

Neutrino-Nucleon Cross-Sections at Energies of Megaton-Scale Detectors

Part II

Askhat Gazizov

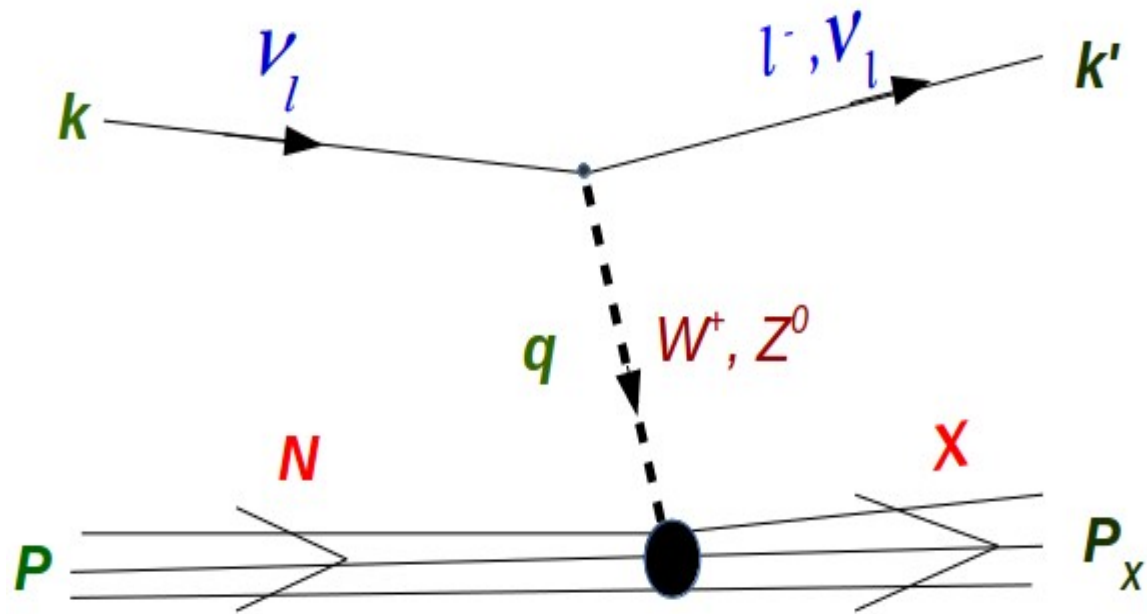
IP NASB, Minsk, Belarus
DESY-Zeuthen, Germany



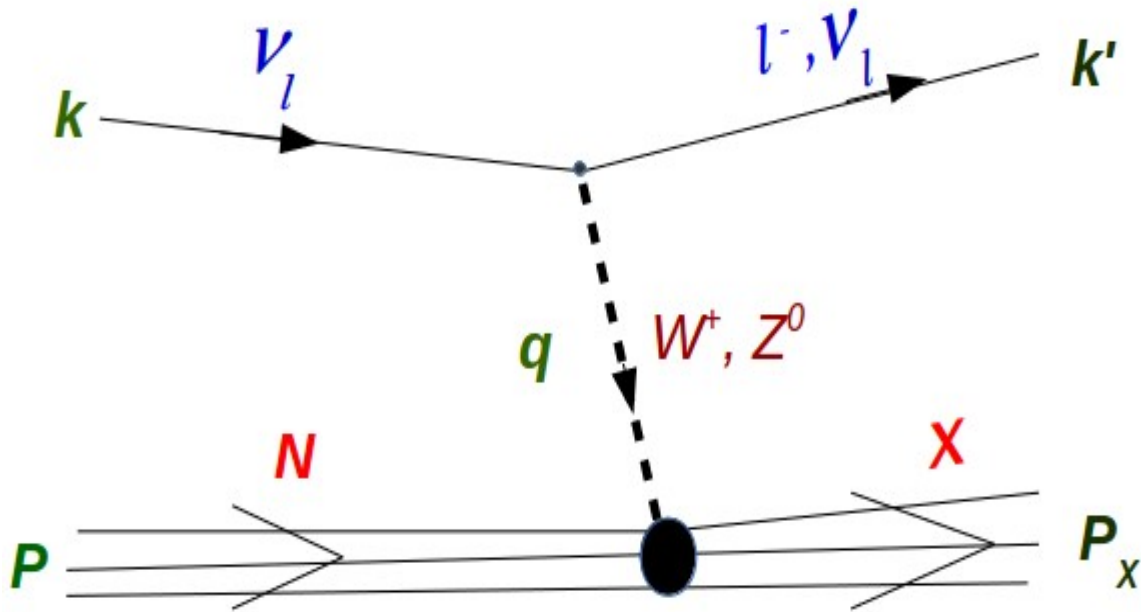
24.06.2015

Kinematics

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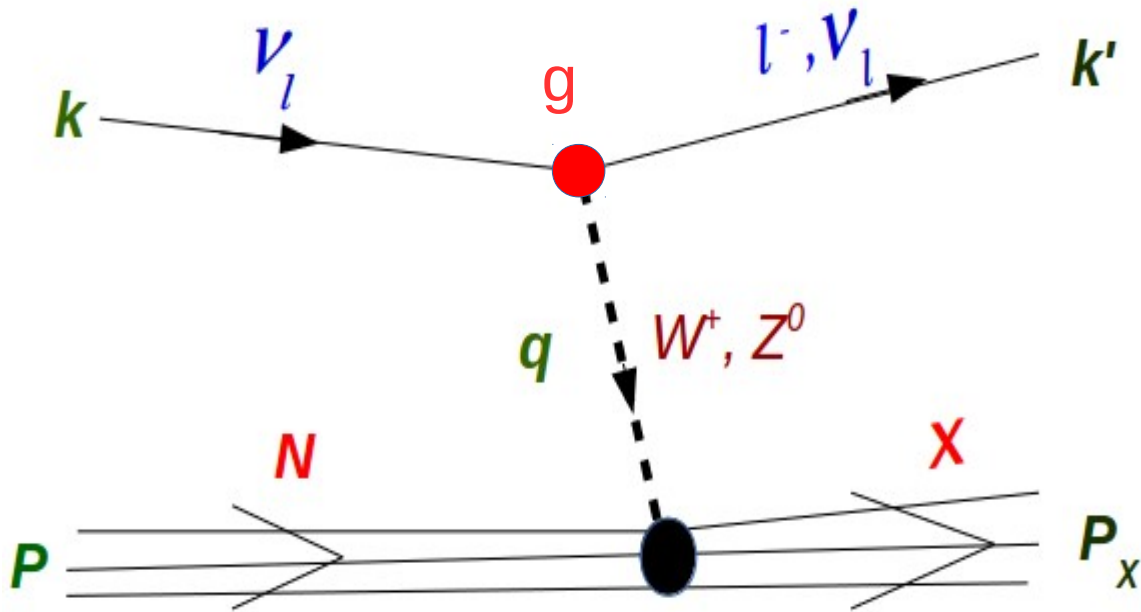
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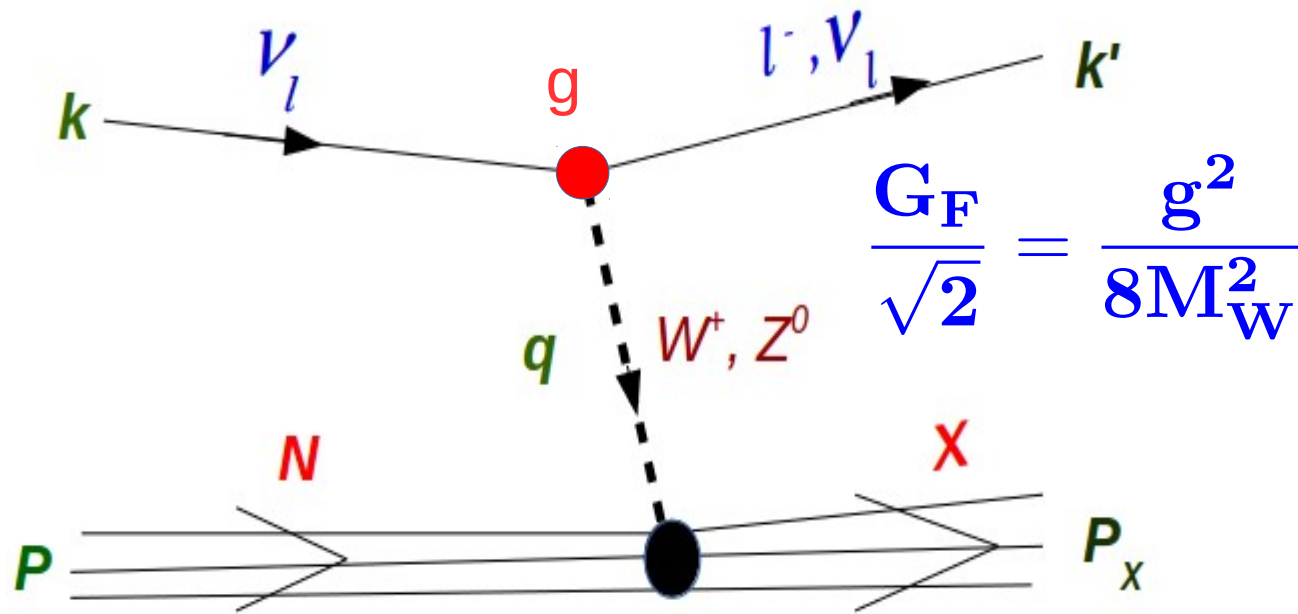
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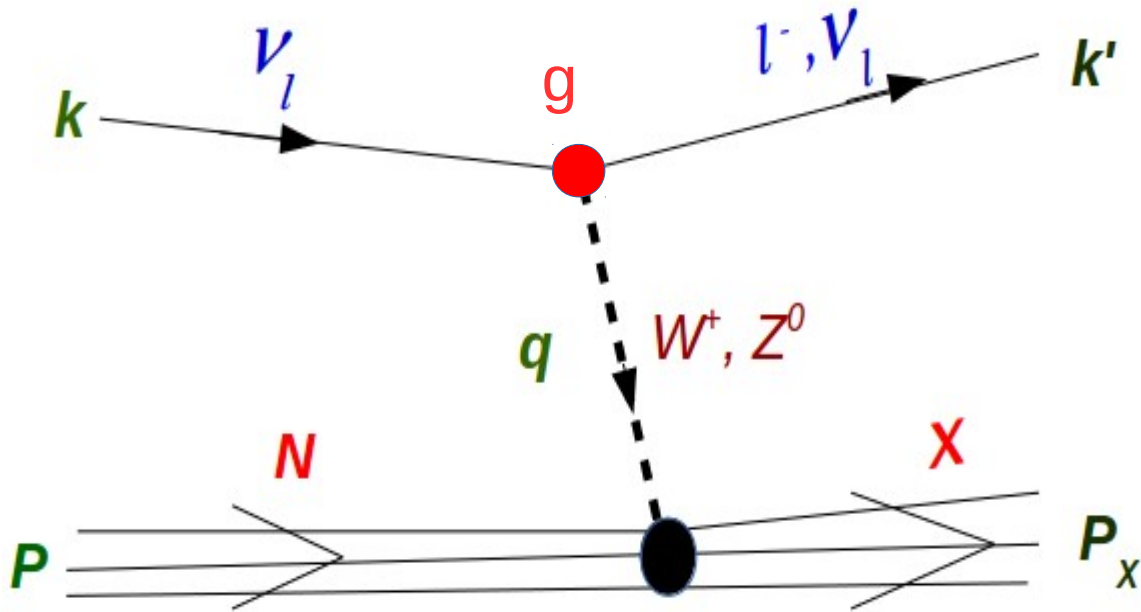
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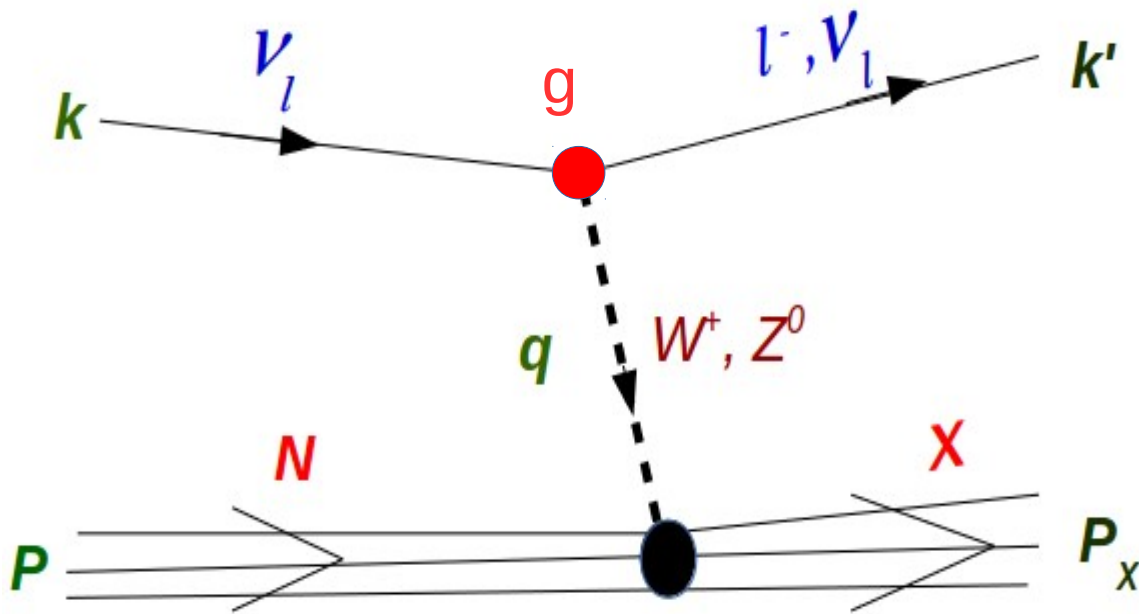
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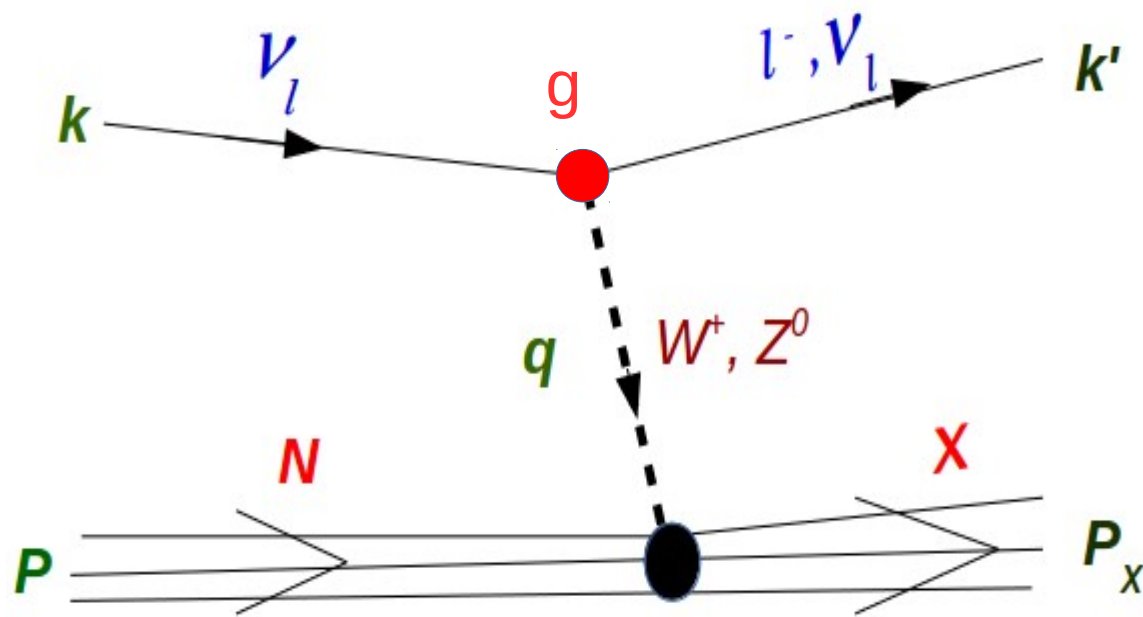
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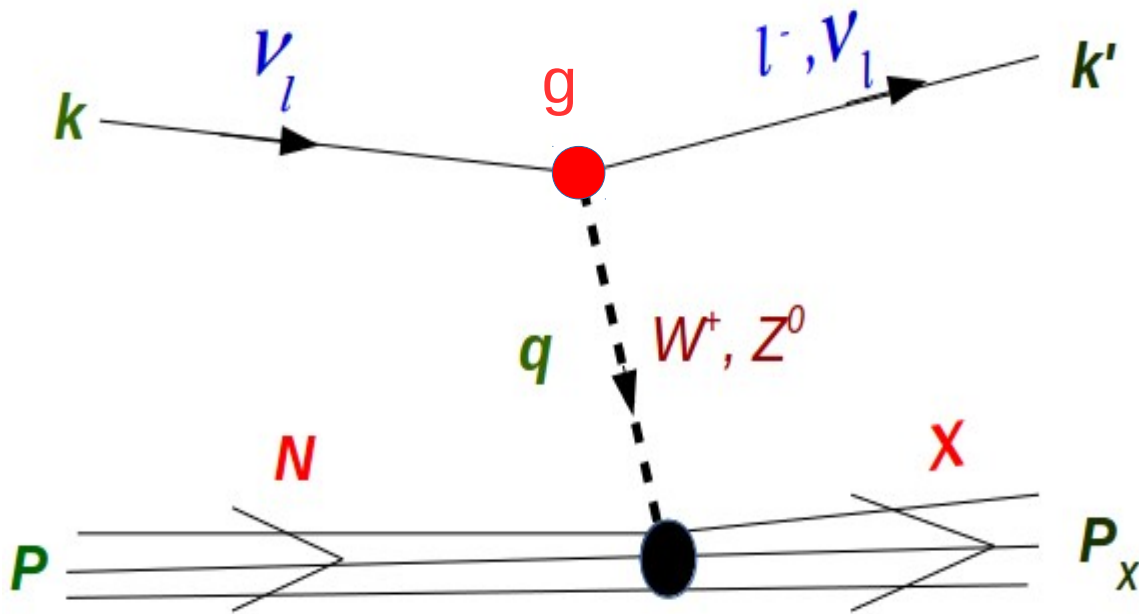
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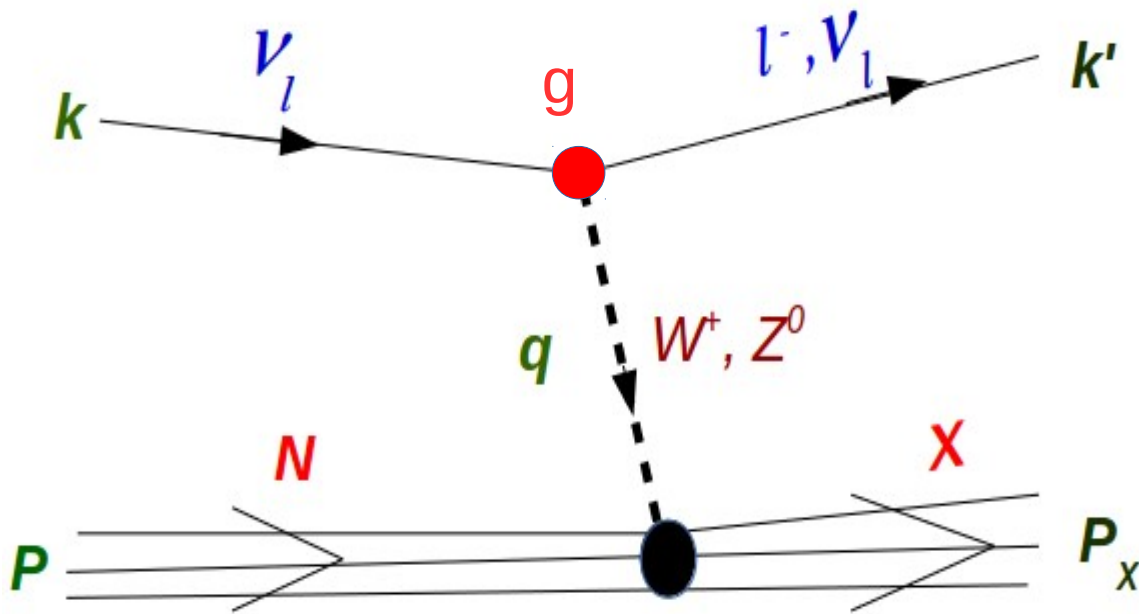
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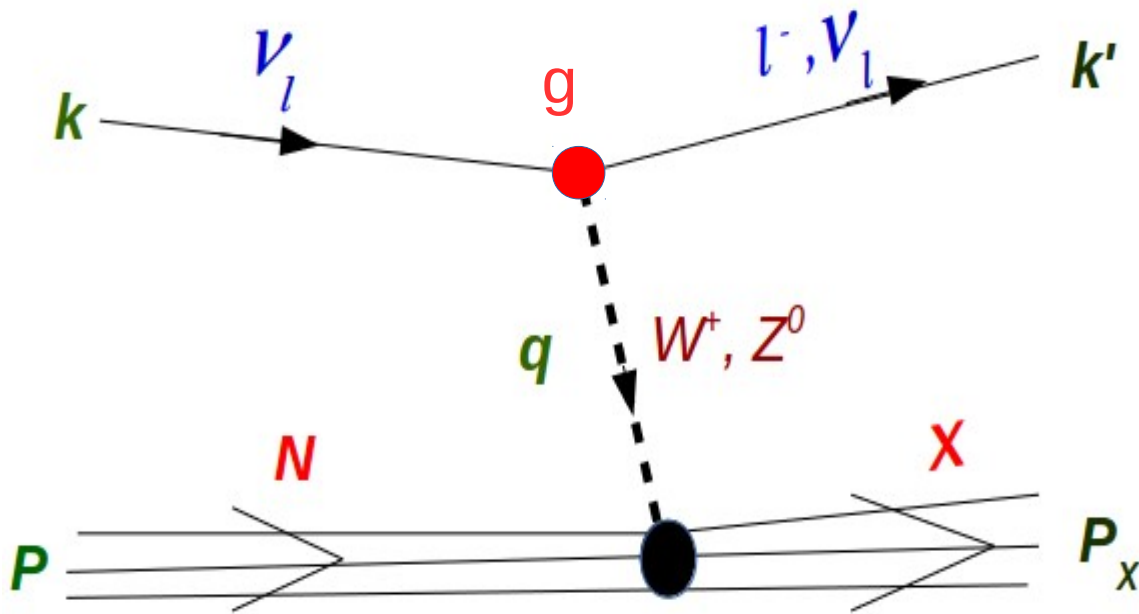
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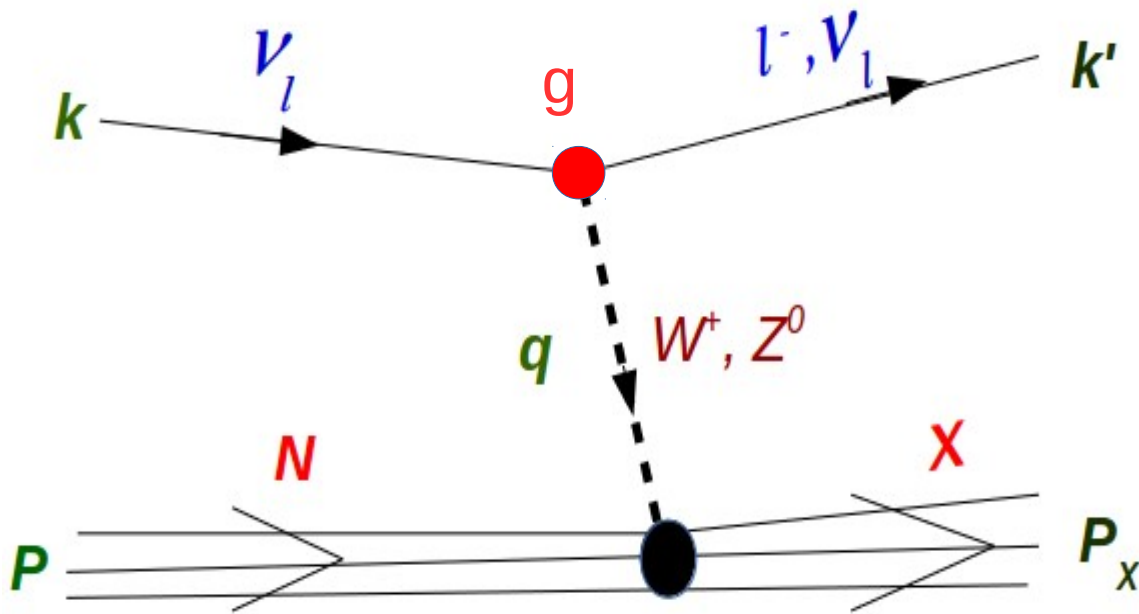
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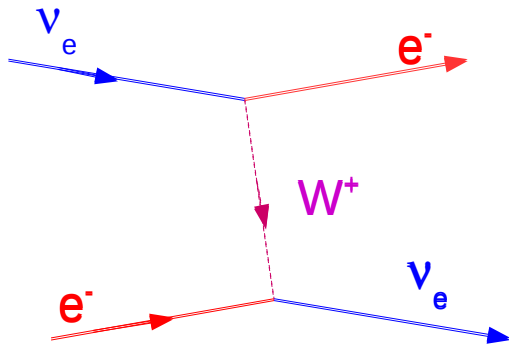
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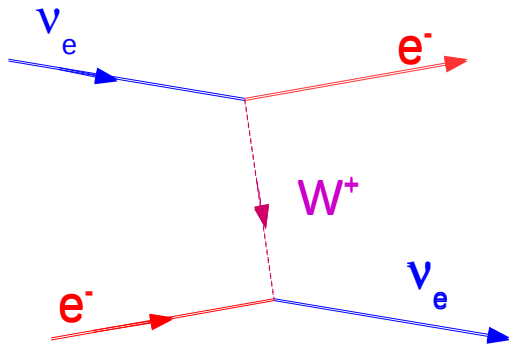
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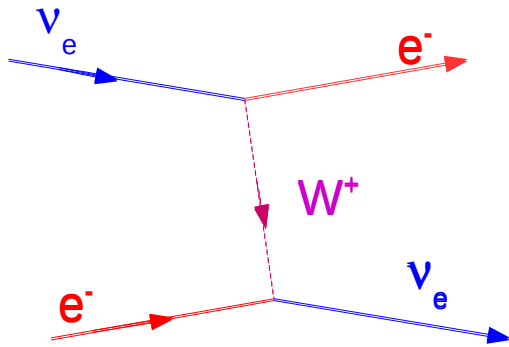


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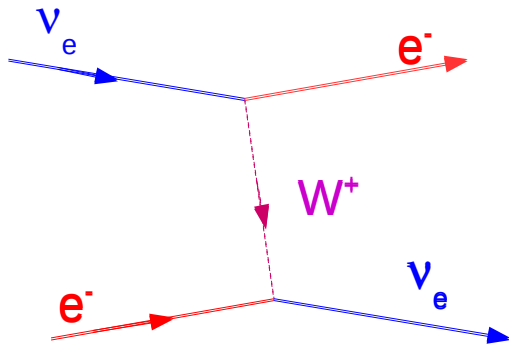
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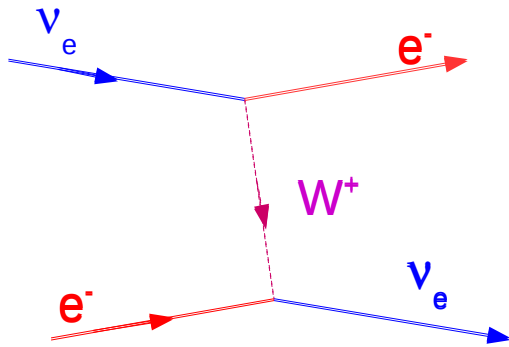


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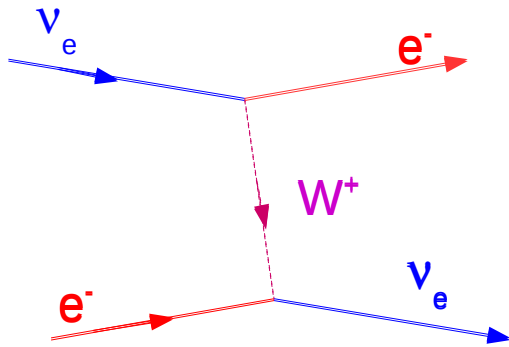
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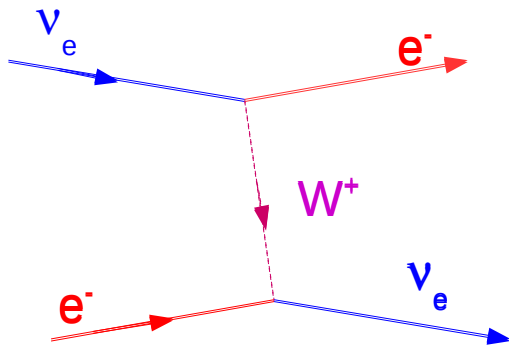
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for $S \gg M_W^2$ $\sigma(E) \rightarrow \frac{\pi G_F^2 M_W^2}{\pi}$

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At LO QCD just put a quark mass m_q instead of the electron for a ν scattering off a nucleon and sum the contributions from all quarks. Valence and sea quarks.

