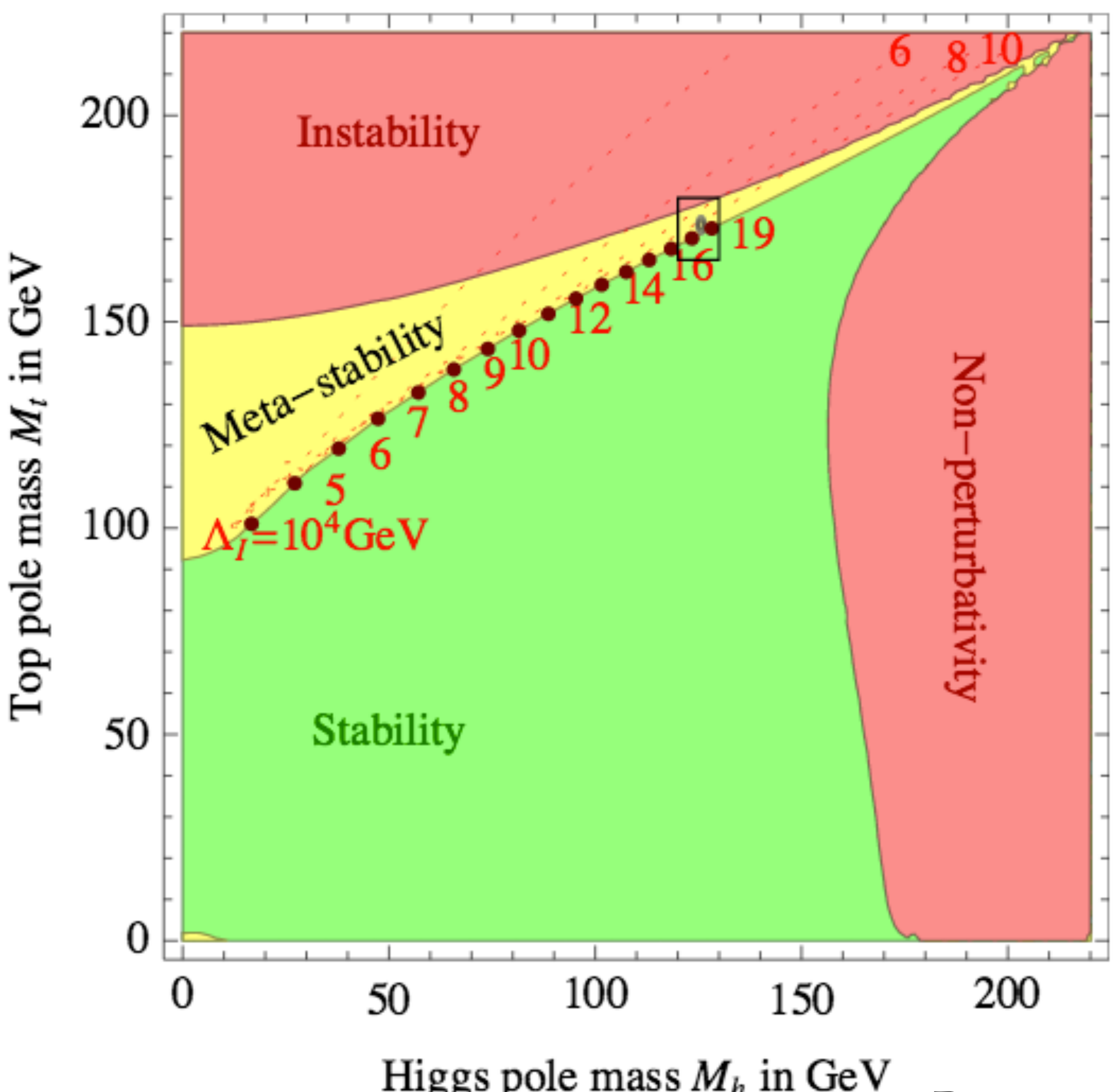
$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}V\,\mathrm{d}t} = \Lambda_B^4 e^{-S_B(\Lambda_B)}$$

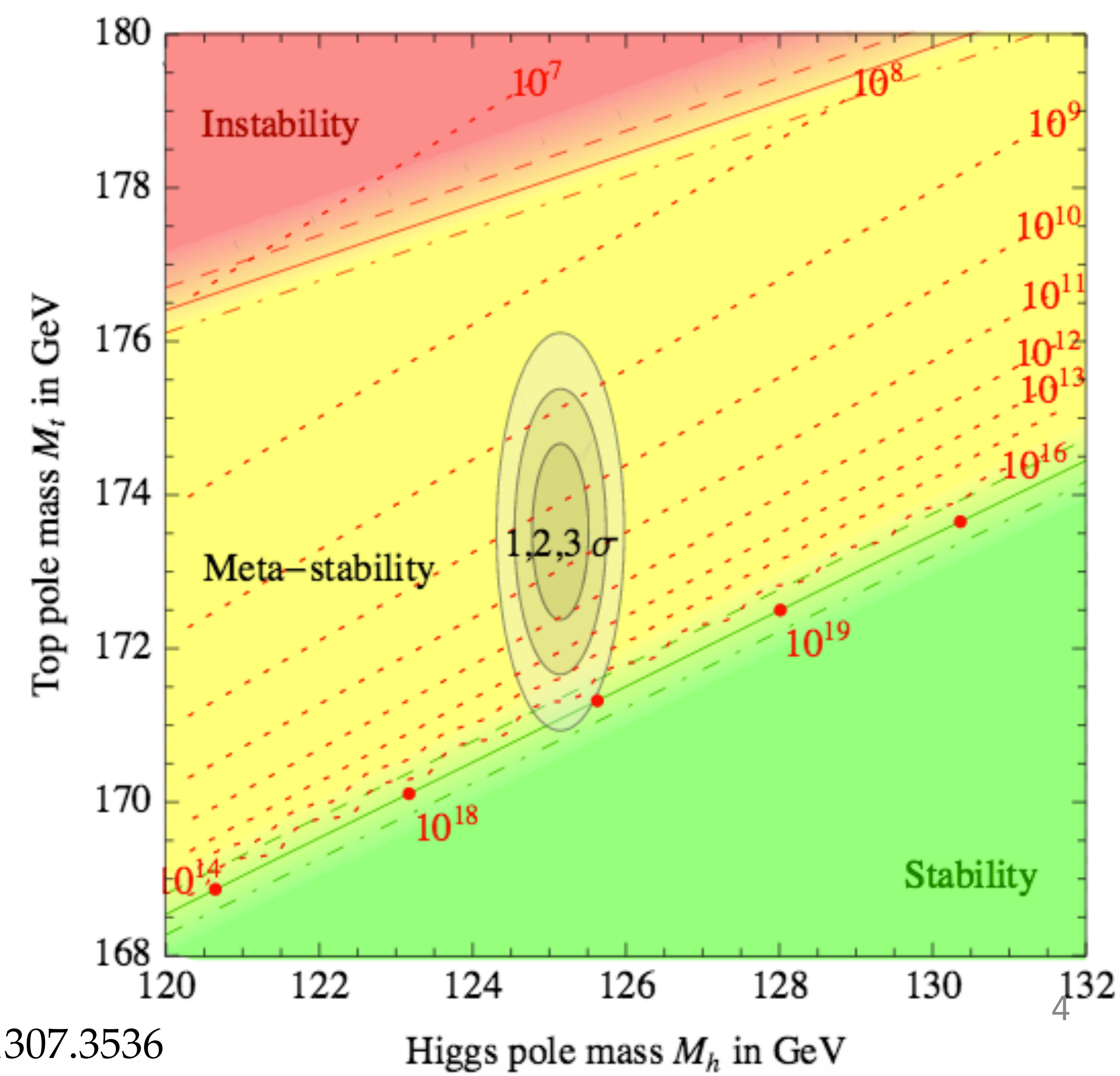
where S_B is the action computed on the bounce solution with radius Λ_B^{-1} that maximises the probability.

The negative exponential suppresses the probability.

The Higgs vacuum instability

Metastability: the values of the SM parameters are very special.
The central values of M_{top} and M_{higgs} are such that the EW vacuum is unstable, but its lifetime is larger than the age of the Universe.
This is highly non trivial, and poses a question about fine-tuning.





Buttazzo & al.1307.3536

Higgs instability in the early universe

In the early universe, many effects can modify the situation:

1. Quantum fluctuations during inflation can trigger the formation of "true vacuum" bubbles

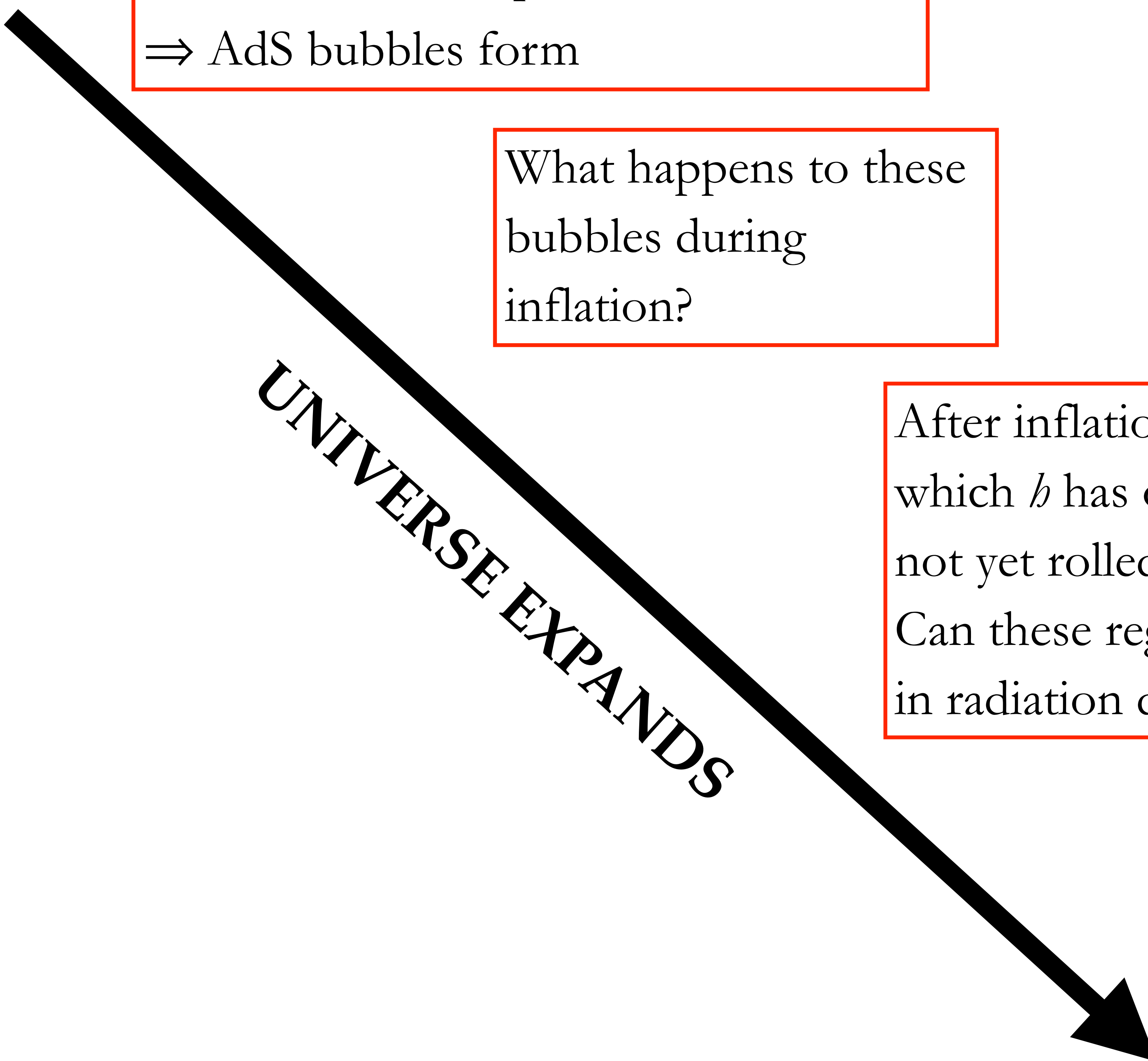
2. A non minimal coupling of h to gravity can induce an effective mass term which stabilises the potential

3. Thermal effects during radiation dominance are twofold: fluctuations can trigger the "jump" of the barrier, while corrections to the effective potential create an additional effective barrier.

"Timeline"

Quantum fluctuations during inflation can overcome the potential barrier
$\Rightarrow$ AdS bubbles form

What happens to these bubbles during inflation?

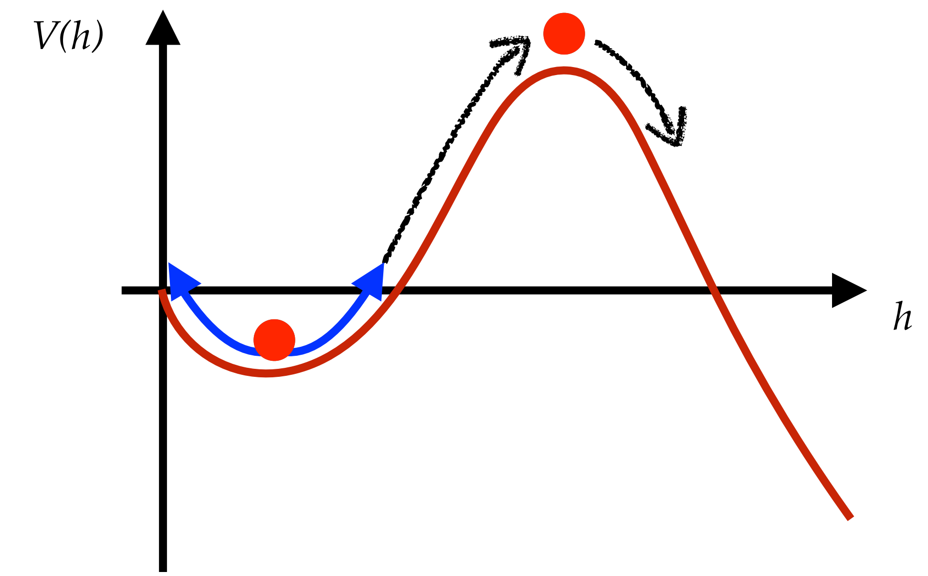


After inflation ends, there will be regions in which h has oscillated over the barrier but has not yet rolled down to its deep minimum. Can these regions be saved by thermal effects in radiation dominance?

What happens during radiation dominance to the bubbles which were formed in the inflationary period?

Higgs fluctuations during inflation

During inflation, quantum fluctuations of long wavelength modes are governed by the value of the Hubble parameter H.
If fluctuations are large enough, h can "jump" over the barrier.



The evolution of h (for long wavelengths) is controlled by a Langevin equation:

$$\frac{dh}{dt} + \frac{1}{3H}\frac{dV(h)}{dh} = \eta(t) \qquad \langle \eta(t)\eta(t')\rangle = \frac{H^3}{4\pi^2}\delta(t-t')$$

This equation is valid only if the effective mass V'' is small enough with respect to H^2. Otherwise all fluctuations are exponentially damped.

