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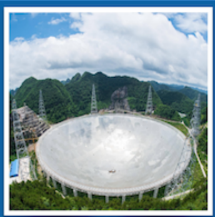
Neutrino Mass Hierarchy

(how to avoid to mess it up ?)

- The present 3 standard neutrino *oscillation* framework
- Mass Ordering (MO), Experiments, Techniques (limited list)
- Issues on the Statistical Analysis
- A robust determination of MO in the near future ?
- Conclusions and Perspectives



Astro@Stats 2017: Sino-Italian Workshop on Astrostatistics

08 September 2017 *Department of Statistical Sciences*
Europe/Rome timezone

The recent Neutrino History

1998
1rst Annus Mirabilis
Oscillations :
- Observation by *SuperK*
- Reactor exclusion by *Chooz*

2002
2nd A. M.
3 x 3 matrix :
- Solar by *SNO*
- Reactor by *KAMLAND*

2012
3rd A. M.
θ_{13}:
- Reactor by *Daya Bay*, *RENO*, *Double Chooz*, *T2K*…

2017-18 ?
4th A. M. ?
- Mass Hierachy by *NOvA/T2K* ?
- CPV by *T2K* ??

≥ 2020 ? —— δ_{CP} ?
- δ_{CP} by *T2K* ?

L. S., Rev. in Phys. 1 (2016) 90

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & +c_{23} & +s_{23}\\ 0 & -s_{23} & +c_{23}\end{pmatrix}\begin{pmatrix}+c_{13} & 0 & +s_{13}e^{-i\delta_{CP}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{CP}} & 0 & +c_{13}\end{pmatrix}\begin{pmatrix}+c_{12} & +s_{12} & 0\\ -s_{12} & +c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

Super-K, K2K, MINOS, OPERA, NOvA, T2K

DChooz, Daya Bay, RENO MINOS, NOvA, T2K

Super-K, SNO, KamLAND

$c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$
(PMNS Neglecting possible Majorana phases)

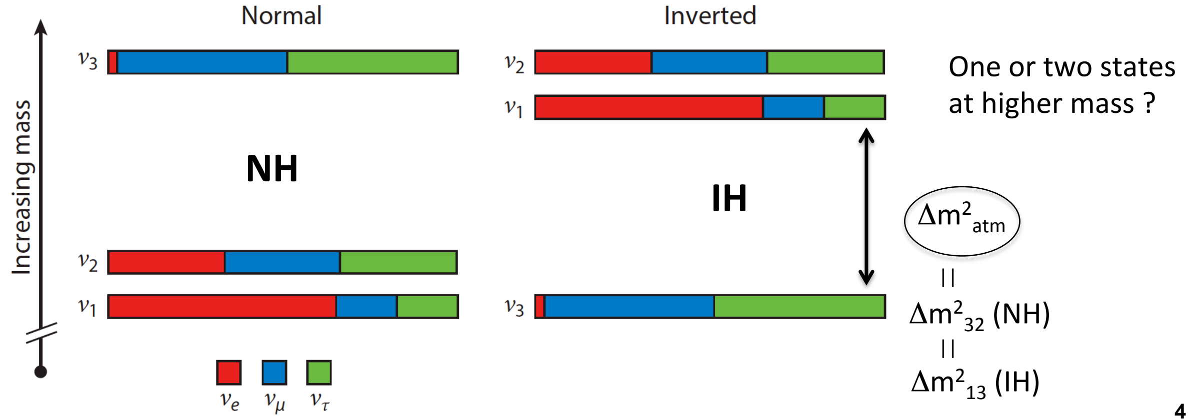
The wonderful frame pinpointed for the 3 standard neutrinos, beautifully adjusted by the θ_{13} measurement, left out some relevant questions:

- Leptonic CP violation, δ_{CP}
- **Mass values**, **MH**
- Dark Matter
- Anomalies and discrepancies in some measurements

The present scenario (cnt.)

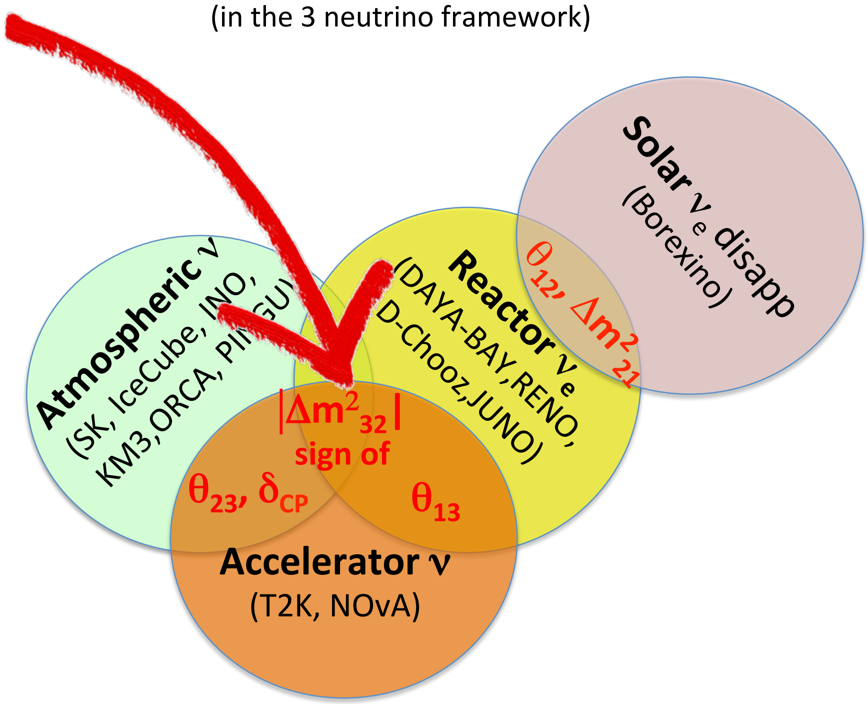
Before really entering in the precision era, there are still 4 results to be obtained, at least at first order :

1) Leptonic CP violation (phase δ_{CP})
2) Mass ordering (MO)
3) (θ_{23} octet)
4) Presence or not of more (sterile ?) neutrinos states



Neutrino Oscillation Industry

(in the 3 neutrino framework)



From a Maxim Gonchar (DLNP) picture

disclaimer: major actors only, not a full list…

First slide on STATISTICS

What is largely accepted in **particle physics** is that:

- Observations are given by a **5 σ** measurement
- Exclusions are given at **95% C.L.** when systematics is well know, and at **3 σ** or more when detectors of new conception are used, or to reject previous results.