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$$\sigma_{\nu N}^{tot} = \sigma_{\nu N}^{(Q)ES} \otimes \sigma_{\nu N}^{1\pi} \otimes \sigma_{\nu N}^{1K} \otimes \sigma_{\nu N}^{DIS}$$

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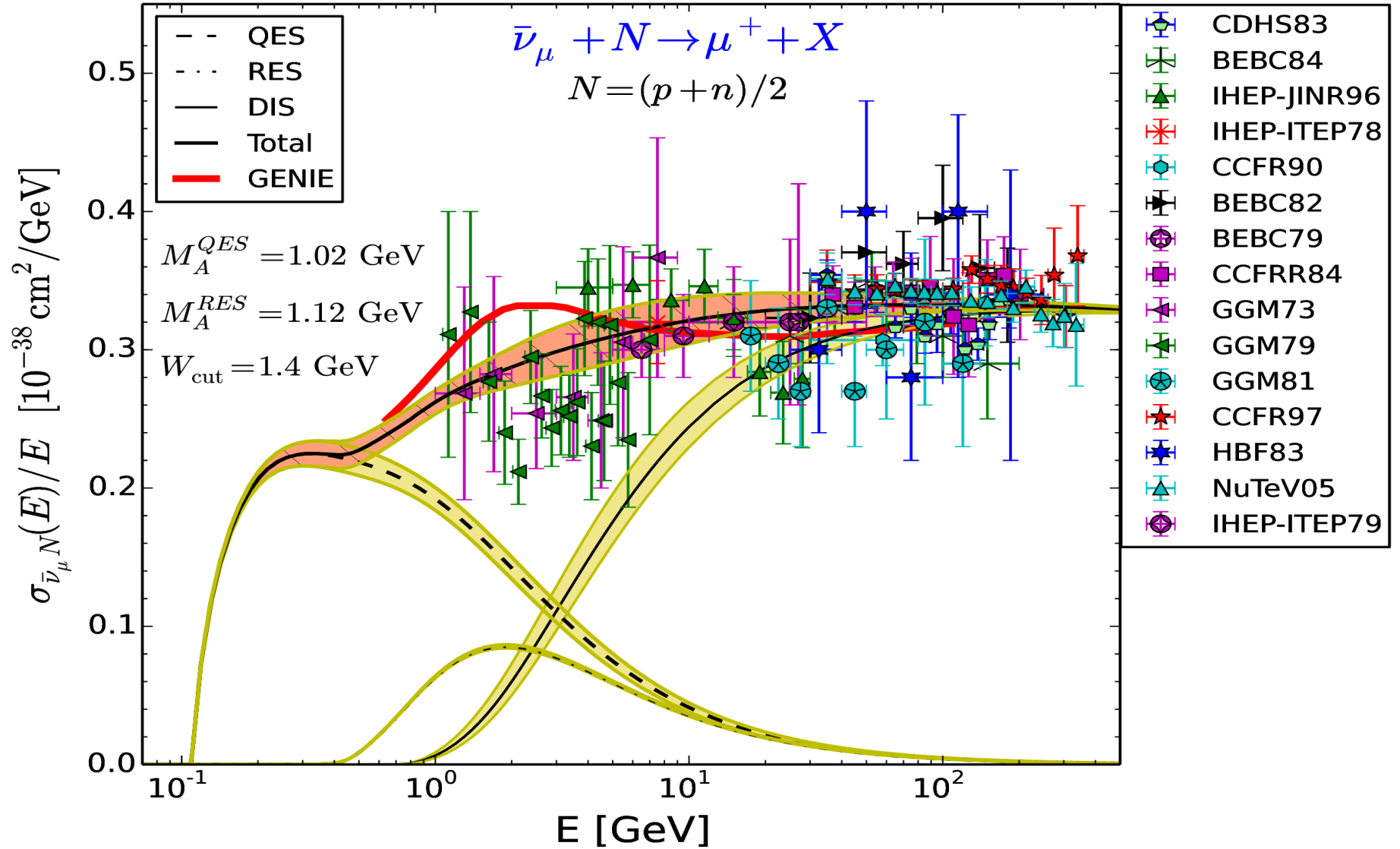
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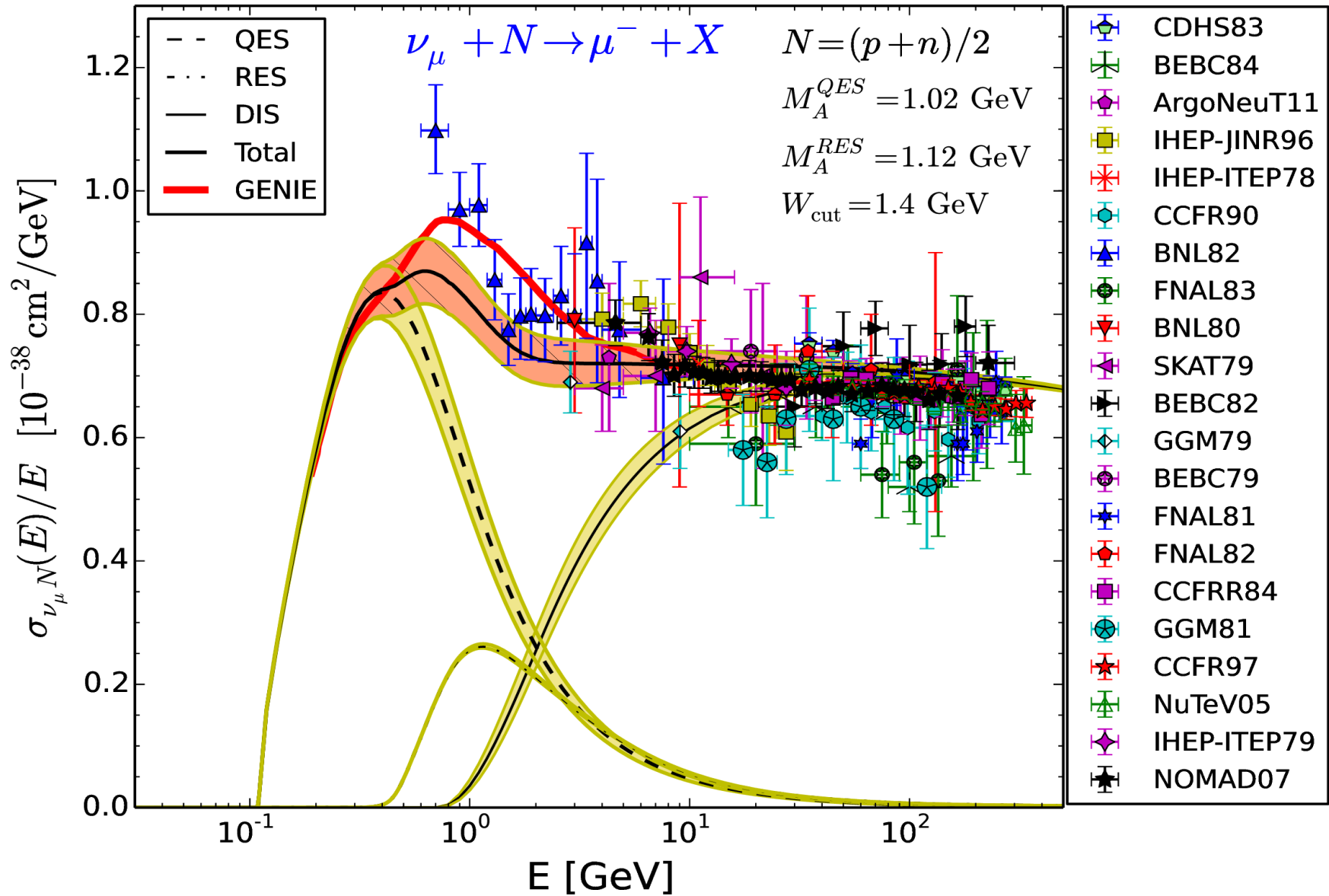
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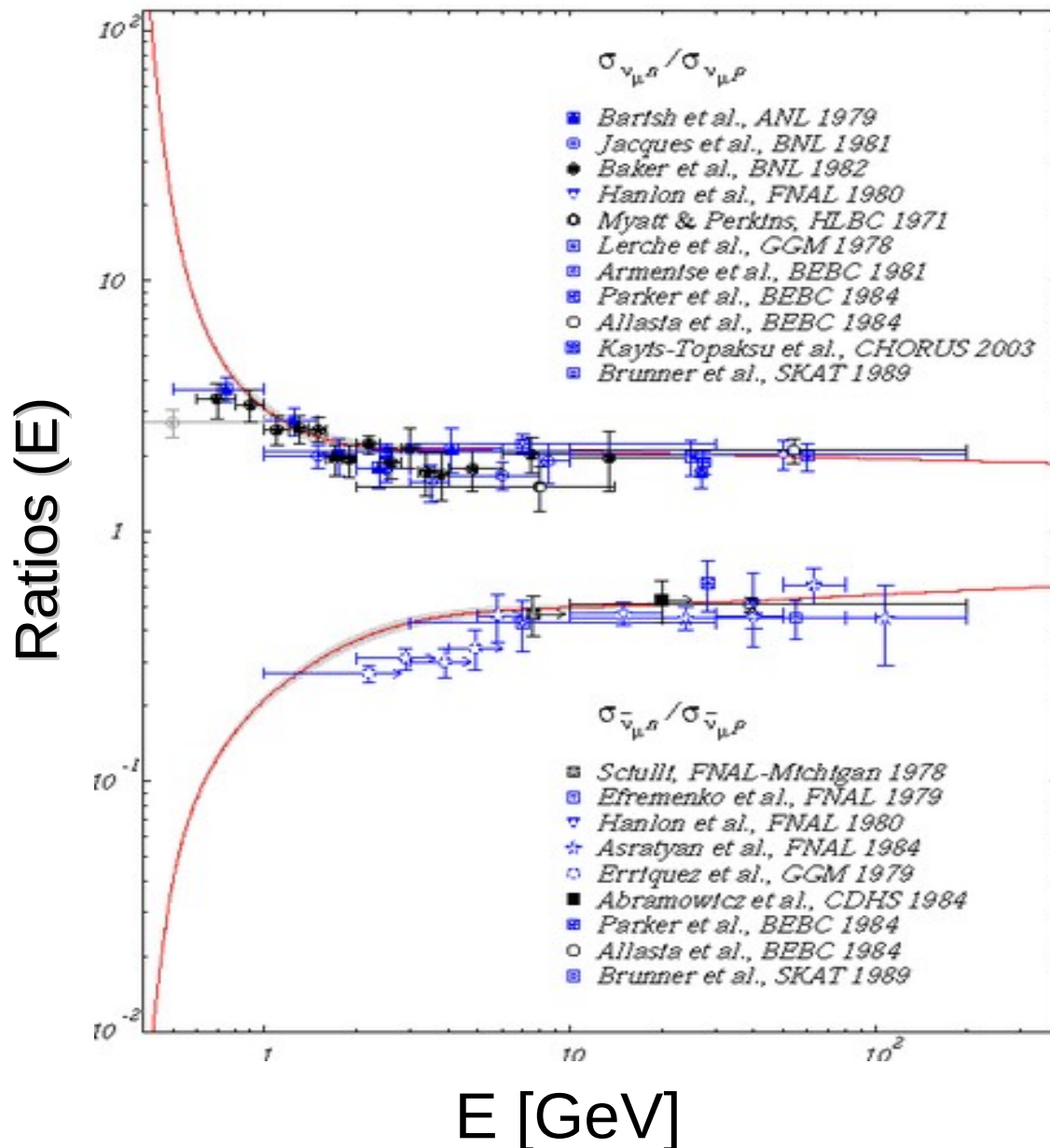
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- Deep inelastic scattering (**DIS**): $> 2\pi$ in the final state **X**

$$\bar{\nu}_\mu + N \rightarrow \mu^+ + X$$



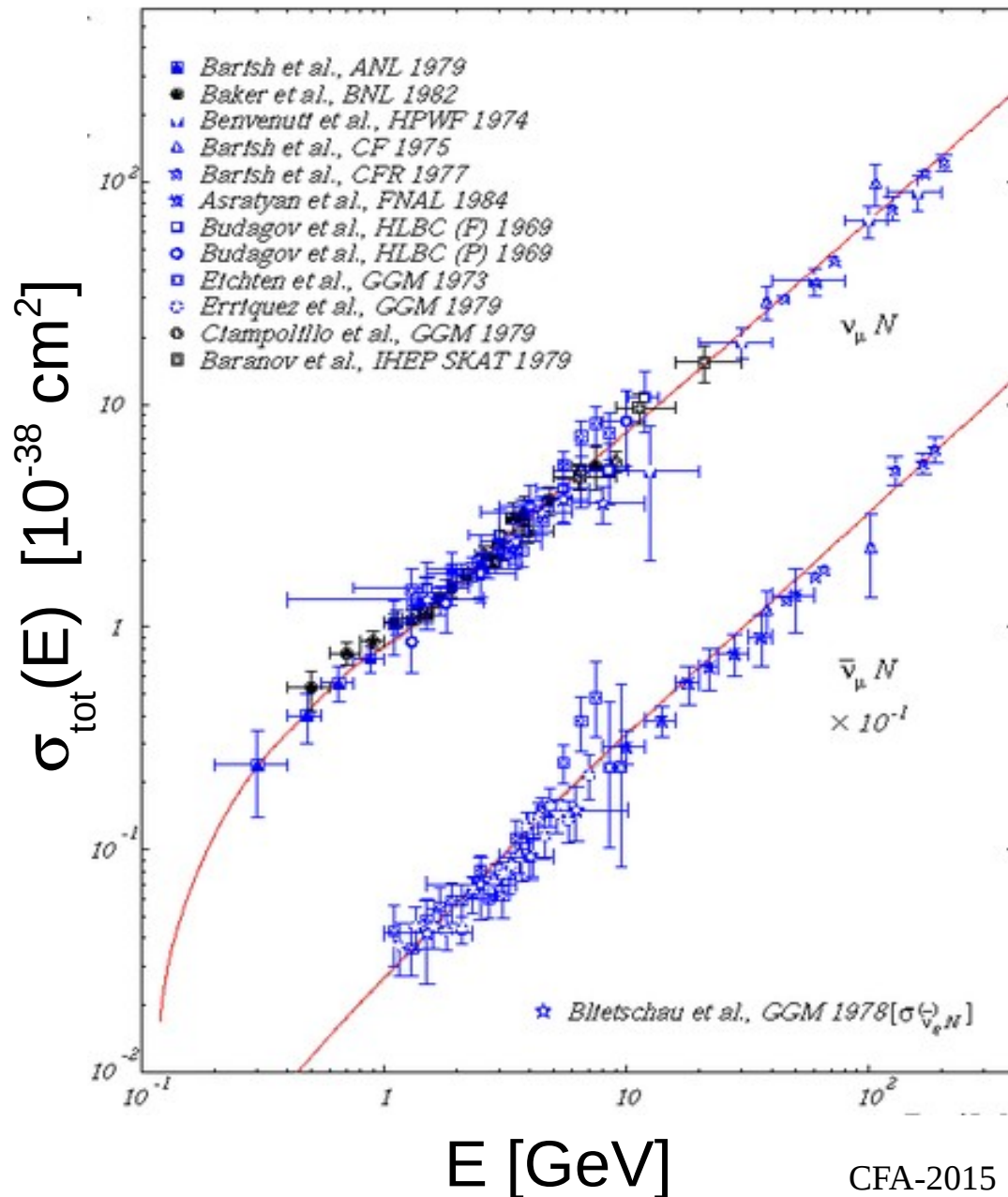


Ratios $\sigma_{\nu n} / \sigma_{\nu p}$ and $\sigma_{\bar{\nu} n} / \sigma_{\bar{\nu} p}$



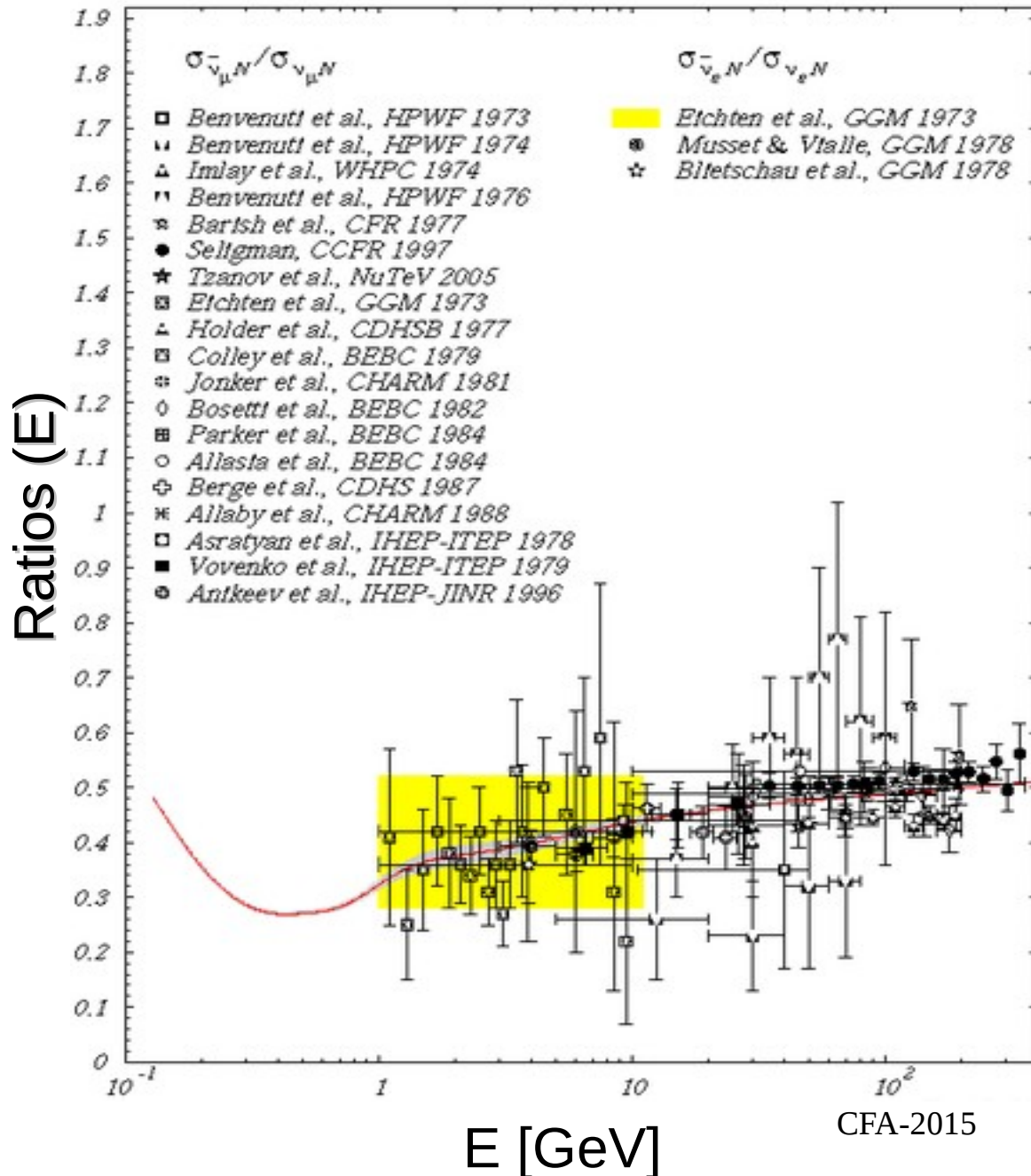
Check model parameterisation using data for such ratios.

Total $\nu_\mu N$ and $\bar{\nu}_\mu N$ Cross-Sections



- Cross-sections grow proportional to E .
- At higher energies just power-law growth due to sea-quarks density increase.

$\bar{\nu}N / \nu N$ Ratios



At low energies ratio is due to kinematics. Both ν and quark are left-handed: $J = 0$. Isotropic distribution.