We can numerically generate random realisations of the Higgs evolution in time (or in the e-folds number N):

$$\bar{h}(N+dN) = \bar{h}(N) - \frac{h_{\max}^2}{3H^2}\bar{V}'(\bar{h})\,dN + r \qquad \sigma = H\sqrt{dN}/(2\pi h_{\max})$$

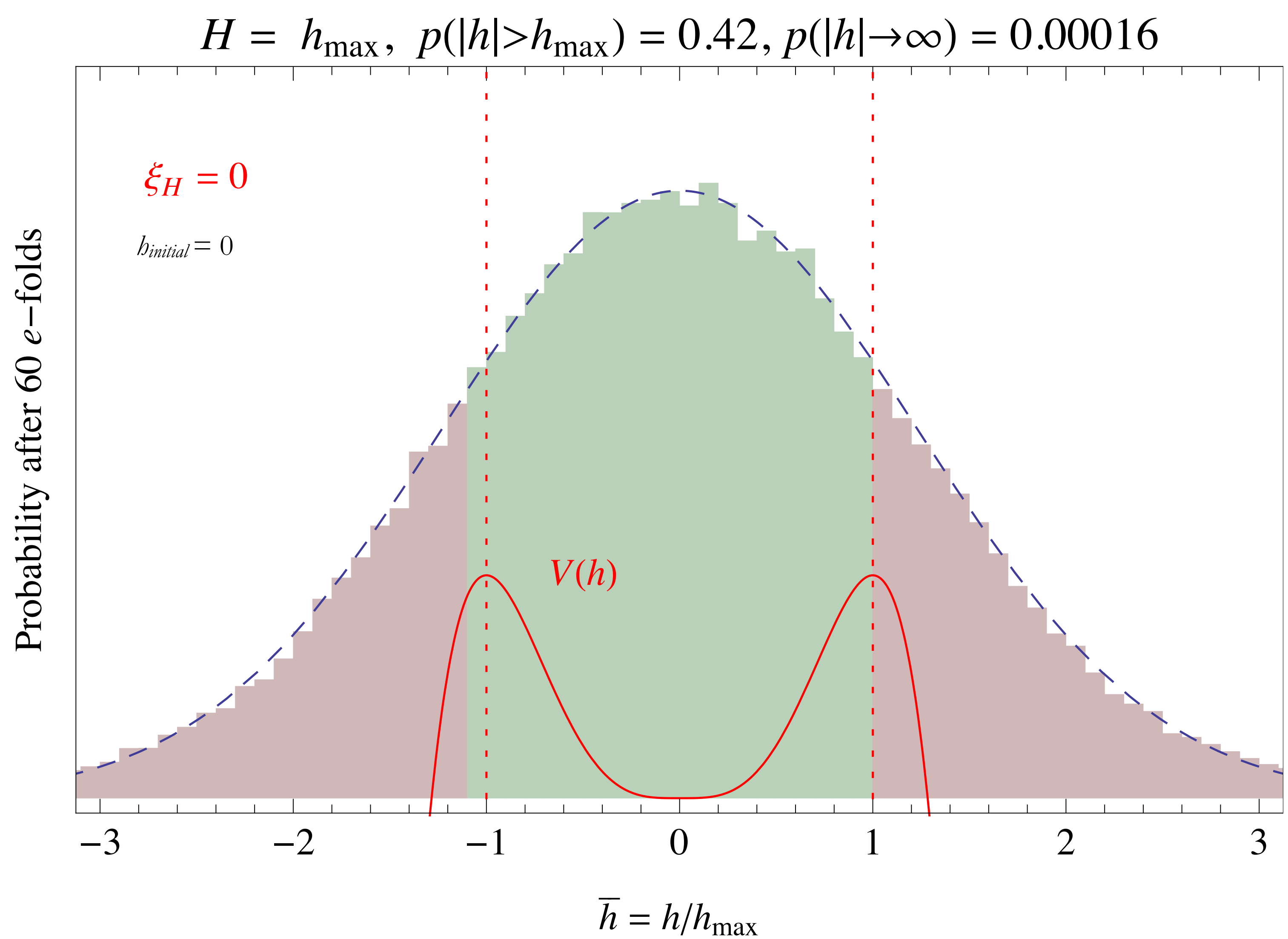
Higgs fluctuations during inflation

- The resulting probability distribution is quasi-gaussian because the evolution is dominated by the quantum noise term;

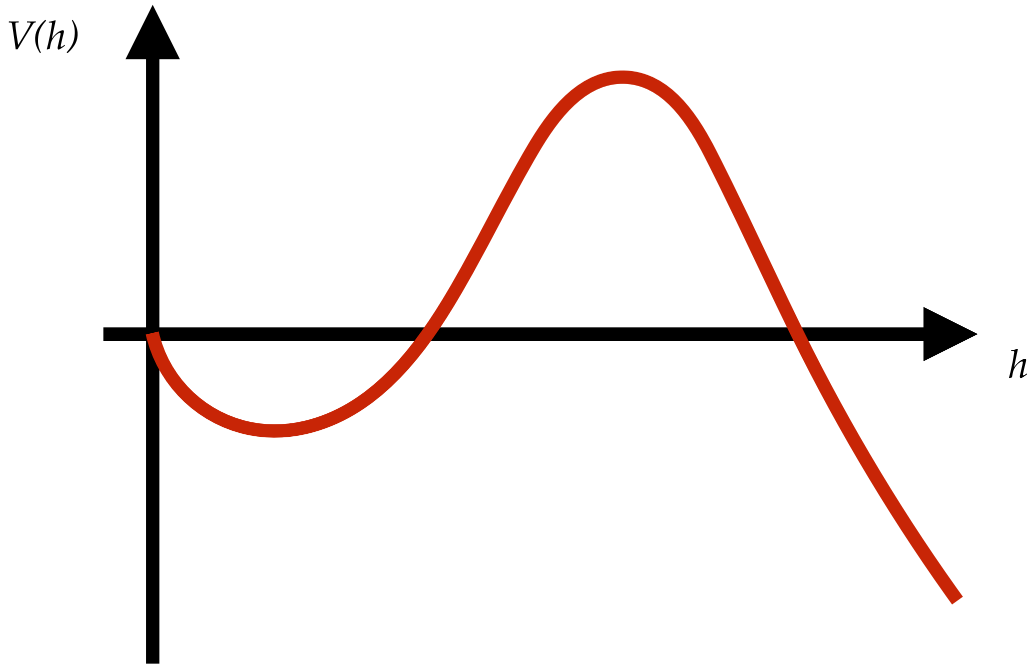
$$\frac{dh}{dt} + \frac{1}{3H}\frac{dV(h)}{dh} = \eta(t)$$

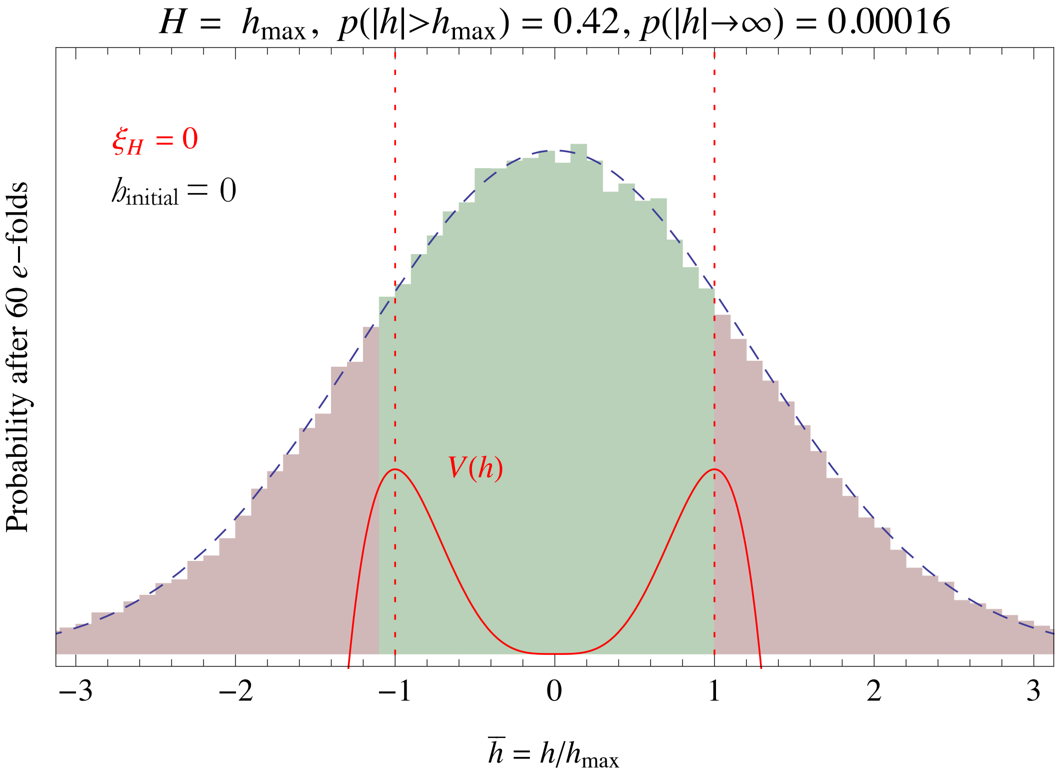
- the potential is only relevant in the very tail of the distribution. Here the evolution becomes classical and the Higgs rolls down towards its true minimum;

$$P(h,N) = \frac{1}{\sqrt{2\pi\langle h^2\rangle}} \exp\left(-\frac{h^2}{2\langle h^2\rangle}\right), \qquad \sqrt{\langle h^2\rangle} = \frac{H}{2\pi}\sqrt{N}.$$