That is established by the last 30 years of experiences in our field.

However, there is a still too large use of the 90% C.L. for exclusion limits

Concerns about the use of 90% C.L.:

- It does not provide the "good experimental feeling" (some observations were just below the "limit")
- Choice for one-sided or two-sided is statistically relevant (and the choice is not often obvious)
- Historical comparison of different measurements is not only a matter of the "median" values

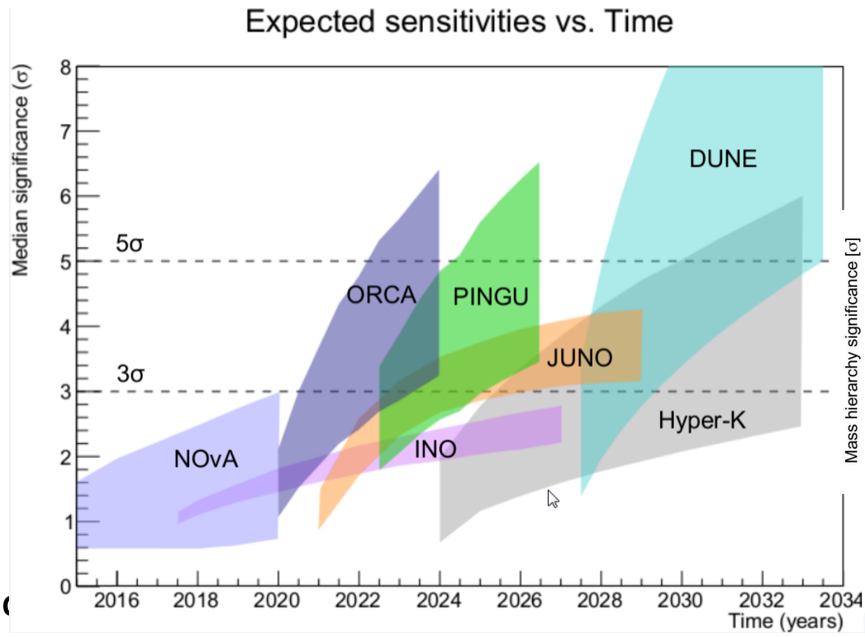
Neutrino community should agree on a more robust statistical framework, since we are entering in the neutrino precision era

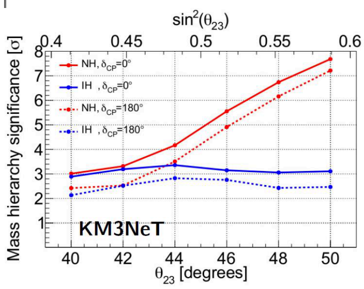
What is the Mass Hierarchy determination?

is it an **exclusion** process or an **observation** one ?

Assume the presence of the signal and compute the compatibility of the data in the parameter space of the signal	Assume no signal and compute the compatibility of the data with the background
(signal rejection)	*(background rejection)*

UPDATE: A. Heijboer, talk at NOW2016, Otranto (Italy), 4-11 September 2016





median sensitivities

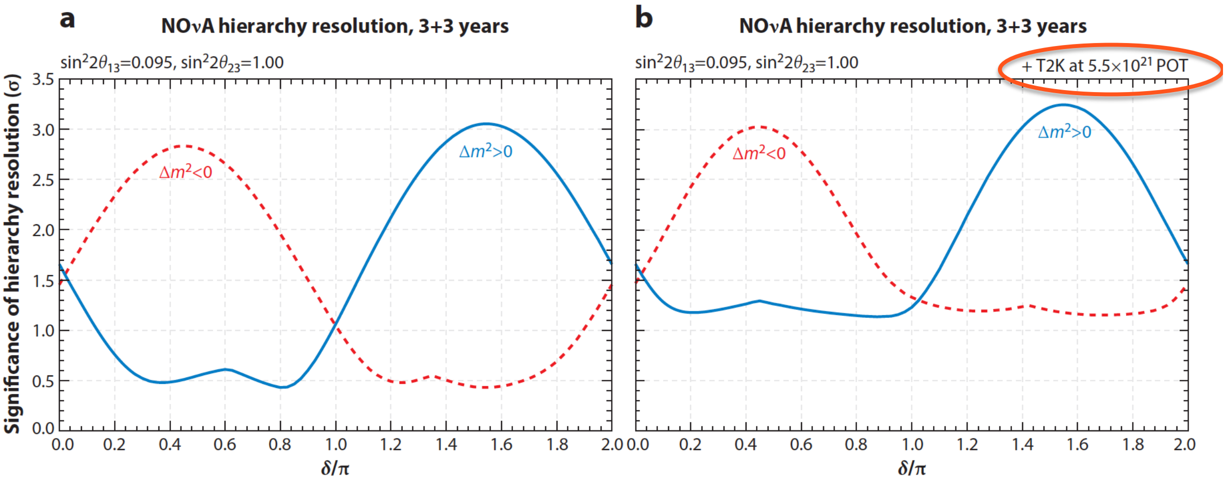
- NOvA: luckiness ?
- PINGU/ORCA: funded ? systematics ?
- INO: really ?
- JUNO: technical challenge on energy resolution ?
- DUNE/HY: ok they will got it, in 10 years from now **IF exploited**