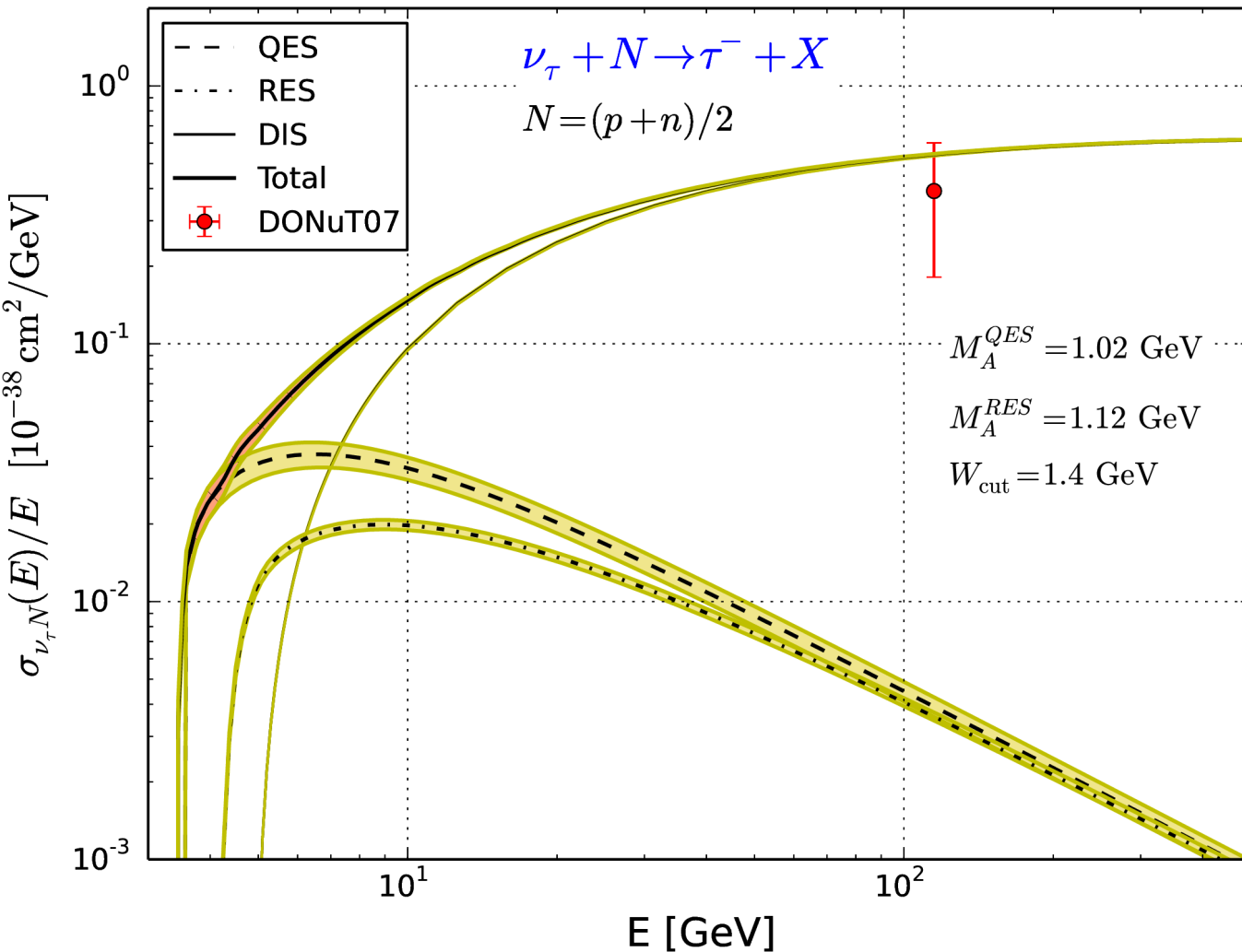
$\bar{\nu}$ is right-handed: $J = 1$,

$$\frac{d\sigma(\cos\theta)}{d\cos\theta} \propto (1 - \cos\theta)^2,$$

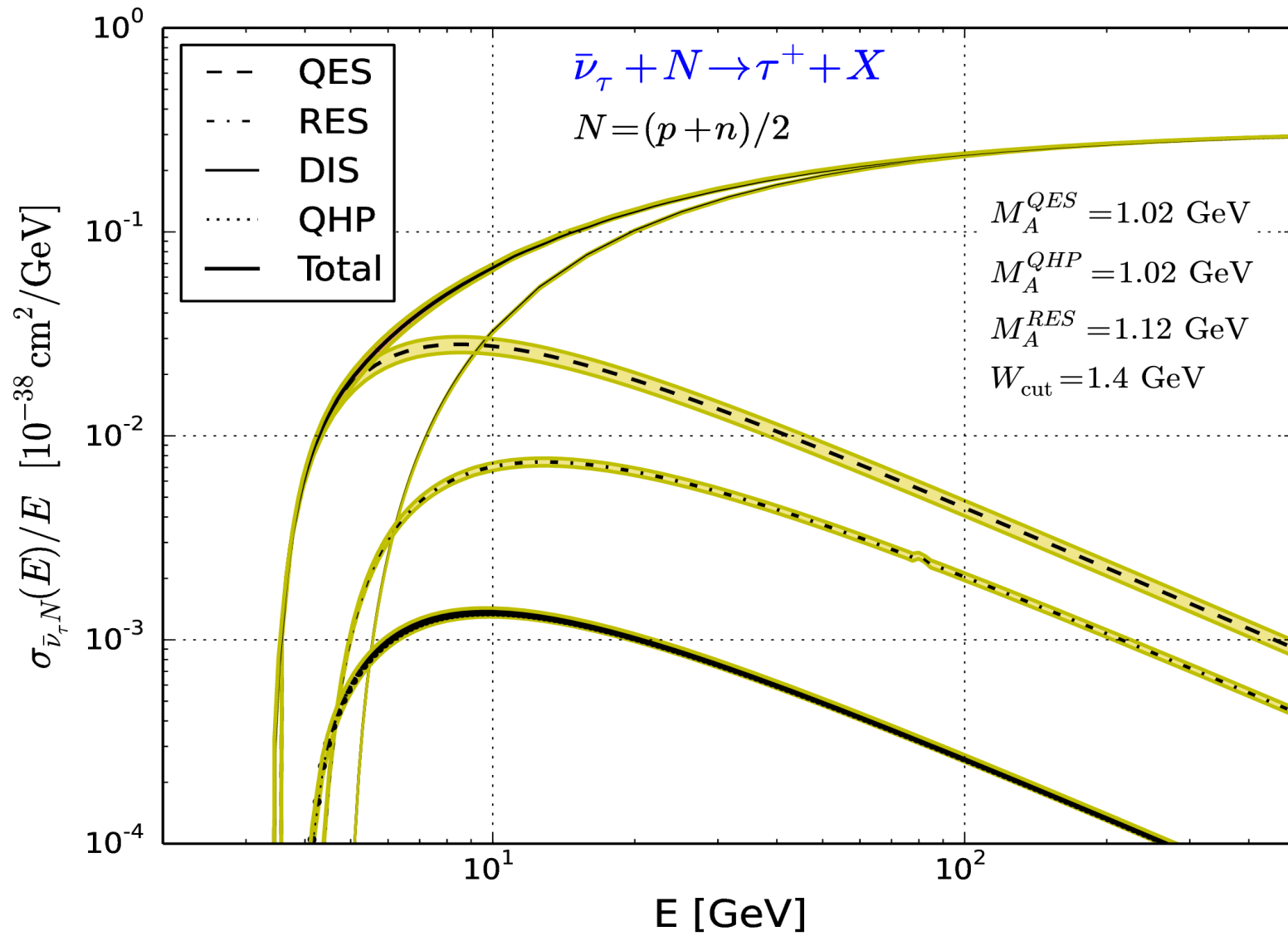
and $\sigma_{\bar{\nu}} / \sigma_{\nu} \approx 1/3$

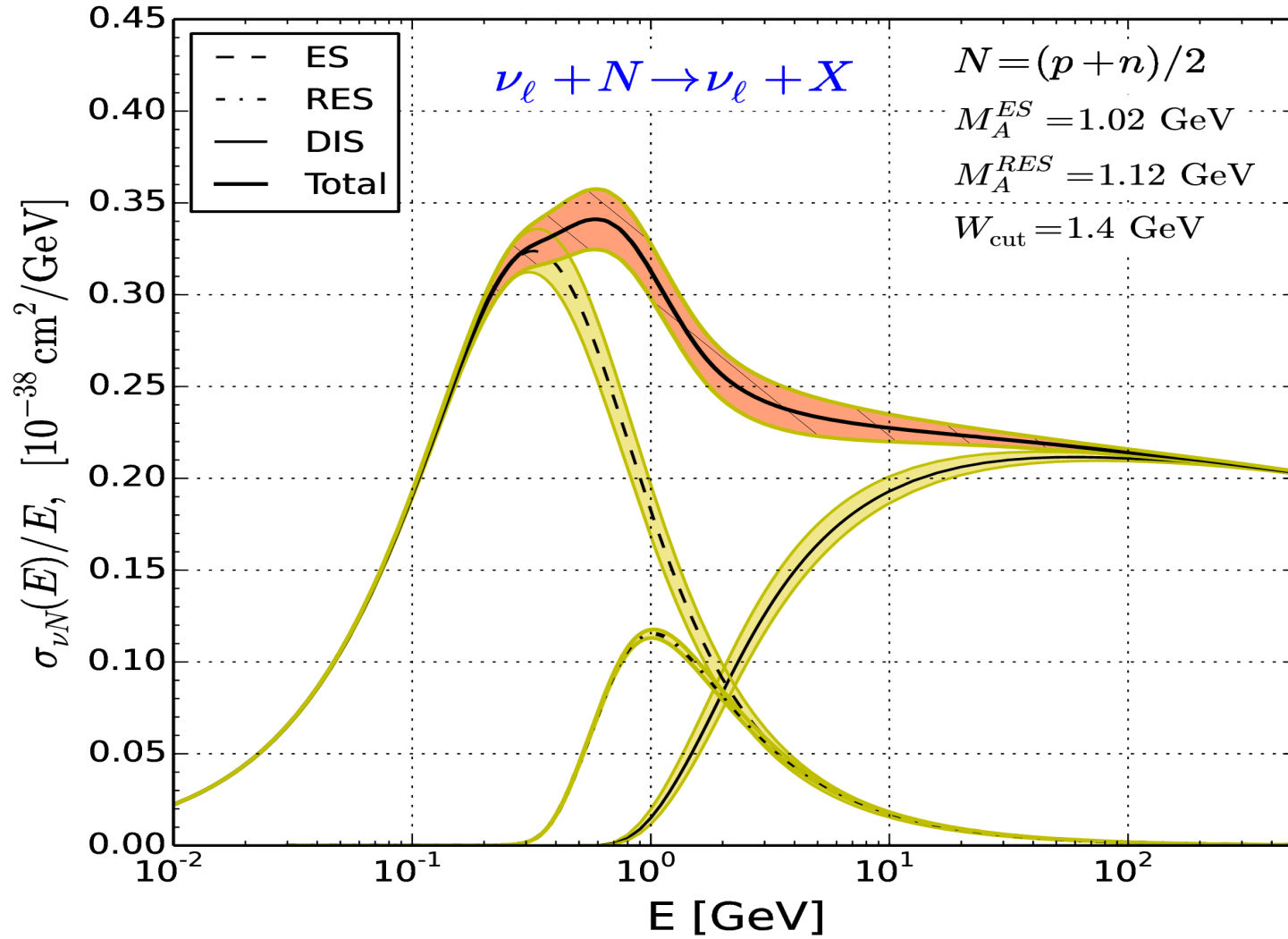


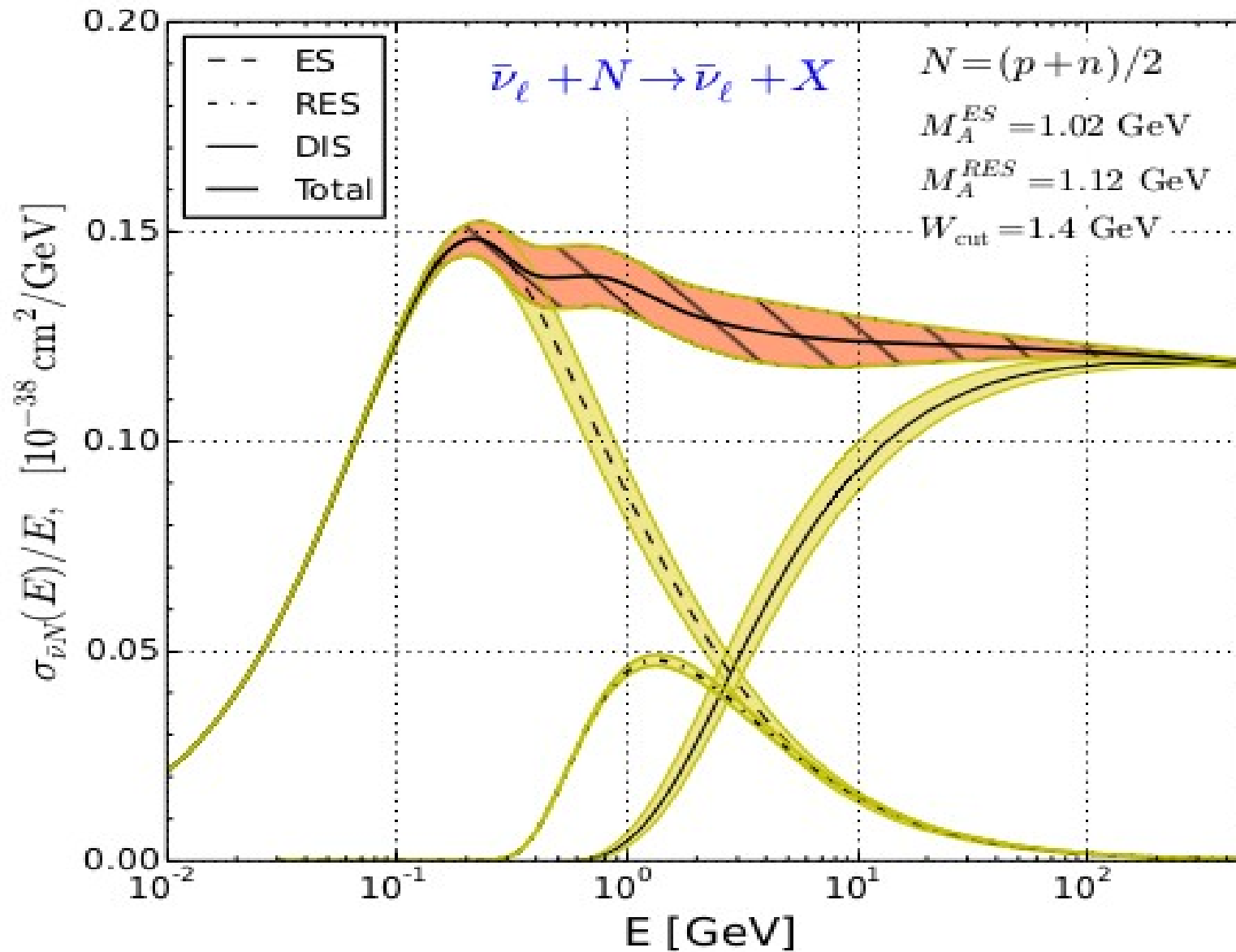
DONUT – Direct Observation of the NU Tau



- Tevatron protons produce charm mesons.
- ν_τ appears in the charm decays.
- ν_τ produces τ 's in the nuclear emulsion.
- 4 τ -events registered, while 0.2 expected if no ν_τ .
- One event with a kink from the τ -decay.

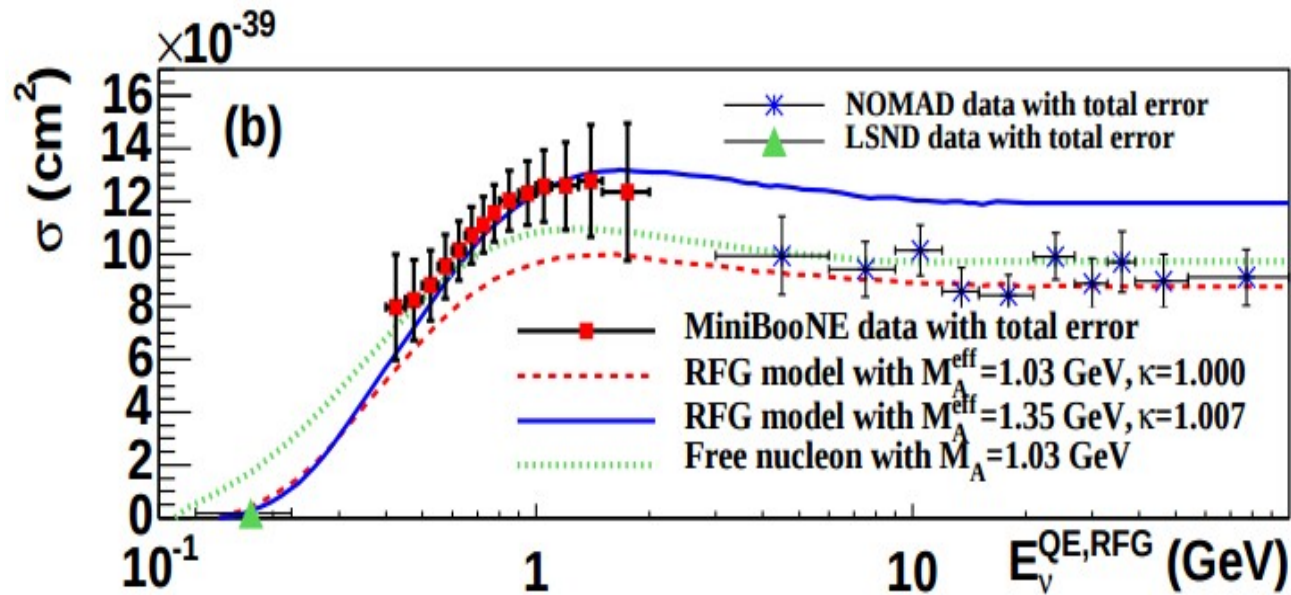
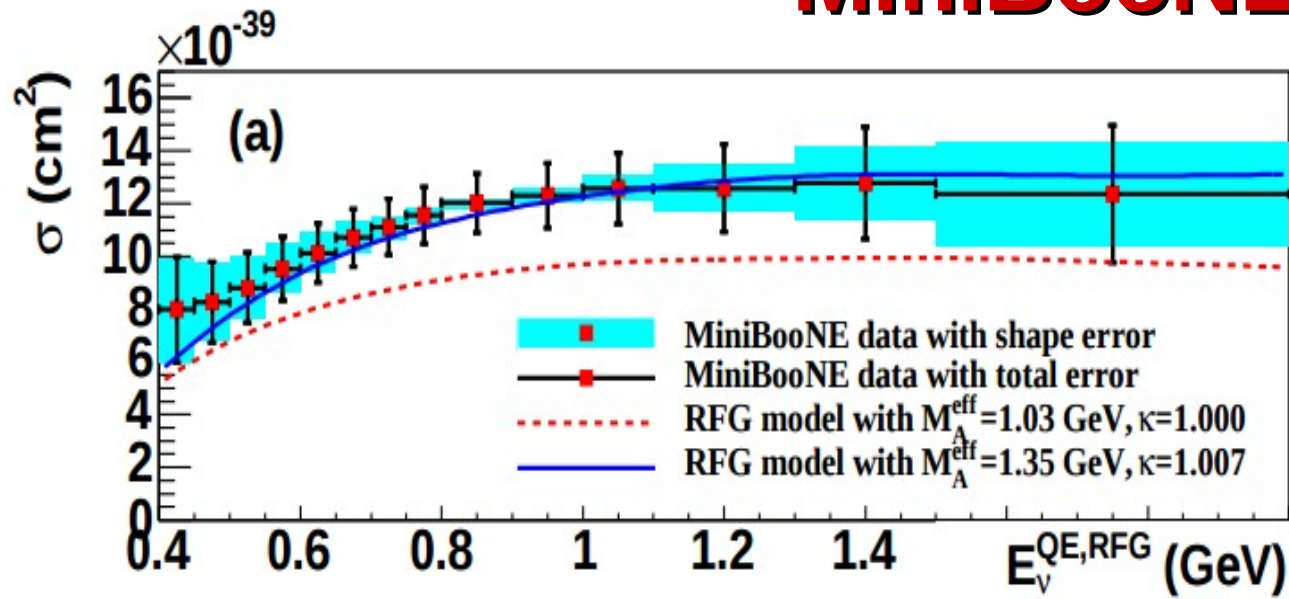






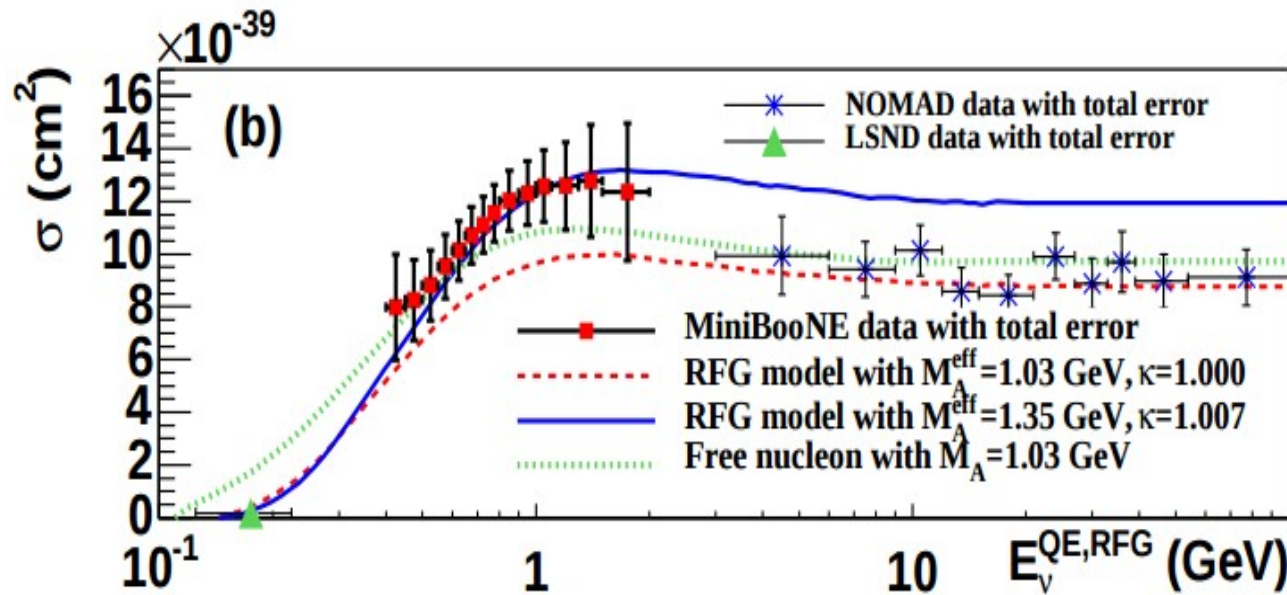
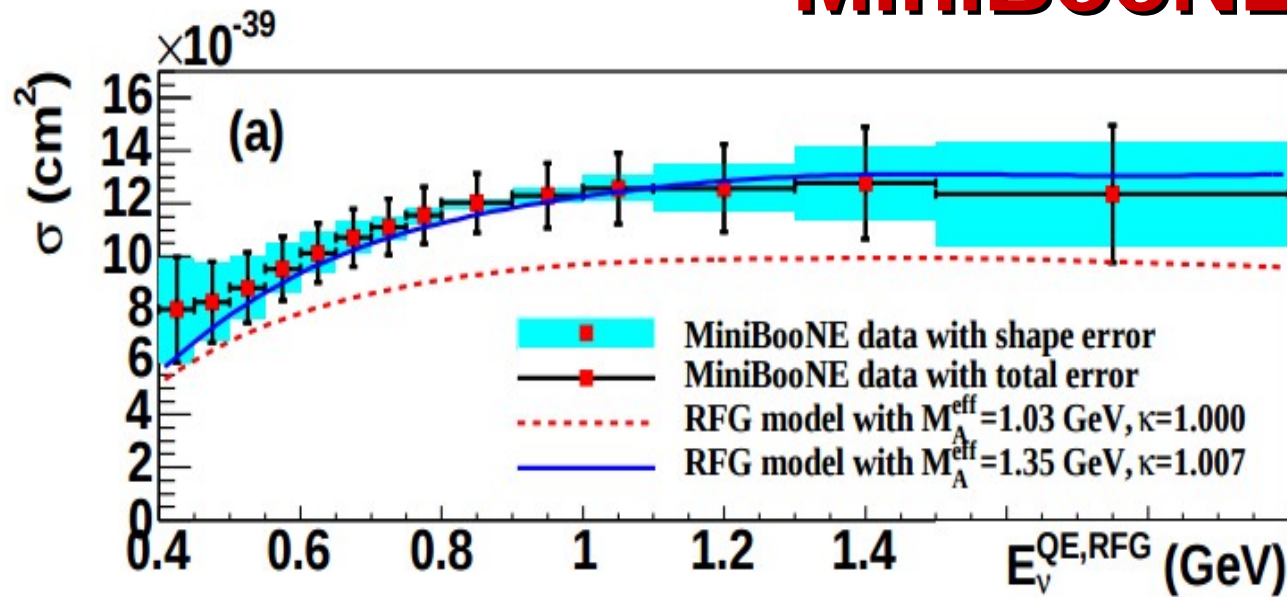
MiniBooNE

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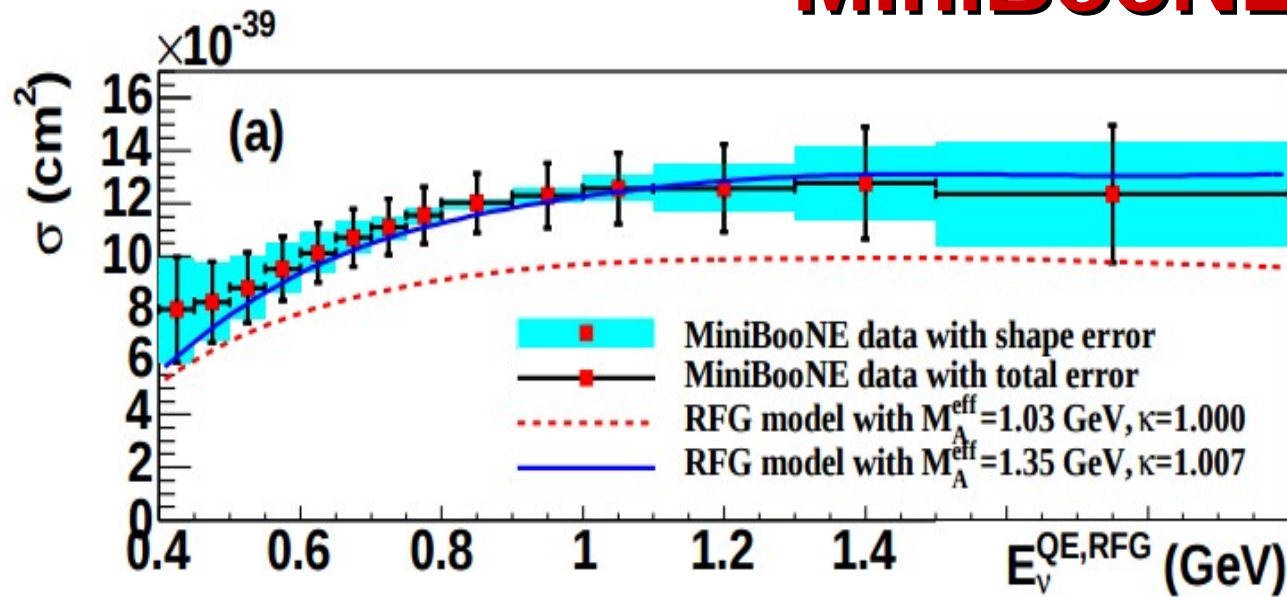


