Modelling the Microbunching Instability for Bunch Compressor Systems

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1. Motivation
2. Self Consistent Vlasov-Maxwell Treatment
3. Microbunching Instability Studies for the FERMI@Elettra Bunch Compressor System
4. Discussion of Some Approximation Schemes

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Self Consistent Vlasov-Maxwell Treatment for Sheet Beam in Beam Frame

Sheet Beam model: $f(Z,X,Y,P_Z,P_X,P_Y;u_0)=\delta(Y)\delta(P_Y)f_L(Z,X,P_Z,P_X;u_0)$.

Approximate beam frame equations of motion

$$z' = -\kappa(s)x, \qquad p_z' = F_{z1}(z,x;s)+p_zF_{z2}(z,x;s),$$
$$x' = p_x, \qquad p_x' = \kappa(s)p_z + F_x(z,x;s),$$

where the self forces are

$$F_{z1} = \frac{q}{P_rc}\mathbf{E}_\parallel(\mathbf{R}(s,x);\frac{s-z}{\beta_r})\cdot\mathbf{t}(s), \qquad F_{z2}\frac{q}{P_rc}\mathbf{E}_\parallel(\mathbf{R}(s,x);\frac{s-z}{\beta_r})\cdot\mathbf{n}(s),$$
$$F_x = \frac{q}{P_rc}[\mathbf{E}_\parallel(\mathbf{R}(s,x);\frac{s-z}{\beta_r})\cdot\mathbf{n}(s)-cB_Y(\mathbf{R}(s,x);\frac{s-z}{\beta_r})].$$

where $\mathbf{R}(s,x)=\mathbf{R}_r(s)+x\mathbf{n}(s)$ and $\mathbf{R}_r(s)=(Z_r(s),X_r(s))^T$.

Associated Vlasov IVP for the evolution of the beam frame phase space density

$$\partial_s f_B + \mathbf{r}'\cdot\nabla_{\mathbf{r}}f_B + \mathbf{p}'\cdot\nabla_{\mathbf{p}}f_B = 0, \qquad f_B(\mathbf{r},\mathbf{p};0) =: f_{B0}(\mathbf{r},\mathbf{p}).$$

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Field Calculation

Formula in free space

$$\mathcal{F}_L(\mathbf{R}_r(s) + M(s)\mathbf{r}, s/\beta_r) = -\frac{1}{4\pi}\int_{u_i}^{s/\beta_r} dv \int_{\theta_m}^{\theta_M} d\theta\, \mathbf{S}[\tilde{\mathbf{R}}(\theta, v; \mathbf{r}, s), v],$$

where $\tilde{\mathbf{R}}(\theta, v; \mathbf{r}, s) = \mathbf{R}_r(s) + M(s)\mathbf{r} + (s/\beta_r - v)\mathbf{e}(\theta)$, $\mathbf{e}(\theta) = (\cos\theta, \sin\theta)^T$ and $M(s) = [\mathbf{t}(s), \mathbf{n}(s)]$.

- θ integration: superconvergent trapezoidal rule (localization in θ for $v \ll s/\beta_r$)
- v integration: adaptive Gauss-Kronrod rule (non uniform behavior in v)

For more details see:

G. Bassi, J.A. Ellison, K. Heinemann and R. Warnock, PRSTAB **12**, 080704 (2009)

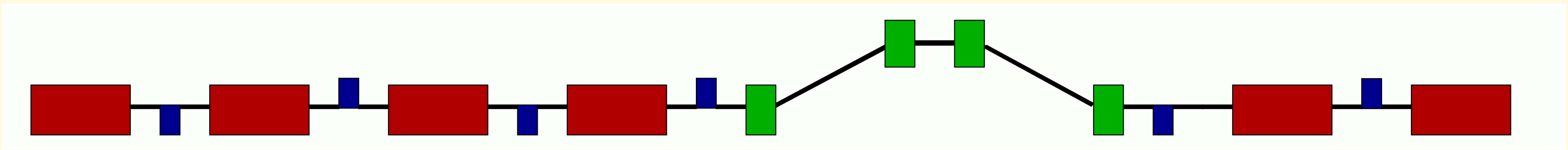
Microbunching in FERMI@Elettra First Bunch Compressor

Microbunching can cause an instability which **degrades** beam quality, i.e. can cause an increase in energy spread and emittance

This is a major concern for free electron lasers where very bright electron beams are required

We discuss numerical results for the FERMI@Elettra first bunch compressor system. See G. Bassi, J.A. Ellison, K. Heinemann and R. Warnock, PRSTAB **12**, 080704 (2009)

This system was proposed as a benchmark for testing codes at the first Workshop on Microbunching Instability held in Trieste in 2007



Layout first bunch compressor system

FERMI@Elettra First Bunch Compressor Parameters

Table 1: Chicane parameters and beam parameters at first dipole

Parameter	Symbol	Value	Unit
Energy reference particle	E_r	233	MeV
Peak current	I	120	A
Bunch charge	Q	1	nC
Norm. transverse emittance	$\gamma\epsilon_0$	1	μm
Alpha function	α_0	0	
Beta function	β_0	10	m
Linear energy chirp	h	-12.6	1/m
Uncorrelated energy spread	σ_E	2	KeV
Momentum compaction	R_{56}	0.057	m
Radius of curvature	ρ_0	5	m
Magnetic length	L_b	0.5	m
Distance 1st-2nd, 3rd-4th bend	L_1	2.5	m
Distance 2rd-3nd bend	L_2	1	m

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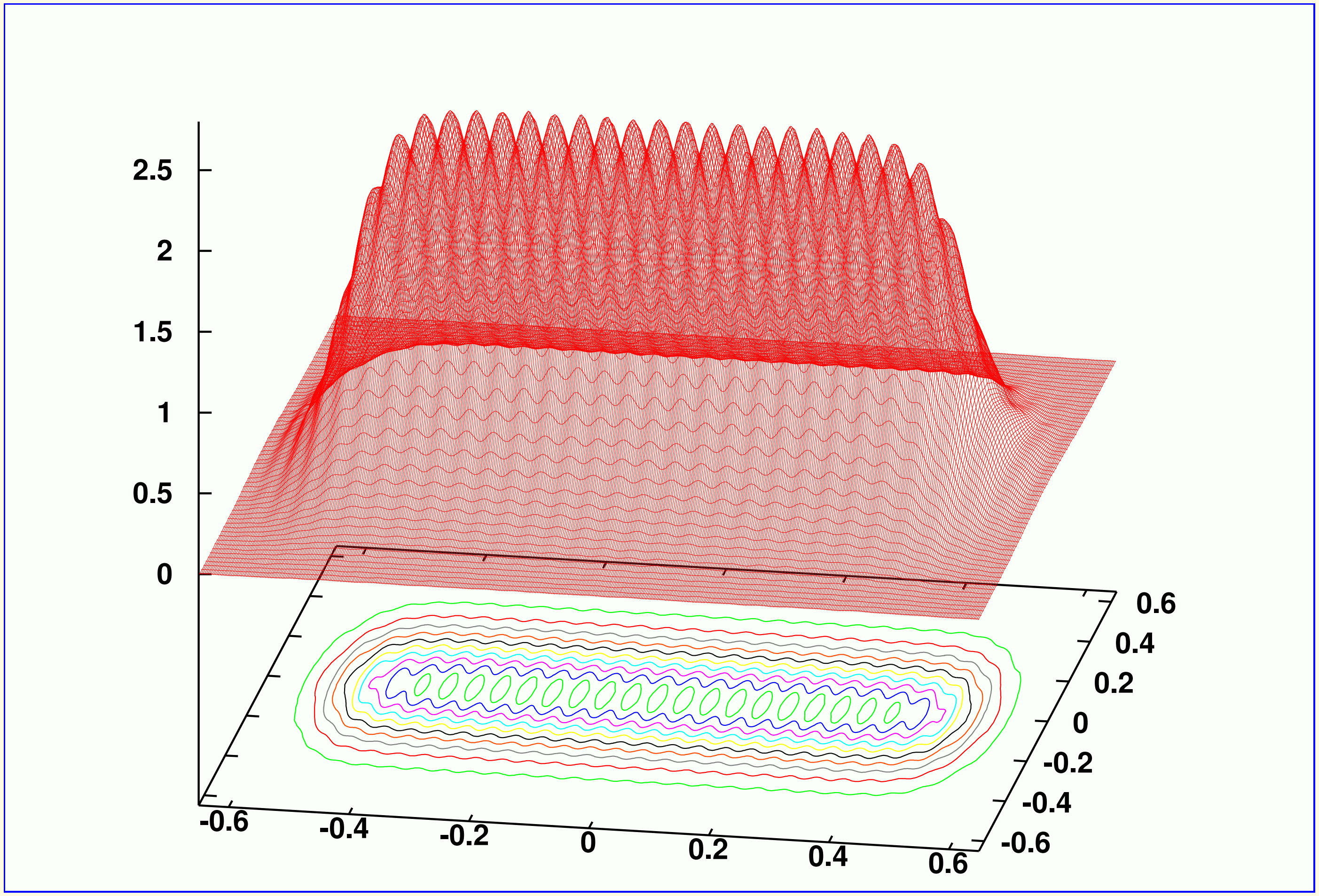
Initial 2D Spatial Density



Initial spatial density in grid coordinates for A=0.05, $\lambda_0 = 100\mu$m.

Init. phase space density $= (1 + A\cos(2\pi z/\lambda_0))\mu(z)\rho_c(p_z - hz)g(x, p_x)$.