Many different ways to define "sensitivity" even if all based on $\chi 2$

Statistical Issues: several studies (Frequentist and Bayesian, Asimov, full simulation…)

They are all based on χ^2 best fits, marginalized over the precisions of 3 ν parameters

➔ Foreseen years and years of data taking and experiments



3.6×10^{21} pot

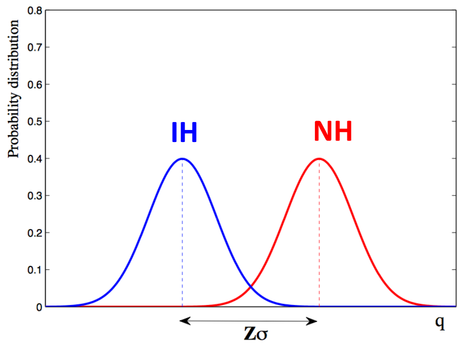
R.B. Patterson, Annu. Rev. Nucl. Part. Sci. 65 (2015) 177

A second slide on STATISTICS

Not all the estimators correspond to the same significance
(Frechet-Cramer-Rao theorem and limit)

For example, the arithmetic mean and the median own two different variances:
Mean ➔ $\sigma/\sqrt{n}$ *(this is also the Cramer-Rao limit)*
Median ➔ $\sqrt{(\pi/2)} \times \sigma/\sqrt{n}$
i.e. the median has an error 25% larger than the mean.

That matters when doing test of hypotheses, determination of NH against IH, etc.



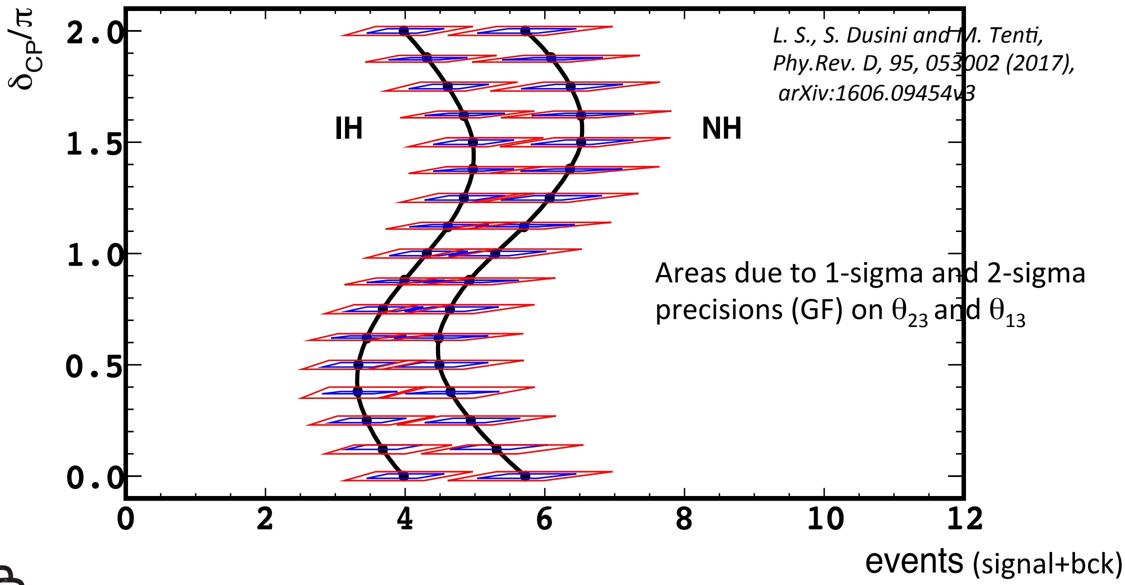
My rule of thumb: only above 5 σ the scientific result of an experiment will NOT depend of the used statistical analysis

An experiment with an estimation of 4 σ but forced to use medians (e.g. due to systematics, outliers etc.) will get 3.2 σ

Many options, some degeneracies, only the χ^2 technique used so far.

Different solutions with best χ^2 for NH at δ_{CP} and for IH at δ_{CP}

NOvA 2015 luminosity + Global Fits + GLoBES simulation*



$\Delta\chi^2=\chi^2_{min}(NH) - \chi^2_{min}(IH)$

Asimov data-set, prob. ratios etc.

My conclusions:

- the χ^2 performs a multi-parametric fit taking into account both uncertainties and degeneracies:
  that is a concern, artificial/artifact solutions may be found

- define a clear procedure for the MH determination

For a discovery sensitivity is much easier to estimate and quote.
Then the right process is to "reject the wrong hierarchy".
A 5 σ result would be needed. Choose an appropriate discriminator !
But that process for MH can be also seen as an exclusion one.
Then a 95% C.L. quotation could be sufficient (*) !

* If
- properly computed with a good estimator and billion (trillion) of simulated events
- to properly evaluate its Probability Density Function and
- to further properly extract its significance

More technical, the χ^2 is defined as: $$\chi^2 = \chi^2_{para} + \chi^2_{sys} + \chi^2_{stat}$$

$$\chi^2_{\rm para} = \left\{\frac{(\sin^2 2\theta_{12})^{\rm fit} - (\sin^2 2\theta_{12})^{\rm input}}{\delta \sin^2 2\theta_{12}}\right\}^2 + \left\{\frac{(\sin^2 2\theta_{13})^{\rm fit} - (\sin^2 2\theta_{13})^{\rm input}}{\delta \sin^2 2\theta_{13}}\right\}^2 + \left\{\frac{(\Delta m^2_{21})^{\rm fit} - (\Delta m^2_{21})^{\rm input}}{\delta \Delta m^2_{21}}\right\}^2 + \left\{\frac{(|\Delta m^2_{31}|)^{\rm fit} - (|\Delta m^2_{31}|)^{\rm input}}{\delta |\Delta m^2_{31}|}\right\}^2 .$$

$$\chi^2_{\rm sys} = \left(\frac{f^{\rm fit}_{\rm sys} - f^{\rm input}_{\rm sys}}{\delta f_{\rm sys}}\right)^2$$

$$\chi^2_{\rm stat} = \sum_i \left(\frac{N_i^{\rm fit} - N_i^{\rm NH(IH)}}{\sqrt{N_i^{\rm NH(IH)}}}\right)^2$$