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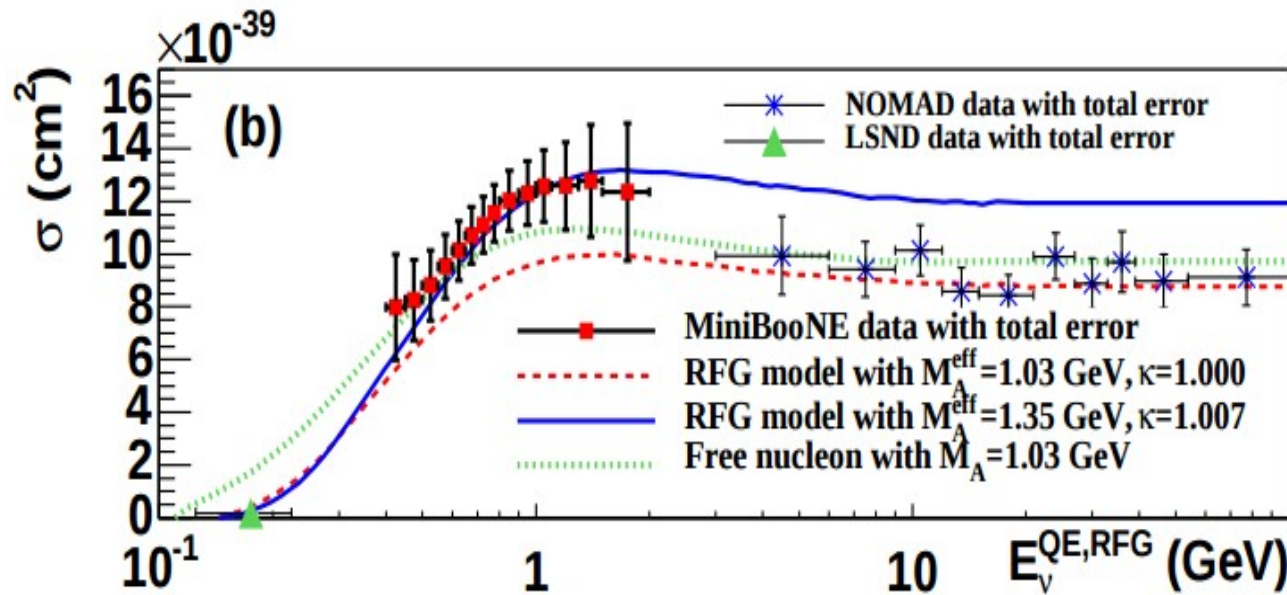
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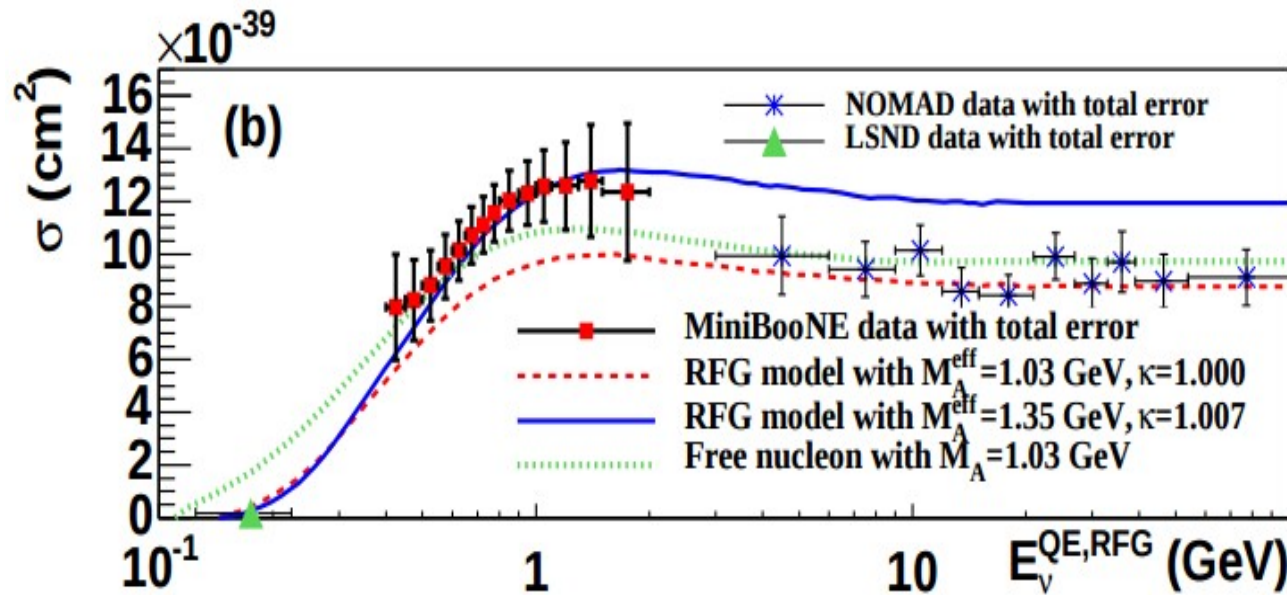
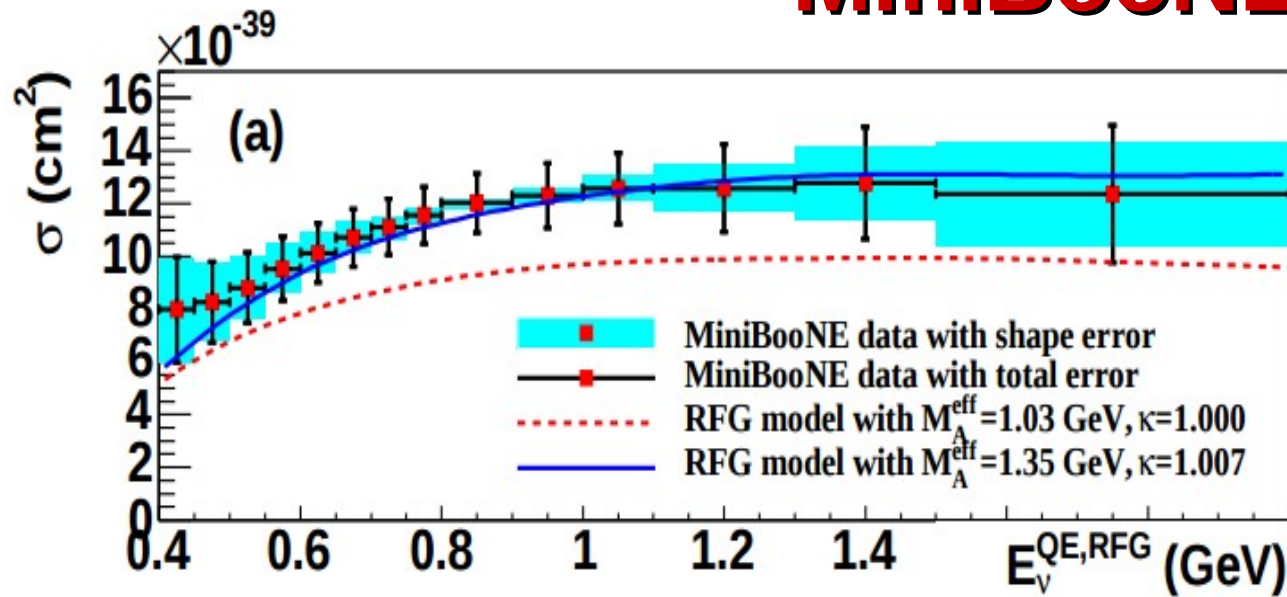
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- MiniBooNE employs the **NUANCE v3** event generator to estimate neutrino interaction rates in the CH_2 target medium.
- The most precision experiment. 2-differential cross-sections.

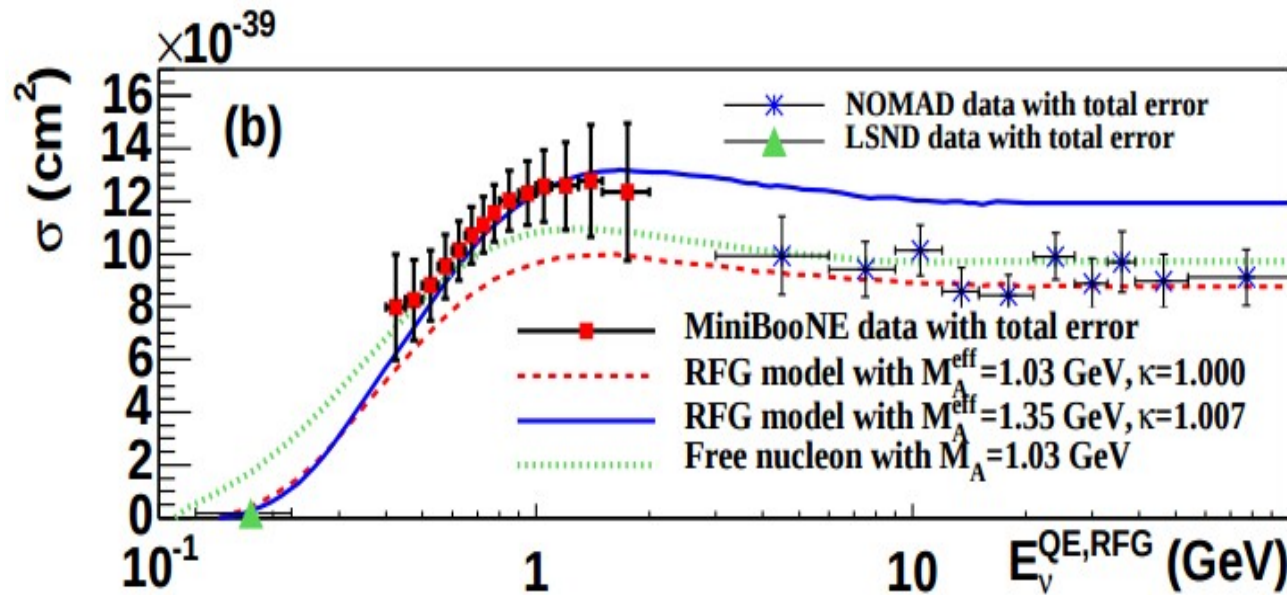
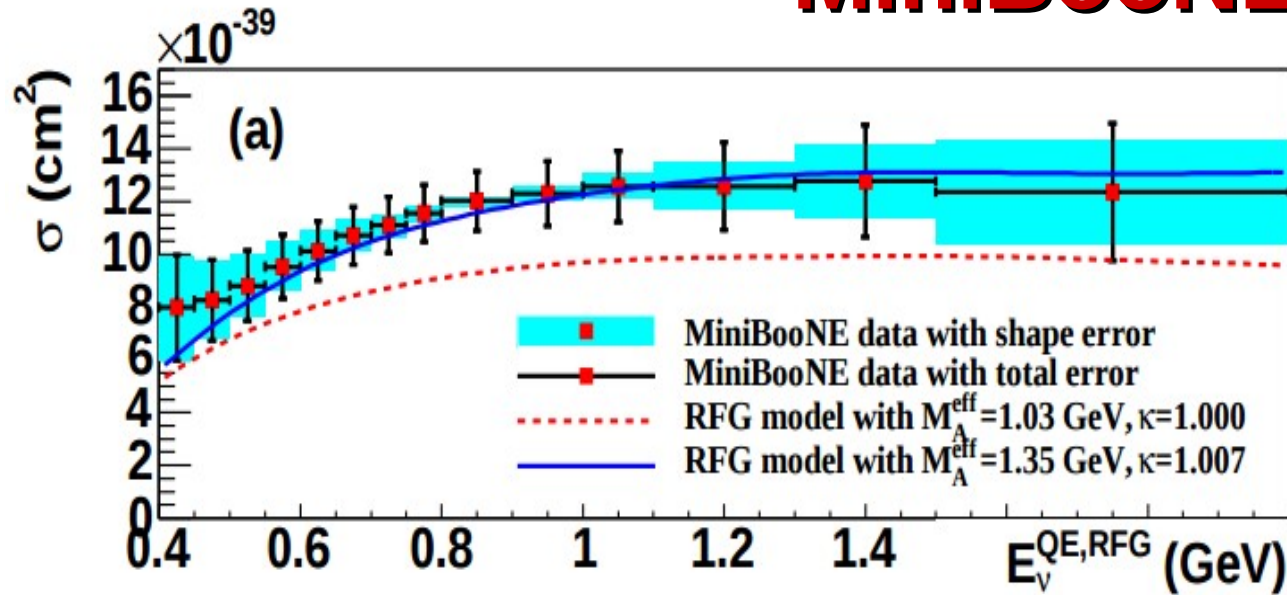


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- Discrepancy between different experiments data is significant.
- We avoided discussion of low energies. Irrelevant for the future PINGU / ORCA megaton experiments.

ES and QES off proton/neutron



- These processes dominate at low energies.
- There is practically no threshold energy for the *ES*.
- The threshold energy for QES with $M_N = m_p$ or m_n
 ($m_n - m_p \approx 1.293 \text{ MeV}$)

$$E_{\nu,th} \geq \frac{(m_l + M_N)^2 - M_N^2}{2 M_N} \simeq m_l \left(1 + \frac{m_l}{2 M_N} \right)$$

- For τ -lepton production with $m_\tau/m_\mu \approx 16.8$, $m_\tau/m_N \approx 1.9$

$$E_{\nu_\tau,th} / E_{\nu_\mu,th} \simeq 30.$$

Theoretical approach to QES

In terms of the Mandelstam variables

$$s = (k + p)^2, \quad t = -Q^2 = (k - k')^2, \quad u = (k - p')^2$$

the differential cross-section of **QES**

$$\frac{d\sigma}{d|t|} = \frac{G_F^2 \cos^2 \theta_C}{2\pi (s - M_N^2)^2} |\mathcal{M}^2|$$

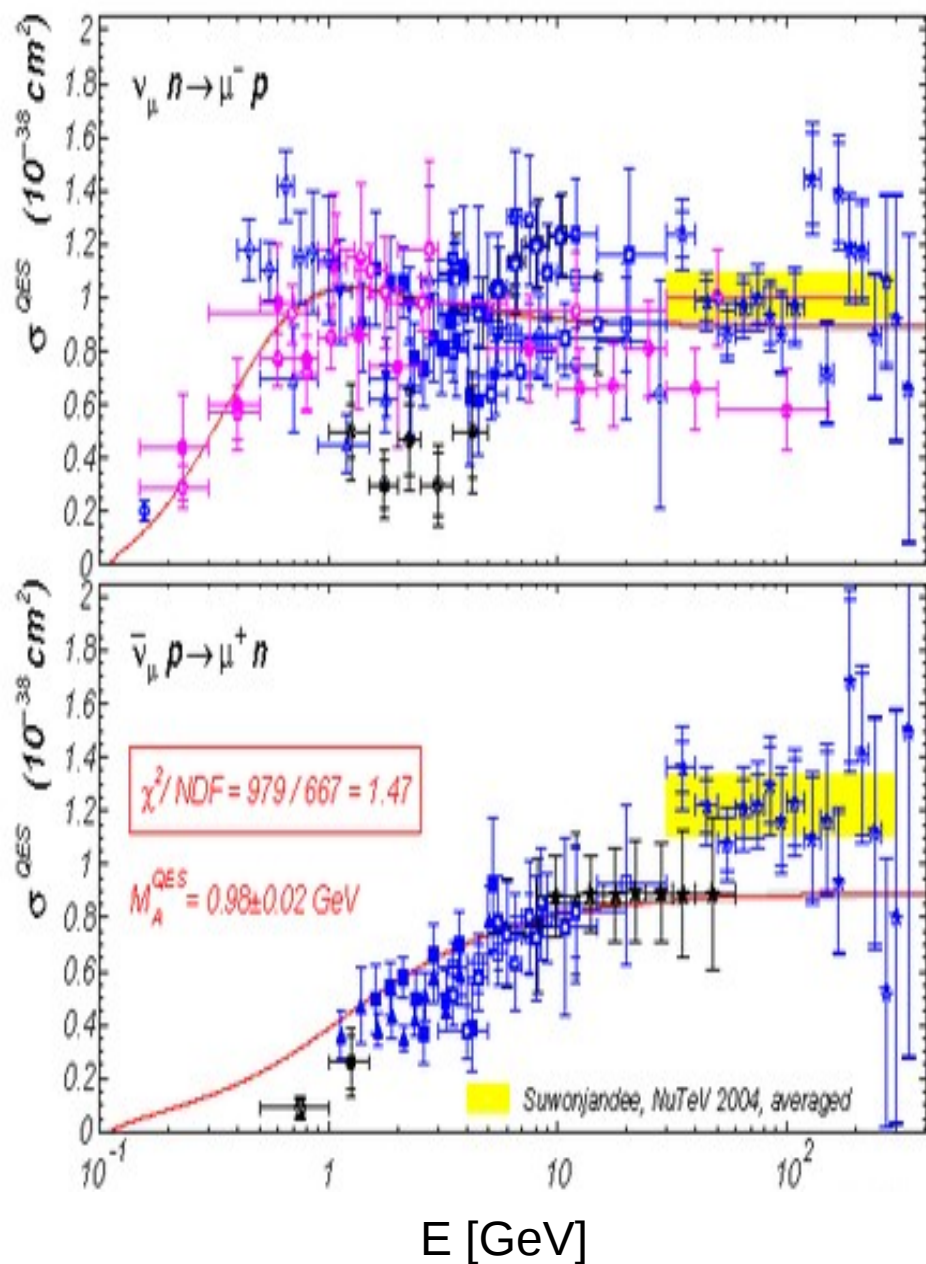
where $|\mathcal{M}^2| = A(t) - (s - u)B(t) + (s - u)^2 C(t)$.

See e.g. A. Strumia & F. Vissani, Phys. Lett. B564:42-54, 2003

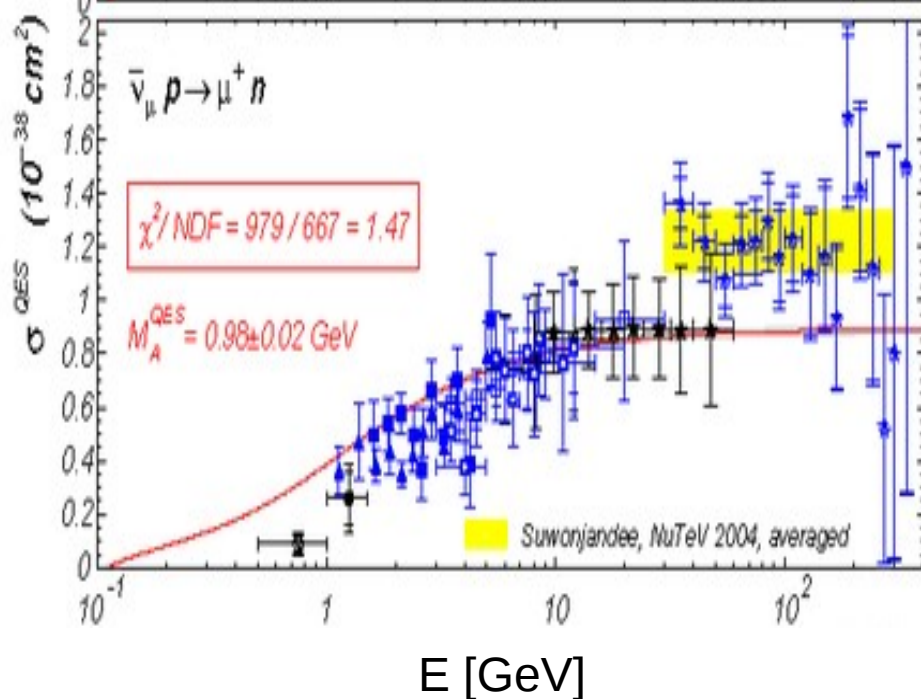
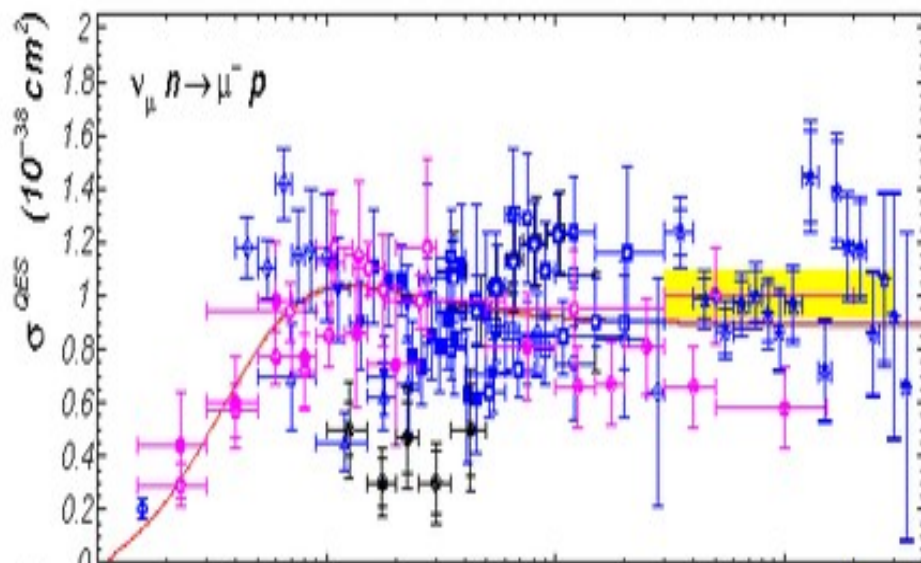
$A(t)$, $B(t)$ and $C(t)$ may be expressed in terms of vector and axial vector form factors. They are parametrized by the vector mass $M_V^2 = 0.71 \text{ GeV}^2$ and the axial-vector mass $M_A^{QES} \approx 1.02 \text{ GeV}$

$$F_A(Q^2) = \frac{F_A(0)}{(1 + Q^2/M_A^2)^2} \quad \text{with} \quad F_A(0) \approx -1.267$$

QES Cross Sections vs. Data

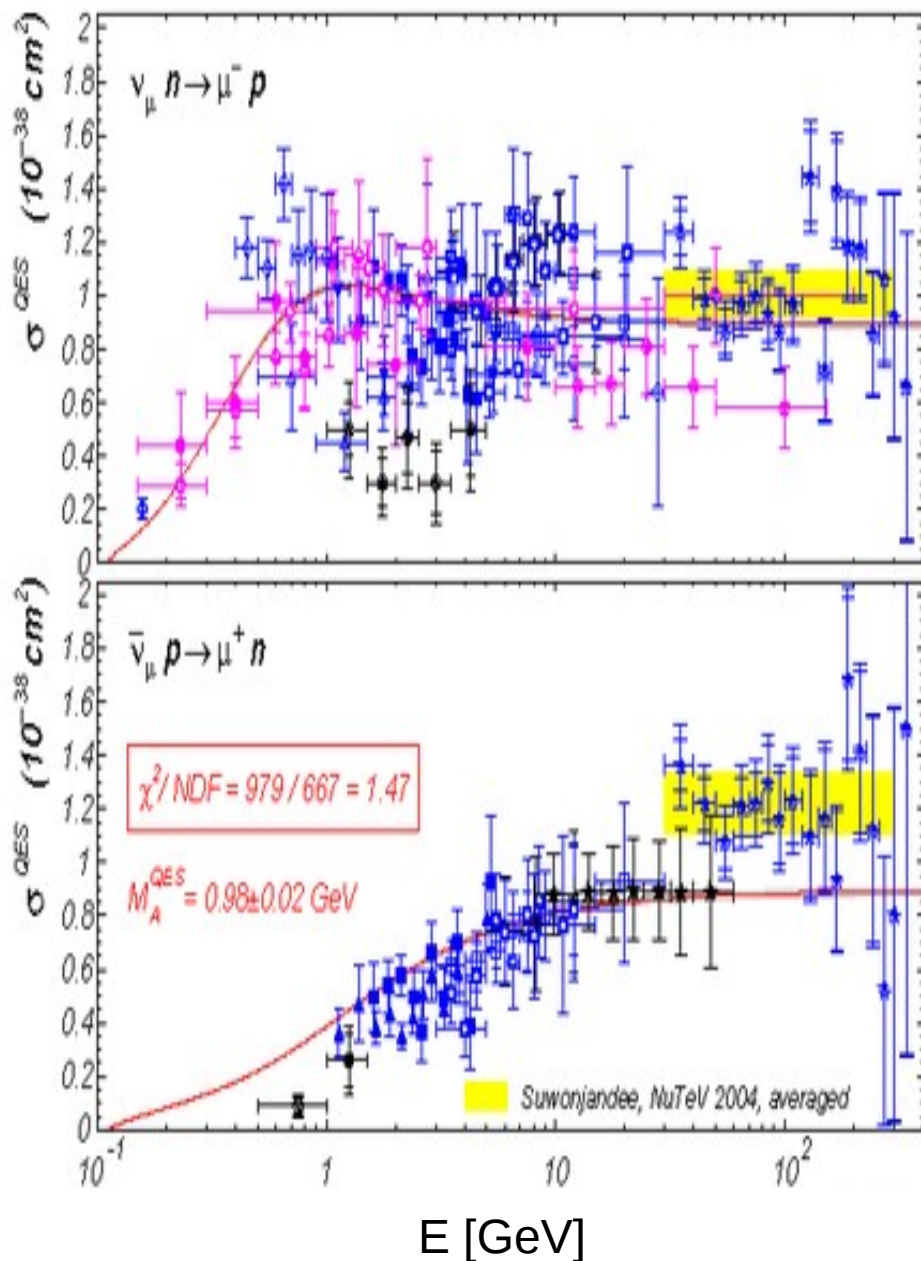


QES Cross Sections vs. Data



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- ◊ Barish et al., ANL 1977 (D₂)
- ◊ Baker et al., BNL 1981 (D₂)
- ◊ Kitagaki et al., FNAL 1983 (D₂)
- ★ Asratyan et al., FNAL 1984 (Ne-H₂)
- ✱ Suwonjandee, NuTeV 2004 (Fe)
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- ◻ Brunner et al., IHEP SKAT 1990 (CF₃ Br)
- ◻ Ammosov et al., IHEP SKAT 1992 (CF₃ Br)

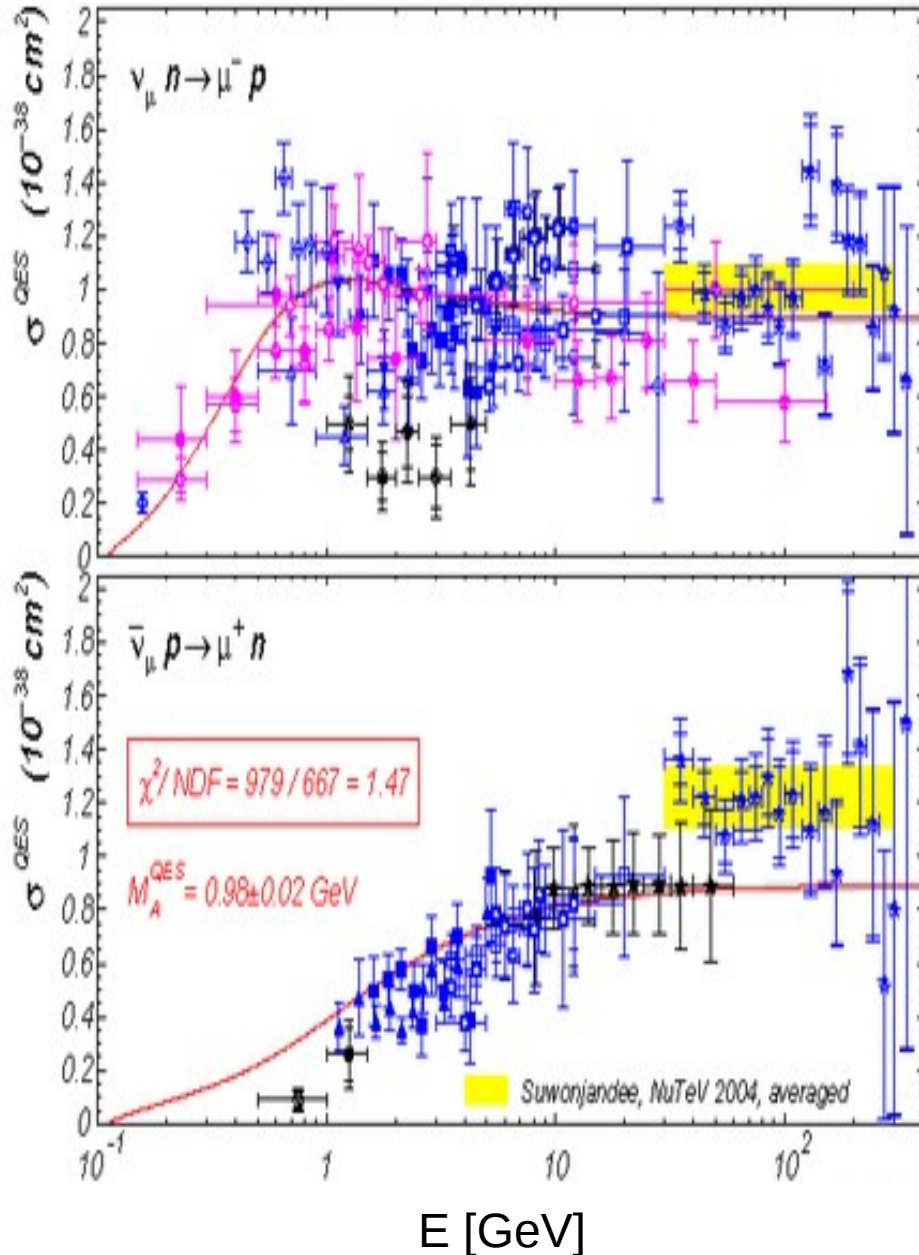
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- Belikov et al., IHEP 1982 (Al)
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● Many experimental data differ.