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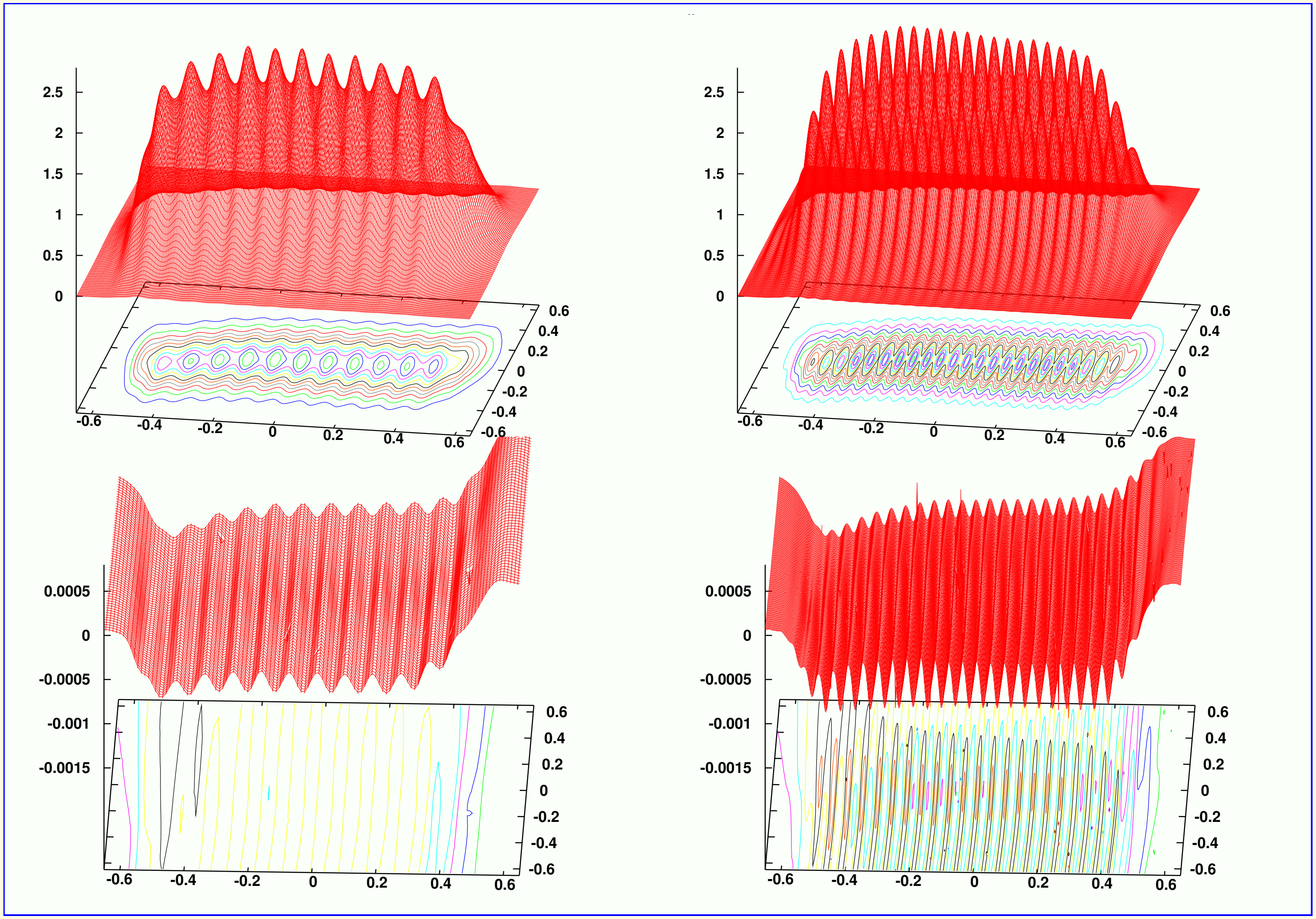
2D Spatial Density and Longitudinal Force F_{z1} at $s = s_f$



$\lambda_0 = 200\mu$m (top left), $\lambda_0 = 100\mu$m (top right), $\lambda_0 = 200\mu$m (bottom left), $\lambda_0 = 100\mu$m (bottom right)

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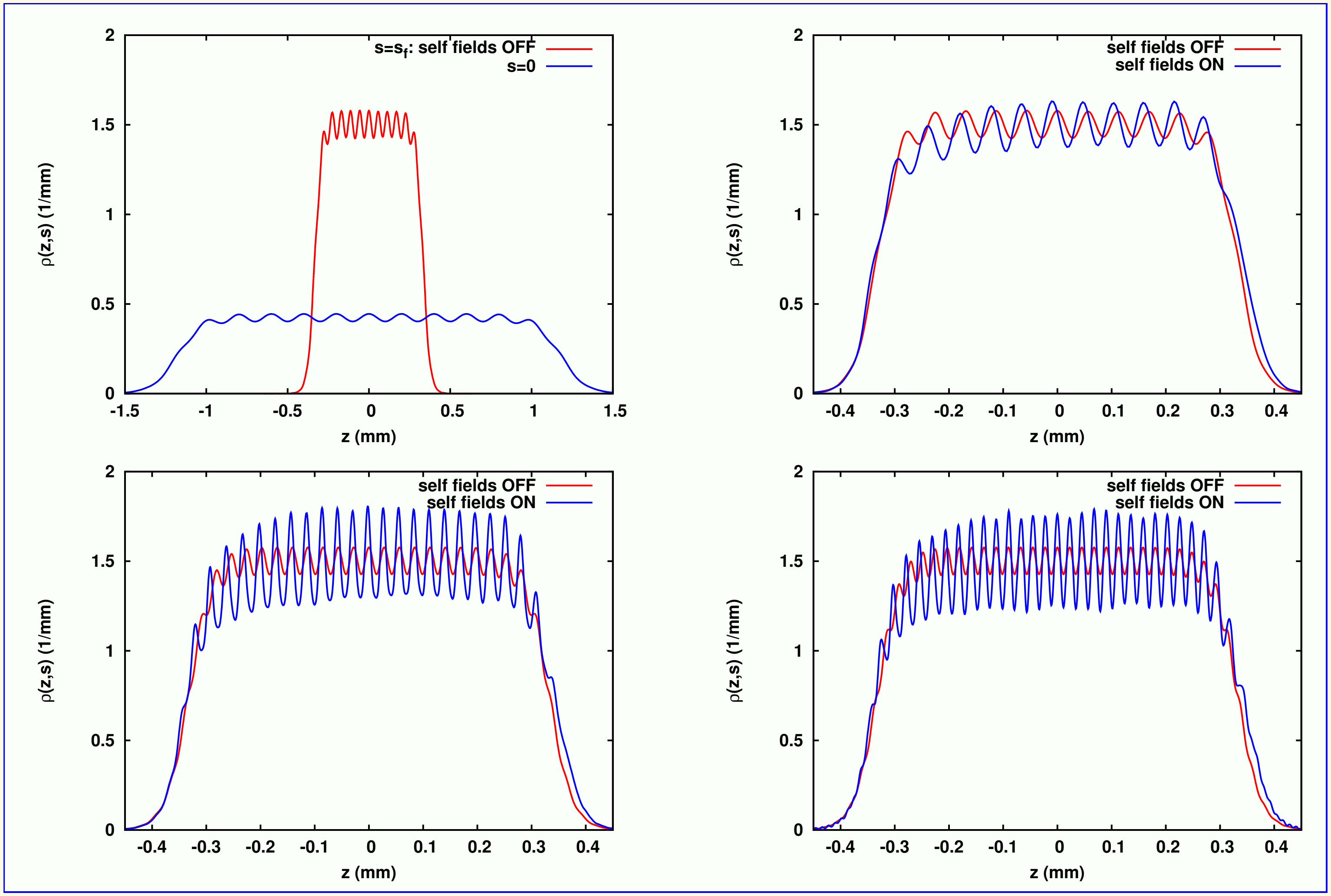
Longitudinal Density



$\lambda_0 = 200\mu$m (top left),
$\lambda_0 = 100\mu$m at $s = s_f$ (bottom left),

$\lambda_0 = 200\mu$m at $s = s_f$ (top right),
$\lambda_0 = 80\mu$m at $s = s_f$ (bottom right).

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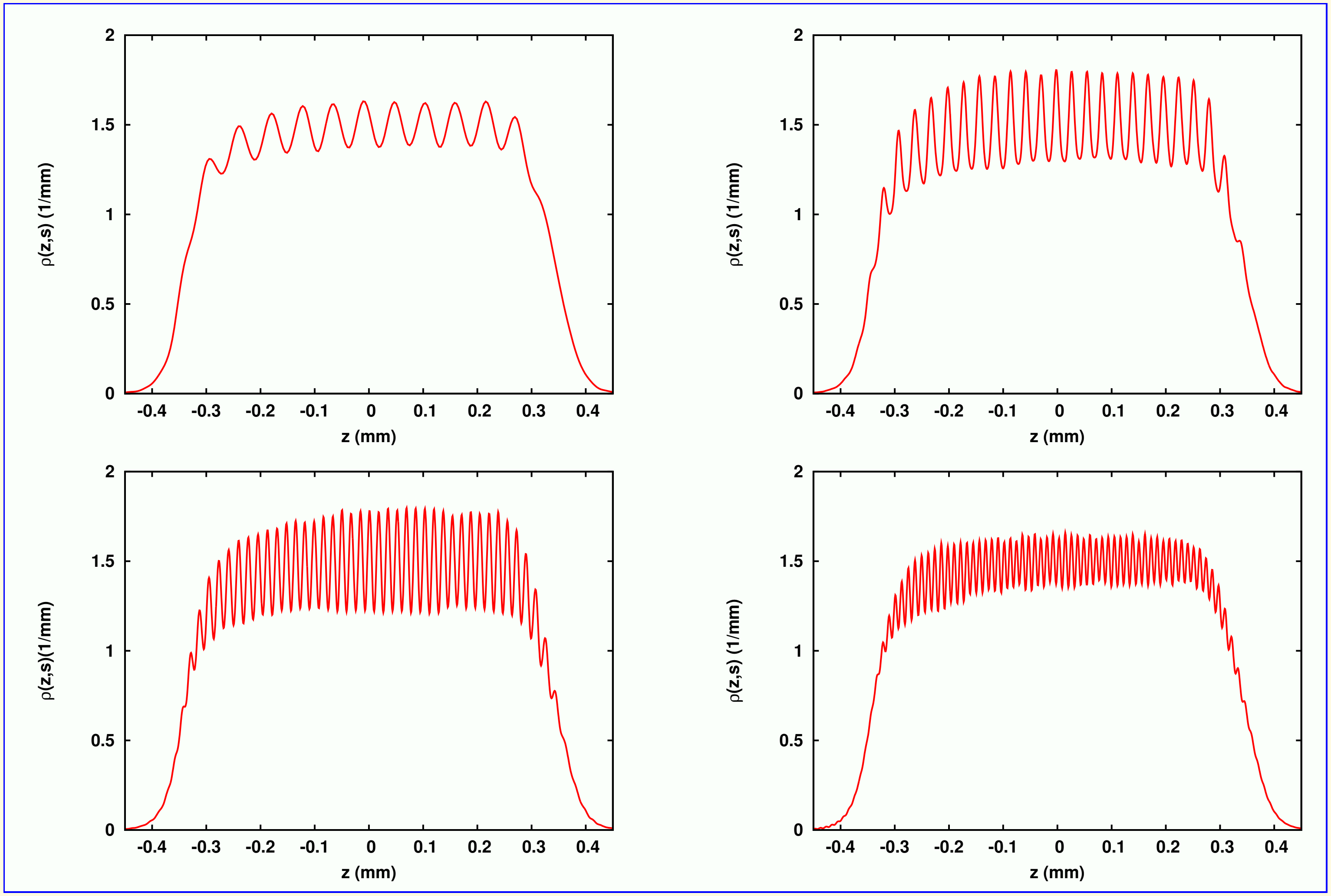
Longitudinal Density (con't)