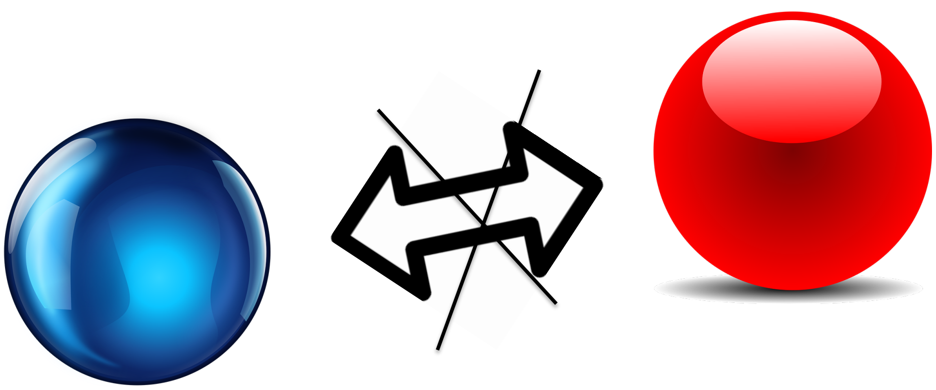
Everything is messing up

penalties

$\theta_{23} + \delta\theta_{23}$ ➔ 1 unit of χ^2

Lum – 2·δLum ➔ 4 units of χ^2

…



Two Universes, not talking each others

A third slide on STATISTICS

Within the chosen hypothesis (either NH or IH) everything looks fine: good Gaussian properties, χ^2 distributions, use of LLR etc.

Problems come when the $\Delta\chi^2$ quantity is used.

Nobody really knows how to properly handle $\boldsymbol{\Delta\chi^2}$ to quote the sensitivity, (and therefore also the measurement)
because $\boldsymbol{\Delta\chi^2}$ is a rather bad estimator:

- not following the Wilk's theorem;
- NH/IH are disjoint and NOT nested hypotheses;
- marginalization or minimization ?
- frequentist or bayesian computation ?

and, additionally (personal thought) it does not correctly approach the observation/rejection issue*,
it deals instead with the fuzzy concept of "separation of two hypotheses" (type I and type II errors)

* *When dealing with $\Delta\chi^2$ the solution is given by the best χ^2. Thus you are trying to define the observation through the right hypothesis itself, and NOT trying to reject the wrong hypothesis.*

The use $\Delta\chi^2$ corresponds to a rough, limited, controversial analysis.

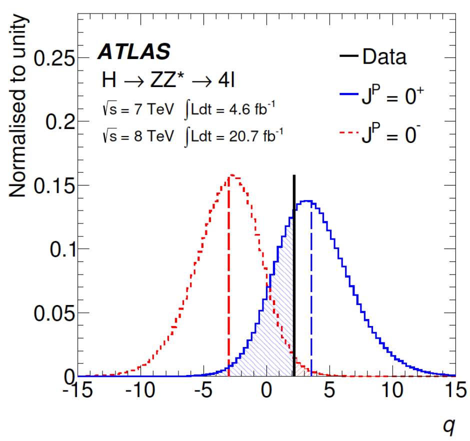
Much better is to individuate an estimator that couples the two hypotheses.
Look at Higgs J^P study, a simple discriminator $\mathcal{D}_{12}$ is used for $\mathcal{P}_1$ and $\mathcal{P}_2$ hypothesis:

$$\mathcal{D}_{12} = \mathcal{P}_1 / (\mathcal{P}_1 + \mathcal{P}_2)$$

and use the likelihhod ratio.q=- ln $\mathcal{L}_1/\mathcal{L}_1$

(Exclusion) sensitivity is given by the (usual at LHC) CLs method:

CLs = p-val(alternate H)/ (1-p-val (0+))



Phy. Lett. B 726 (2013) 120, arXiv:1307.1432

➔ new method *(it means new estimator)*

L. S., S. Dusini and M. Tenti, Phy.Rev. D, 95, 053002 (2017), arXiv:1606.09454v3

1 Take the Poisson distributions, f_{IH} and f_{NH} and the new estimators, q_{IH} and q_{NH}, as function of a threshold n, nb. of events, and of δ_{CP}:

$$q_{IH}(n,\delta_{CP}) = \frac{\sum_{n_i^{IH}\geq n} f_{IH}(n_i^{IH};\mu_{IH}|\delta_{CP})}{\sum_{n_i^{NH}\geq n} f_{NH}(n_i^{NH};\mu_{NH}|\delta_{CP})}$$

$$q_{NH}(n,\delta_{CP}) = \frac{\sum_{n_i^{NH}\leq n} f_{NH}(n_i^{NH};\mu_{NH}|\delta_{CP})}{\sum_{n_i^{IH}\leq n} f_{IH}(n_i^{IH};\mu_{IH}|\delta_{CP})}$$

q_{IH} and q_{NH} are discrete random variables in [0,1]