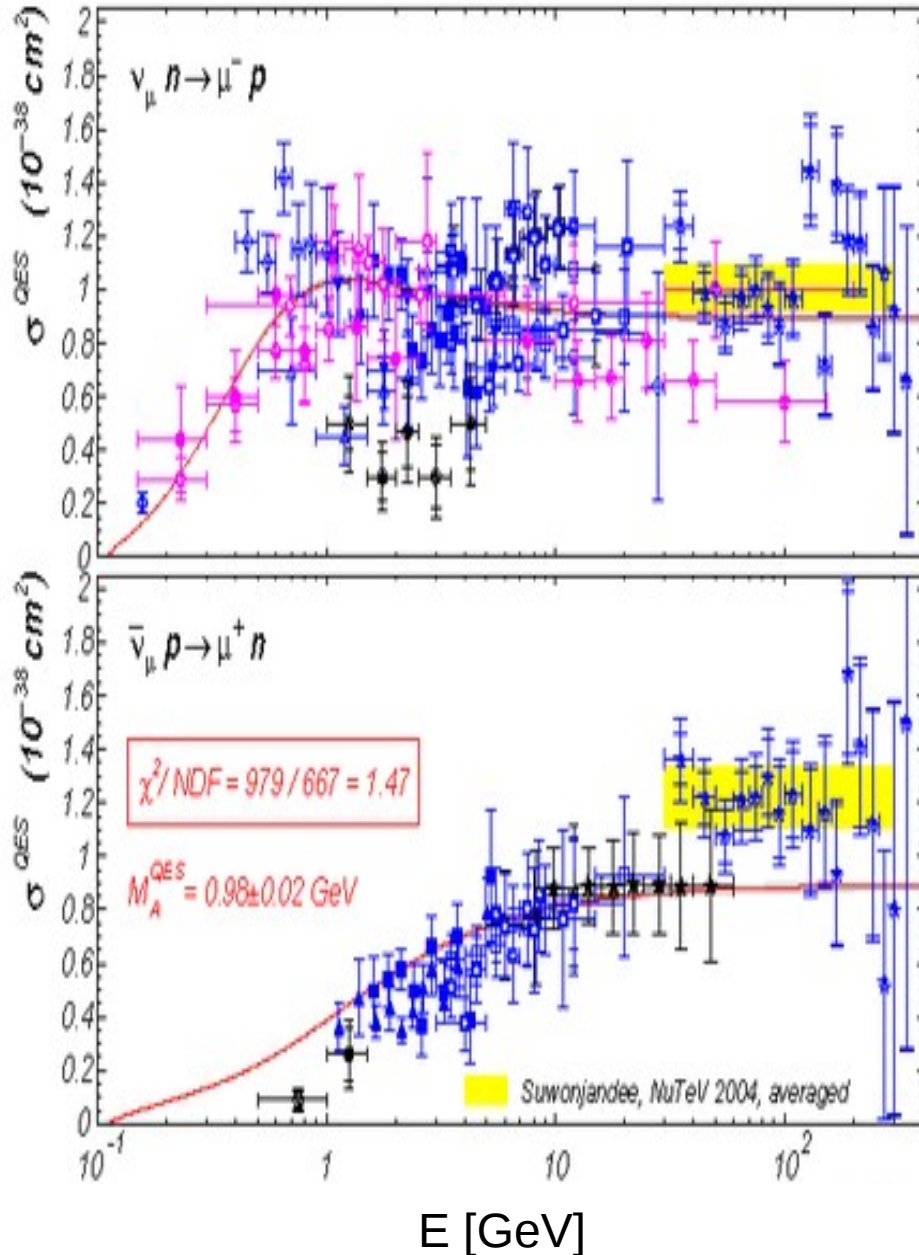
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- Many experimental data differ.
- Different targets, different ν -fluxes, different methods.

QES Cross Sections vs. Data

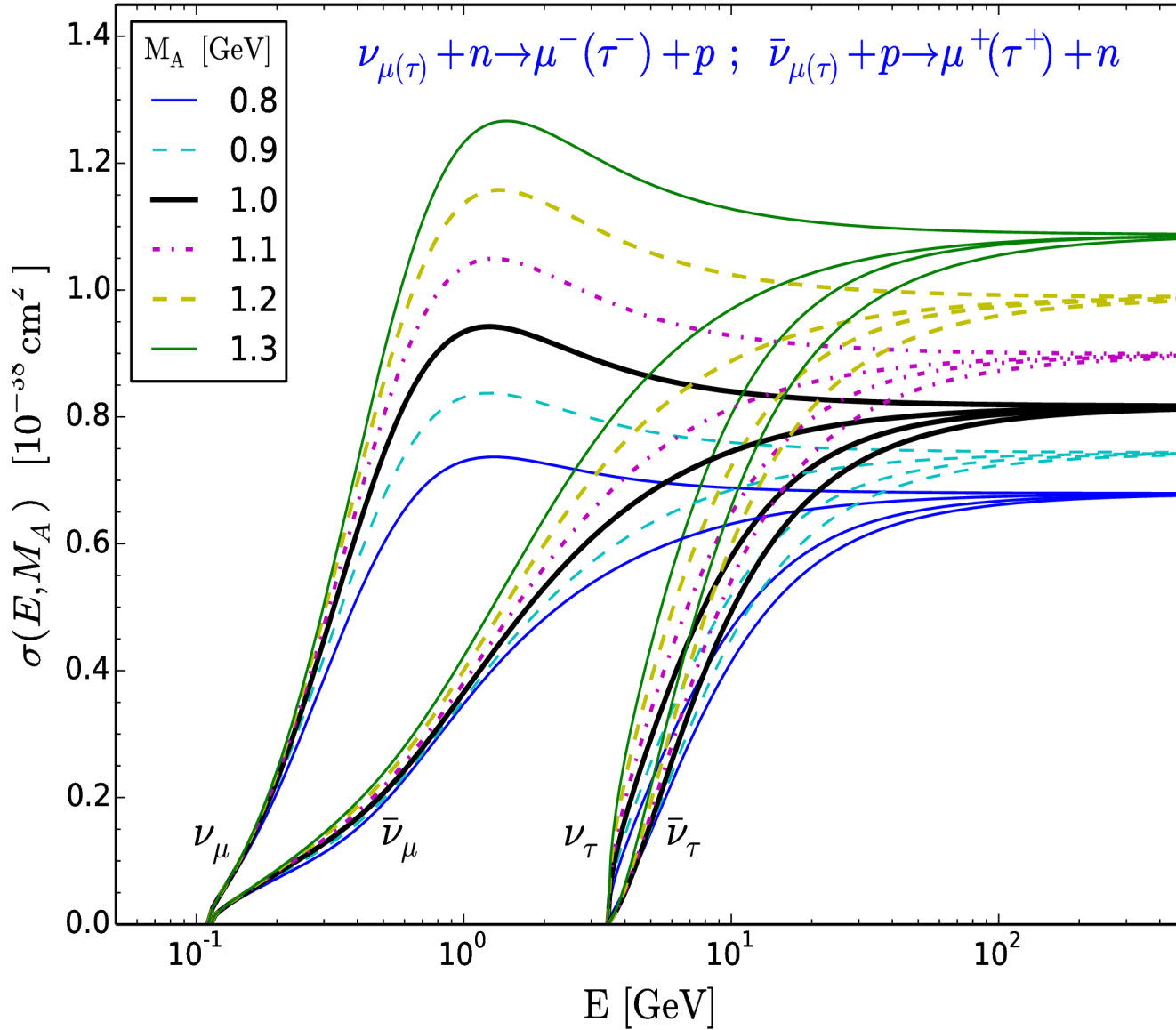


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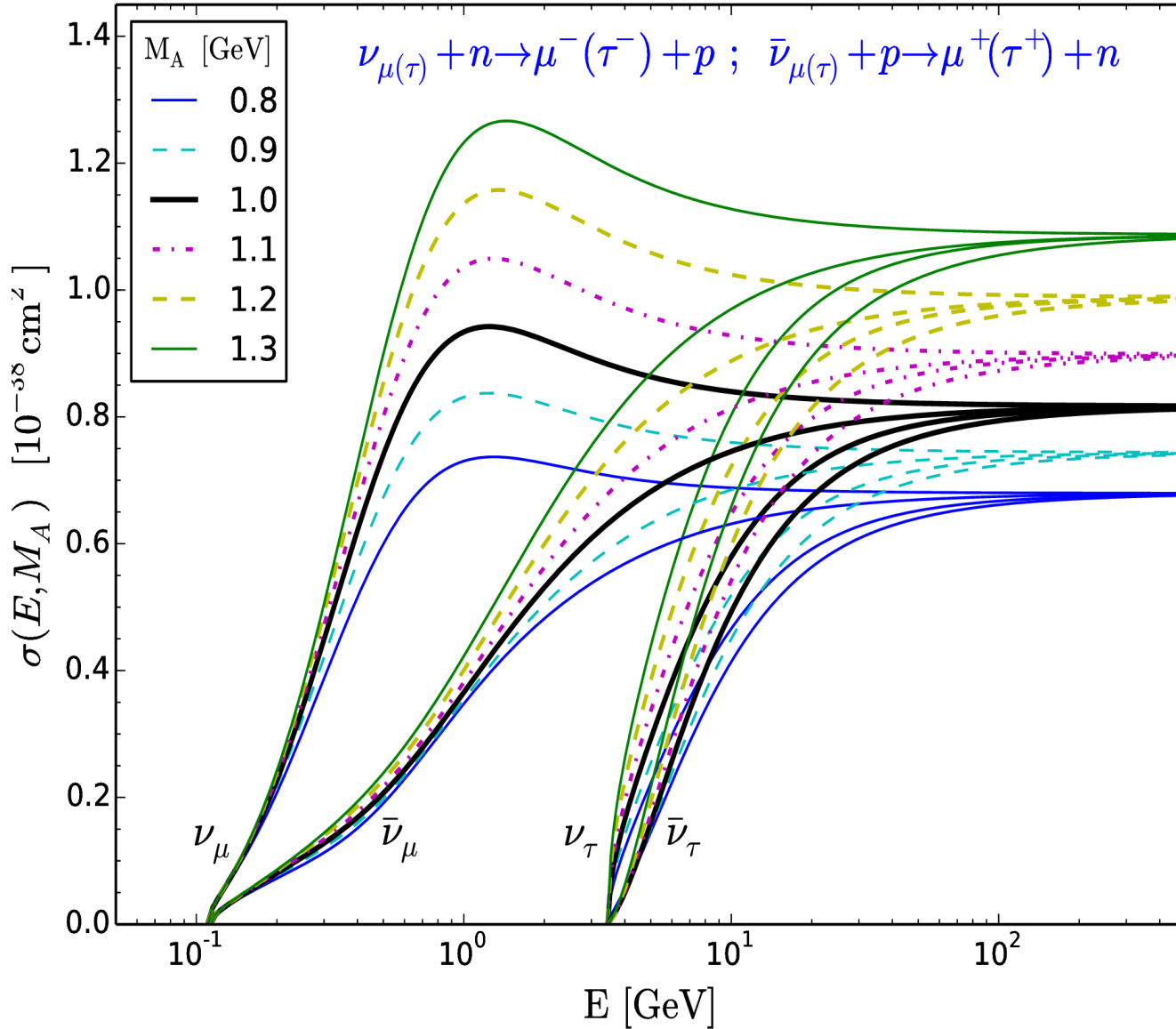
- Many experimental data differ.
- Different targets, different ν -fluxes, different methods.
- A global fit needs a careful choice of data to be included.

Sensitivity of QES to M_A

Sensitivity of QES to M_A

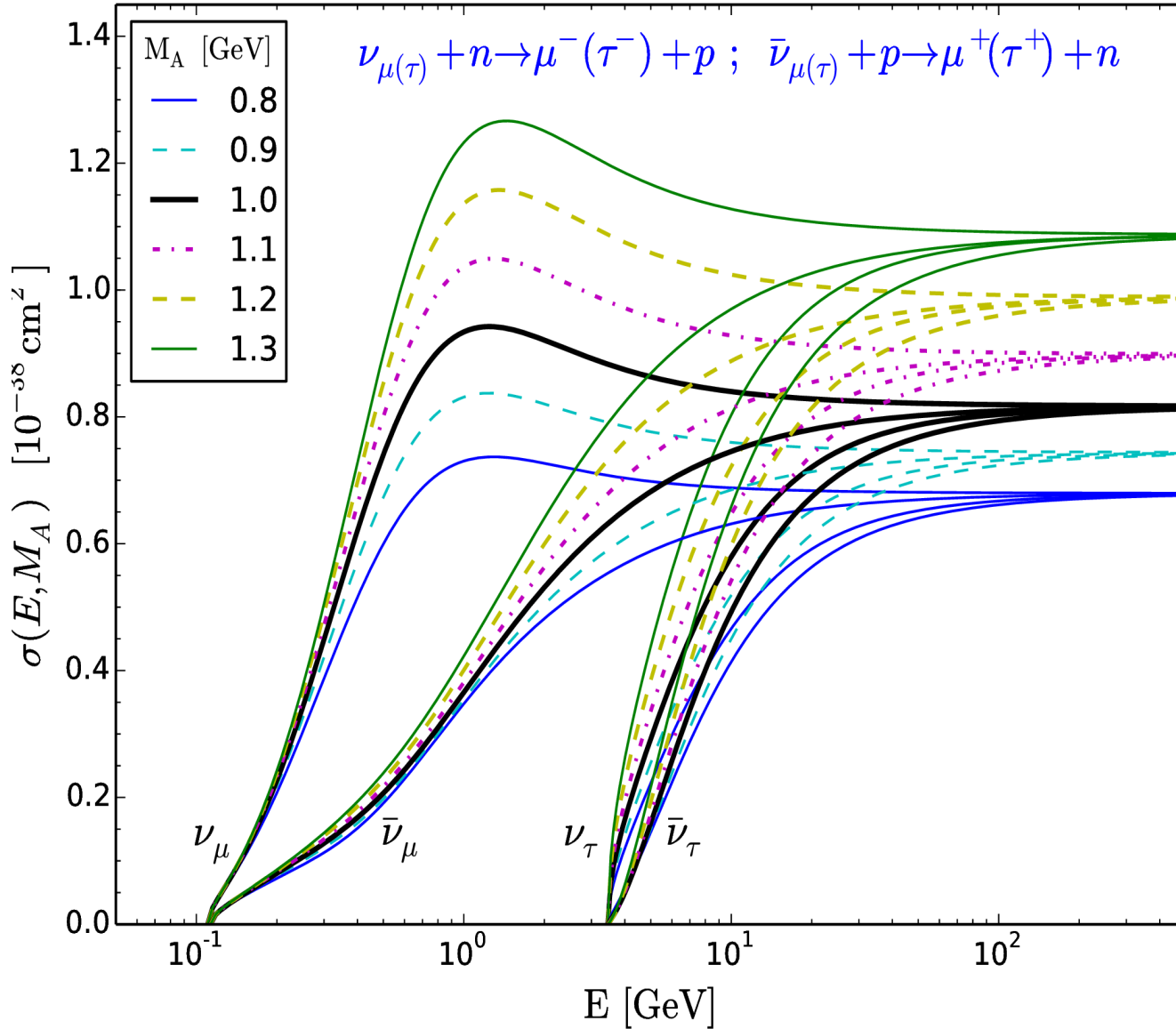


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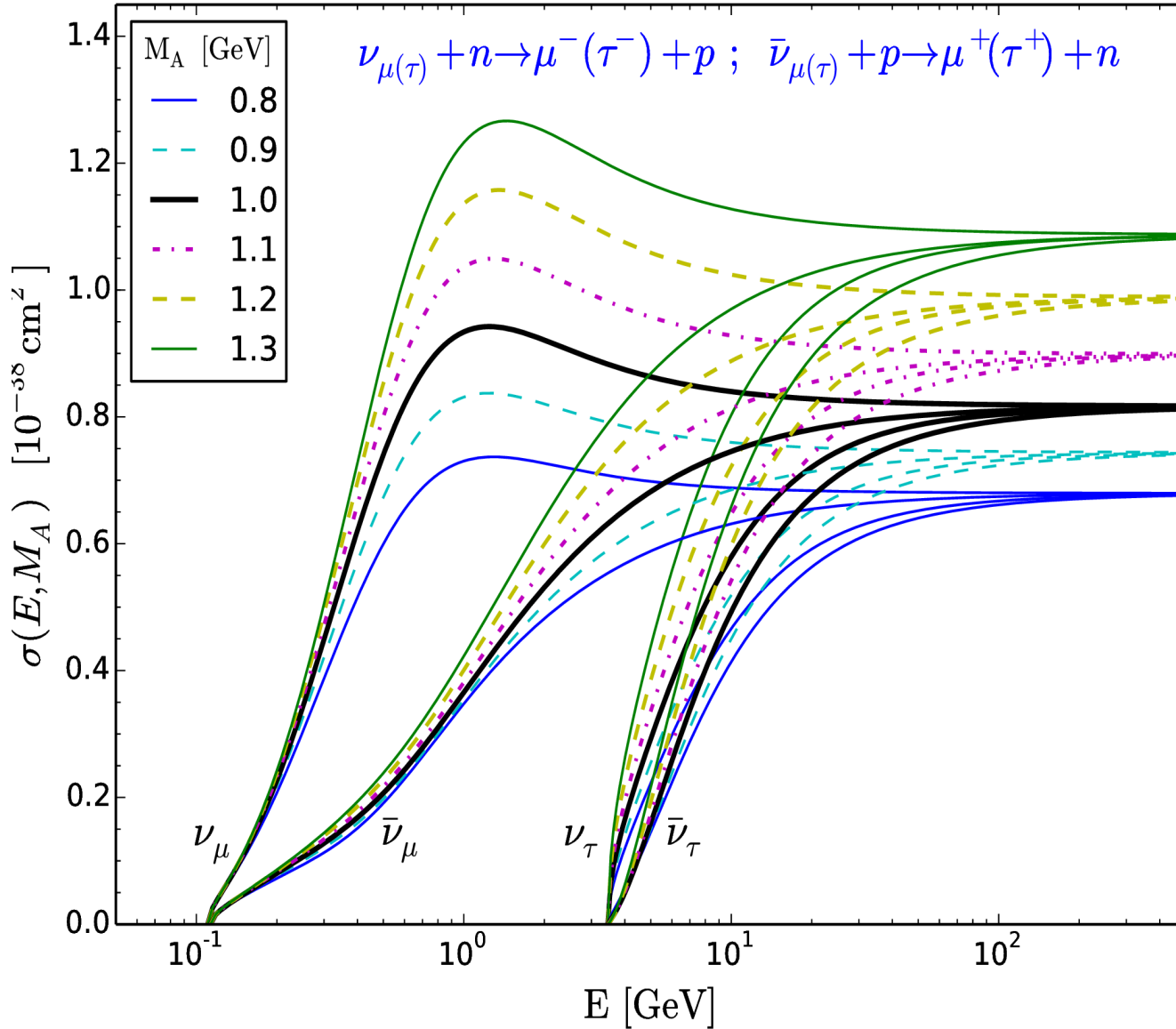
- QES cross-sections grow with axial mass M_A increasing.

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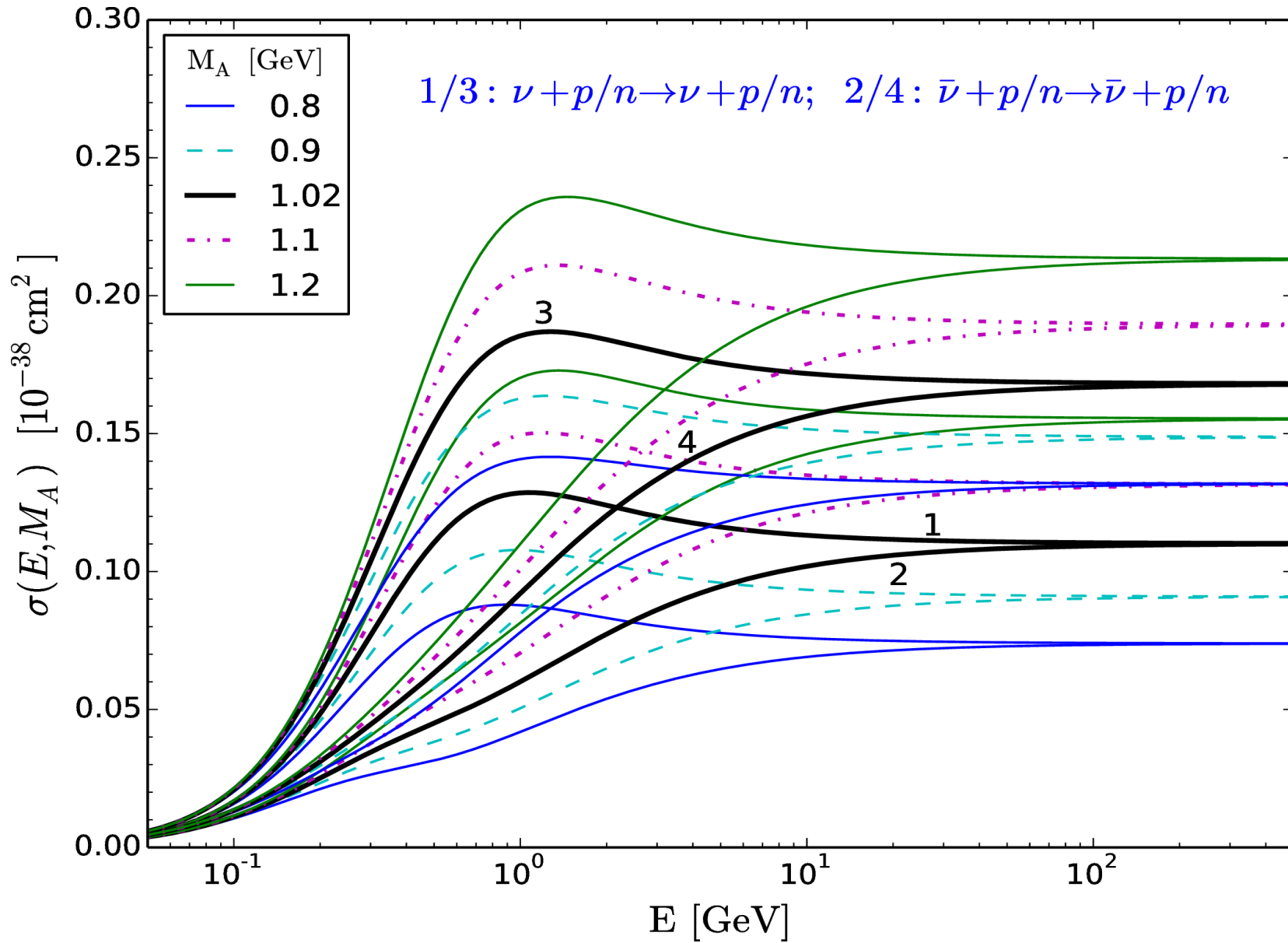
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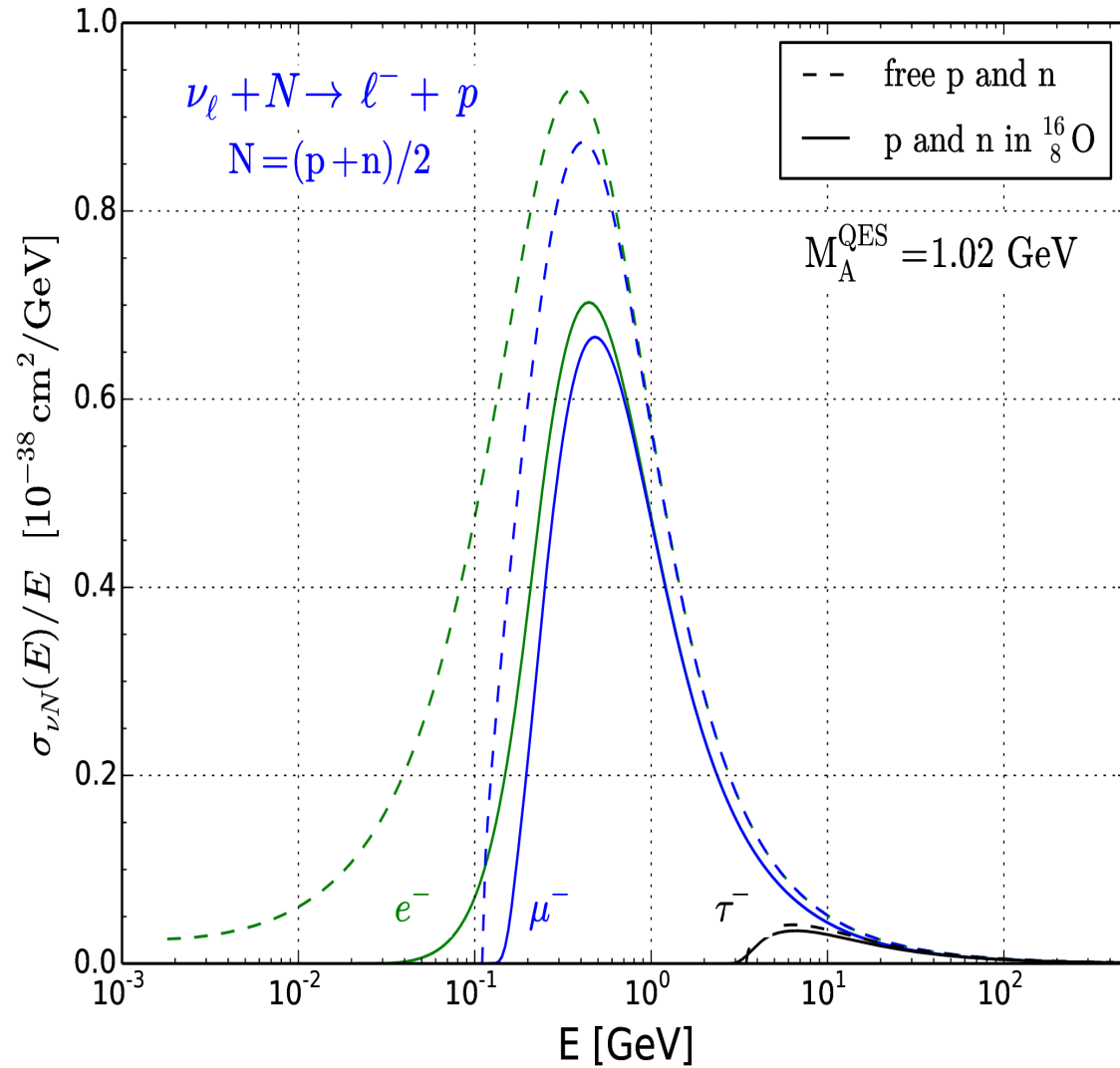
- QES cross-sections grow with axial mass M_A increasing.
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- At high energies QES cross-sections for ν and $\bar{\nu}$ coincide.