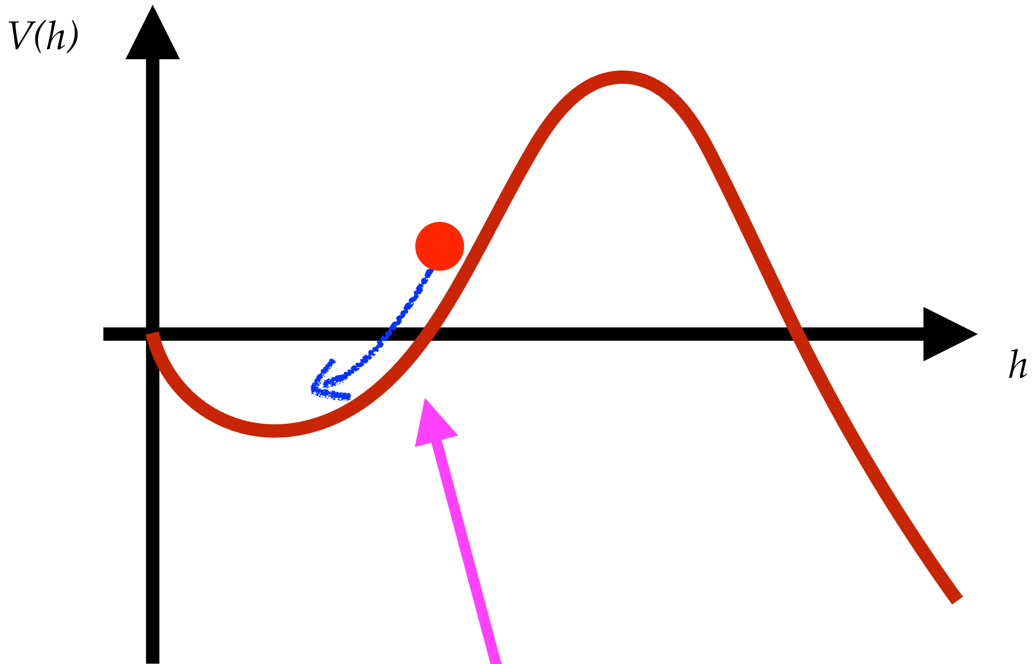


Higgs fluctuations during inflation

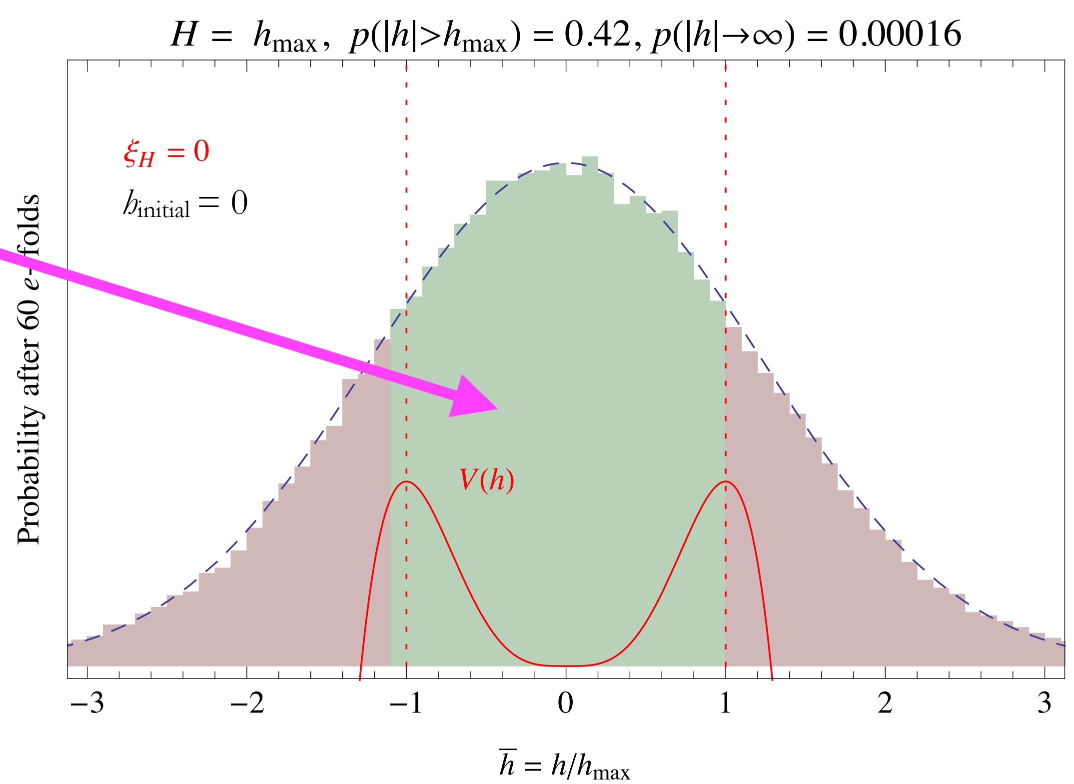




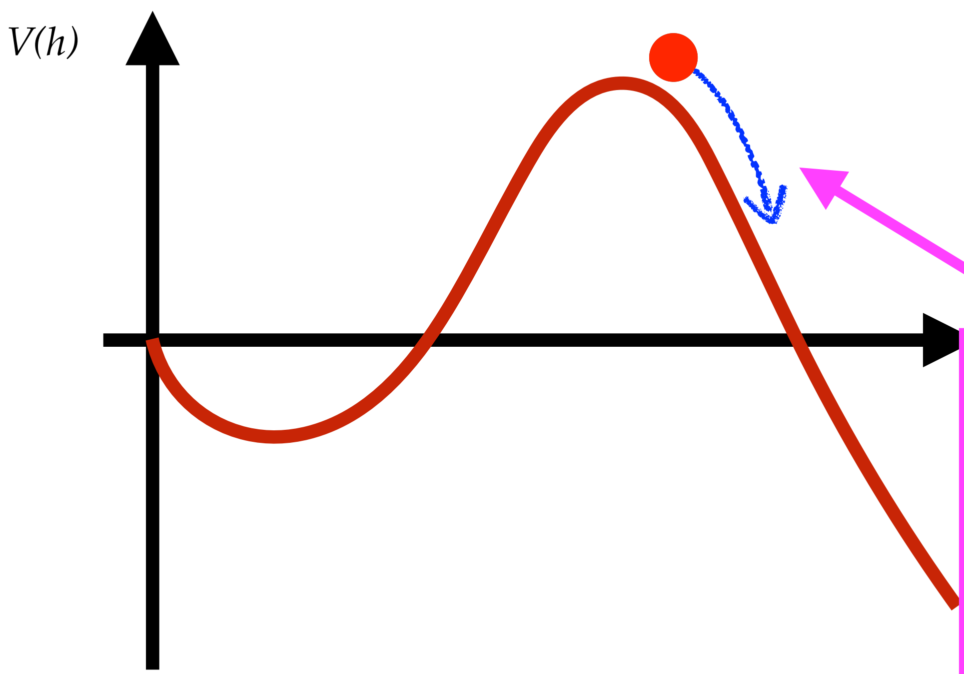
Higgs fluctuations during inflation



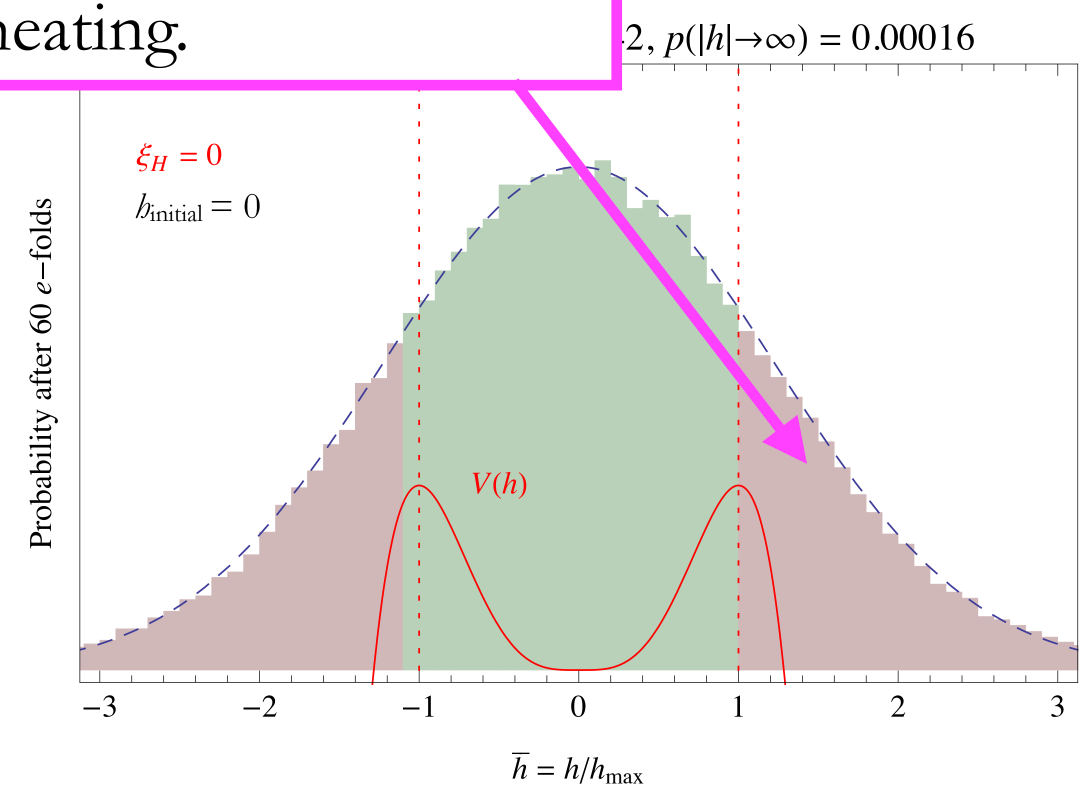
If, at the end of inflation, the Higgs is below the barrier, then it will roll down to the EW vacuum.
No bubble forms.



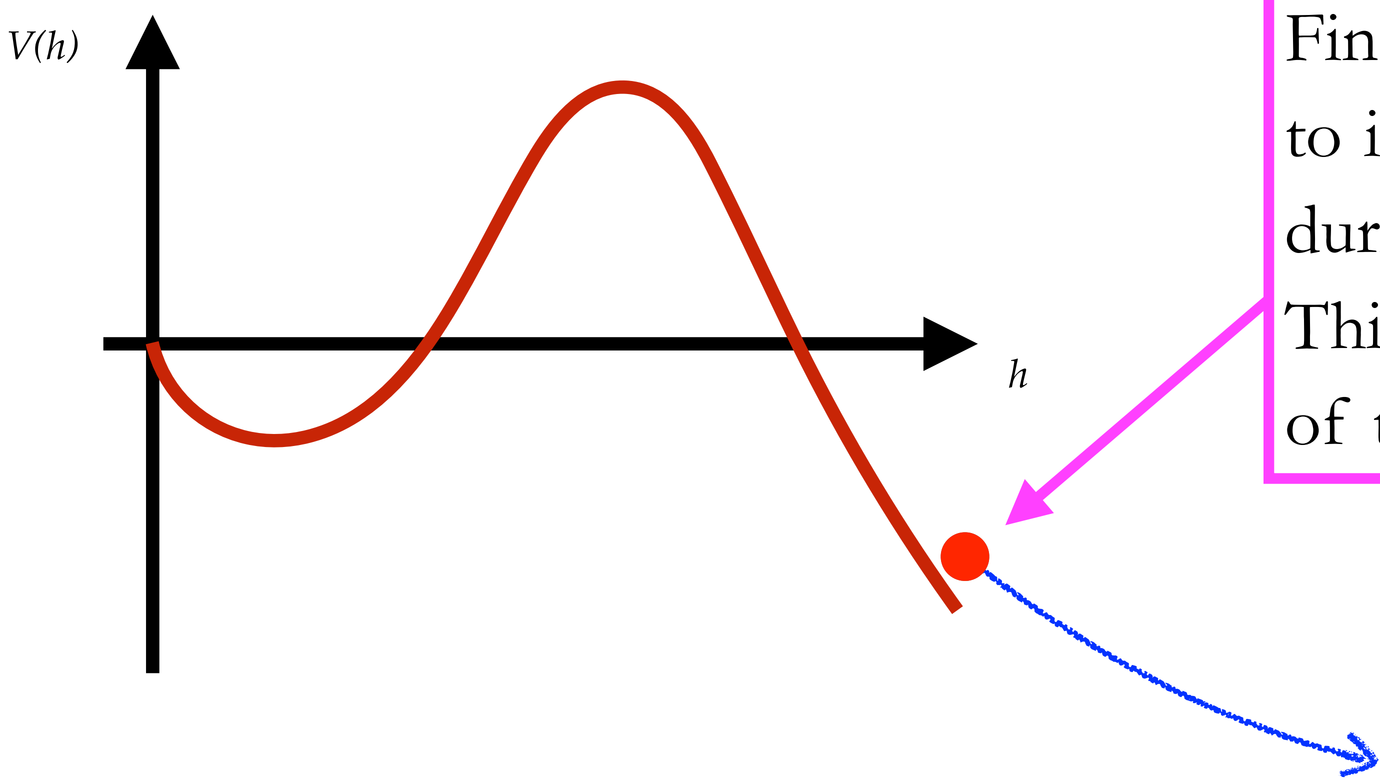
Higgs fluctuations during inflation



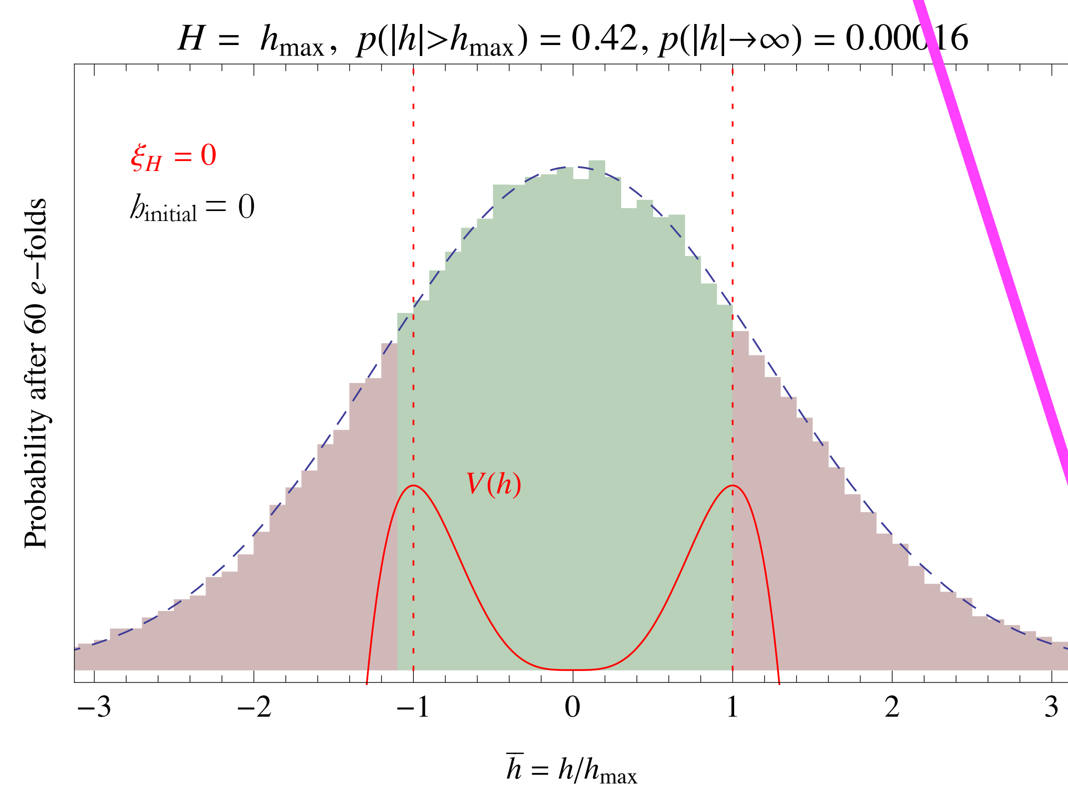
The Higgs can be above the barrier, but without having fallen into the true minimum.
This region can be saved by thermal effects during re-heating.



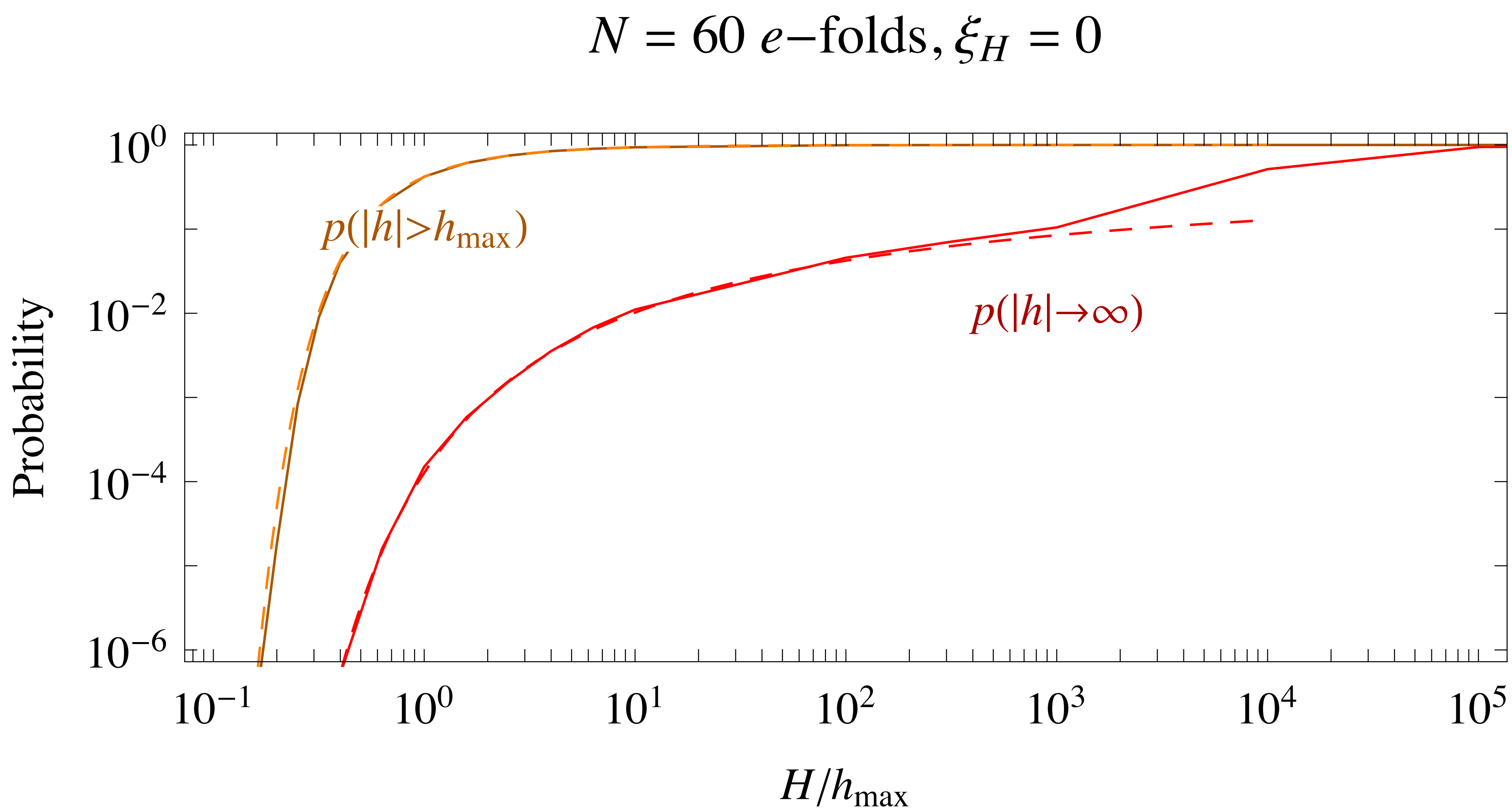
Higgs fluctuations during inflation



Finally, if the Higgs has fallen to its minimum, bubbles form during inflation.
This is the non gaussian tail of the distribution.