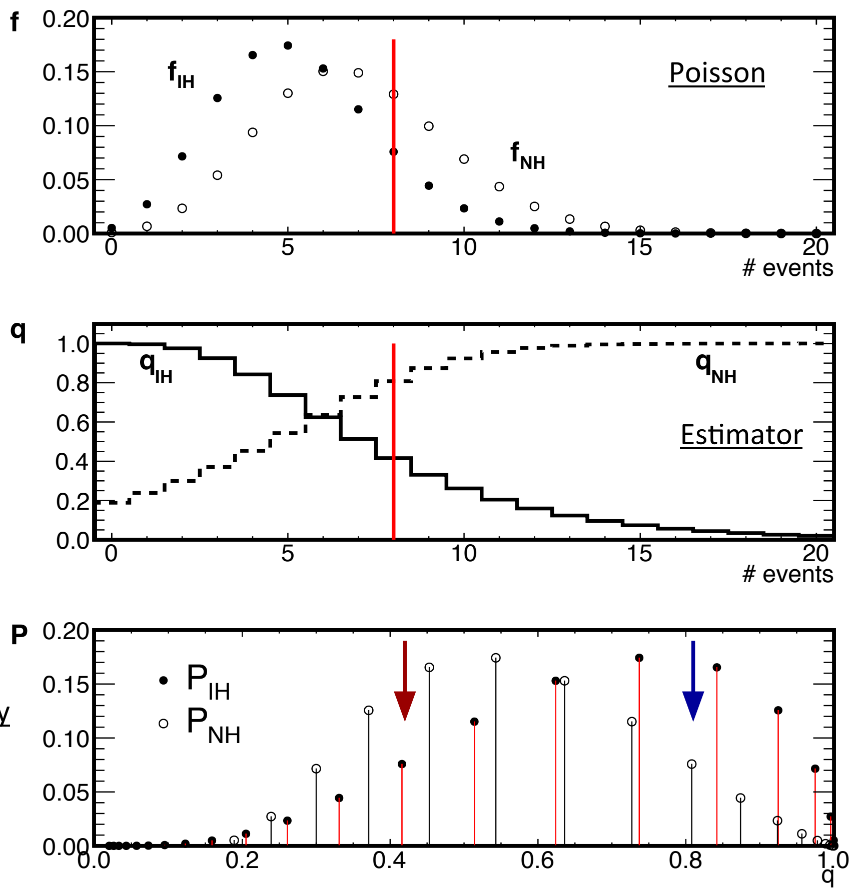
2 Compute probability mass functions P_{IH} and P_{NH} of q_{IH} and q_{NH}

Assume one Hierarchy, compute its p-value for the corresponding estimator

Compute n-sigma with the one-sided option

Example for
$\delta_{CP} = 3/2\pi$ and
2.74×10^{20} p.o.t. ,
$n = 8$

- f_{IH} (for test of IH against NH)
- f_{NH} (for test of NH against IH)



Probability

4 Assume one Hierarchy, compute its ***p***-value for the corresponding estimator

$$p_{IH}(n_D, \delta_{CP}) = \sum_{q'_{IH} \leq q_{IH}(n_D)} P_{IH}(q'_{IH}|\delta_{CP})$$

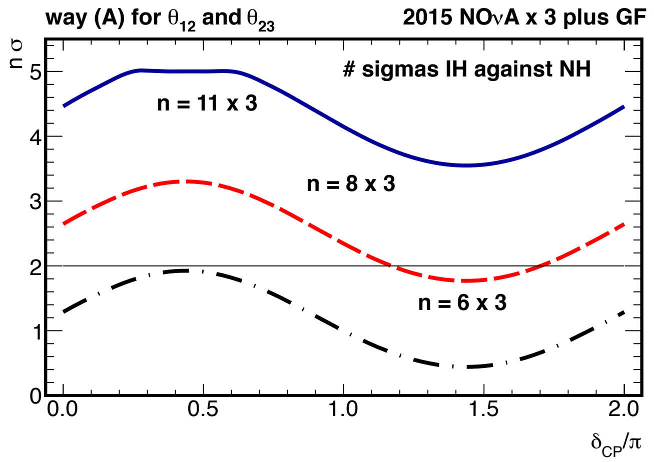
$$p_{NH}(n_D, \delta_{CP}) = \sum_{q'_{NH} \leq q_{NH}(n_D)} P_{NH}(q'_{NH}|\delta_{CP})$$

5 Compute n-sigma with the one-sided option
(***p***=0.5 ➔ 0 sigma)

➔ new method/procedure for NOvA

L. S., S. Dusini and M. Tenti, Phy.Rev. D, 95, 053002 (2017), arXiv:1606.09454v3

Suppose to have a factor 3 more of 2015 NOvA luminosity*:



(* with the same analysis conditions)

Gaining Factors

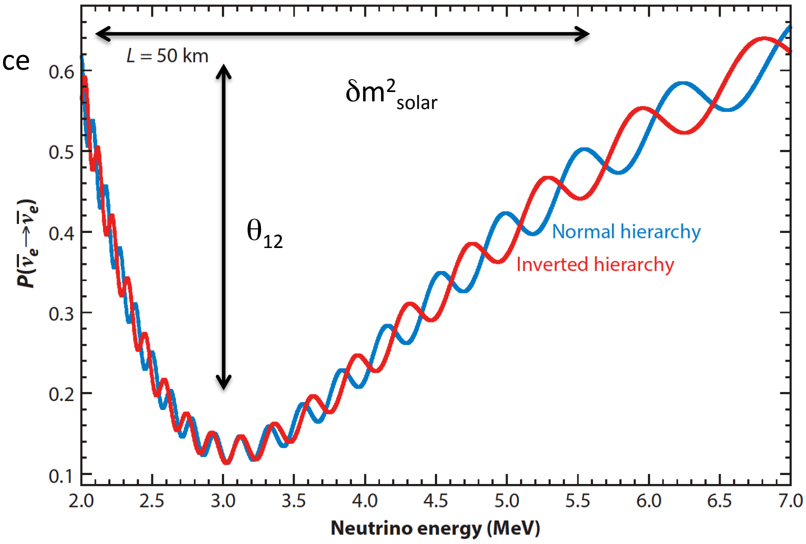
median	average on δ	spread
100% prob	2.27	+0.18-0-12
32% prob.	2.75	+0.51-0.19
10% prob.	3.78	+0.79-0.39