Sensitivity of ES to M_A

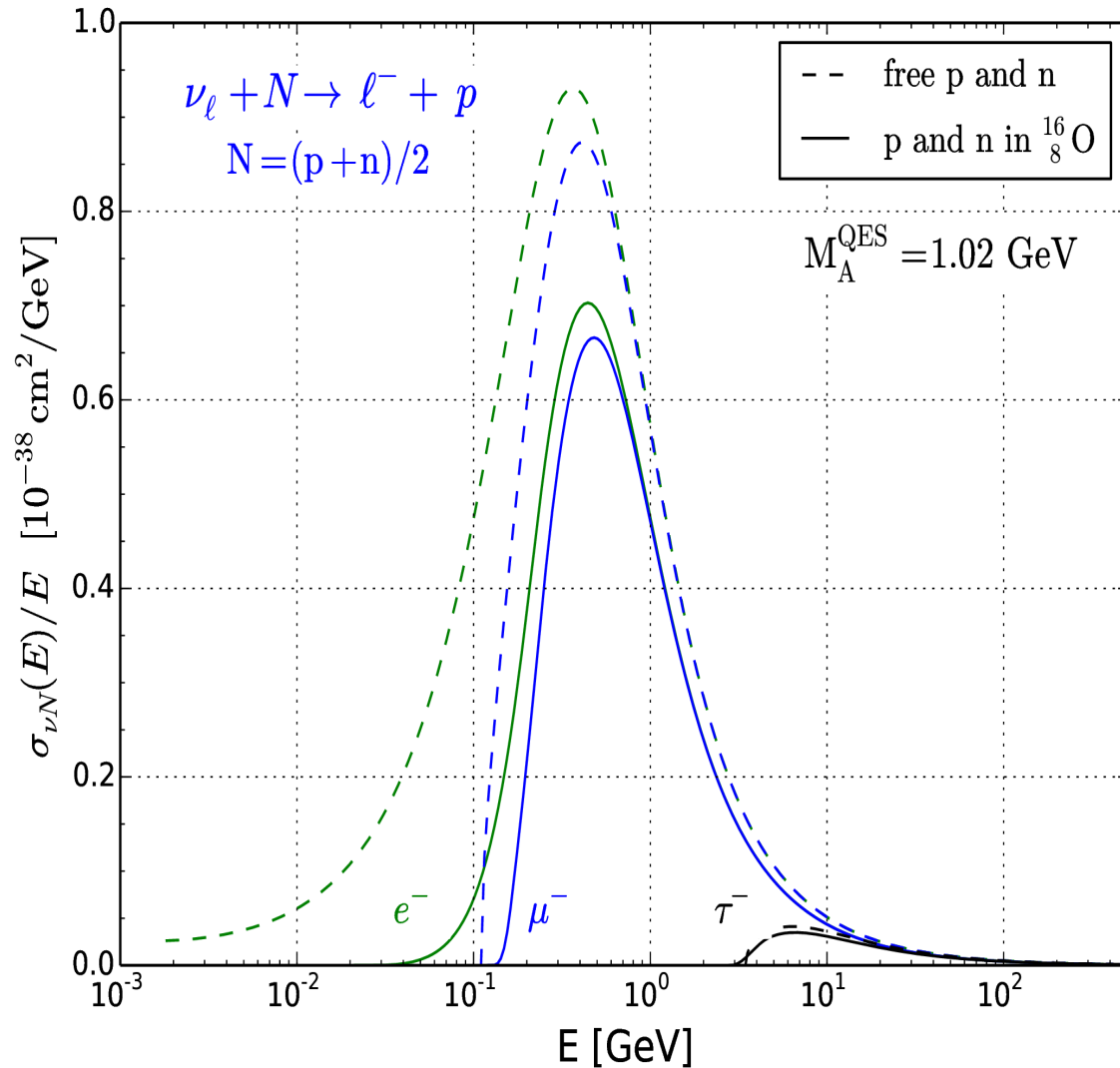


QES off Oxygen Target

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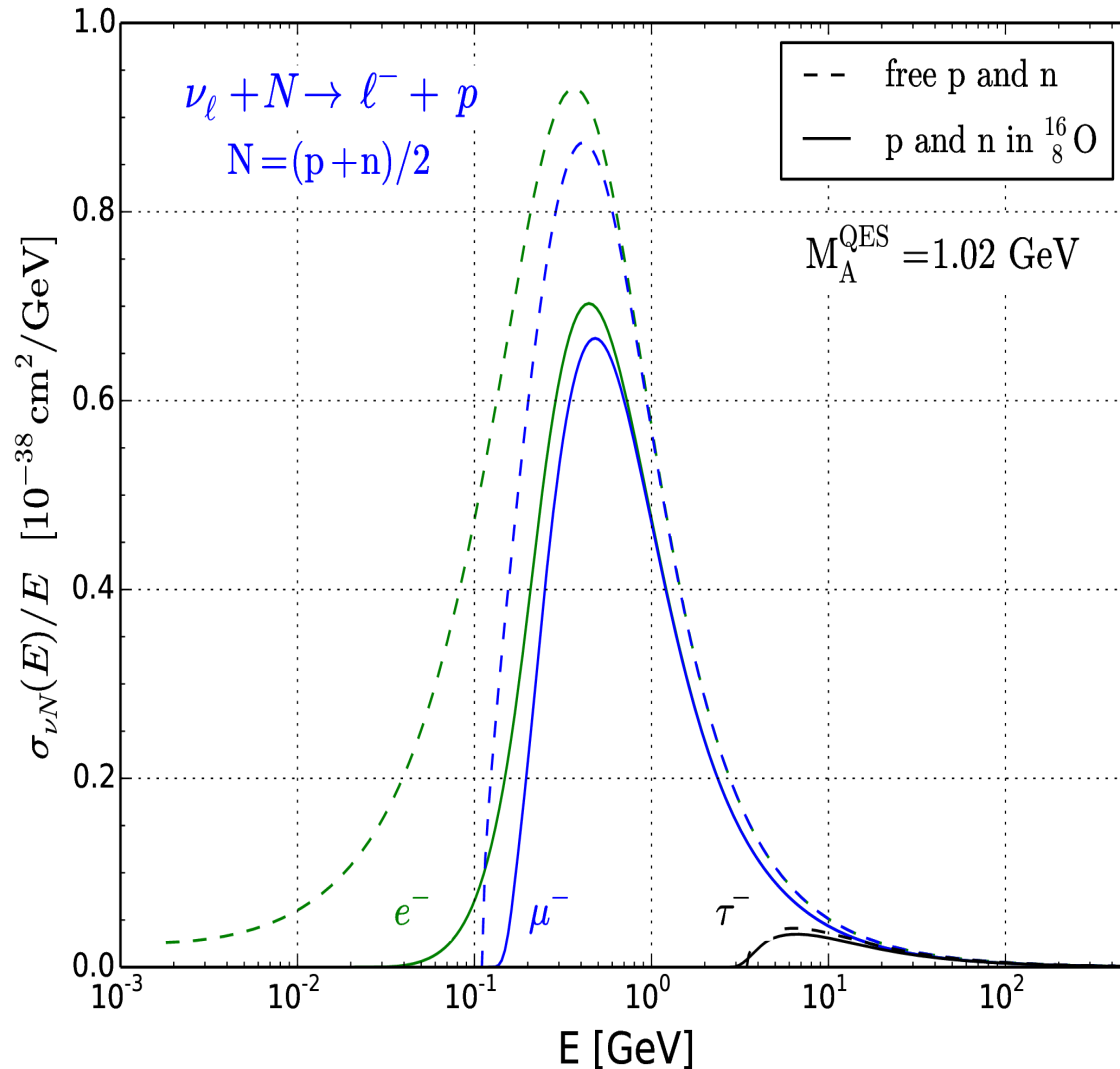


QES off Oxygen Target



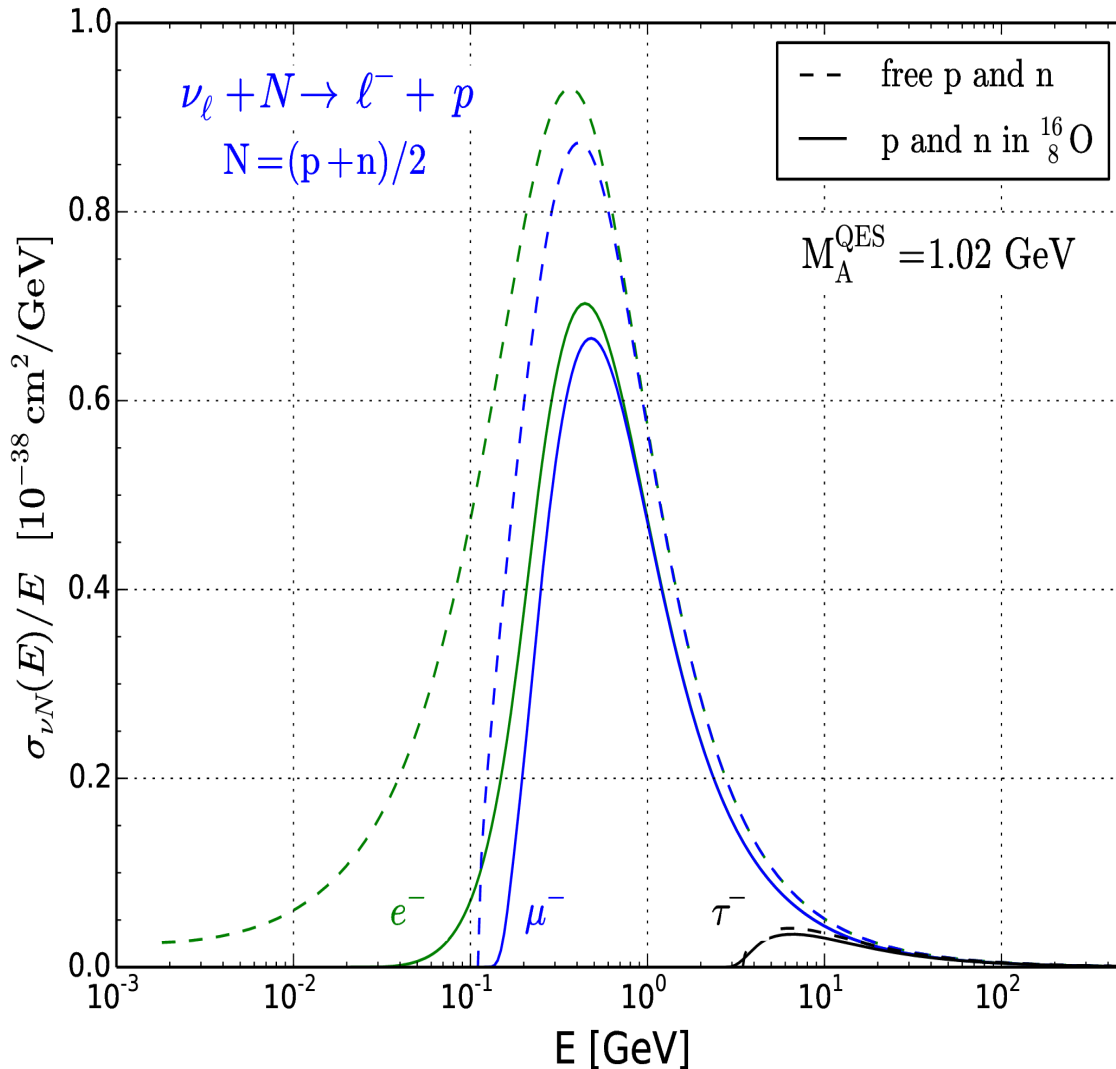
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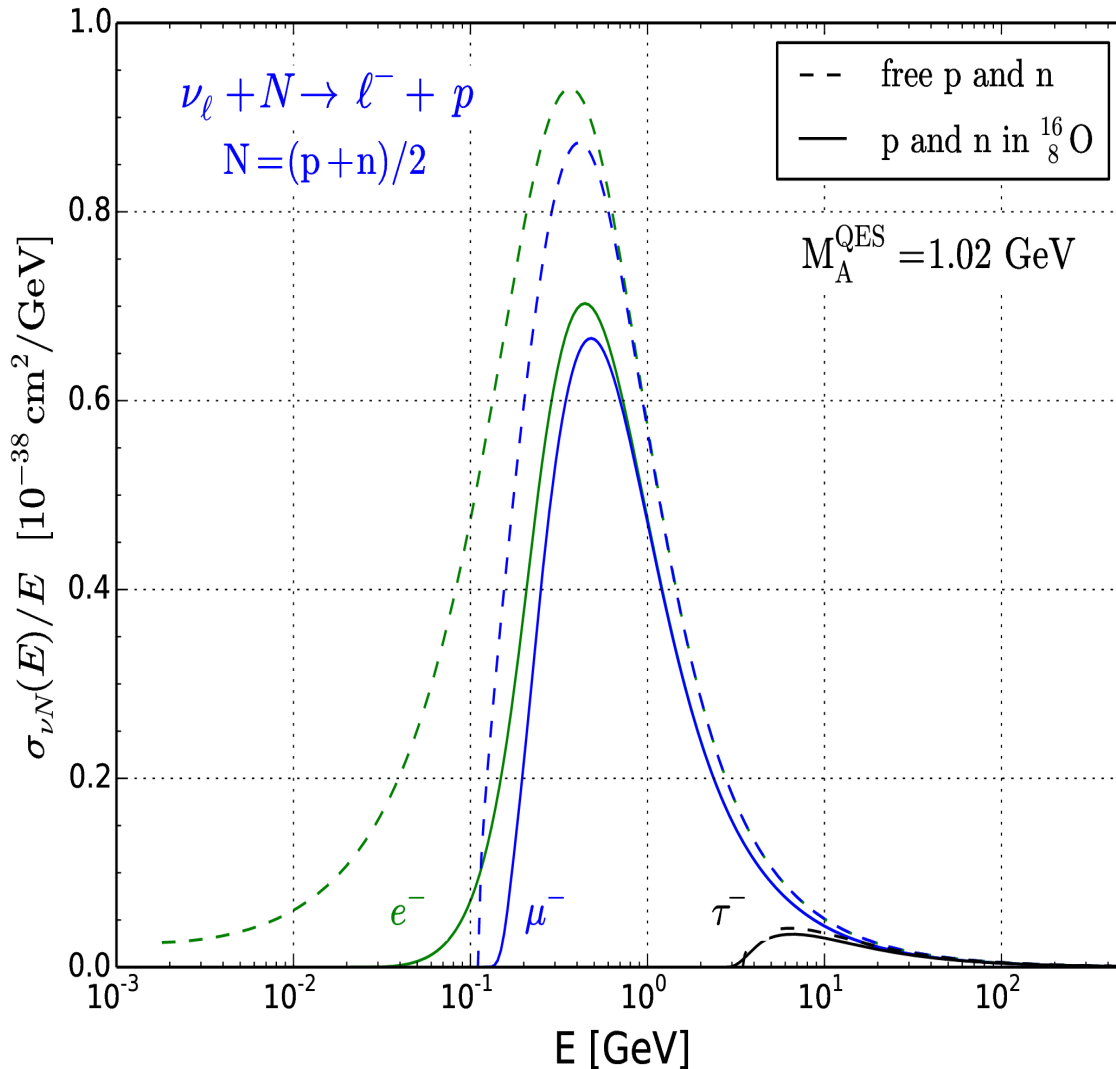
- At small energies nuclear corrections are important.
- Shown calculations using standard RFG model.

QES off Oxygen Target



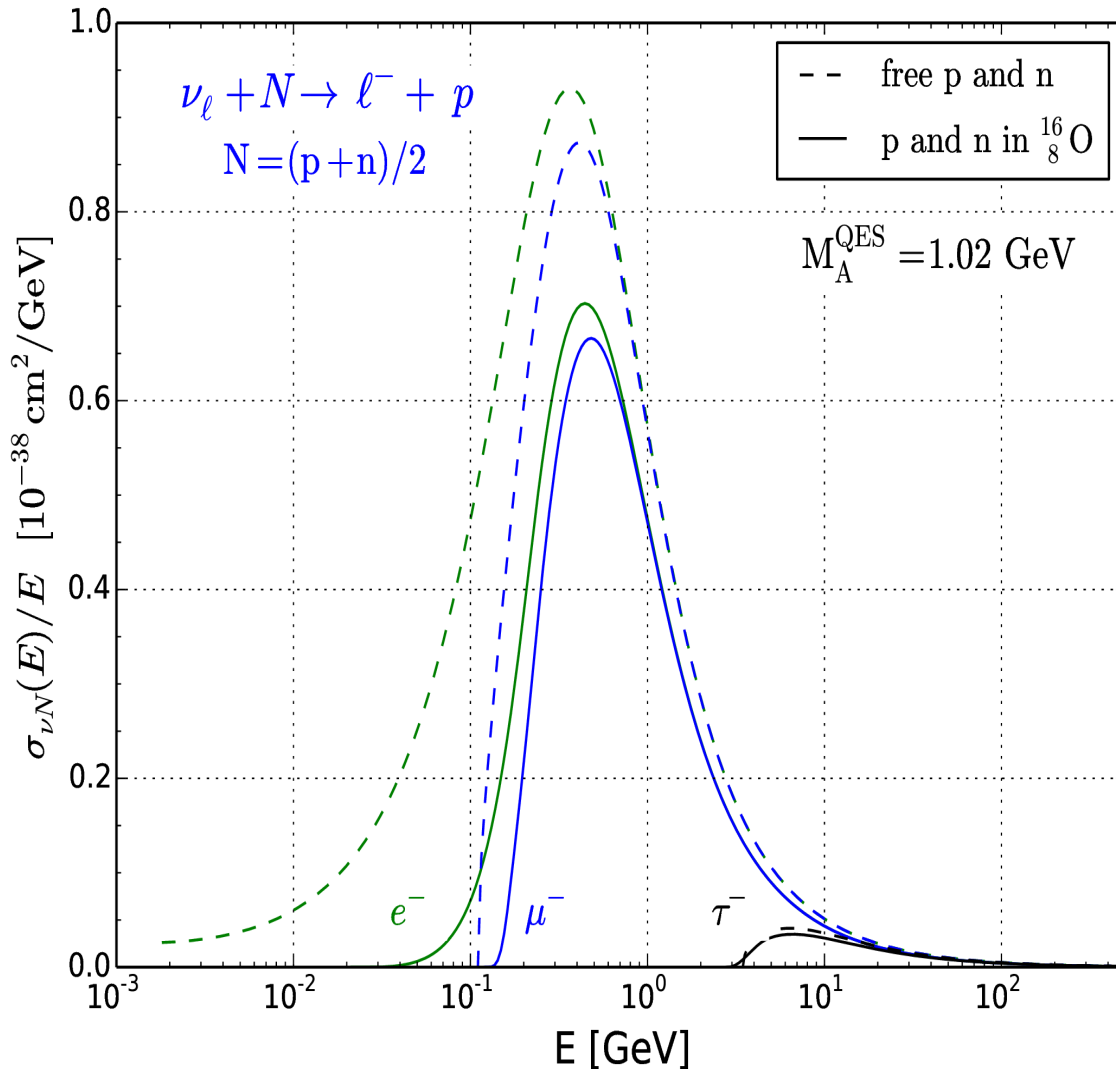
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- Note that such targets do not exist. But H_2O is close.

QES off Oxygen Target



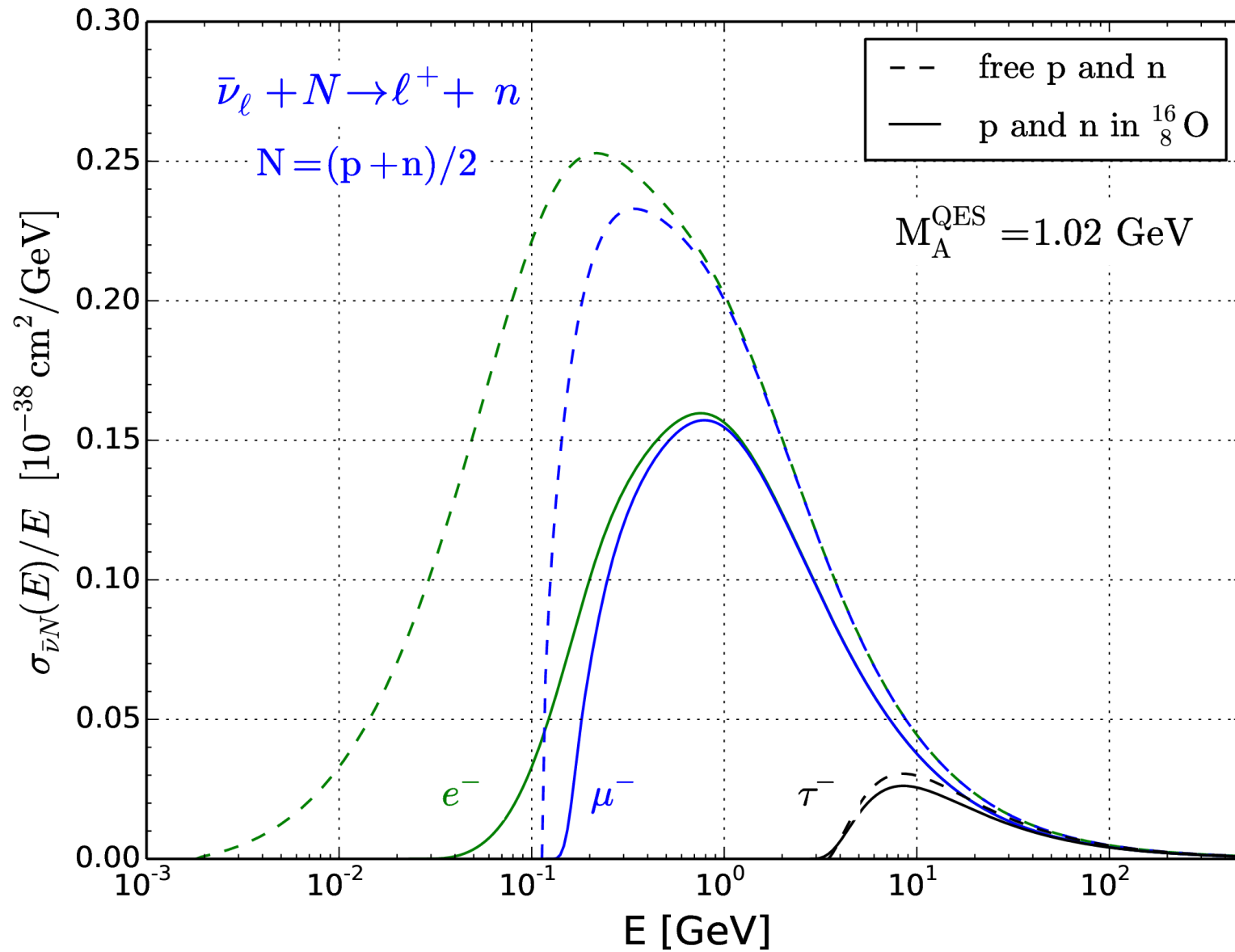
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- Shown calculations using standard RFG model.
- Note that such targets do not exist. But H_2O is close.
- The difference is just at small energies lower than interesting for PINGU.
- But for future HyperK lower energies are important.

ES off Oxygen Target



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- The **RS** model was extended (ExRS) to account for the finite mass of secondary leptons in the case of CC-scattering.