Higgs fluctuations during inflation



Imposing that no bubble formed in the e^{3N} Hubble patches that form our visible Universe, we obtain bounds on the values of inflationary parameters:

$$p(|h| > h_{\rm max}) < e^{-3N} \Longrightarrow \frac{H}{h_{\rm max}} < \sqrt{\frac{2}{3}\frac{\pi}{N}} \approx 0.04$$

$$p(|h| \to \infty) < e^{-3N} \Longrightarrow \frac{H}{h_{\rm max}} < \sqrt{\frac{2}{3}\frac{\pi}{N} e^{\pi^2 k/2bN^3}} \approx 0.045$$

Effective mass term

Higgs fluctuations during inflation can get damped if the Higgs doublet Φ_H during inflation acquires an effective mass m.

- Quartic coupling to the inflaton:

$$\lambda_{h\phi}|\Phi_H|^2\phi^2 \longrightarrow m^2 = \lambda_{h\phi}\phi^2$$

- Inflaton decays into SM particles could generate a non-vanishing temperature during inflation, inducing a thermal mass term

$$m^2 \approx H^2$$

- Non-minimal coupling to gravity:

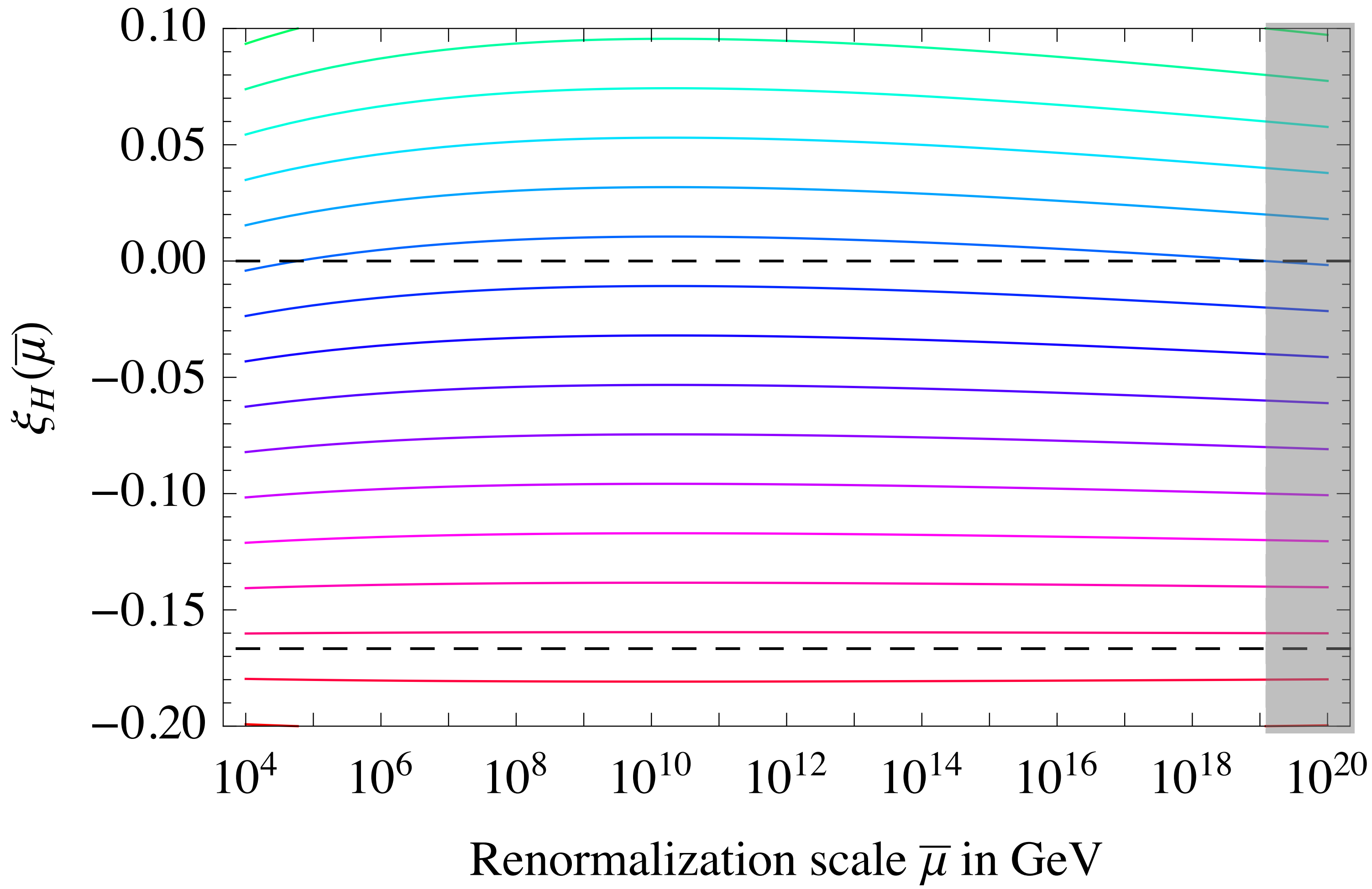
$$-\xi_H|\Phi_H|^2 R \longrightarrow m^2 = \xi_H R = -12\xi_H H^2$$

Coupling to gravity

$$S = \int d^4x \sqrt{g}\left[-\frac{\bar{M}_{\rm Pl}^2}{2} R - \xi_H |\Phi_H|^2 R + |D_\mu \Phi_H|^2 - V + \cdots \right]$$

A non minimal coupling to gravity is unavoidable, because it's generated by quantum corrections:

$$\frac{d\xi_H}{d\ln\bar{\mu}} = \frac{\xi_H + 1/6}{(4\pi)^2}\left(6y_t^2 - \frac{9}{2}g_2^2 - \frac{9}{10}g_1^2 + 12\lambda_H\right) + \cdots$$



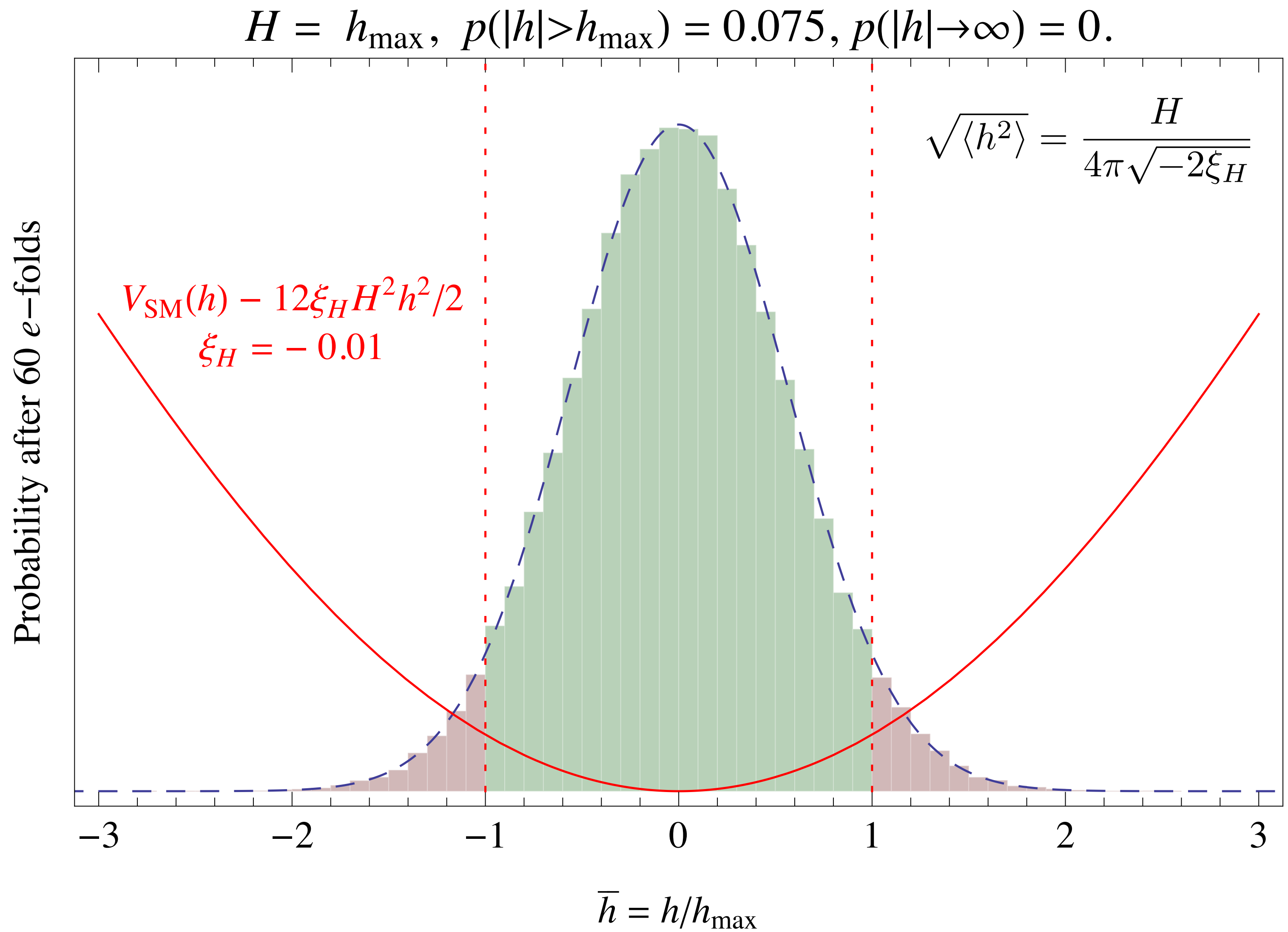
Coupling to gravity

If ξ_H<0 the potential is stabilised by the effective mass term

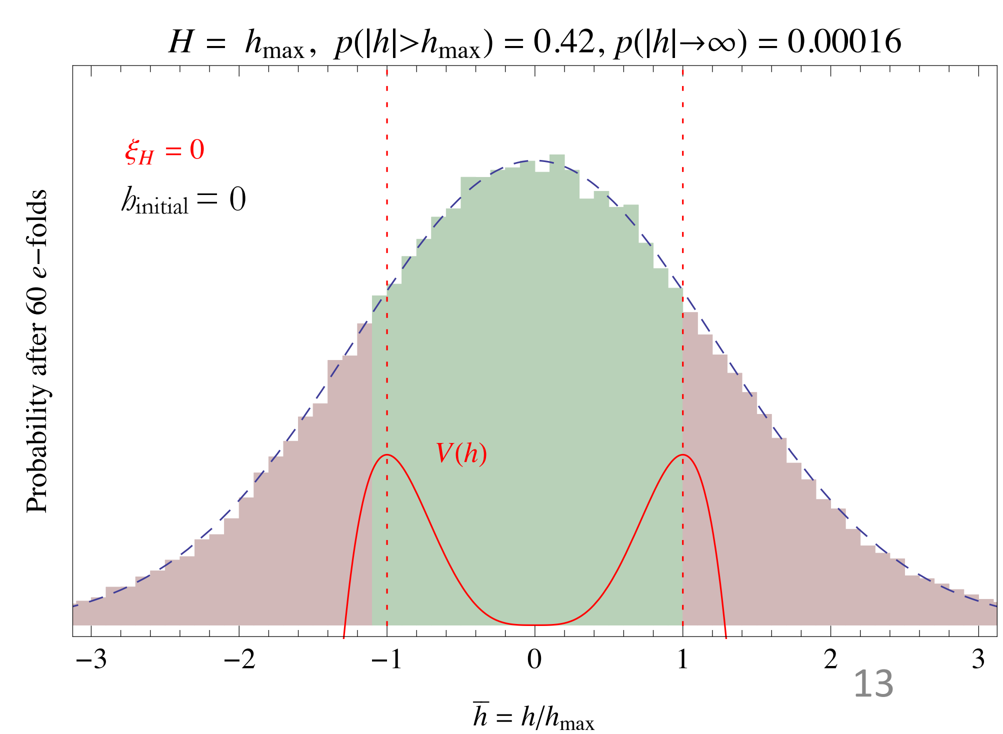
$$m^2 = -12\xi_H H^2$$

If $m^2 > (9/4)H^2$ (*i.e.* $\xi_H < -3/16$), then fluctuations are damped.

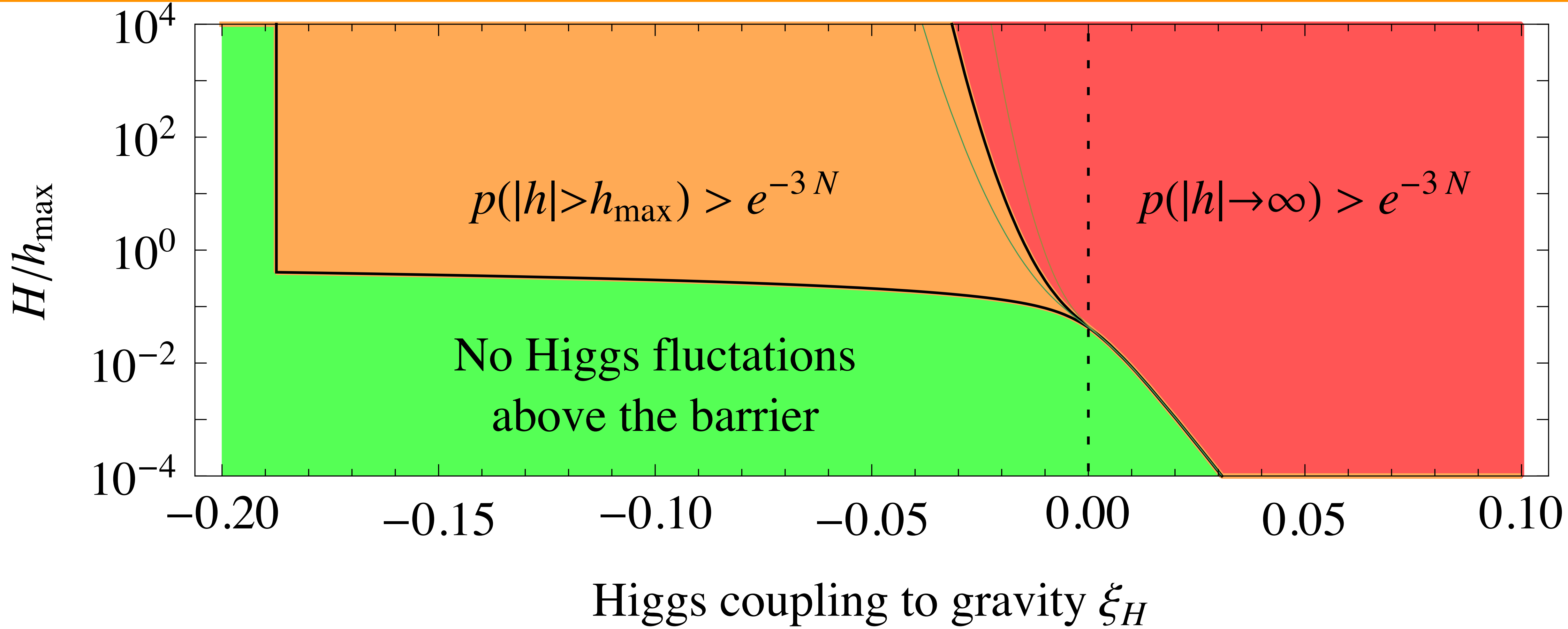
If $-3/16 < \xi_H < 0$ the distribution is quasi-gaussian, narrower than for ξ_H=0