$\lambda_0 = 200\mu$m at $s = s_f$ (top left),
$\lambda_0 = 60\mu$m at $s = s_f$ (bottom left),
$\lambda_0 = 100\mu$m at $s = s_f$ (top right),
$\lambda_0 = 40\mu$m at $s = s_f$ (bottom right).

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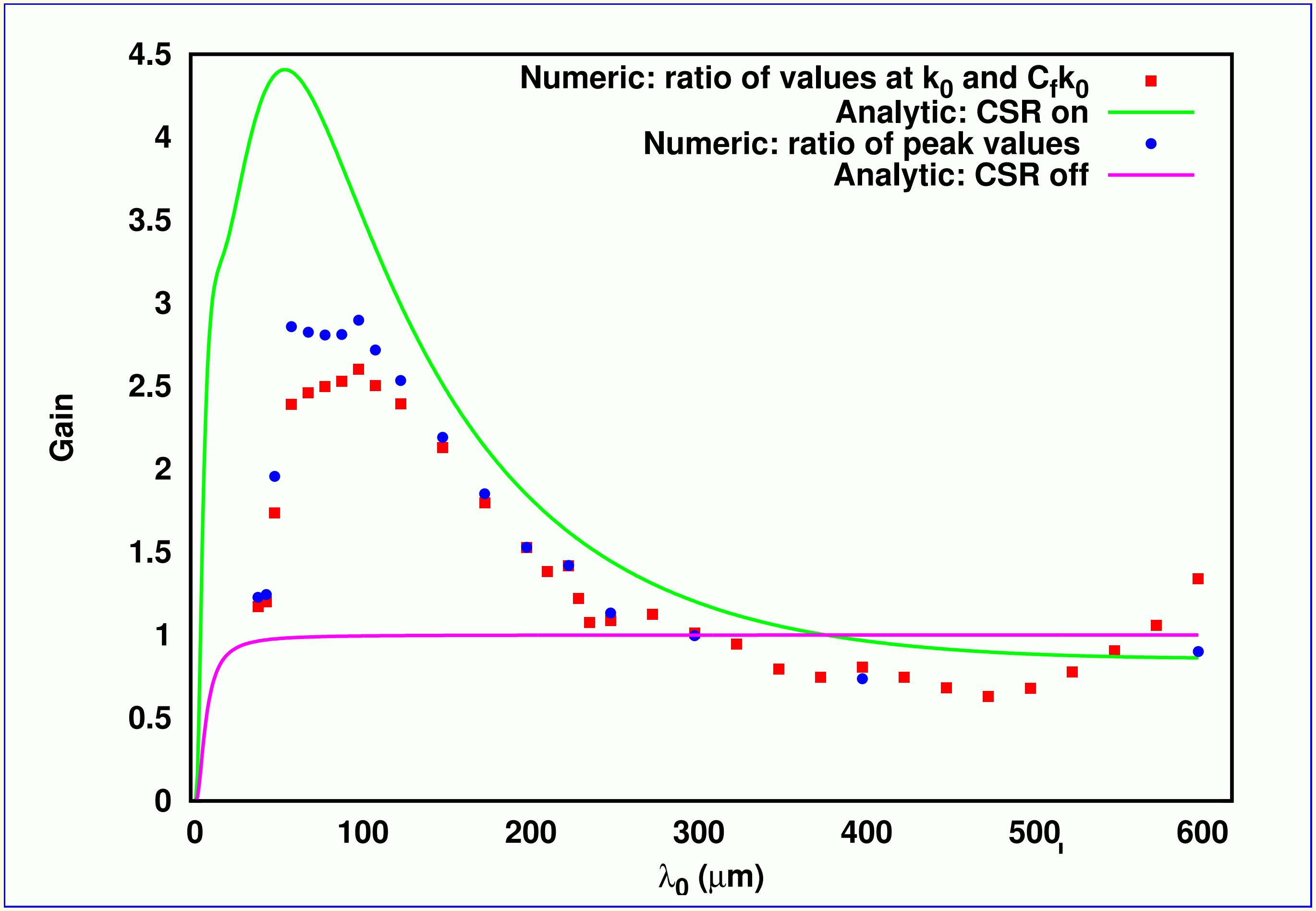
Gain factor



Gain $:= |\tilde{\rho}(k_f, s_f)/\tilde{\rho}(k_0, 0)|$, $\tilde{\rho}(k, s) = \int dz \exp(-ikz)\rho(z, s)$ and $k_f = C(s_f)k_0$ for $\lambda_0 = 2\pi/k_0$. Here $C(s_f) = 1/(1 + hR_{56}(s_f)) = 3.54$, $s_f = 8.029$m.

H. Huang and K. Kim, PRSTAB **5**, 074401, 129903 (2002); S. Heifets, G. Stupakov and S. Krinsky, PRSTAB **5**, 064401 (2009); G. Bassi, J.A. Ellison, K. Heinemann and R. Warnock, PRSTAB **12**, 080704 (2009).

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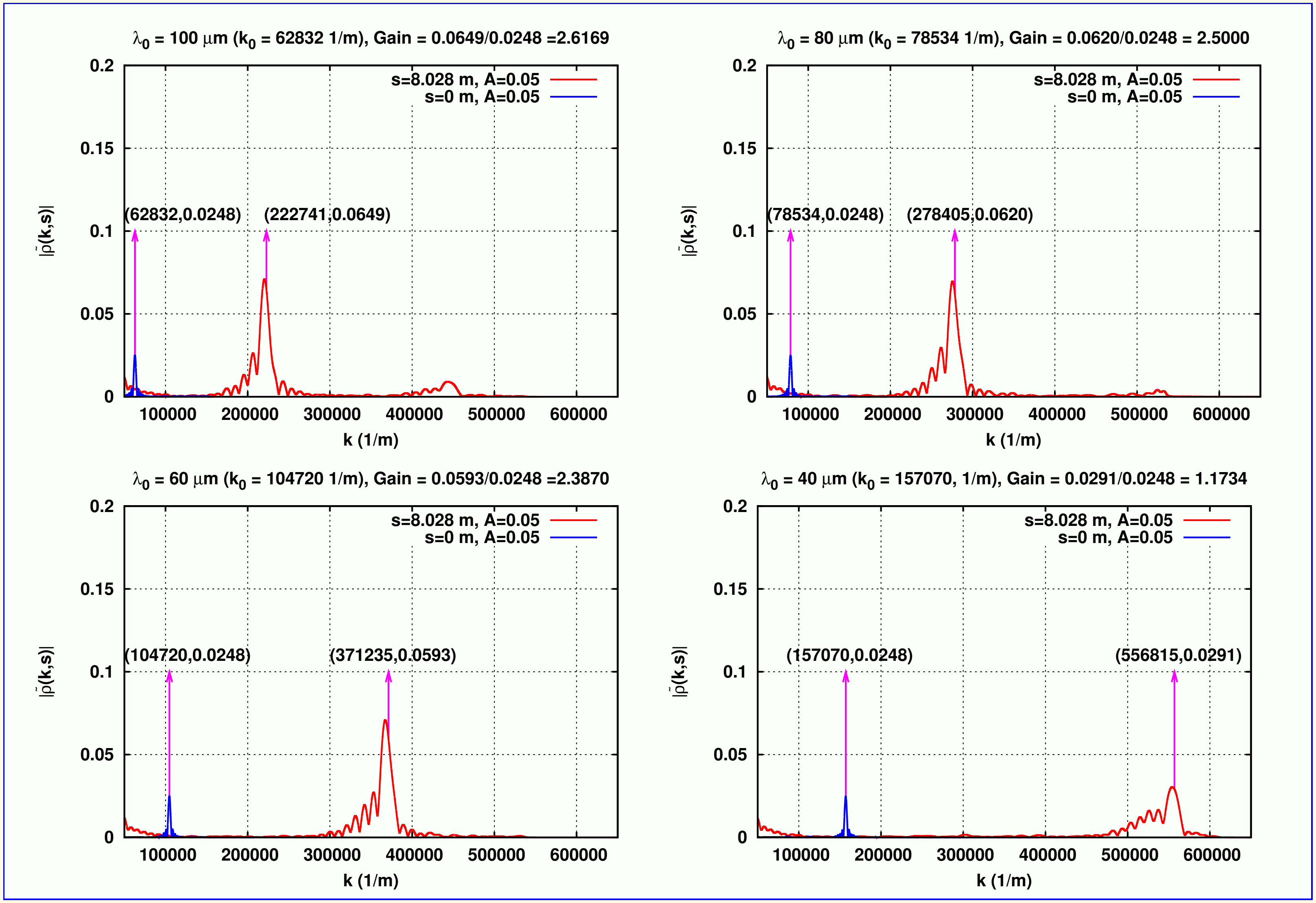
Spectra Longitudinal Density