Nucleon Resonances with $M < 2 \text{ GeV}$

according to PDG-2014

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1	2	3	4	5	6	7
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D_{13}	$N(1520)$	$[70, 1^-]_1$	1510–1520 (1515)	100–125 (120)	55–65 (60.0)	–
S_{11}	$N(1535)$	$[70, 1^-]_1$	1525–1545 (1535)	125–175 (150)	35–55 (45.0)	–
S_{11}	$N(1650)$	$[70, 1^-]_1$	1645–1670 (1655)	110–170 (140)	50–90 (70.0)	+
D_{15}	$N(1675)$	$[70, 1^-]_1$	1670–1680 (1675)	130–165 (150)	35–45 (40.0)	+
F_{15}	$N(1680)$	$[56, 2^+]_2$	1680–1690 (1685)	120–140 (130)	65–70 (67.5)	+
D_{13}	$N(1700)$	$[70, 1^-]_1$	1650–1750 (1700)	100–250 (150)	7–17 (12.0)	–
P_{11}	$N(1710)$	$[70, 0^+]_2$	1680–1740 (1710)	50–250 (100)	5–20 (12.5)	+
P_{13}	$N(1720)$	$[56, 2^+]_2$	1700–1750 (1720)	150–400 (250)	8–14 (11.0)	+
F_{17}	$N(2190)$	$[70, 2^+]_2$	2100–2200 (2190)	300–700 (500)	10–20 (15.0)	+
P_{33}	$\Delta(1232)$	$[56, 0^+]_0$	1230–1234 (1232)	114–120 (117)	99.4	+
P_{33}	$\Delta(1600)$	$[56, 0^+]_2$	1500–1700 (1600)	220–420 (320)	10–25 (17.5)	+
S_{31}	$\Delta(1620)$	$[70, 1^-]_1$	1600–1660 (1630)	130–150 (140)	20–30 (25.0)	+
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P_{33}	$\Delta(1920)$	$[56, 2^+]_2$	1900–1970 (1920)	180–300 (260)	5–20 (12.5)	+
F_{37}	$\Delta(1950)$	$[56, 2^+]_2$	1915–1950 (1930)	235–335 (285)	35–45 (40.0)	+

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Nucleon Resonances with $M < 2 \text{ GeV}$

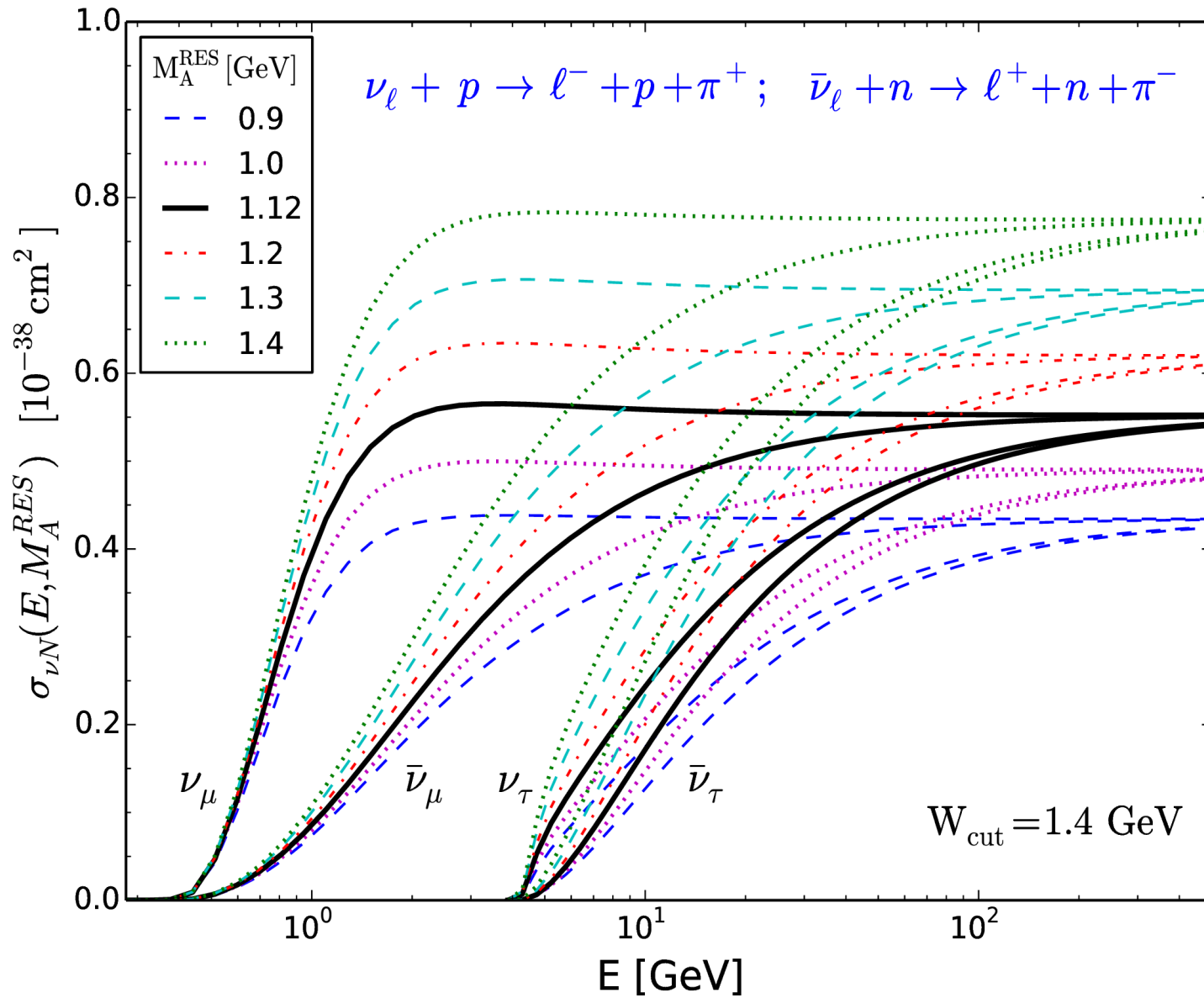
according to PDG-2014

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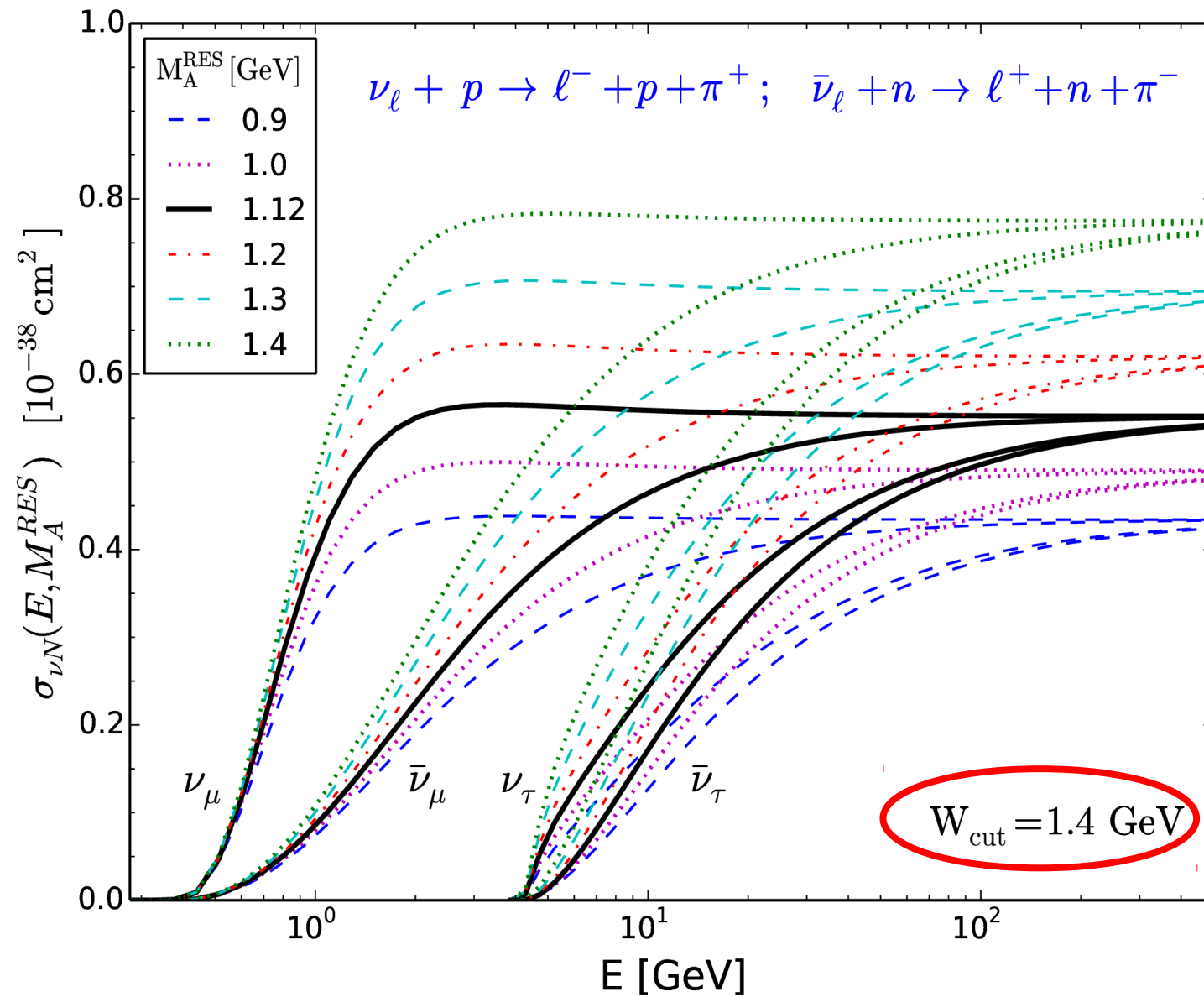
Single Pion Production (RES), cont.

- Besides resonances, a non-resonance background is to be added (general problem!).
- Relative normalization of contributions from different resonances is a problem, though $\Delta(1232)$ dominates.
- To avoid a double counting with DIS, the maximum mass of the bound πN state is to be defined – the W_{cut} .
- We find $W_{\text{cut}} = 1.4 \text{ GeV}$ to be the best fit in our approach.
- It's choice is a general problem as well. Experiments also present data for different W_{cut} from 1.1 GeV up to 2 GeV.
- It will be discussed later.

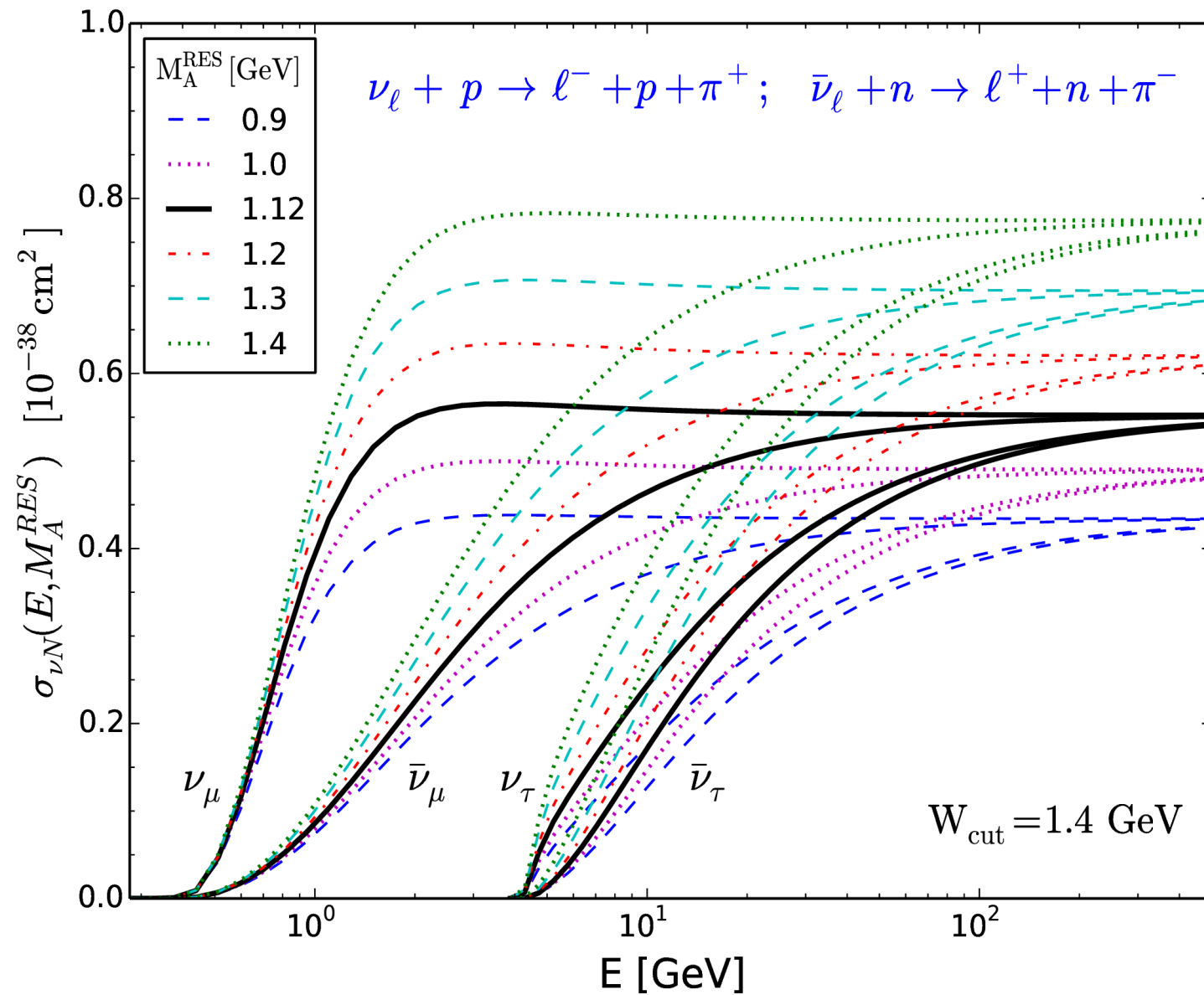
Sensitivity of RES to M_A (CC 1)



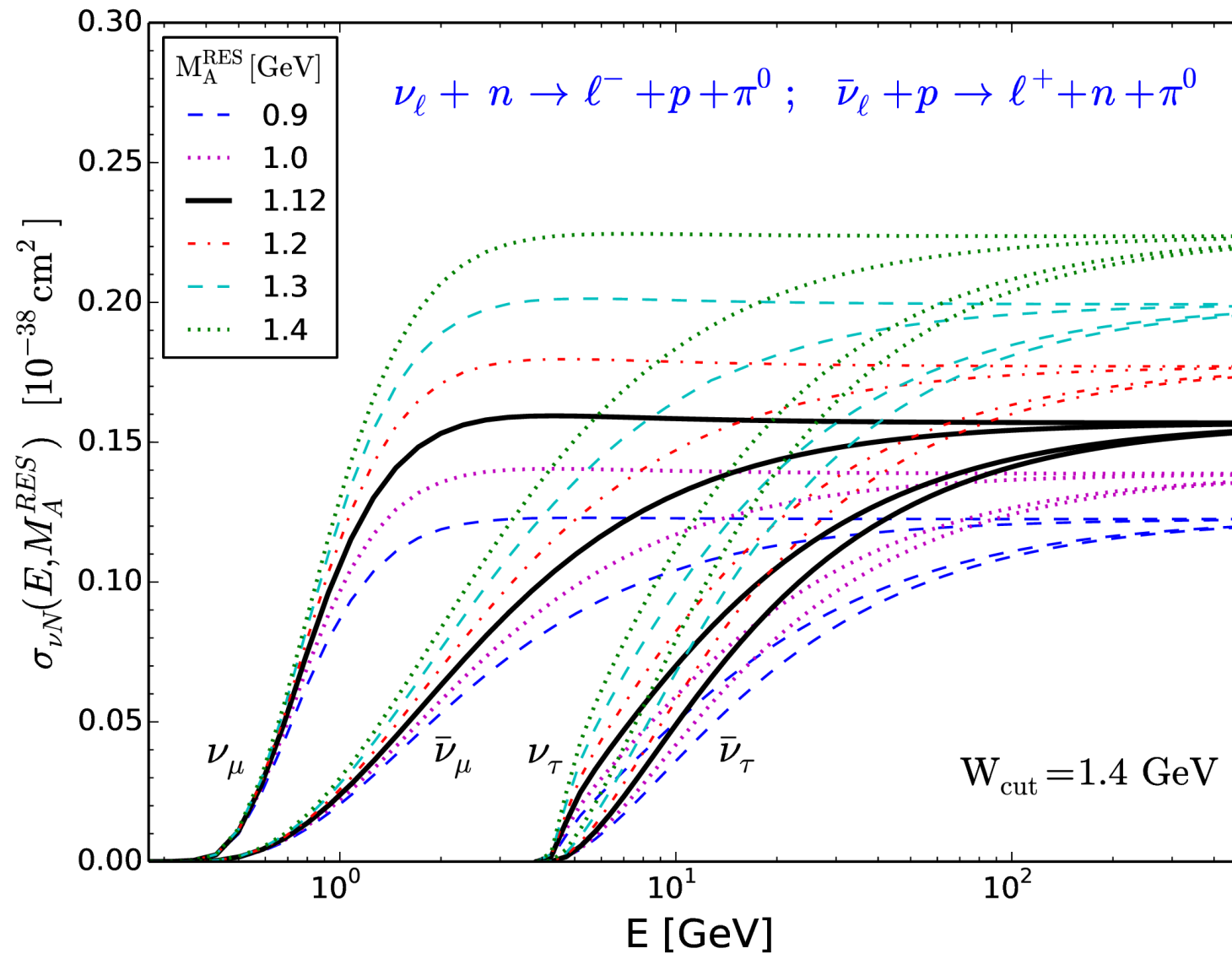
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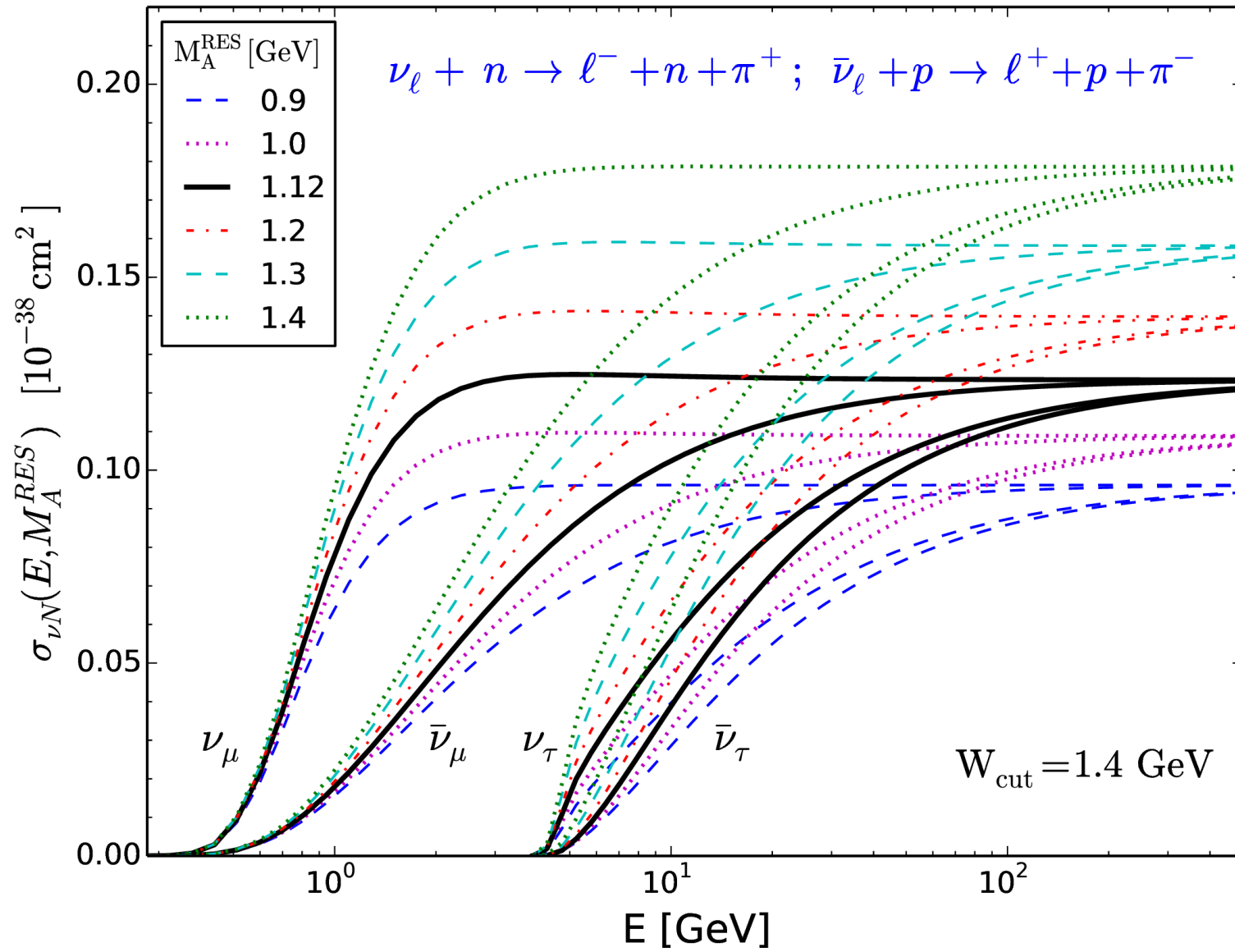
Sensitivity of RES to M_A (CC 1)



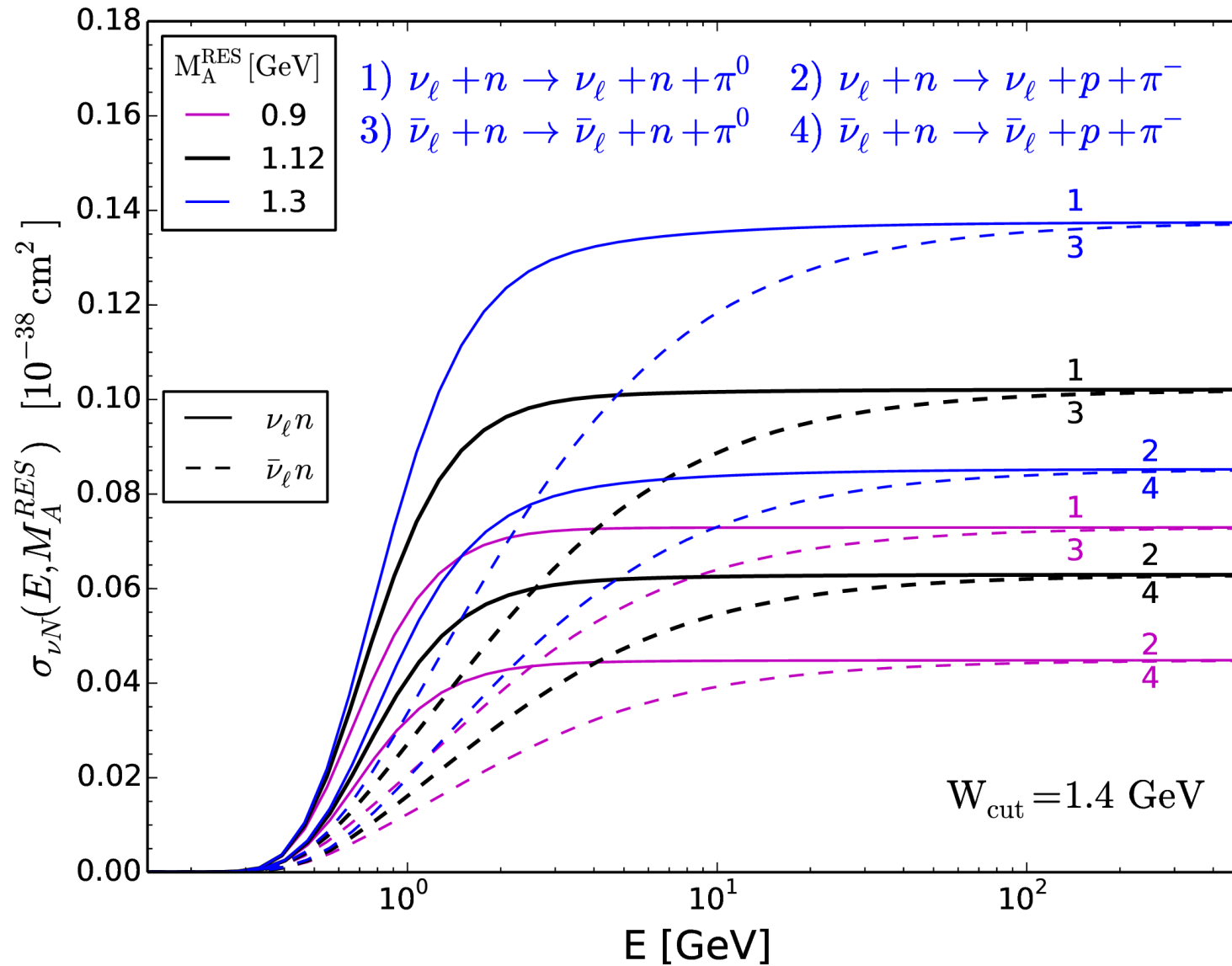
Sensitivity of RES to M_A (CC 2)