Let us look at **JUNO** perspectives

One possibility to access MO: interference of atmospheric and solar oscillations in reactors

(as originally pointed out by M.Piai&S.Petcov in 2002, PLB)

$\bar{\nu}_e$ disappearance



JUNO example

wiggles correspond to the subdominant atmospheric oscillation

F. An et al. (JUNO), J. Phys. G 43 (2016) 030401

Jiangmen Underground Neutrino Observatory

the first multi-kton liquid scintillator detector ever

JUNO current effort to “**ascertain the true hierarchy**”
*(is this really what we want ? A better approach would be **to reject the wrong hierarchy**)*

Stat. discrim, power of **4σ** for:
- Baseline 53 km,
- Fiducial Volume 20 kt,
- Therm. React. Power 36 GW,
- Exposure Time 6 years,
- Energy resolution 3%

→ **3σ** add syst. (position) → **4σ** Δm^2_{23} with 1% unc.

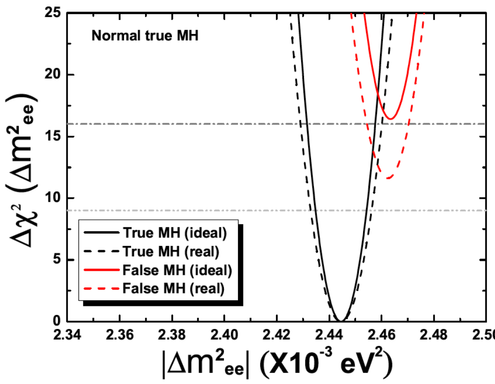
HOWEVER:
In vacuum oscillations, it holds

$$|\Delta m^2_{31}| = |\Delta m^2_{32}| \pm \delta m^2_{21}$$

$$|\Delta m^2_{31}| = (256.2 \pm 3.5)\cdot 10^{-5}\, eV^2$$

$$\delta m^2_{21} = (7.8 \pm 0.2)\cdot 10^{-5}\, eV^2$$

$$\rightarrow \quad |\Delta m^2_{31}| = |\Delta m^2_{31}|\cdot(1 \mp 0.03)$$



$$\Delta m^2_{ee} = \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}.$$

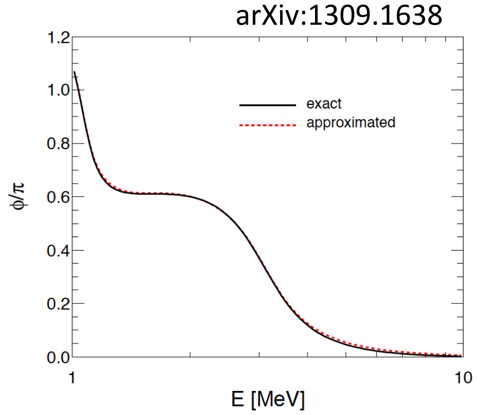
→not possible to achieve more than 4 σ using the $\Delta\chi^2$ method at leading order

VERY STRONG dependence on Δm^2_{atm}

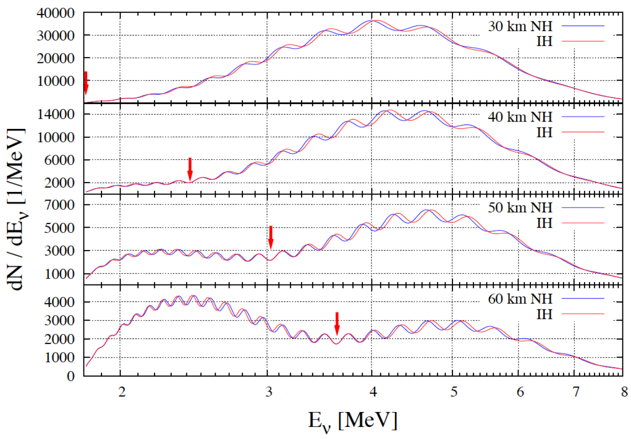
More relevant: artificial degeneracies are introduced !

JUNO: $$P^{3\nu}_{\rm vac} = c^4_{13}P^{2\nu}_{\rm vac} + s^4_{13} + 2s^2_{13}c^2_{13}\sqrt{P^{2\nu}_{\rm vac}}\cos(2\Delta_{ee} + \alpha\varphi)$$ α=±1 (MH)

arXiv:1309.1638



arXiv:1210.8141

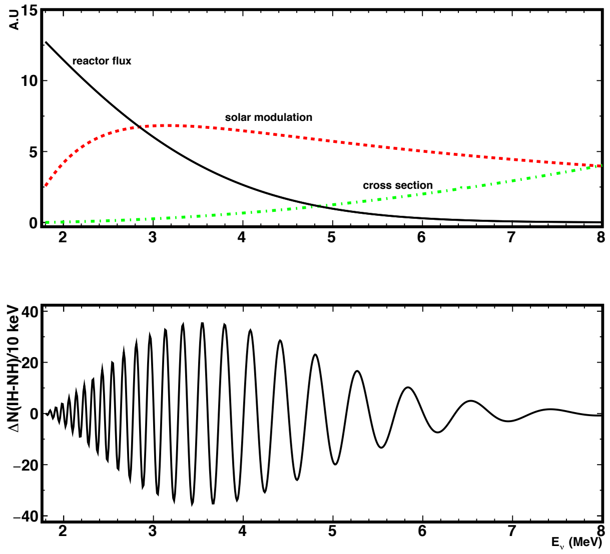


Rewrite formulas,
introduce the phase φ,
no dependence on L/E,
be sensitive only to ± φ
that breaks the original Δm^2 deg
introducing a new one due to
different choices of Δm^2 for MH