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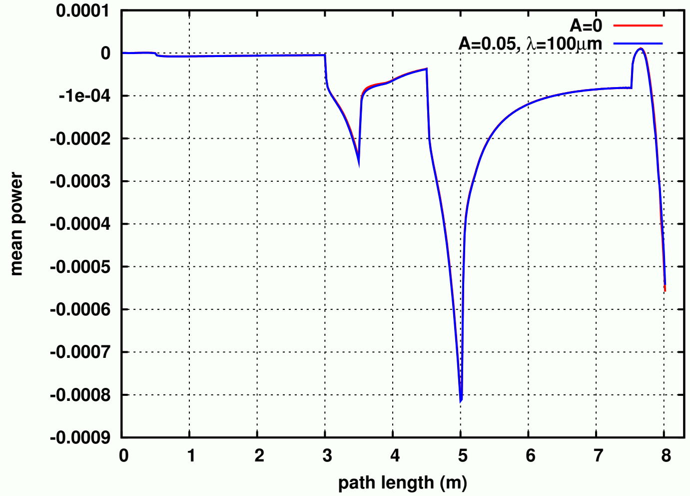
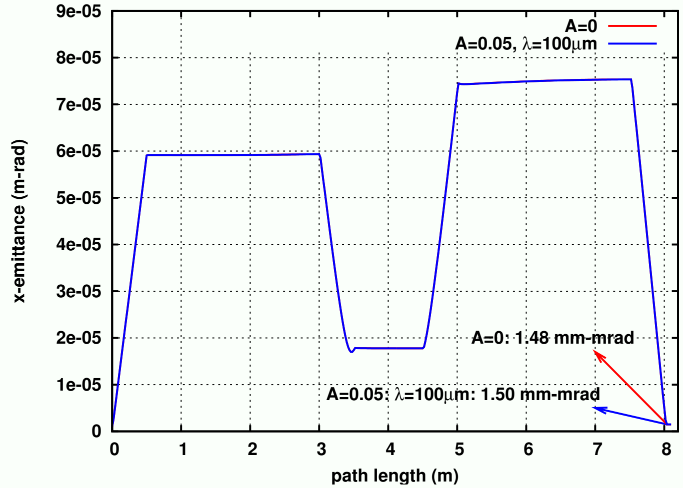
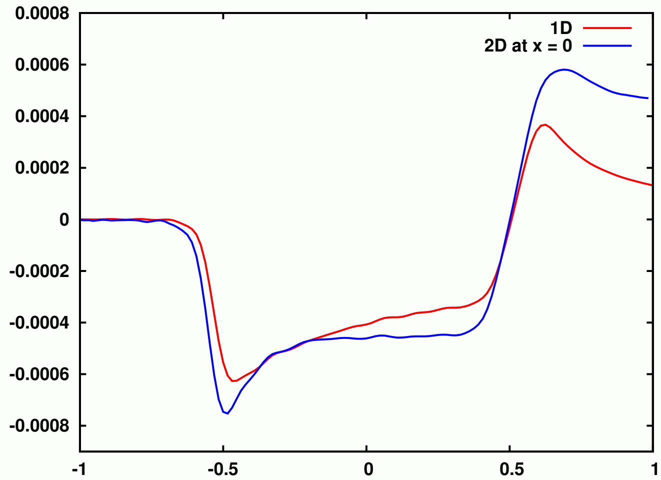
Mean Power, Transverse Emittance and 2D Longitudinal Force F_{z1} vs 1D Wake







Mean power (top left), transverse emittance (top right), 2D longitudinal force F_{z1} (section at $x = 0$) vs. 1D (steady state CSR wake) at $s = s_f$ (bottom left).

1D steady state CSR wake:

$$W(z,s) = \frac{2Nr_e}{\gamma(3R)^{1/3}} \int_{-\infty}^{z} \frac{1}{(z-z')^{1/3}} \frac{\partial \lambda(z',s)}{\partial z'}.$$

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Discussion Microbunching Studies for FERMI@Elettra BC1

- FERMI@Elettra microbunching studies at $\lambda_0 \geq 40\mu$m:
  - Very small effect of μBI on mean power and transverse emittance
  - Gain factor at long wavelengths shows breakdown coasting beam assumption
  - Gain factor at short wavelengths indicates deviations from analytical gain formula
- Work in progress and future work:
  - Study wavelengths shorter than $\lambda_0 = 40\mu$m
  - Study dependence on the amplitude of the initial modulation and on the uncorrelated energy spread
  - Study initial perturbation with more than one frequency
  - Study energy modulations

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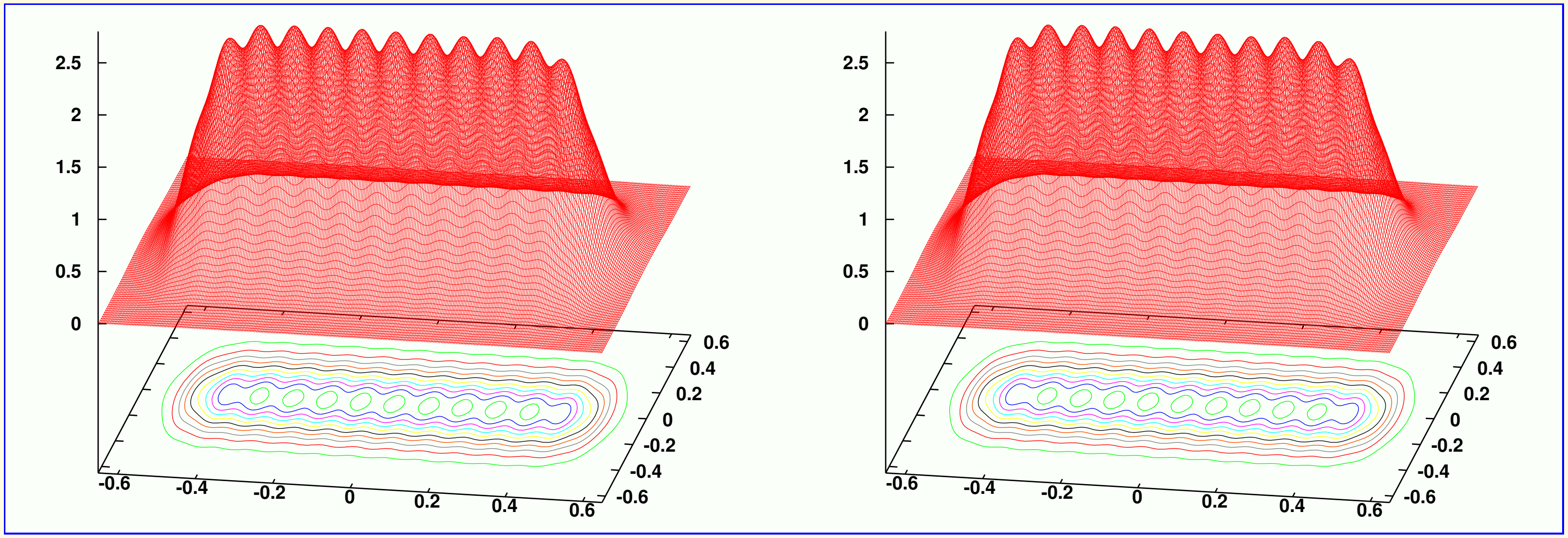
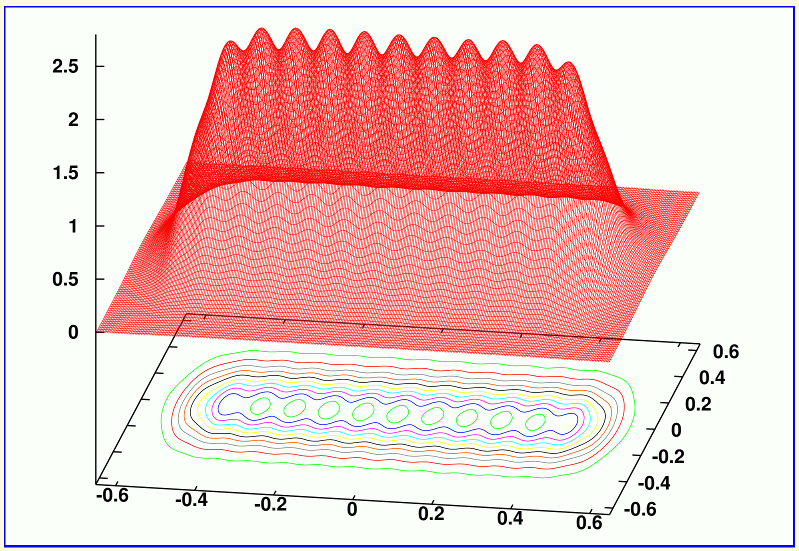
Stationary Grid





2D spatial density without self fields for $\lambda_0 = 200\mu$m at $s = 0.5s_f$ (top left), $s=0.75s_f$ (top right), $s=s_f$ (bottom left). In the limit $\sigma_u \to 0$ and $\sigma_{px_0} \to 0$ stationary grid defined by:

$$\begin{aligned} z &= (1 + hR_{56}(s))\tilde{z} - D'(s)\tilde{x} \\ x &= hD(s)\tilde{z} + \tilde{x} \end{aligned}$$