…compare with



Coupling to gravity

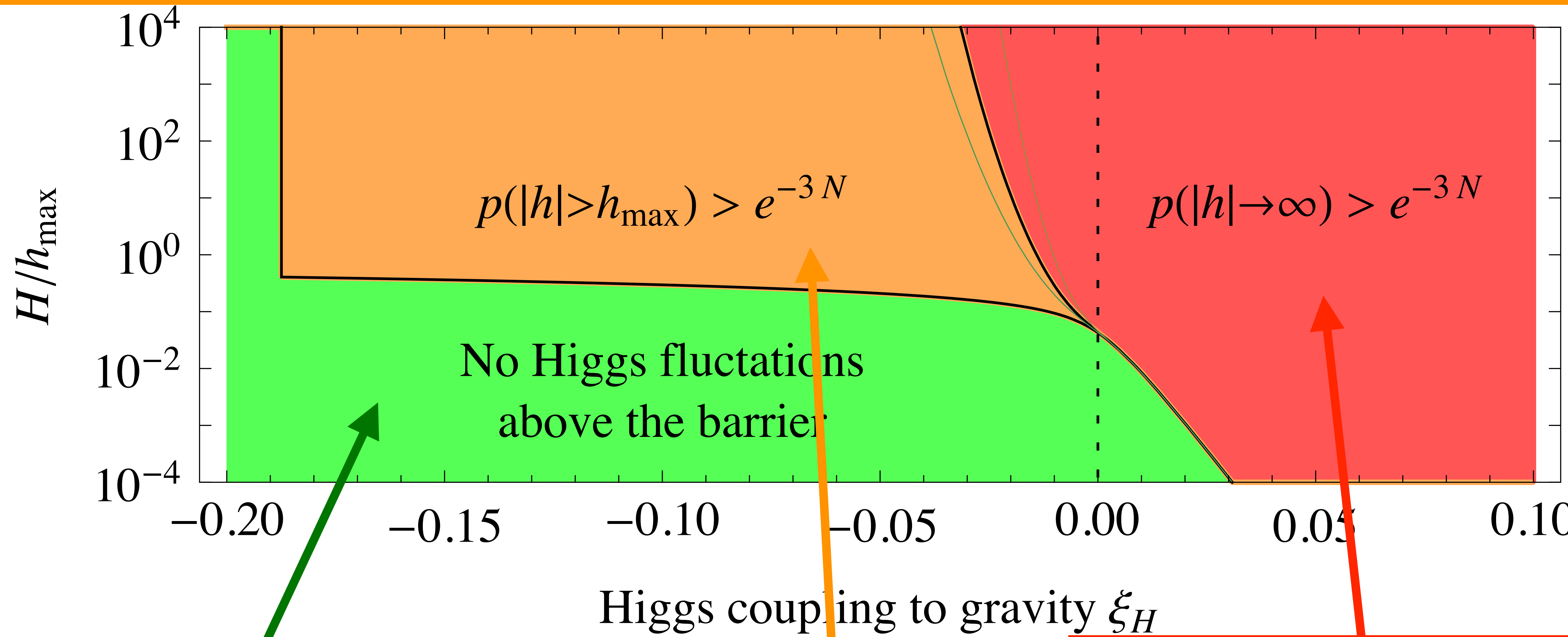


Bounds on H now depend on ξ_H:

$$p(|h| > h_{\max}) < e^{-3N} \Rightarrow \frac{H}{h_{\max}} < 4\pi\sqrt{\frac{-\xi_H}{3N}}$$

$$p(|h| \to \infty) < e^{-3N} \Rightarrow \frac{H}{h_{\max}} < 4\pi\sqrt{\frac{-\xi_H}{3N}}e^{32\pi^2\xi_H^2/bN}$$

Coupling to gravity



GREEN Region:
The instability is avoided

RED Region:
The Higgs falls into its instability during inflation

ORANGE Region:
The Higgs fluctuates beyond its instability without falling into its deep minimum

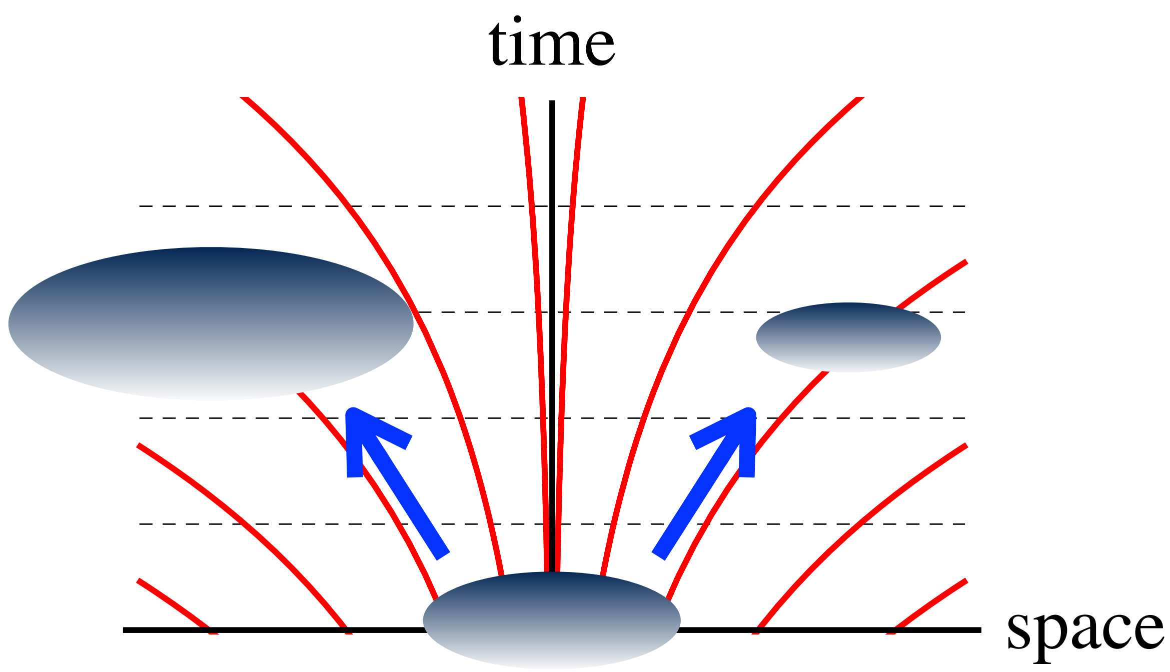
Evolution of AdS bubbles

When the Higgs falls into its deep (negative) minimum, that patch of the universe receives an Anti-de Sitter metric.

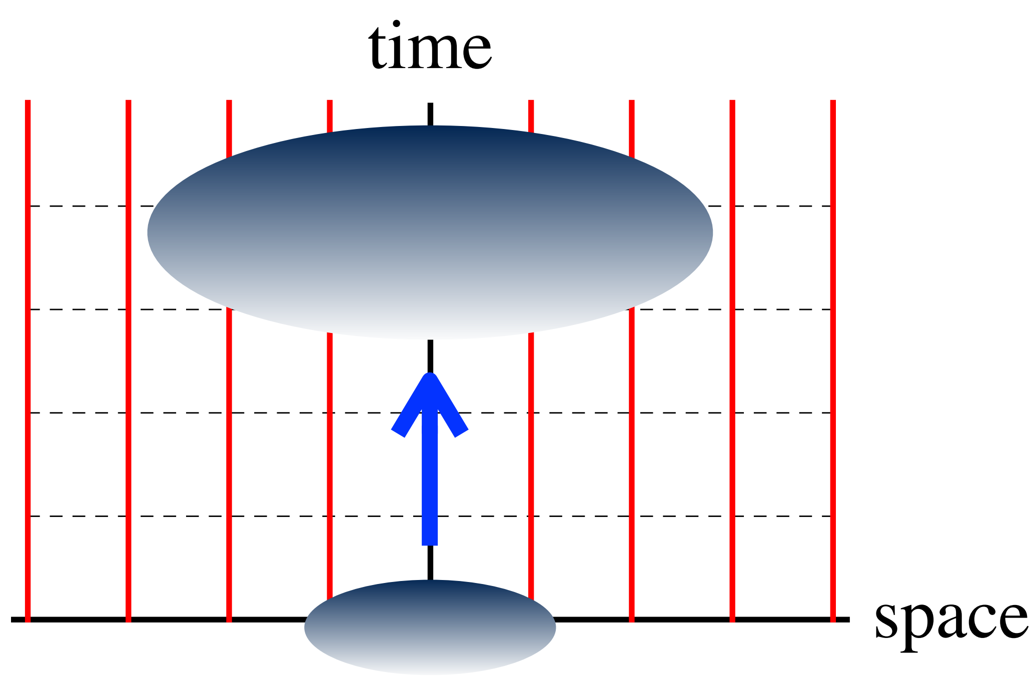
Understanding what happens afterward requires a delicate GR calculation:

- approximation: spherical bubble with a thin wall;
- relevant parameters: bubble size, initial wall velocity, internal energy, surface tension.

Evolution of AdS bubbles

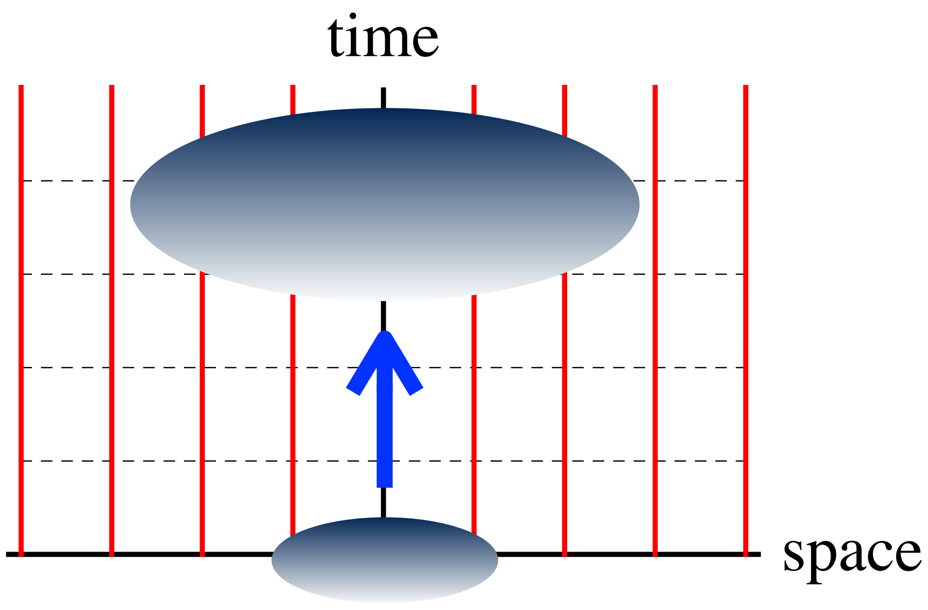


During inflation (**de Sitter exterior**) an anti - de Sitter bubble can have a variety of possible behaviours.
In general, even if bubbles expand, exponential expansion wins and they are hidden behind a de Sitter horizon.



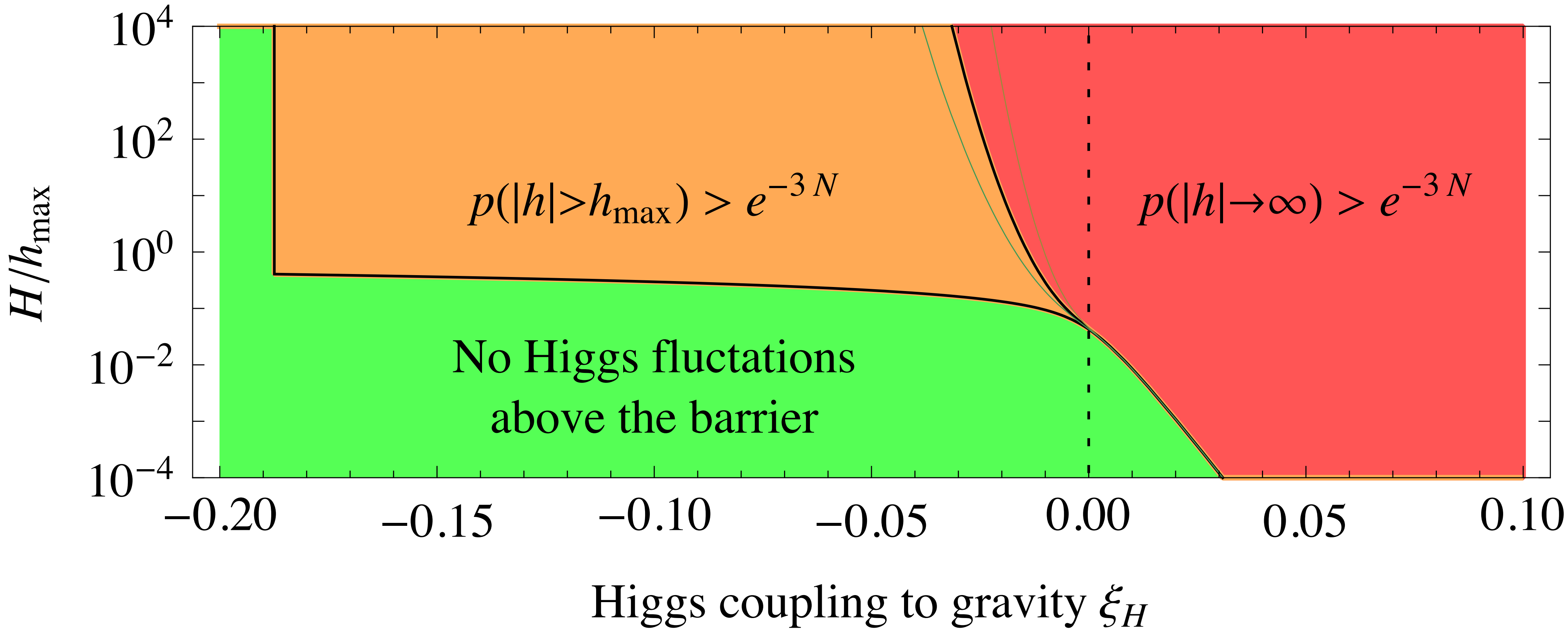
After inflation ends (**quasi-Minkowski exterior**) expanding bubbles continue their growth faster than the expansion rate of the universe, and eat all space.