LS, G. Salamanna, C. Sirignano, F. Sawy, arXiv:1707.07651

New procedure, which couples NH/IH, new discriminant, no fit

Amount of difference in events for 180 GW•year



$$\Delta N(E_\nu)_{IH-NH} = T \times \phi(E_\nu) \times \sigma_{\bar{\nu}_e p} \times \sin^2 2\theta_{13} \cos 2\theta_{12} \times \sin \frac{L \delta m^2_{sol}/2}{2E_\nu} \times \sin\left[\frac{L}{2E_\nu}\left(\Delta m^2_{atm} - \delta m^2_{sol}/2\right)\right]$$

New estimator: F

$$F = \sum_j \Delta_j^+ + \sum_j \Delta_j^-$$

$\Delta^+ = n_{obs} - n^{IH}_i$ when $n_{obs} > n^{IH}_i$ in I^+

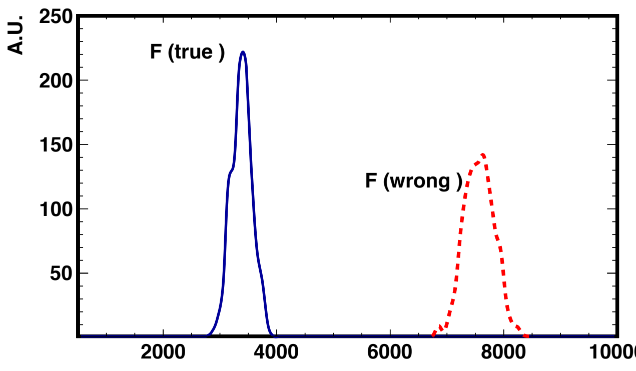
$\Delta^- = n^{IH}_i - n_{obs}$ when $n_{obs} < n^{IH}_i$ in I^-

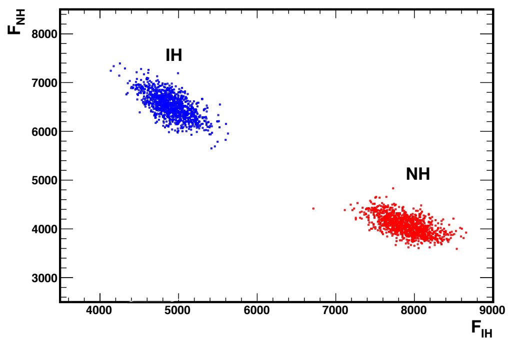
I^+ intervals when $n^{IH}_i < n^{NH}_i$

I^- intervals when $n^{IH}_i > n^{NH}_i$

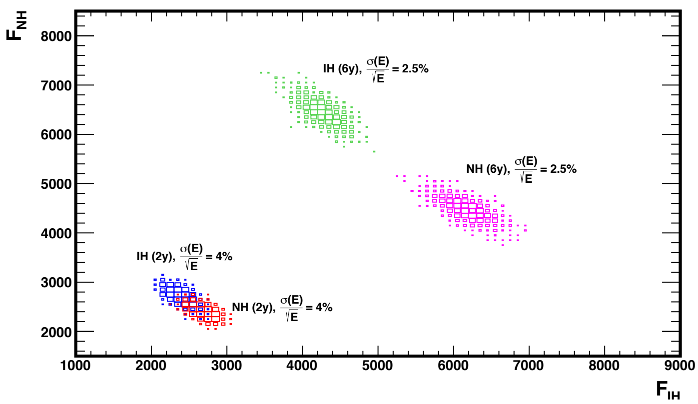
➔ *a possible complementary analysis*

F discriminator is dominated by statistical fluctuations and energy resolution





plus energy resolution

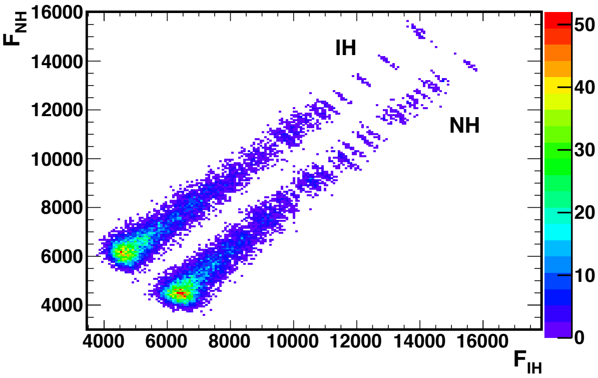


How F deals with the degeneracy

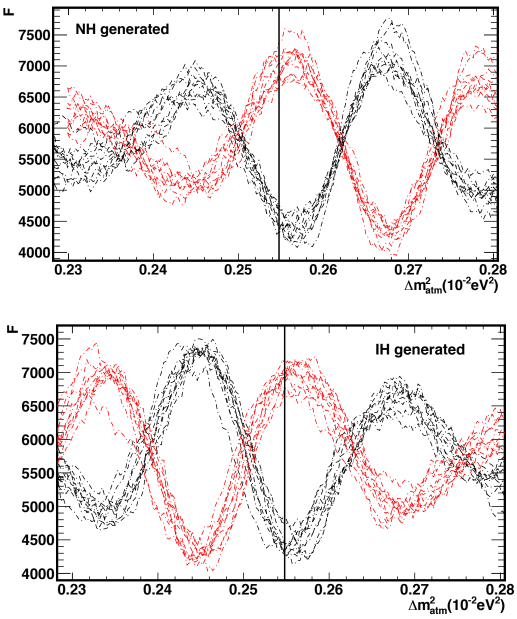
No strong dependence on parameters other than Δm^2_{23} (technically they are marginalized)

TABLE III. The parameters used in the large simulation. For each of them the chosen central values and their uncertainties are quoted. They are allowed to vary contemporary, each following a Gaussian distribution. The two Δm^2_{atm} have been excluded and studied separately. The baselines uncertainties follows a ± 5 m uniform distribution. Central values for flux and cross-sections are considered from the computations described in the text. The cross-section uncertainty is not realistic. It has been included just to show its possible correlation with the F estimator.