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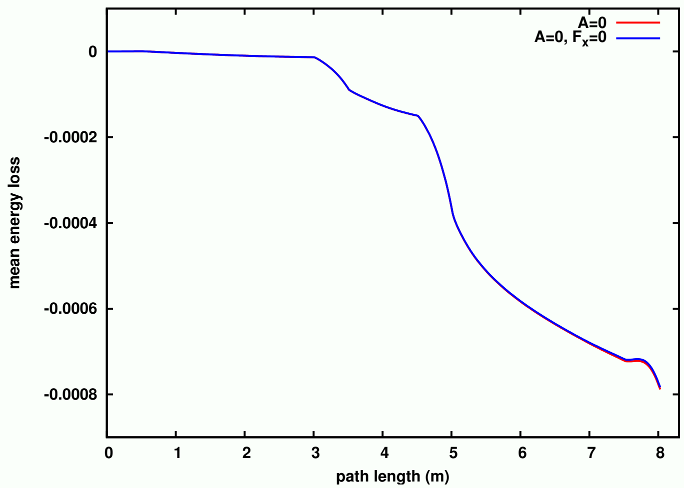
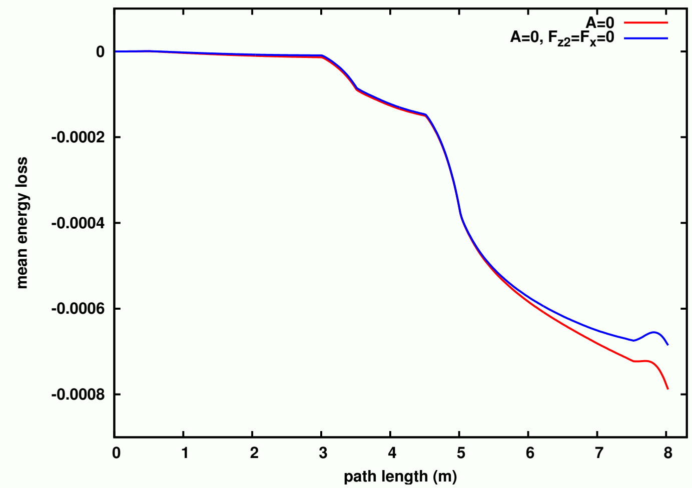
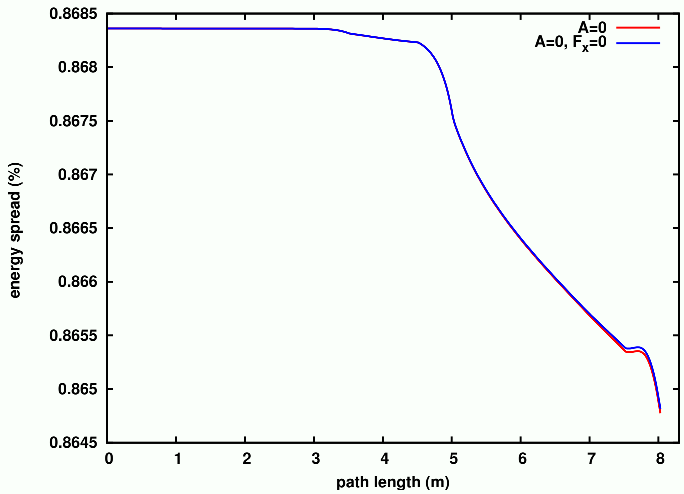
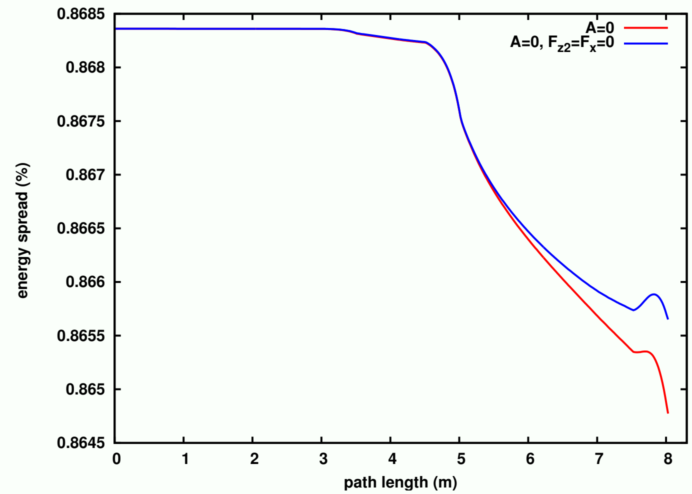
Discussion of F_{z2} and F_x for $A = 0$









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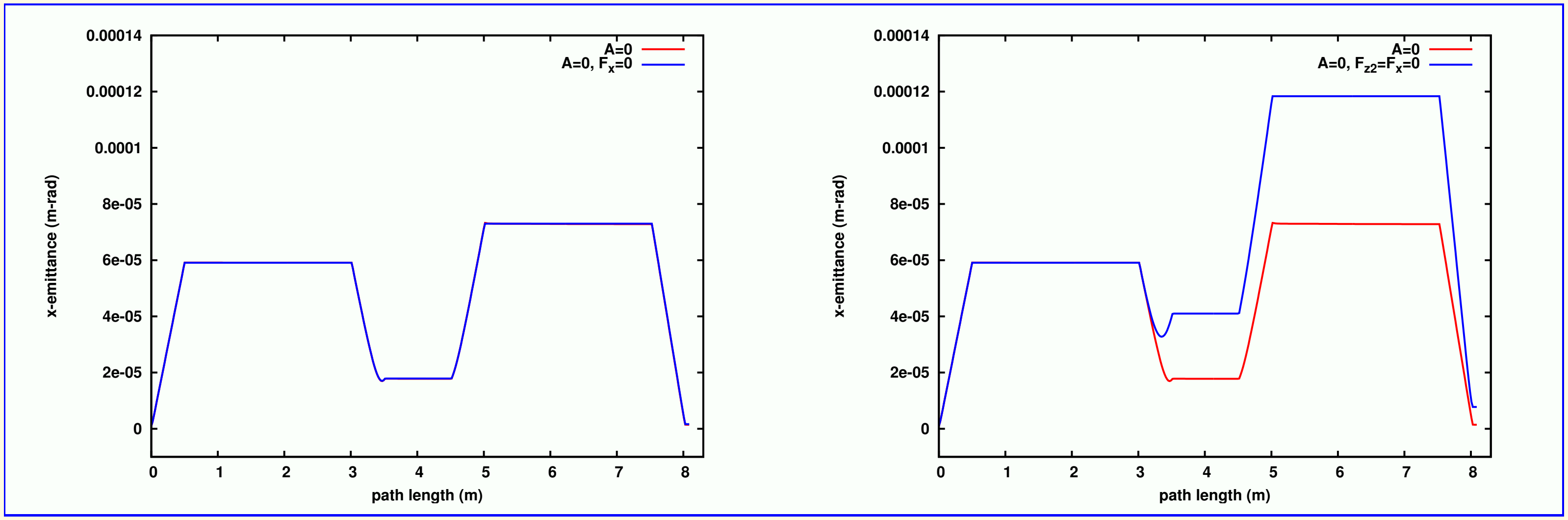
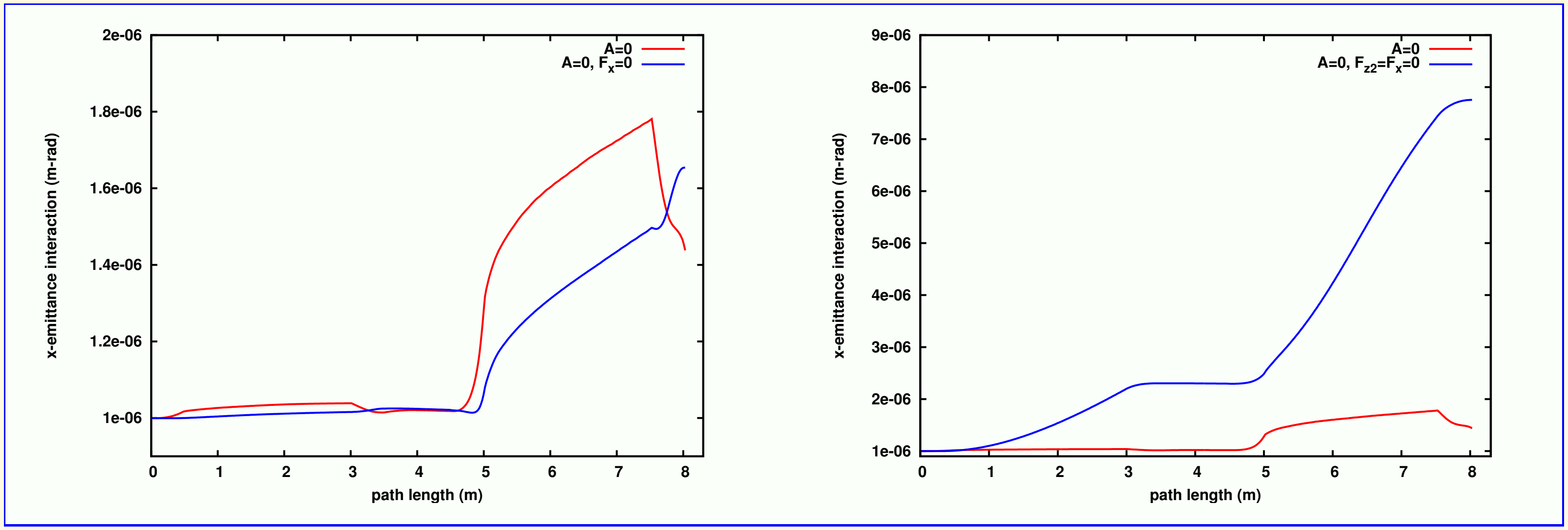
Discussion of F_{z2} and F_x for $A = 0$ (con't)