Evolution of AdS bubbles

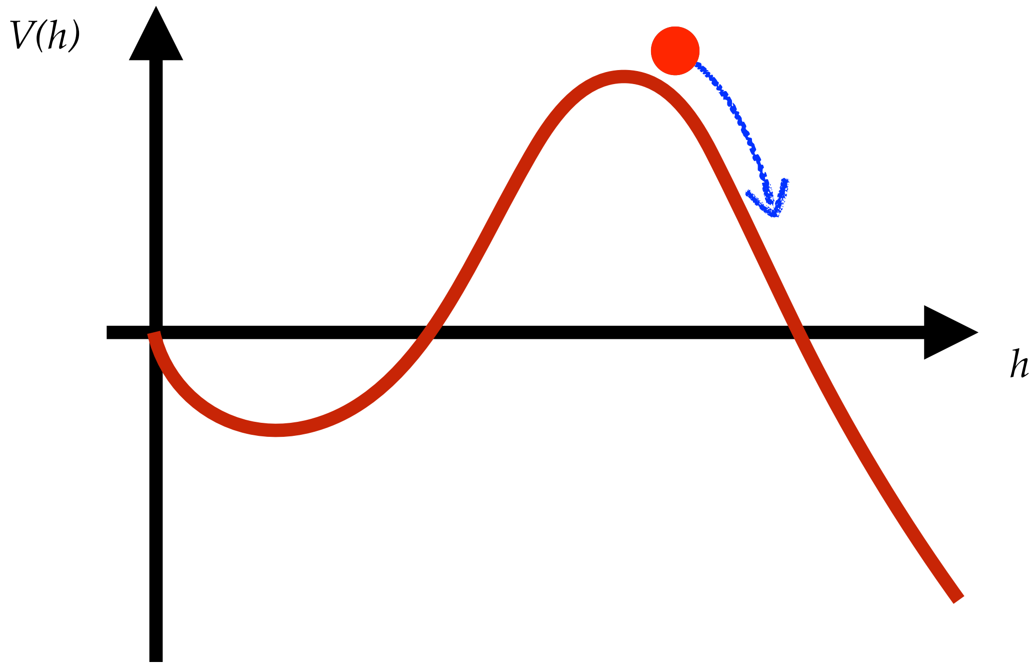


General conclusion: no GR effect prevents AdS bubbles from "eating" all the visible universe.

We must impose that
no bubble nucleation happens during inflation.
The "red" region is excluded.



Higgs evolution after inflation



ORANGE Region:
The Higgs fluctuates beyond its instability without falling into its deep minimum

Even if we avoid bubble creation during inflation, we will be left with regions in which the Higgs fluctuated over the barrier without falling into the deep minimum.
Can these regions be saved by the post-inflationary dynamics?

Higgs evolution after inflation

After inflation ends:

1. **pre-heating:** inflaton oscillates, curvature scalar R decreases, matter-dominated phase $\to$ coupling to gravity

2. **reheating:** inflaton decays into SM, temperature rises $\to$ thermal effects

Reheating

The reheating phase can be described by

$$\begin{cases} \dfrac{d\rho_\phi}{dt} &= -3H_r\rho_\phi - \Gamma_\phi\rho_\phi \\ \dfrac{d\rho_R}{dt} &= -4H_r\rho_R + \Gamma_\phi\rho_\phi \end{cases}$$

with

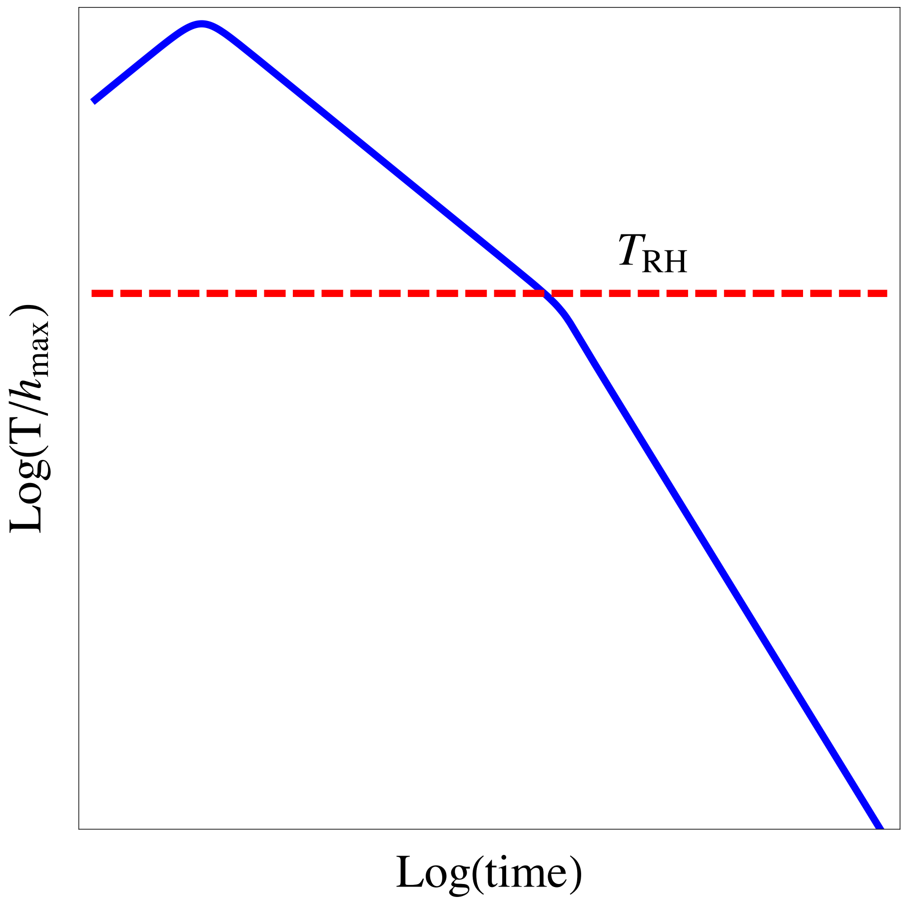
$$H_r = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi}{3}\frac{\rho_\phi + \rho_R}{M_{\rm Pl}^2}}$$

The temperature evolution is given by

$$T \approx 1.3\; T_{\rm max}\; a^{-3/8}(1 - a^{-5/2})^{1/4}$$

with

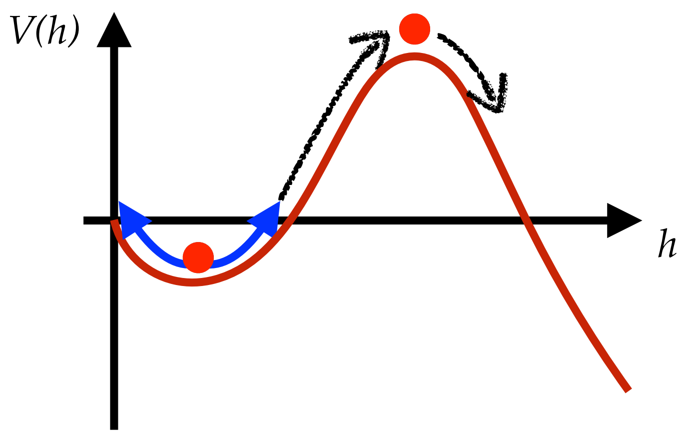
$$T_{\rm max} = 0.54\left(\frac{H M_{\rm Pl} T_{\rm RH}^2}{g_*^{1/2}}\right)^{1/4}$$



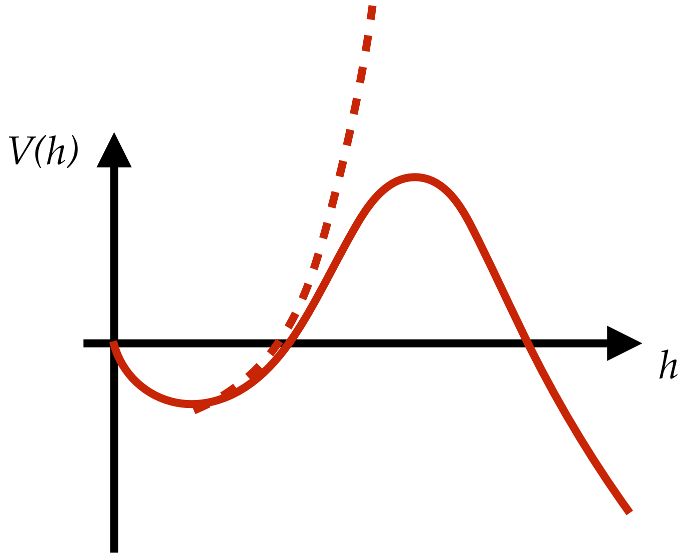
Thermal effects

When the inflaton decays into SM particles, a rising temperature generates (reheating phase). Two contrasting effects:

1. If thermal fluctuations are large enough, h can jump over the barrier



2. Thermal corrections to the potential create an additional effective barrier that stabilises low values of h



This second effect is known to be dominant: thermal corrections stabilise low energy values of the field *(Espinosa, Giudice & Riotto, 0710.2484)*

Effect of the thermal barrier

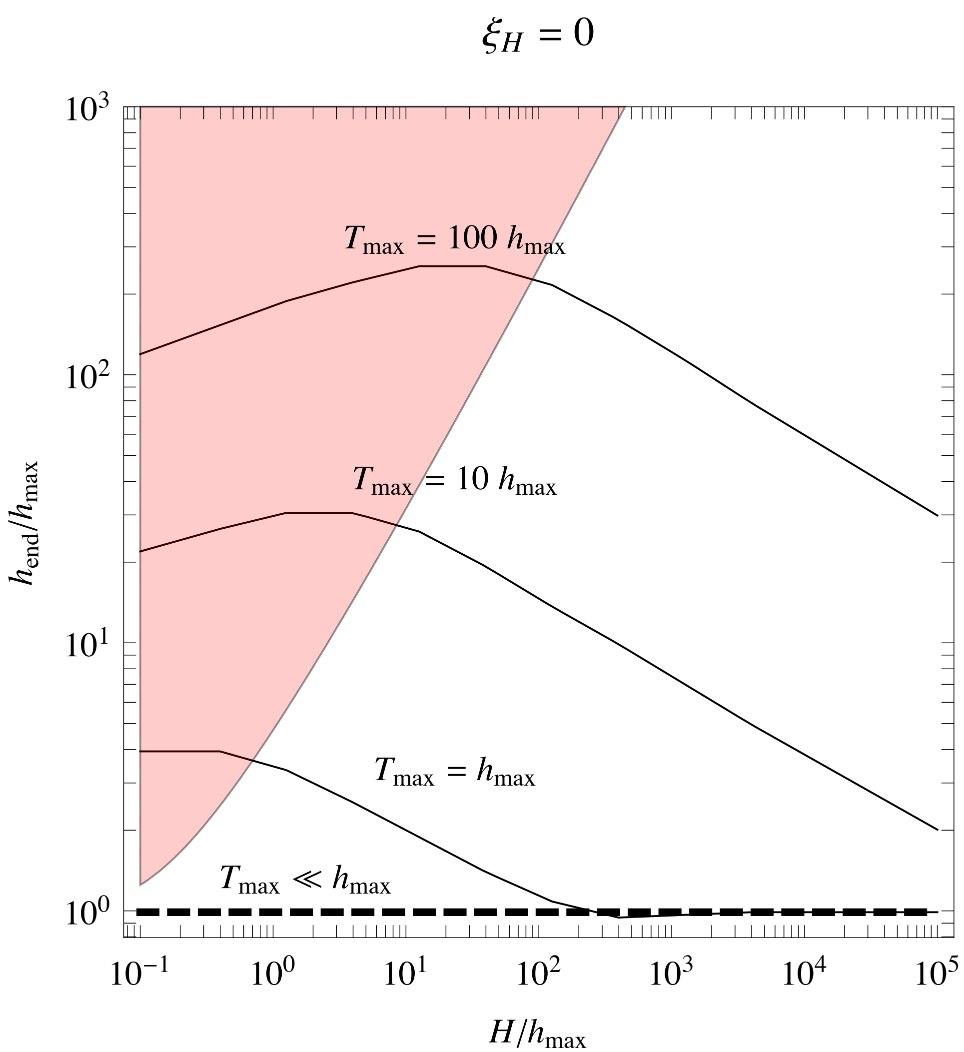
Because of thermal corrections, the Higgs potential receives an extra mass term $m^2 \sim T^2$ valid up to $h \lesssim 2\pi T$.

The thermal correction to the potential can be approximated as

$$V_T \approx \left(0.21 - 0.0071 \log_{10} \frac{T}{\text{GeV}}\right) T^2 \frac{h^2}{2} e^{-\frac{h^2}{(2\pi T)^2}}$$

In order for the Higgs to be "saved" by thermal effects it must be

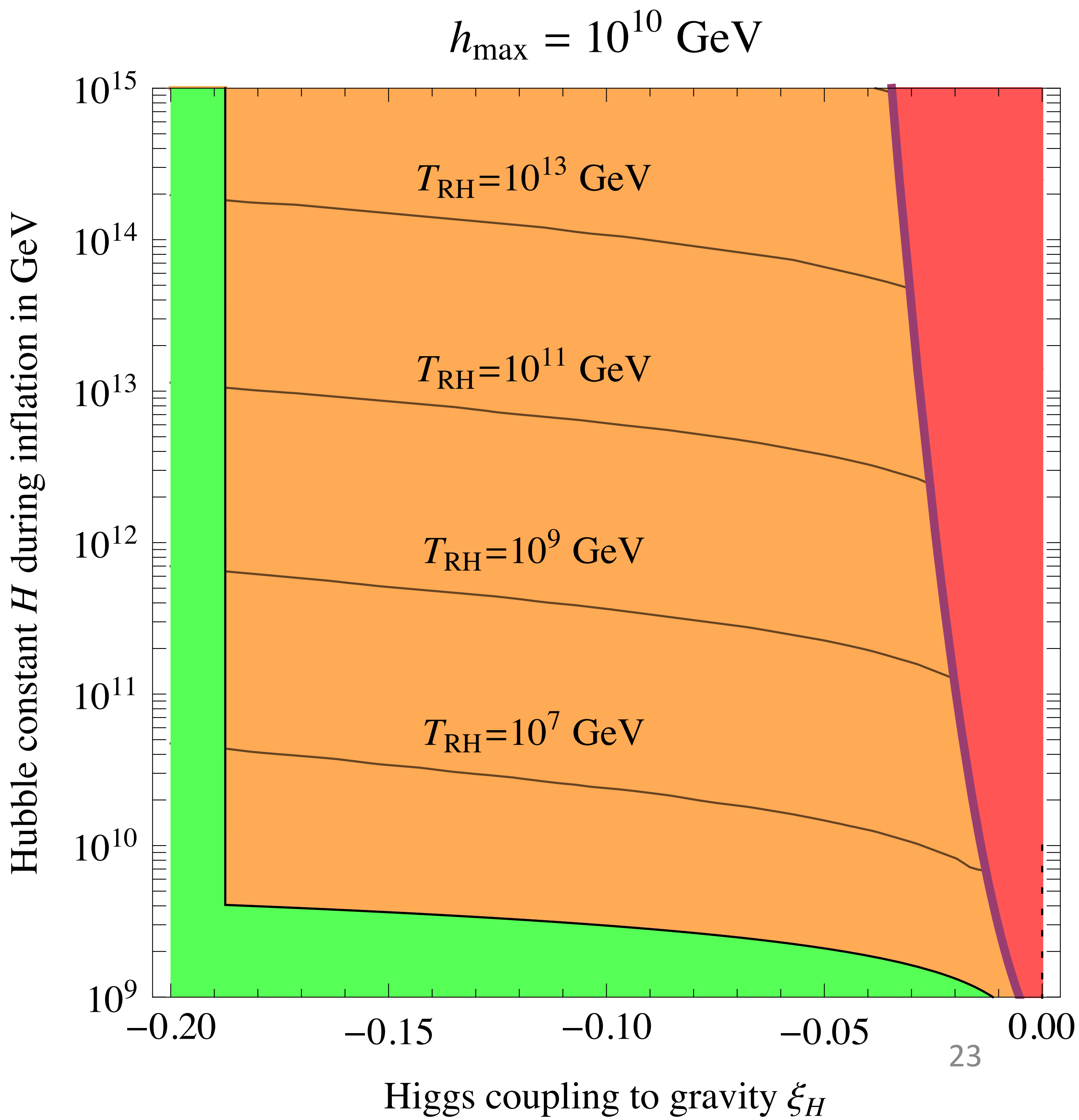
$$h_{\text{end}} \lesssim \left(\frac{8}{45}\right)^{1/6} \frac{H^{-1/3}\, T_{\max}^{4/3}}{\sqrt{|\lambda|}}$$



Higgs evolution after inflation

Putting together the effects of ξ_H and of temperature, we find the minimal reheating temperature that can save the largest fluctuation that we expect to happen somewhere in our visible universe, *i.e.* the one for which $p(h_{\rm end} > h_*) = e^{-3N}$

The ORANGE Region in which the Higgs fluctuates beyond its instability without falling into its deep minimum can be saved by thermal effects for a high enough reheating temperature.