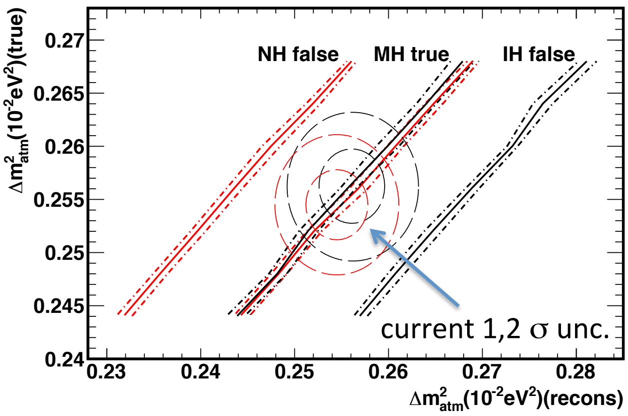
oscillation parameter	best fit	1 σ range
$\sin^2\theta_{12}$	0.297	$\pm$0.017
$\sin^2\theta_{13}$	0.0215	$\pm$0.0007
δm^2_{sol}	7.37×10^{-5}	0.16×10^{-5}
$\Delta m^2_{31}(NH)$	256×10^{-5}	$\pm 4\times10^{-5}$
$\Delta m^2_{23}(IH)$	256×10^{-5}	$\pm 4\times10^{-5}$
flux		$\pm$3%
cross-section		$\pm$ 1%
baselines		$\pm$ 5 m



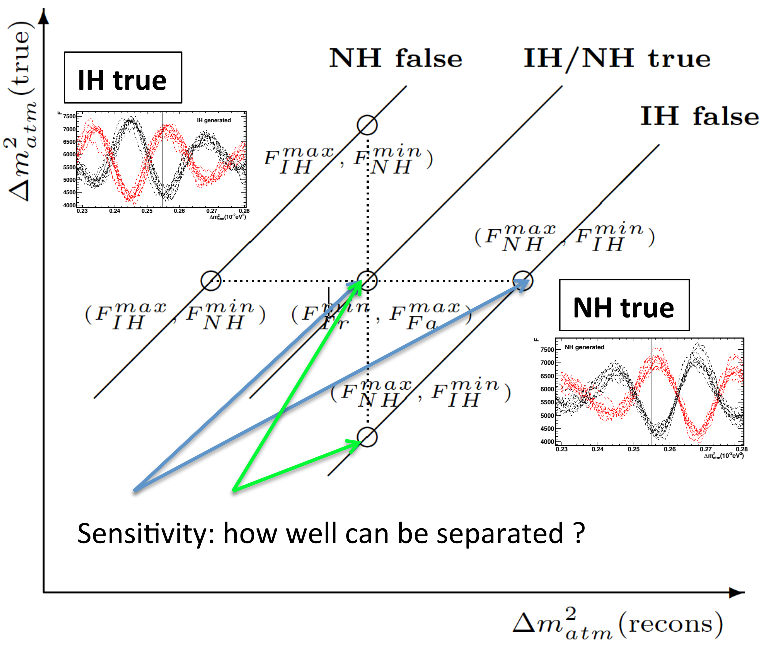
500 different configuration sets, each with 20 JUNO-like toys



Two solutions are individuated, far away in Δm^2_{23} ,
excellent Δm^2_{23} precision

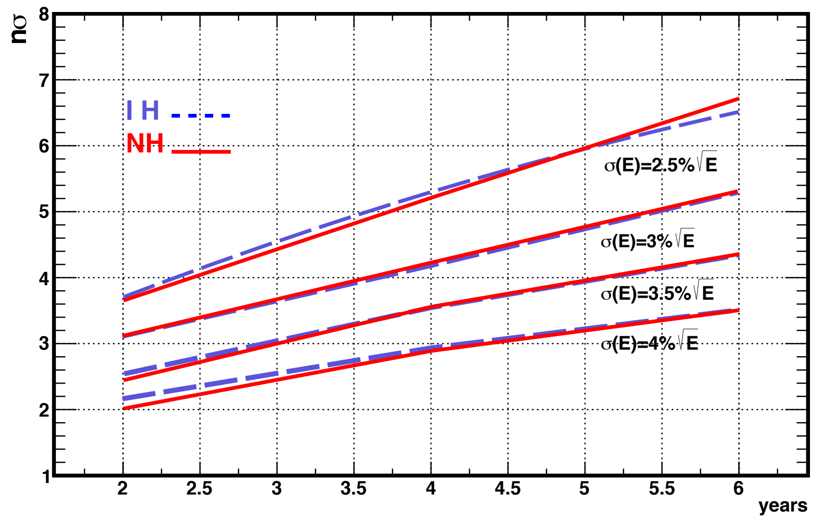


The (MH, Δm^2_{23}) degeneracy when the F discriminator is used:



Analysis update, instead of using F_{IH},F_{NH} extract F_{IH}(min),F_{NH}(max) (undergoing)

Event simulation, 3% energy resolution, 10+2 reactor cores



more than 5 σ in less than 6 years
(and improvements in analysis is foreseen)

WOW!

Conclusions and perspectives

- The MASS ORDERING is one of the 3 missing measurements to complete the 3ν oscillation framework and start the precision era
- Hierarchy is strongly dependent of CP angle at accelerators, of Δm^2_{atm} at reactors exps (degeneracy IS an issue)

- Statistical analysis IS an issue since we are dealing with "sensitivities" below 5 σ
- New statistical methods have been introduced for NOvA (PRD95, 1606.09454) and for JUNO (arXiv:1707.07651)
- With the new methods NOvA may quite soon set a comfortable exclusion level for IH (or NH), JUNO will be able to achieve more than 5 σ observation.
- Deep discussion in the community about the statistical analysis

- Look forward the neutrino programs and investments