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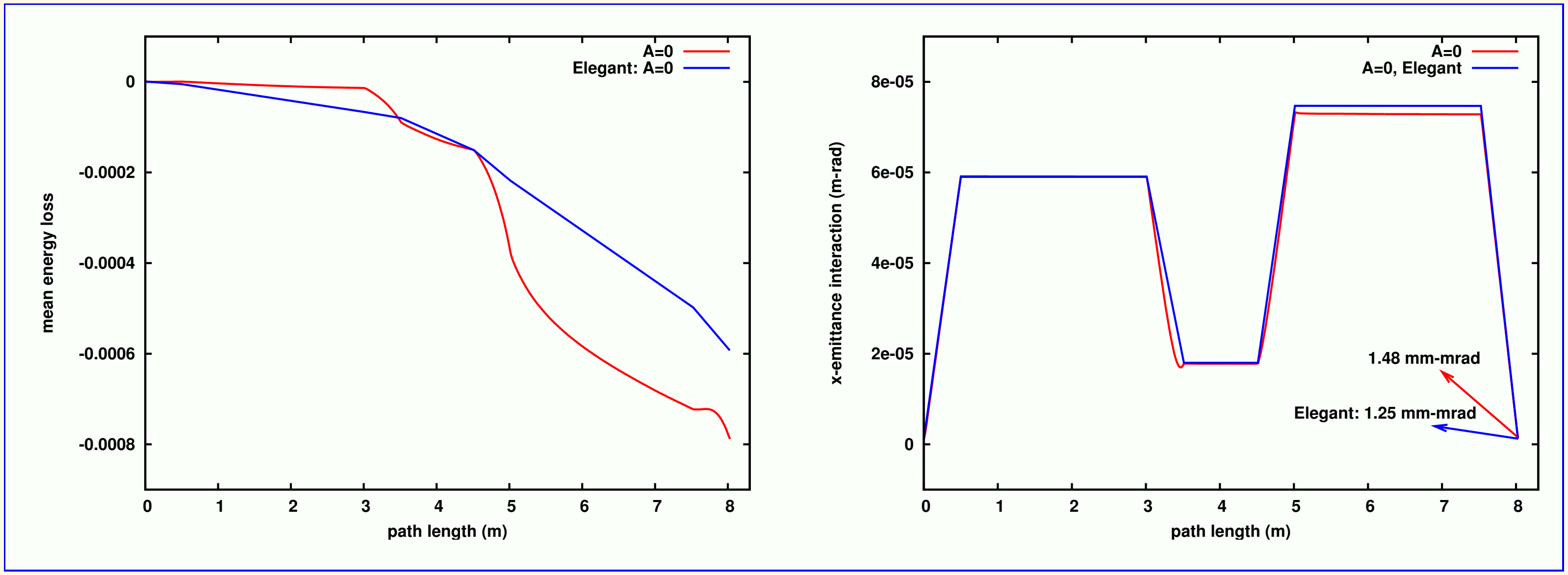
Comparison with Elegant for $A = 0$



Courtesy of Deepa Angal-Kalinin

- Preliminary results, planning to calculate gain factor

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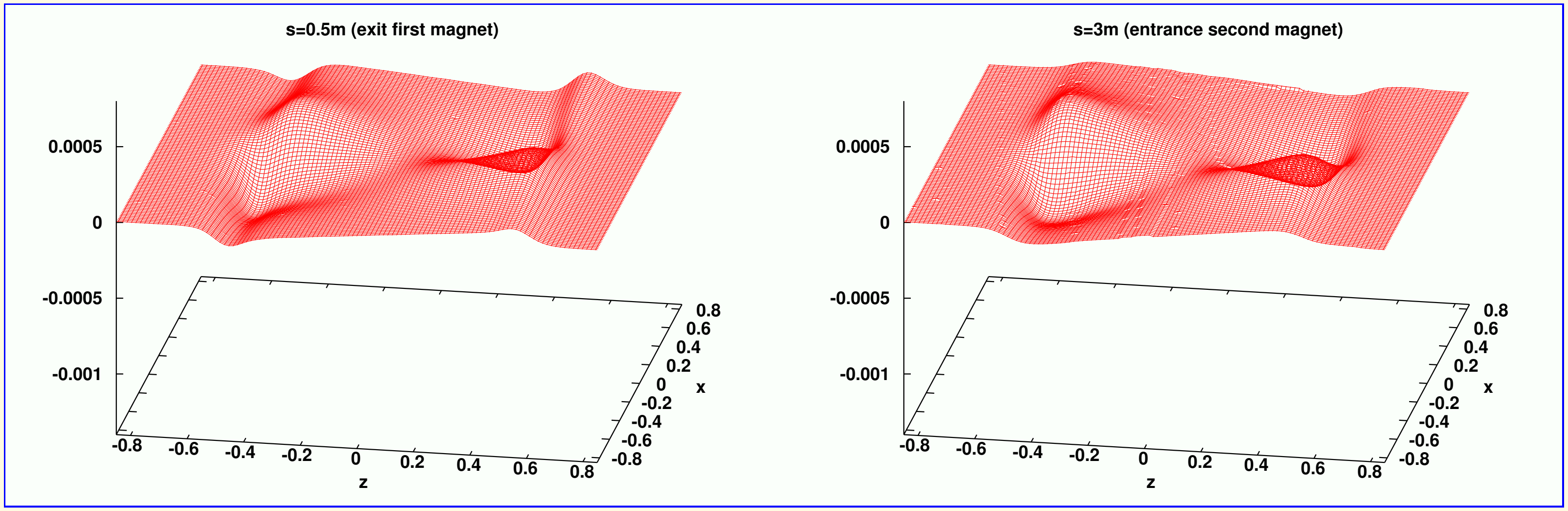
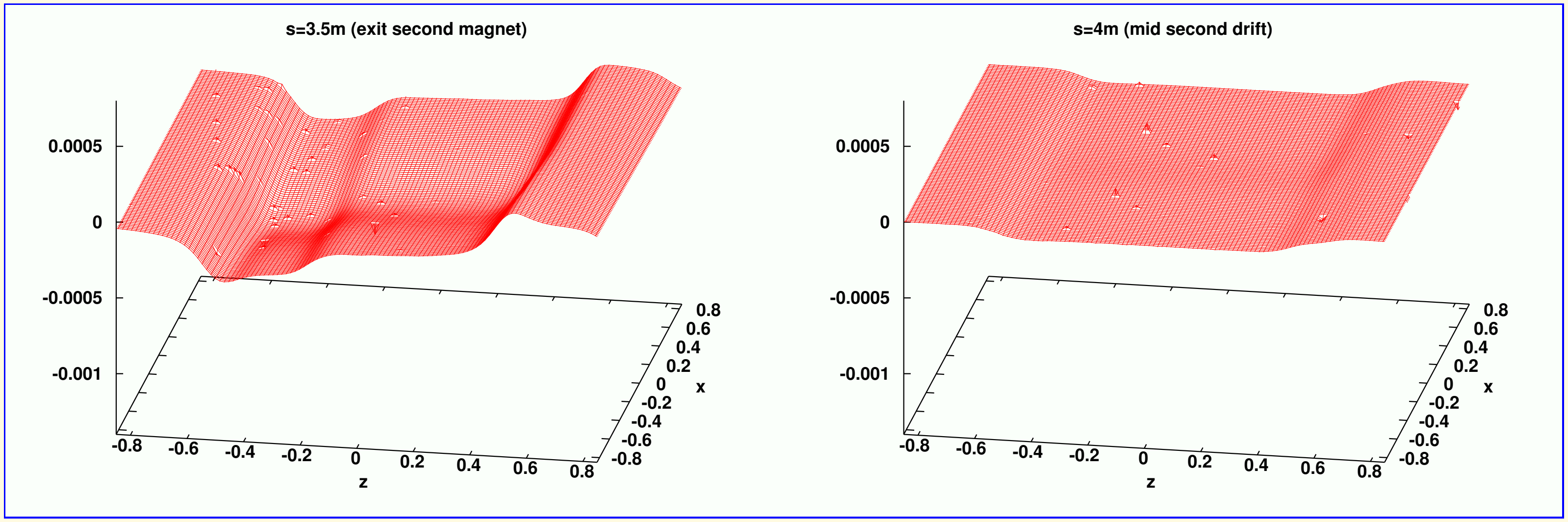
2D longitudinal Force F_{z1} for $A = 0$





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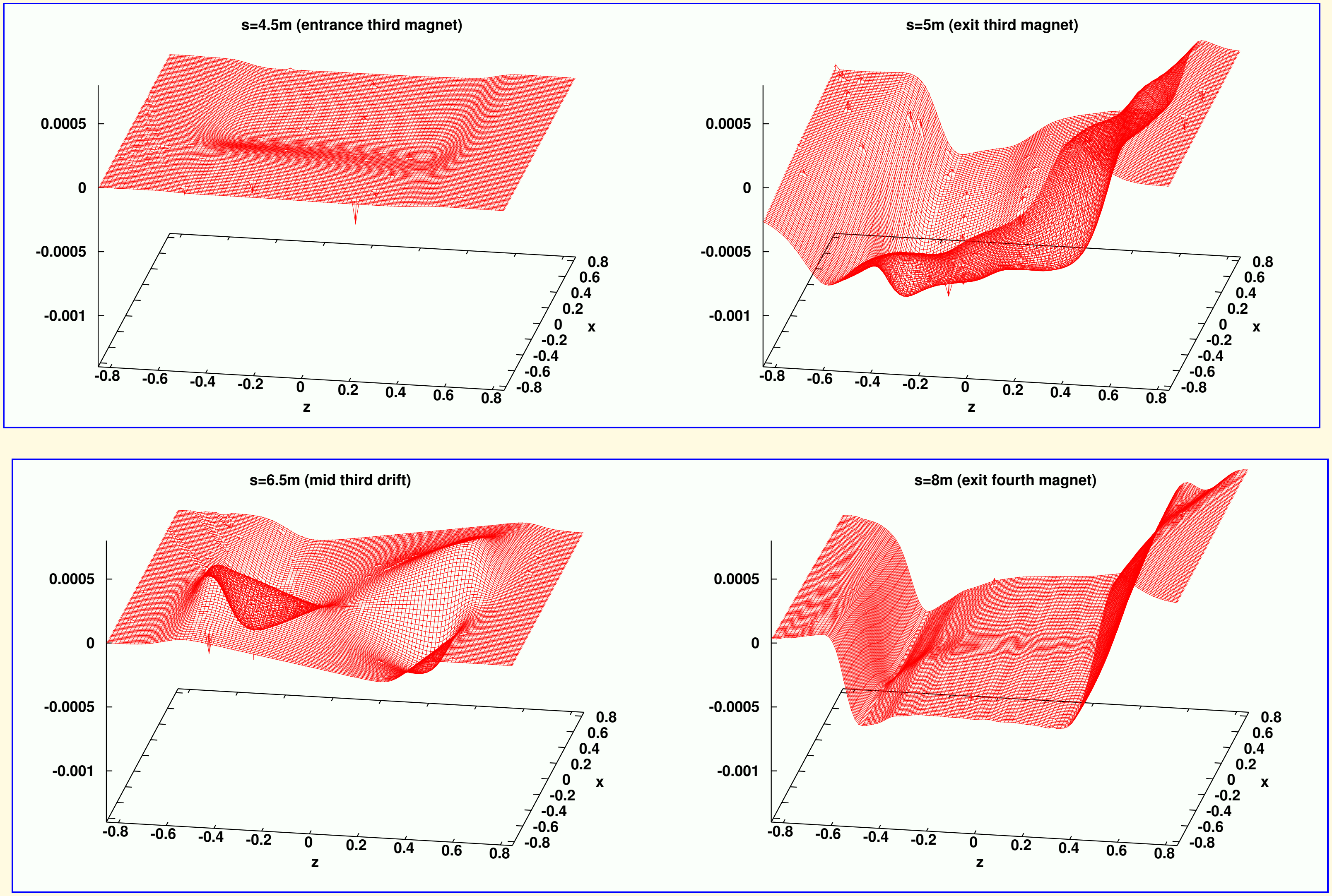
2D Longitudinal Force F_{z1} for $A = 0$ (con't)