Summary

- We studied the evolution of the Higgs field and its instability, during inflation and during the early phases of radiation dominance.
- Whenever the Higgs falls in its deep minimum, a bubble of AdS forms, (possibly) expands and eventually eats all the visible universe.
- We can put bounds on inflationary parameters by requiring that no bubble forms during inflation (RED region excluded)
- Thermal effects after inflation and induced Higgs mass terms (e.g. non minimal coupling to gravity) play a key role in "saving" the EW vacuum

New directions

- ⋆ Is a fully gauge-invariant treatment possible? What are the gauge invariant quantities related to the instability and/or to the bubble nucleation process?

- ⋆ Is metastability related to quantum-gravity, as a way to escape from stable de Sitter?

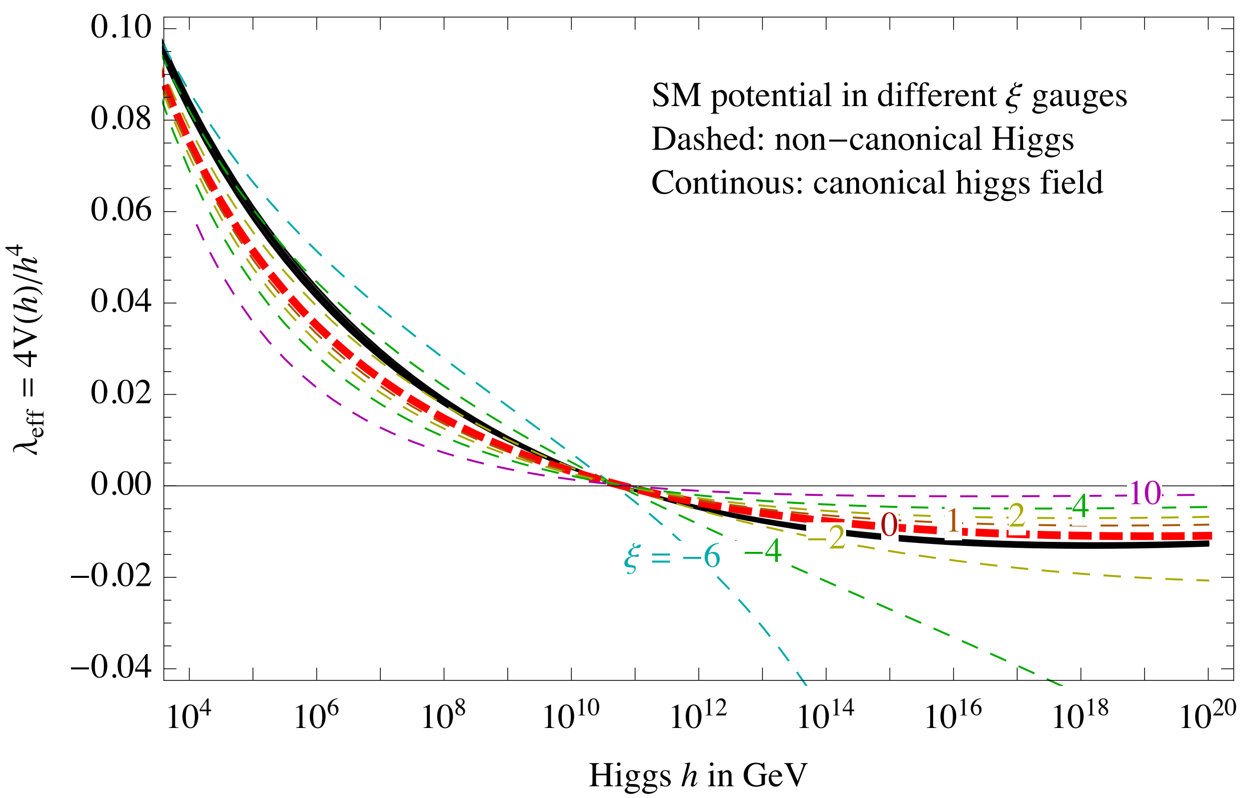
- ⋆ Avoiding the instability can be used as a guidance to put bounds on specific models of inflation (different couplings to gravity, coupling Higgs-inflaton *etc.*) and/or reheating.

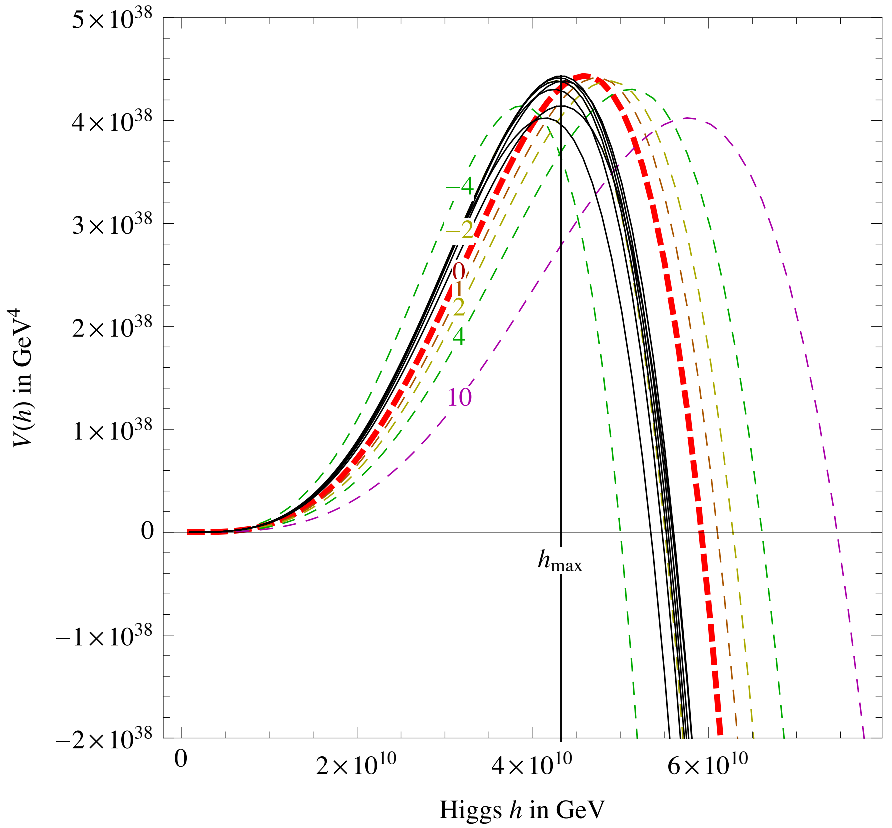
- ⋆ …

Gauge (in-)dependence

The effective action and the effective potential $V(h)$ are known to be gauge-dependent. Their ξ-dependence is determined by the Nielsen identity:

$$\xi \frac{\partial S_{\text{eff}}}{\partial \xi} = - \int d^4x \; K[h(x)] \, \frac{\delta S_{\text{eff}}}{\delta h(x)}$$





Can we obtain physical gauge-independent quantities (bubble nucleation rates, survival probability, …) from a gauge-dependent potential?

Gauge (in-)dependence

We adopt a "practical" solution to this problem.
Starting from the Nielsen identity, and expanding in the number of derivative, we obtain a simple expression for the ξ-dependence of the effective potential:

$$\begin{cases} S_{\text{eff}}[h] = \int d^4x \left[-V(h) + \frac{1}{2} Z(h)(\partial_\mu h)^2 + \mathcal{O}(\partial^4) \right] \\ K[h] = C(h) + D(h)(\partial_\mu h)^2 - \partial^\mu [\tilde{D}(h) \partial_\mu h] + \mathcal{O}(\partial^4) \\ \frac{\delta S_{\text{eff}}}{\delta h} = -V' + \frac{1}{2} Z'(\partial_\mu h)^2 - \partial^\mu [Z(h) \partial_\mu h] + \mathcal{O}(\partial^4) \end{cases} \longrightarrow \quad \xi \frac{\partial V}{\partial \xi} + C(h) V' = 0$$

which tells us that the explicit ξ-dependence of the effective potential could be compensated by an implicit ξ-dependence of the field h:

$$\xi \frac{dh}{d\xi} = C(h) \quad \longrightarrow \quad \frac{dV}{d\xi} = 0$$

Gauge (in-)dependence

To describe the Higgs fluctuations during inflation, we will need the Langevin and the Fokker-Planck equations:

$$\textsc{Langevin}[h] \equiv \sqrt{Z}\frac{dh}{dt} + \frac{1}{3H\sqrt{Z}}V' - \eta = 0$$

$$\textsc{FokkerPlanck}[P(h,t)] \equiv \frac{1}{\sqrt{Z}}\frac{\partial}{\partial h}\left\{\frac{1}{\sqrt{Z}}\left[\frac{\partial}{\partial h}\left(\frac{H^3}{8\pi^2}\frac{P}{\sqrt{Z}}\right) + \frac{1}{3H}\frac{PV'}{\sqrt{Z}}\right]\right\} - \frac{1}{\sqrt{Z}}\frac{\partial P}{\partial t} = 0$$

It turns out that both them and the equation of motion of the Higgs become gauge independent if we assume the same ξ-dependence of the Higgs field that works for the Nielsen identity:

$$\xi\frac{dh}{d\xi} = C(h) \longrightarrow \begin{cases} \frac{d}{d\xi}\textsc{Langevin}[h]\Big|_{h=h_L} = 0 \\ \frac{d}{d\xi}\textsc{FokkerPlanck}[P(h,t)]\Big|_{P=\bar{P}} = 0 \end{cases}$$

Gauge (in-)dependence

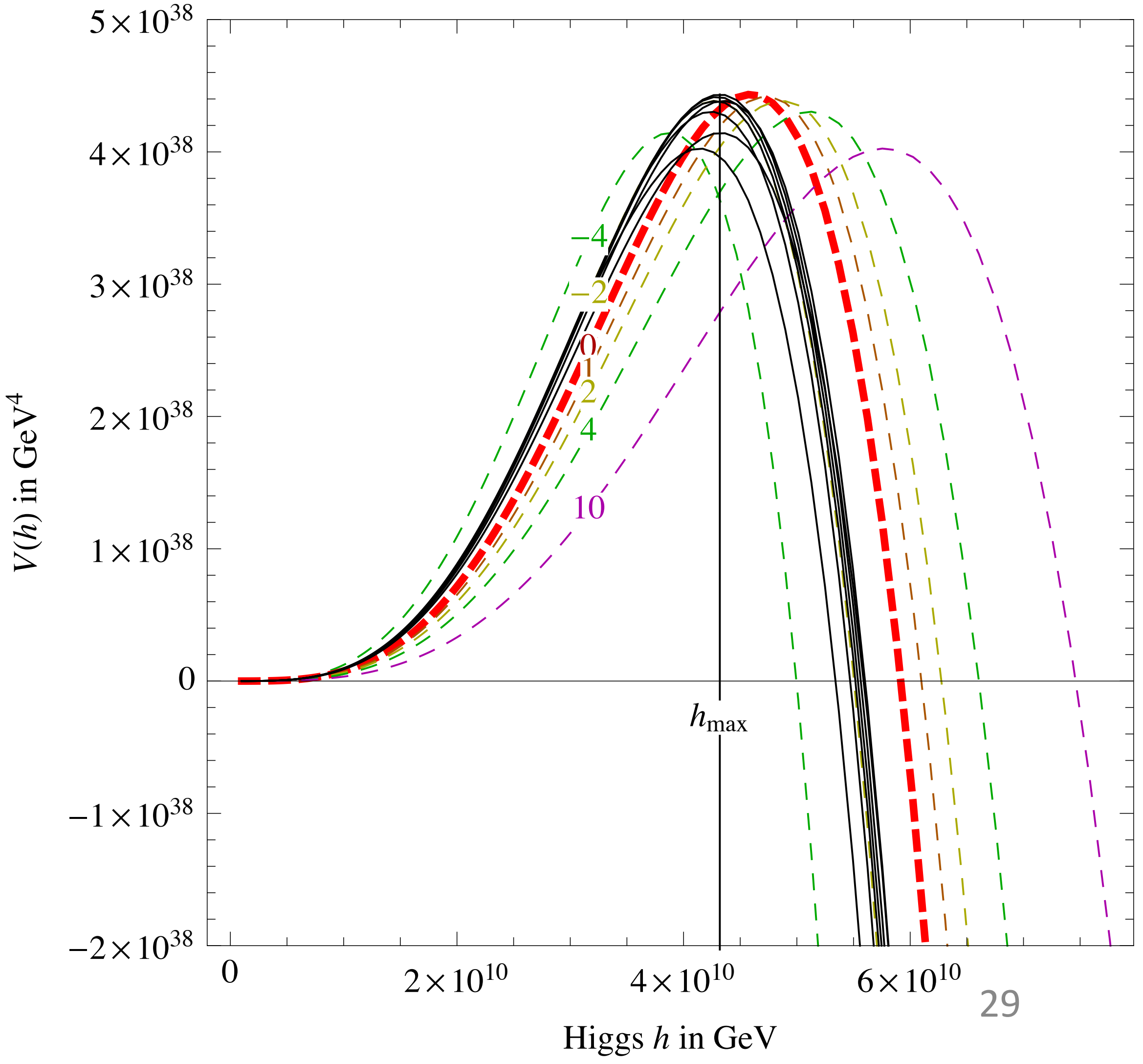
Consider the SM effective lagrangian

$$\mathcal{L}_{\rm eff} = Z(h,\xi)\frac{(\partial_\mu h)^2}{2} - \lambda_{\rm eff}(h,\xi)\frac{h^4}{4} + \cdots$$

It's useful to rewrite it in terms of the canonically normalised field $h_{can}(h,\xi)$

$$\frac{dh_{\rm can}}{dh} = Z^{1/2} \longrightarrow \mathcal{L}_{\rm eff} = \frac{(\partial_\mu h_{\rm can})^2}{2} - \lambda_{\rm can}(h_{\rm can},\xi)\frac{h_{\rm can}^4}{4} + \cdots$$

With this field redefinition, the gauge dependence of the effective potential is reduced.
The reason is that, at leading order, the transformation that makes the field canonical is precisely the one dictated by the Nielsen identity.