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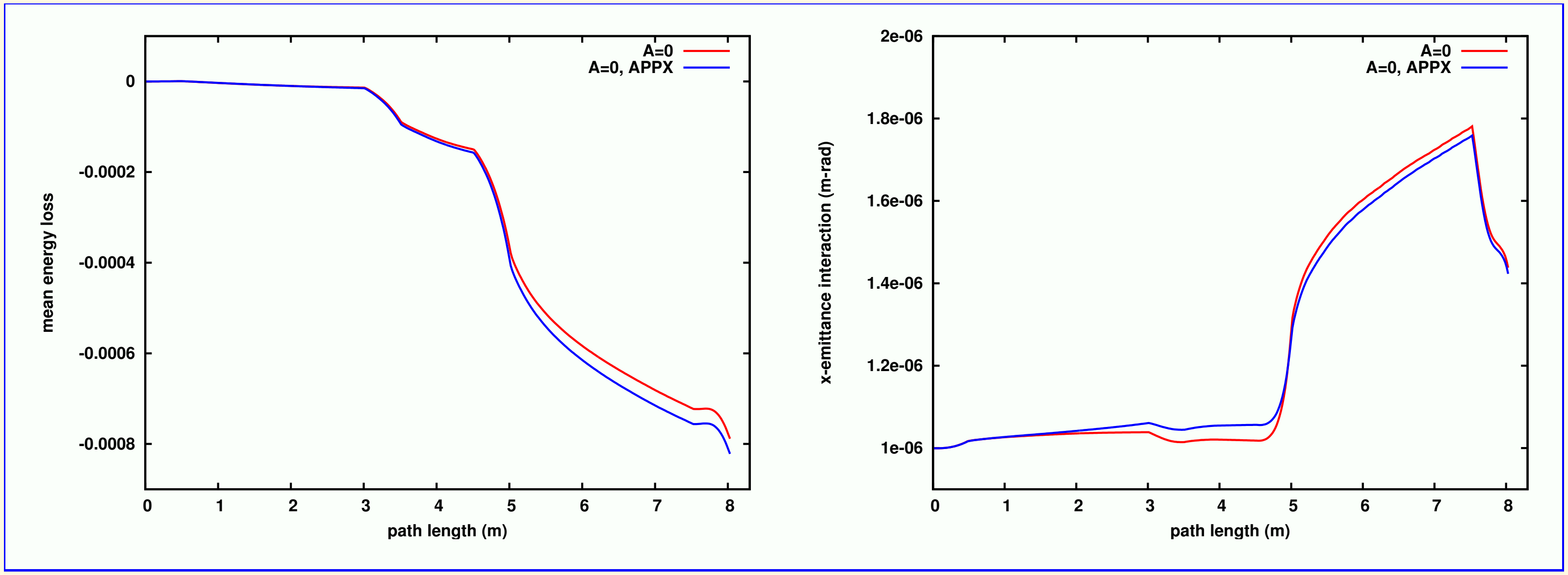
Fast Calculation: Kick Particles According to Fields at $x = 0$



Fast calculation: kick particles at s according to fields calculated at $(z, 0)$: $F_{z1}(z, x, s) \rightarrow F_{z1}(z, 0, s)$.

Computational cost for simulations done on Franklin at NERSC:
For $\lambda_0 = 50\mu$m, N procs = 1600, N particles = 5×10^8, CPU time:
- 2D calculation: 8 hours
- Fast calculation: 10 minutes

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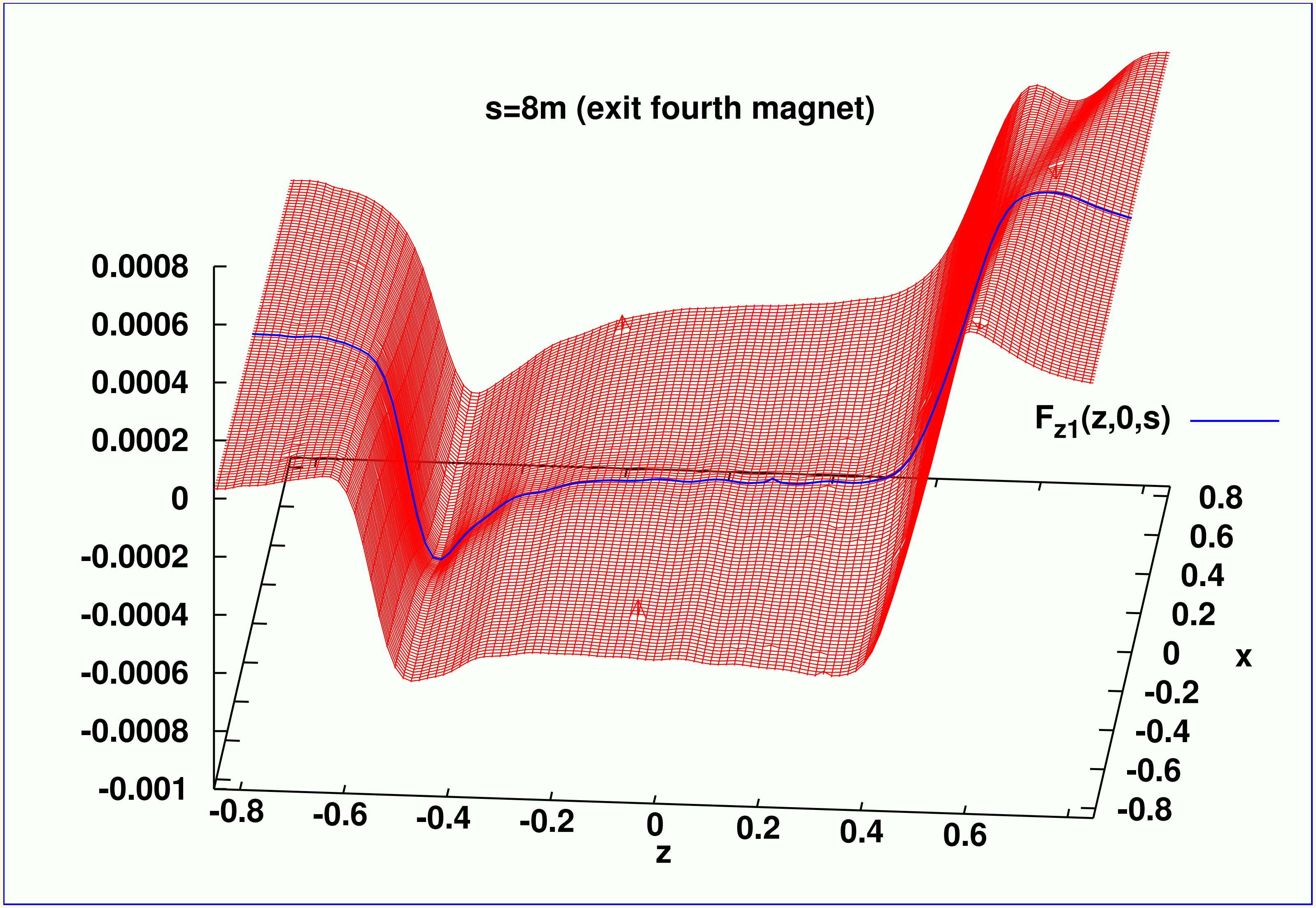
Fast Calculation: 2D Longitudinal Force F_{z1} for $A = 0$ at $s = 8$m



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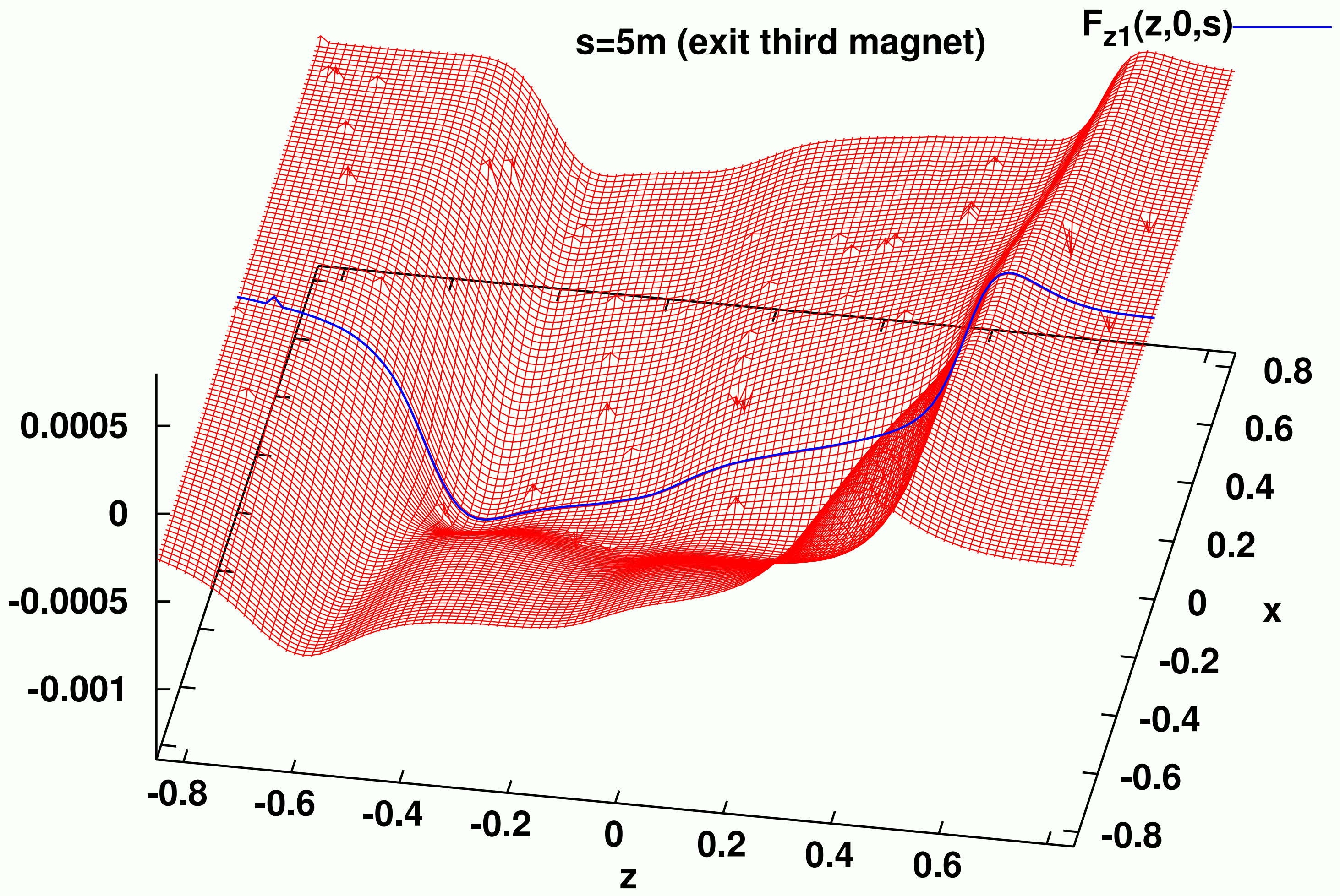
Fast Calculation: 2D Longitudinal Force F_{z1} for $A = 0$ at $s = 5$m