Gauge (in-)dependence

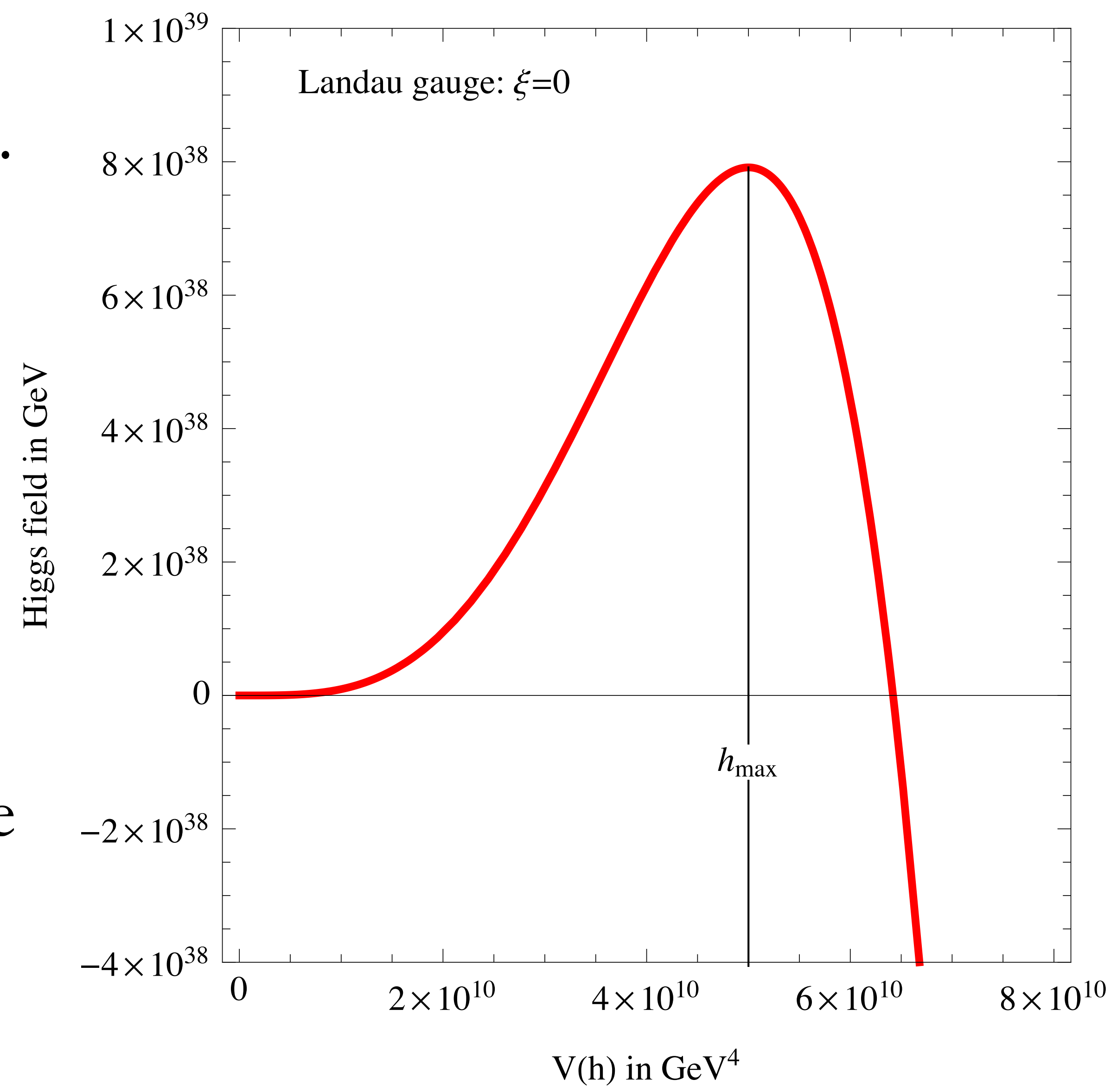
We choose to work in Landau gauge (ξ=0) and with canonically normalised Higgs field. The effective potential at NNLO accuracy can be approximated by

$$V_{\text{eff}}(h) \approx -b \ln\left(\frac{h^2}{h_{\text{max}}^2 \sqrt{e}}\right) \frac{h^4}{4}$$

h_{max} is not a gauge invariant quantity, but the results will be gauge invariant.

In Landau gauge, for the present best fit value of the SM parameters,

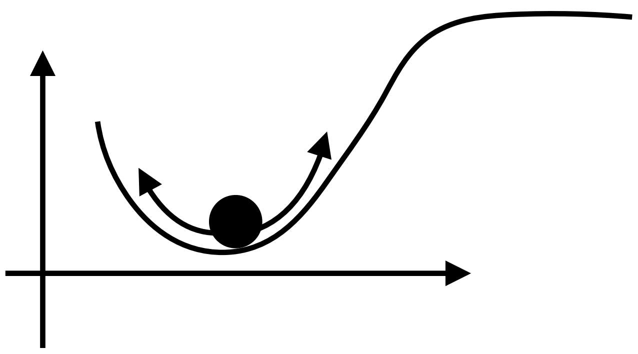
$$h_{\text{max}} = 5 \text{ x } 10^{10} \text{ GeV}$$



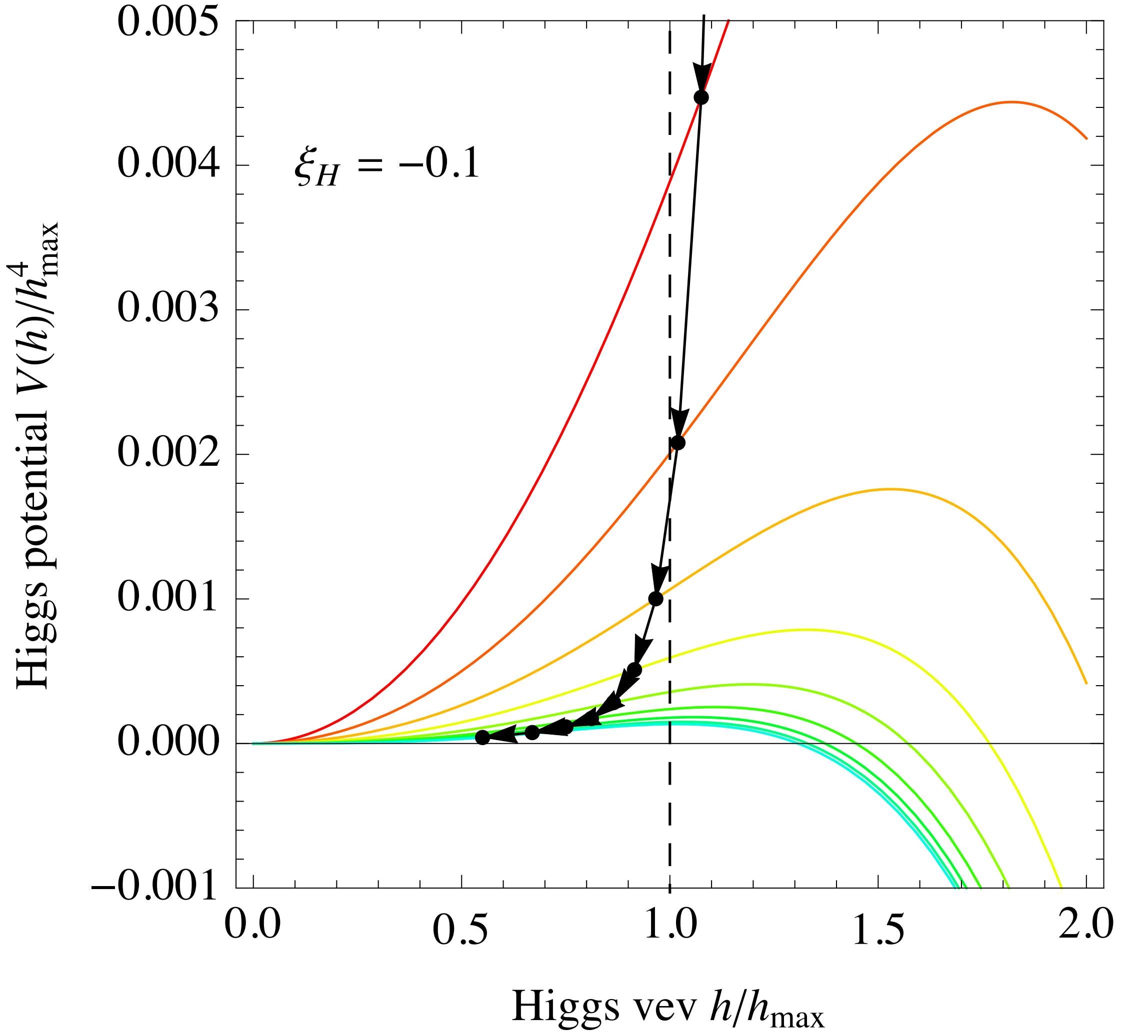
Coupling to gravity after inflation

As the inflaton oscillates the Hubble rate decreases as

$$H_{\rm m} = H/a^{3/2}$$



and so does the mass term $\quad m^2 = -12\xi_H H_{\rm m}^2$

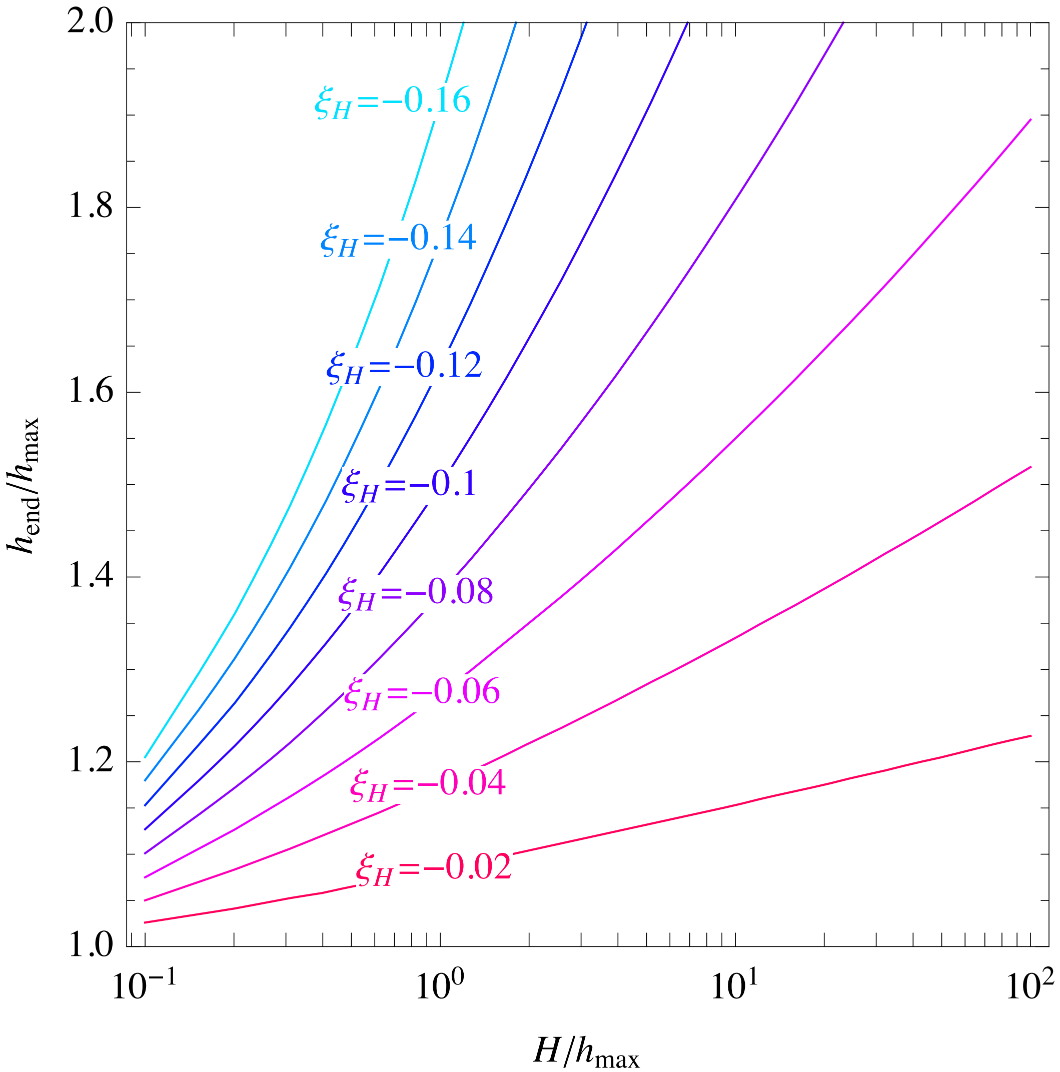


As the induced barrier height decreases, h rolls down the potential and, depending on its value at the end of inflation $h_{\rm end}$ and on the initial height, it can safely land in the EW vacuum.

Coupling to gravity after inflation

Solving for the classical evolution of the Higgs field in a matter-dominated background, one can find the maximum value of $h_{\rm end}$ that can be "saved" by the mass term:

$$\frac{d^2h}{dt^2} + 3H_{\rm m}(t)\frac{dh}{dt} + \frac{\partial V}{\partial h} = 0 \longrightarrow \frac{d^2h}{da^2} + \frac{5}{2a}\frac{dh}{da} + \frac{a}{H^2}\frac{\partial V}{\partial h} = 0$$



The instability is avoided if

$$\boxed{h_{\rm end} \lesssim h_{\rm max}\, a_{\rm max}^{-2\xi_H}}$$

where $\quad a_{\rm max}^3 \approx -\frac{12\xi_H H^2}{b h_{\rm max}}$