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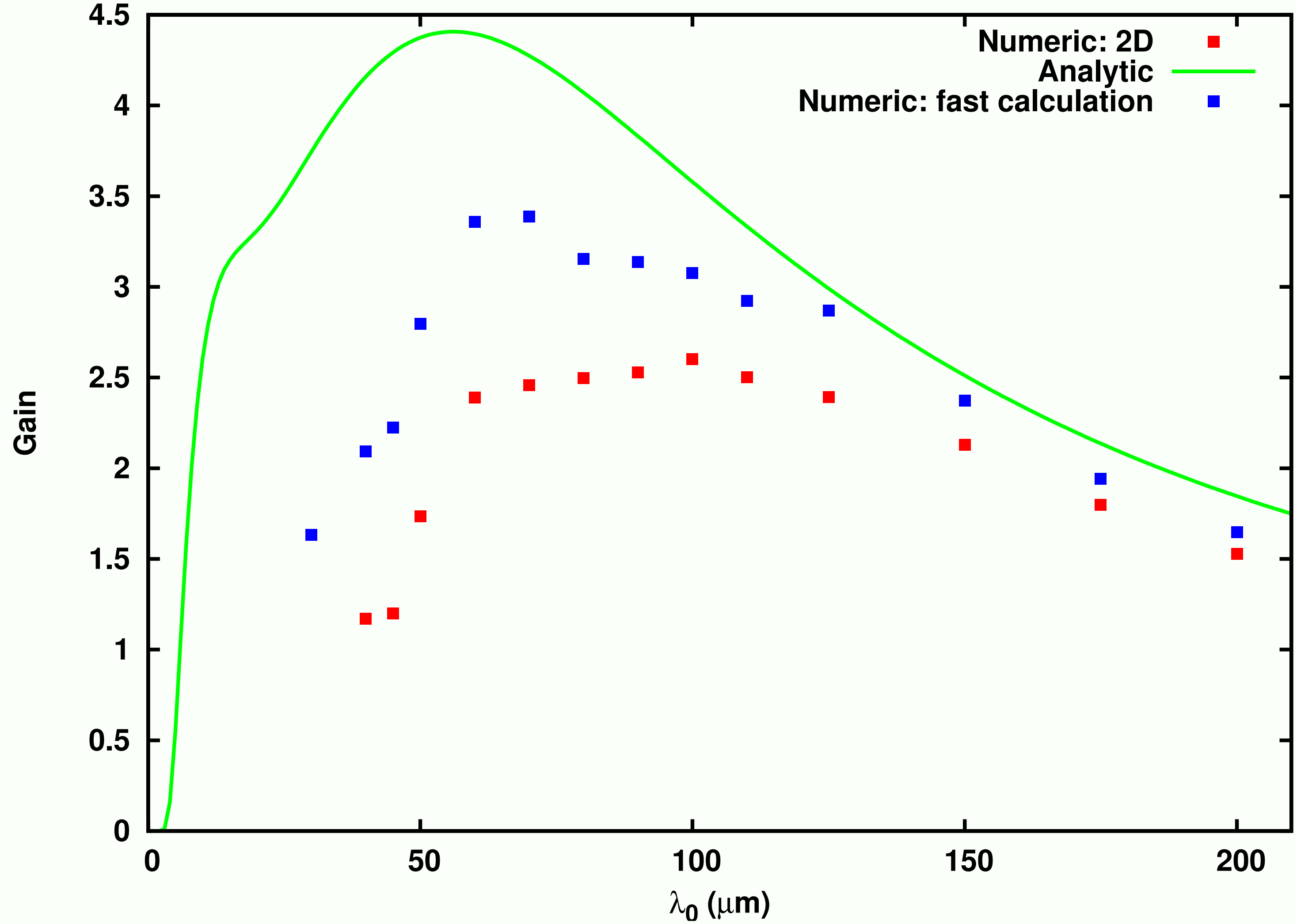
Fast Calculation: Gain Factor



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Back-up Slides

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Fields in Terms of Beam Frame Density and Causality Issue

To solve the beam frame equations of motion we need (ignoring shielding)

$$\mathcal{F}_L(\mathbf{R}_r(s)+x\mathbf{n}(s);(s-z)/\beta_r)=-\int_{\mathbb{R}^2} d\mathbf{R}' \frac{\mathbf{S}[\mathbf{R}';(s-z)/\beta_r-|\mathbf{R}'-(\mathbf{R}_r(s)+x\mathbf{n}(s))|]}{4\pi|\mathbf{R}'-(\mathbf{R}_r(s)+x\mathbf{n}(s))|},$$

To compute this we need $\rho_L[\mathbf{R}';(s-z)/\beta_r-|\mathbf{R}'-(\mathbf{R}_r(s)+x\mathbf{n}(s))|]$, as $\mathbf{R}'$ varies over the support of ρ_L in $\mathbb{R}^2$, given $\rho_B(\cdot;s')$ for $0\le s'\le s$.

There is a causality issue here, since the calculation of ρ_L requires values ρ_B for s' slightly outside the range $0\le s'\le s$.

This issue can be easily resolved with the following slowing varying approximation

$$f_B(\mathbf{r},\mathbf{p};s)\approx f_B(\mathbf{r},\mathbf{p};s+\Delta)$$

where Δ is of the order of the bunch size.

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Density Estimation: Search for Improvement

- Cloud in cell charge deposition followed by computation of the Fourier coefficients of the truncated Fourier series by a simple quadrature (already implemented).

  The computational effort is $O(\mathcal{N}) + O(N_z N_x J_z J_x)$, where $\mathcal{N}$ is the number of simulated particles, N_z and N_x are the number of grid points in z ans x respectively, and J_z and J_x the number of Fourier coefficients in z and x respectively.
  For $N_z = 1000$, $N_x = 128$, $J_z = 150$ and $J_x = 50$, $O(N_z N_x J_z J_x) = O(10^9)$.

- Kernel density estimation using standard kernels like bivariate Gaussians or bivariate compact support kernels (e.g. Epanechnikov kernels).

  The computational effort is $O(\mathcal{N}\tilde{N}_z\tilde{N}_x)$, where $\mathcal{N}$ is the number of simulated particles and $\tilde{N}_z\tilde{N}_x$ is the number of grid points inside the circle of radius h (bandwidth) centered at the scattered particle position z, x.
  For $\mathcal{N} = 5 \times 10^8$ and $\tilde{N}_z = \tilde{N}_x = 4$, $O(\mathcal{N}\tilde{N}_z\tilde{N}_x) = O(10^{10})$.

- Wavelets-denoising (G. Bassi, B. Terzić, PAC09)

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Interaction Picture

- Interaction picture in beam frame ($\zeta = (z, p_z, x, p_x)$) to isolate CSR dynamics
  From $F_z = F_x = 0 \implies \zeta = \Phi(s|0)\zeta_0$
  $\therefore \zeta_0' = \Phi(0|s)F, \qquad F = (0, F_z, 0, F_x)$.
- In component form

$$
\begin{aligned}
z_0' &= -R_{56}(s)F_z - D(s)F_x, & p_{z0}' &= F_z, \\
x_0' &= (sD'(s) - D(s))F_z - sF_x, & p_{x0}' &= -D'(s)F_z + F_x,
\end{aligned}
$$

where $D(s) = \int_0^s \kappa(\tau)d\tau$ and $R_{56}(s) = -\int_0^s D(\tau)\kappa(\tau)d\tau$.

Here the principal solution matrix is

$$
\Phi(s|0) = \begin{pmatrix} 1 & R_{56}(s) & -D'(s) & D(s) - sD'(s) \\ 0 & 1 & 0 & 0 \\ 0 & D(s) & 1 & s \\ 0 & D'(s) & 0 & 1 \end{pmatrix}.
$$

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Computational Issues

- Intensive memory requirement and expensive computational cost:
  - Typical simulations done on the parallel clusters ENCANTO in New Mexico and NERSC at LBNL: N procs = 200-1000, N particles = 2×10^7-5×10^8, few hours of CPU time
  - Memory requirement: for $\lambda_0 = 50\mu$m store 3D array of dimension $1500 \times 128 \times 200$ on master processor (to avoid massive communications between slave processors)
- To reduce storage/computational cost:
  - Analytical work + state of the art numerical techniques: integration, interpolation, density estimation
  - Parallel computing

Spectra Longitudinal Density II