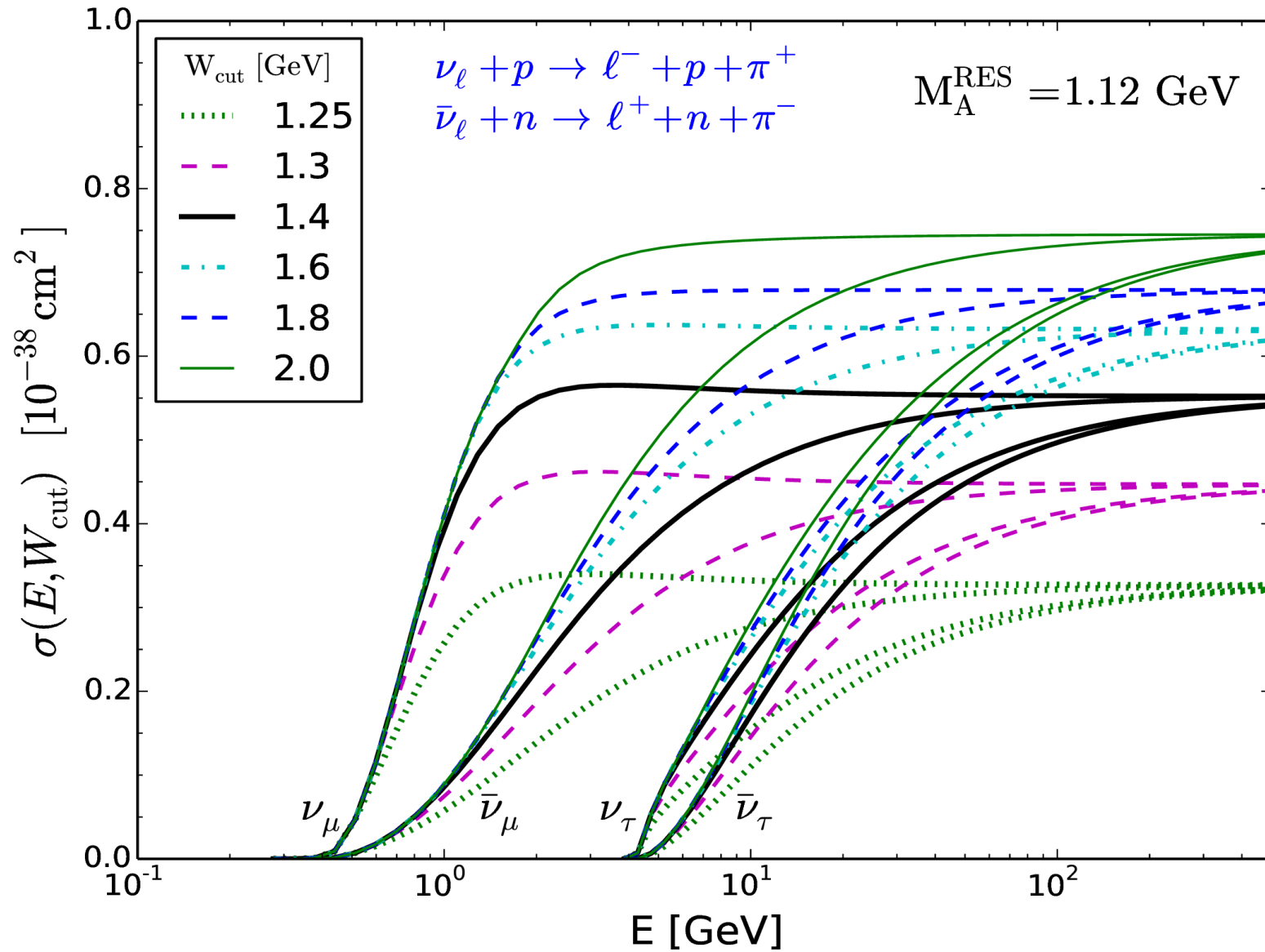
Sensitivity of RES to M_A (CC 3)



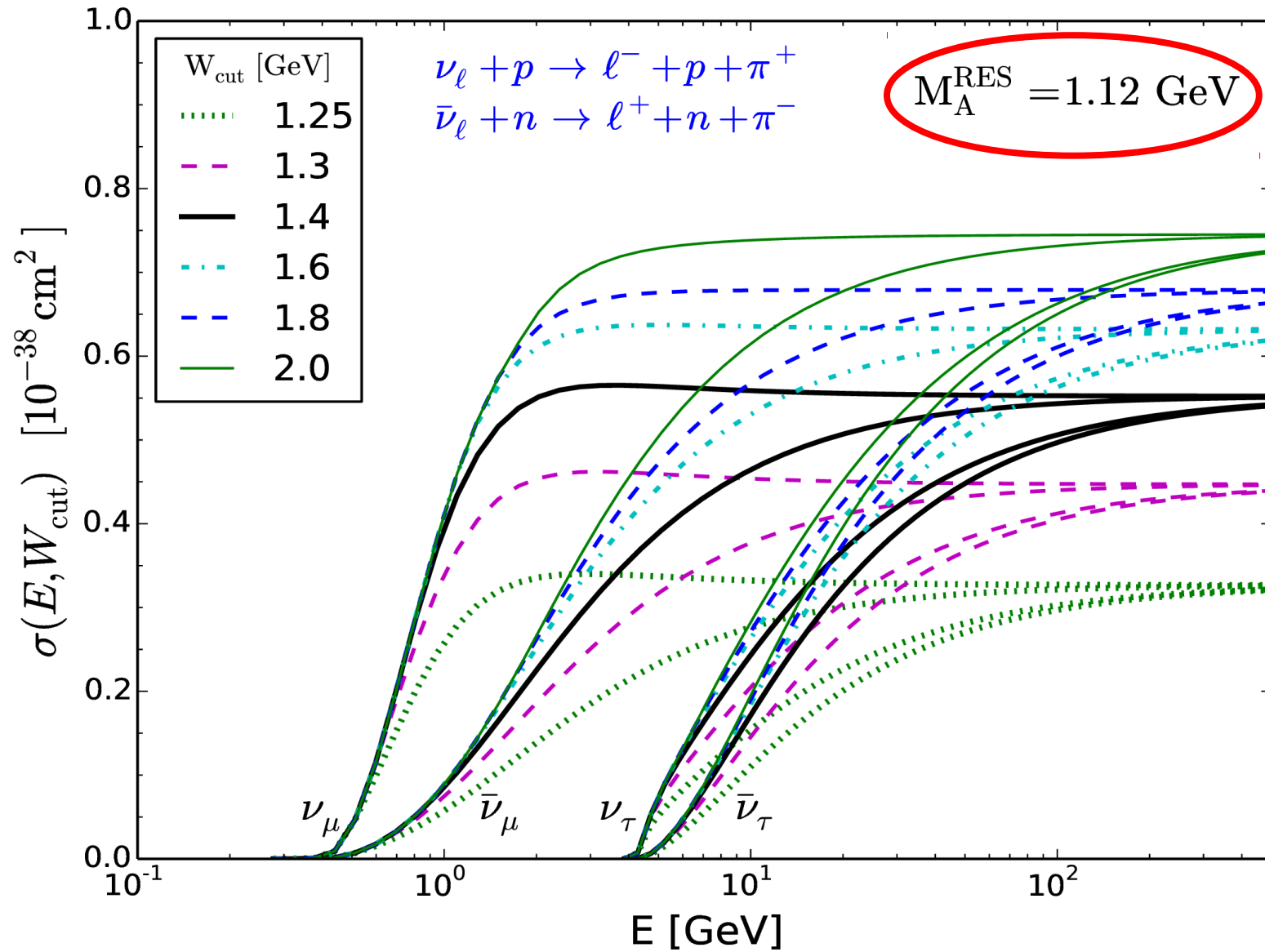
Sensitivity of RES to M_A (NC)



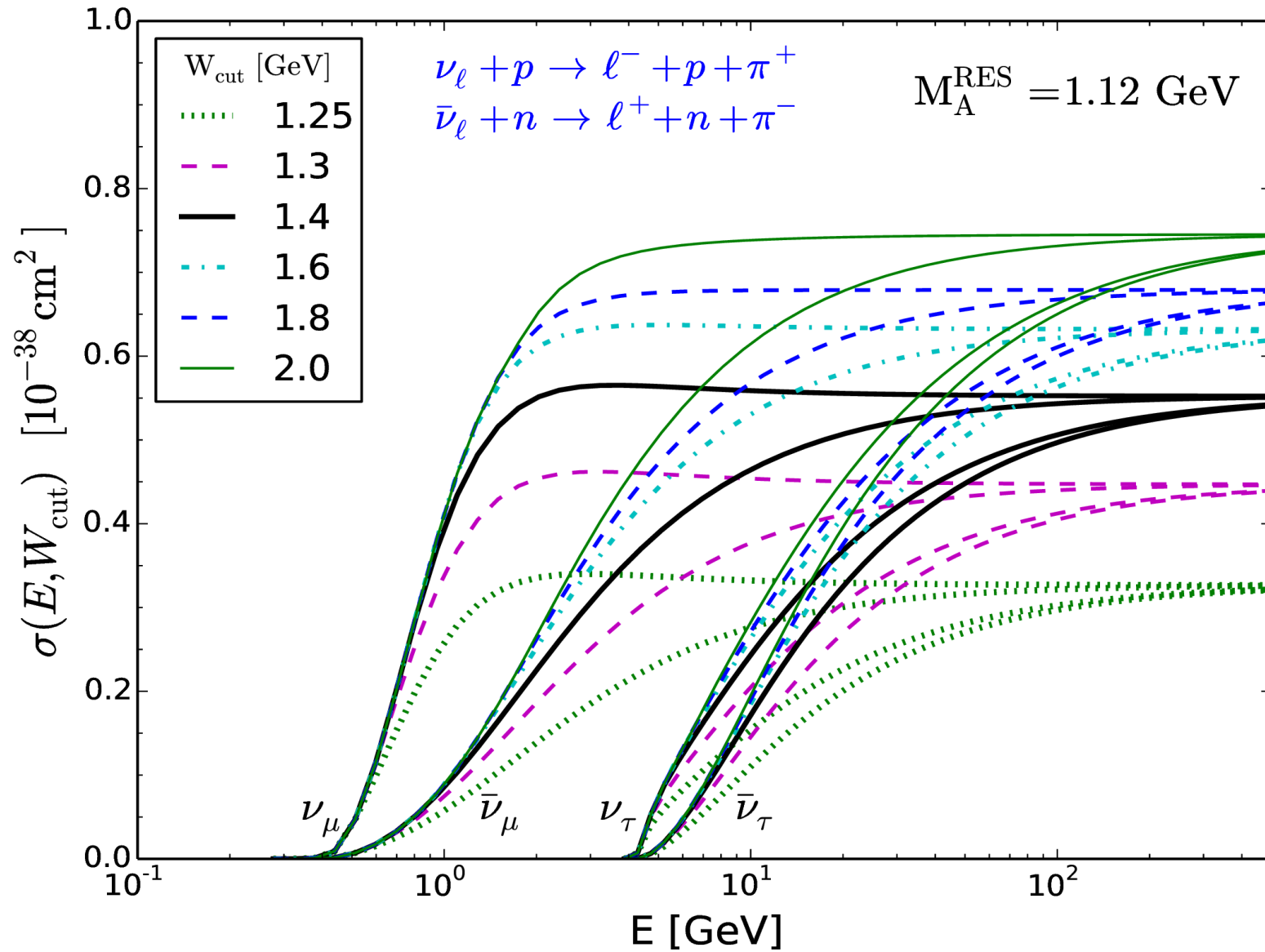
Sensitivity of CC RES to W_{cut}



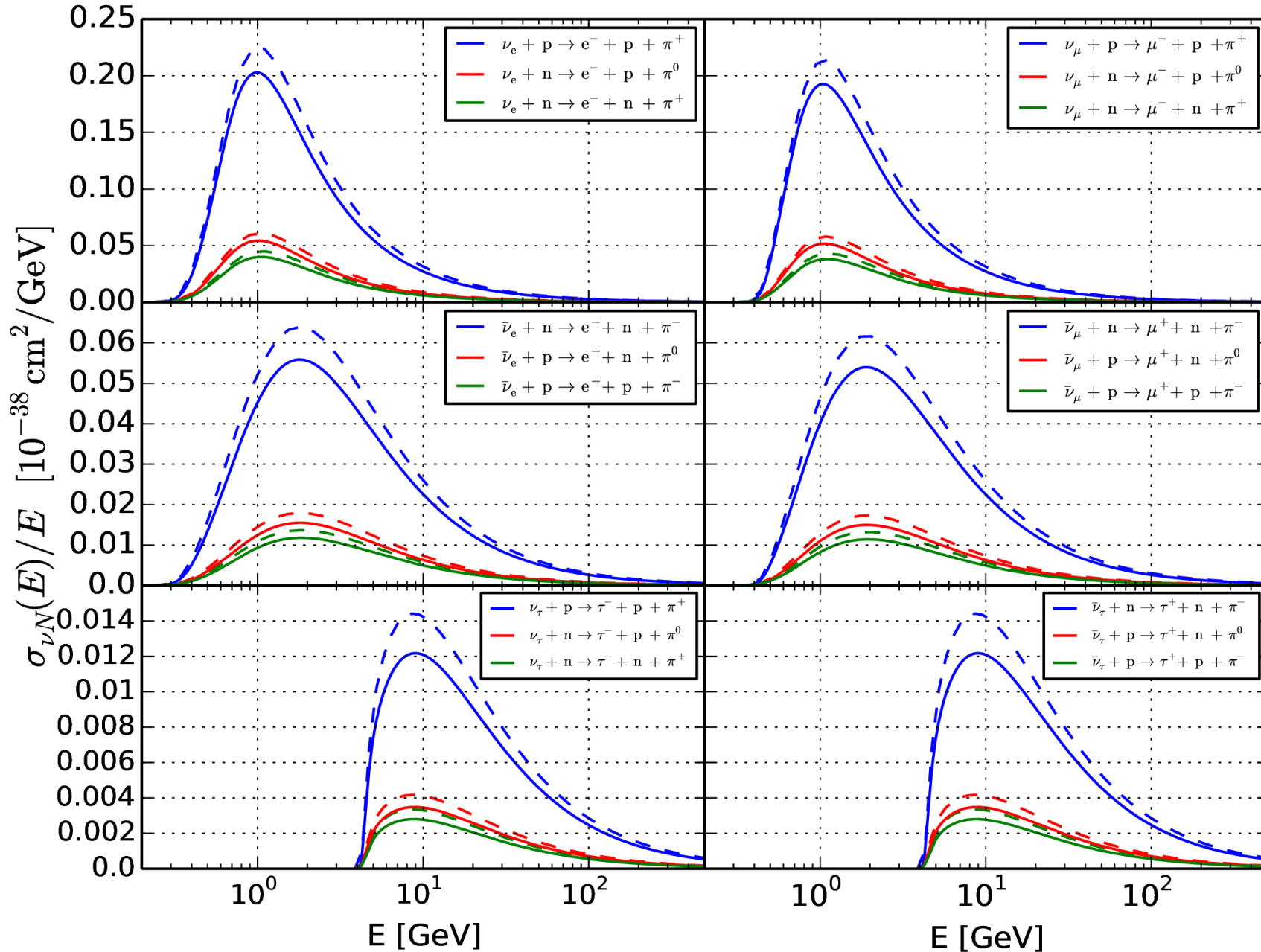
Sensitivity of CC RES to W_{cut}



Sensitivity of CC RES to W_{cut}



RES off Oxygen Target



Deep Inelastic Scattering

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- **DIS** is the sum of cross-sections on all quarks in a nucleon.

QCD and Structure Functions

- At LO of perturbative QCD nucleon consists of valence quarks, “sea”-quarks and gluons.
- Sea quarks are $q\bar{q}$ -pairs. They arise from qg - and gg -interactions.
- For unpolarized nucleons – 5 Sfs.

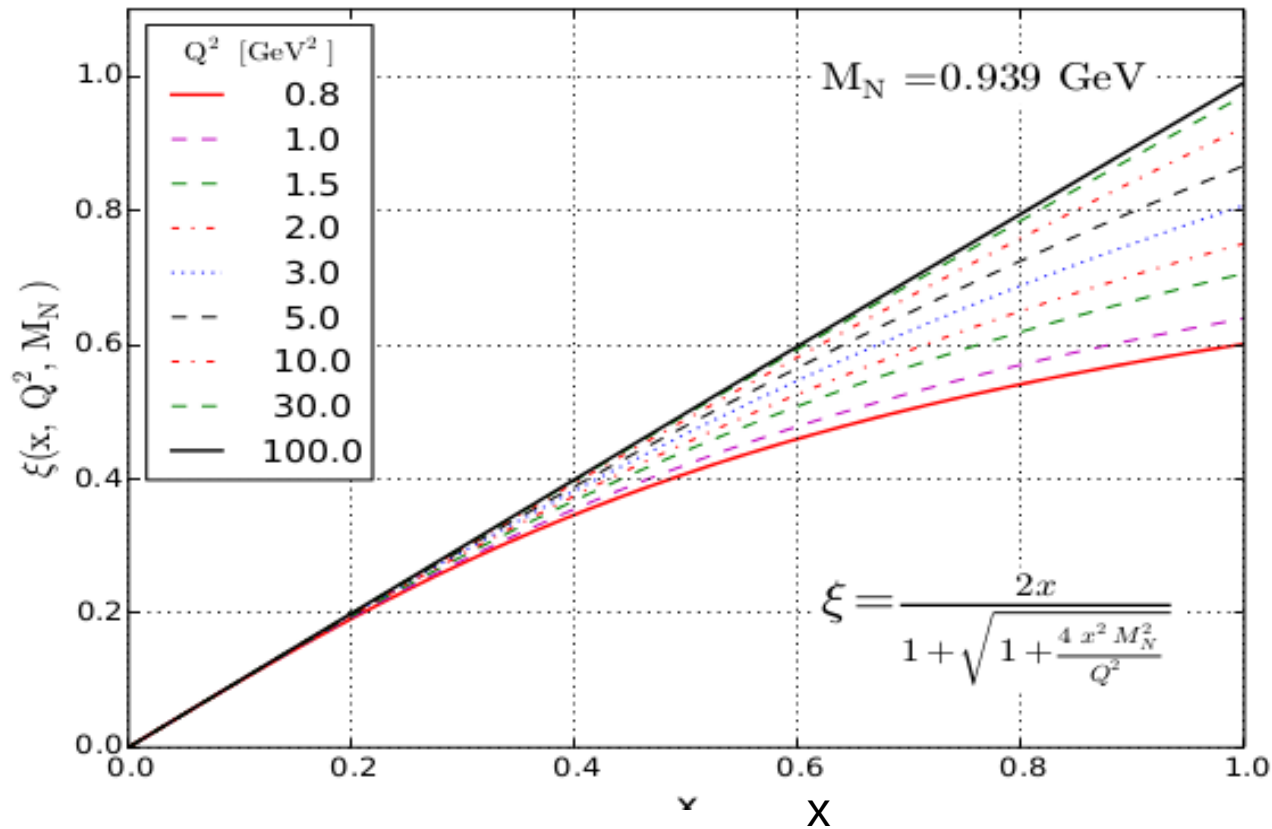
$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx dy}(E_\nu, x, y) = \frac{G_F^2 M_N E_\nu}{\pi(1+Q^2/M_W^2)^2} \times \sum_{i=1}^5 a_i(E_\nu, x, y, M_N, m_\ell) F_i^{TMC}(x, Q^2),$$

$$a_1 = y^2 x + \frac{m_\ell^2 y}{2E_\nu M_N}, \quad a_2 = 1 - \frac{m_\ell^2}{4E_\nu^2} - \left(1 + \frac{M_N x}{2E_\nu}\right) y,$$

$$a_3 = \pm \left[xy \left(1 - \frac{y}{2}\right) - \frac{m_\ell^2 y}{4E_\nu M_N} \right],$$

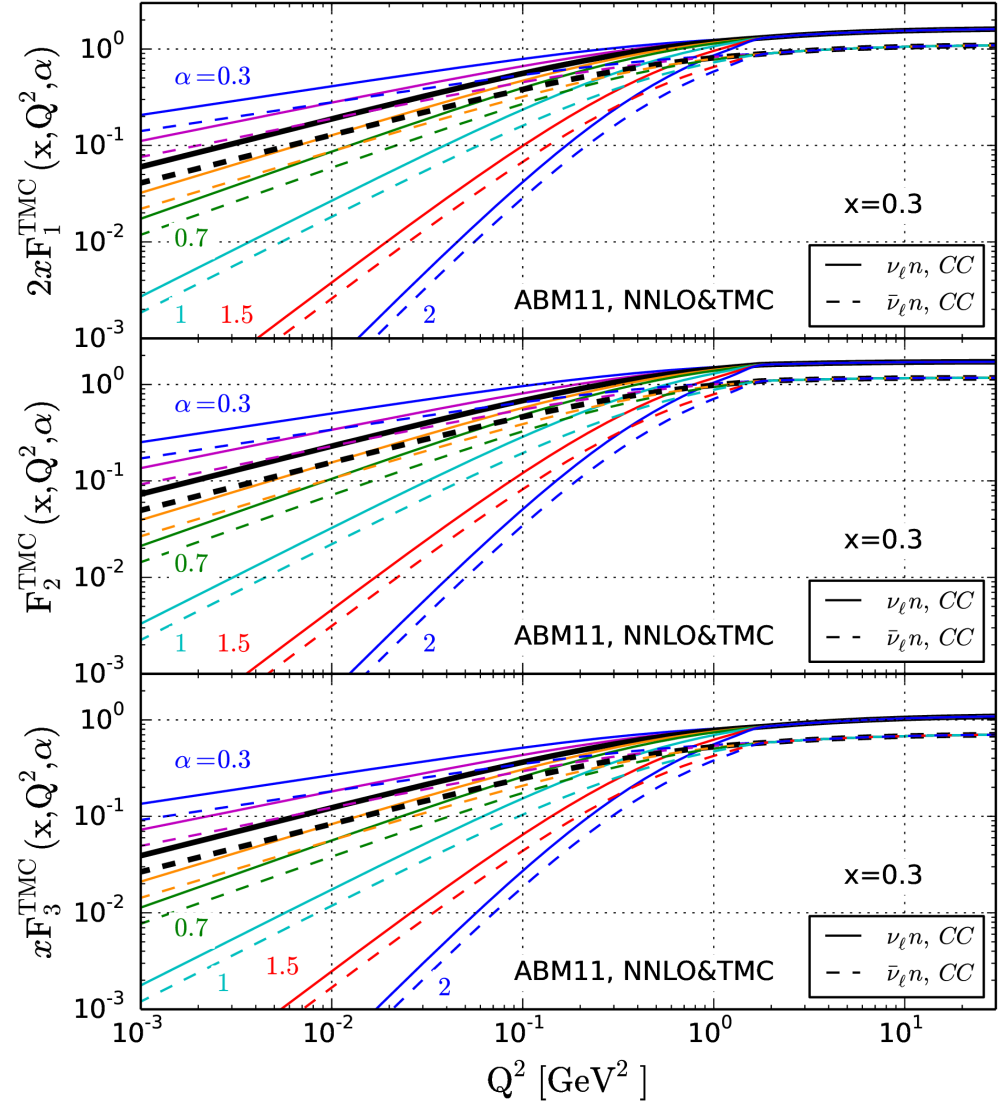
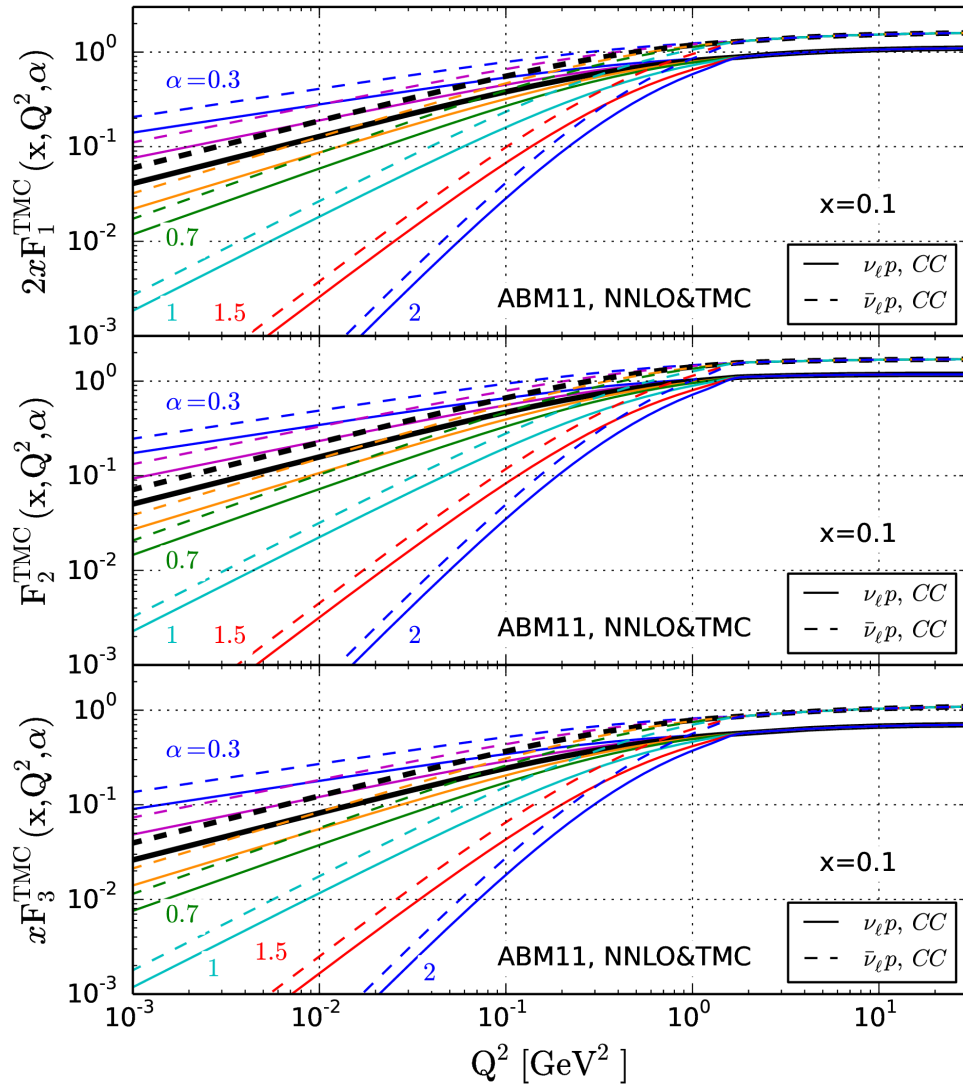
$$a_4 = \frac{m_\ell^2(m_\ell^2 + Q^2)}{4E_\nu^2 M_N^2 x}, \quad a_5 = -\frac{m_\ell^2}{E_\nu M_N}.$$

Nachtmann Variable

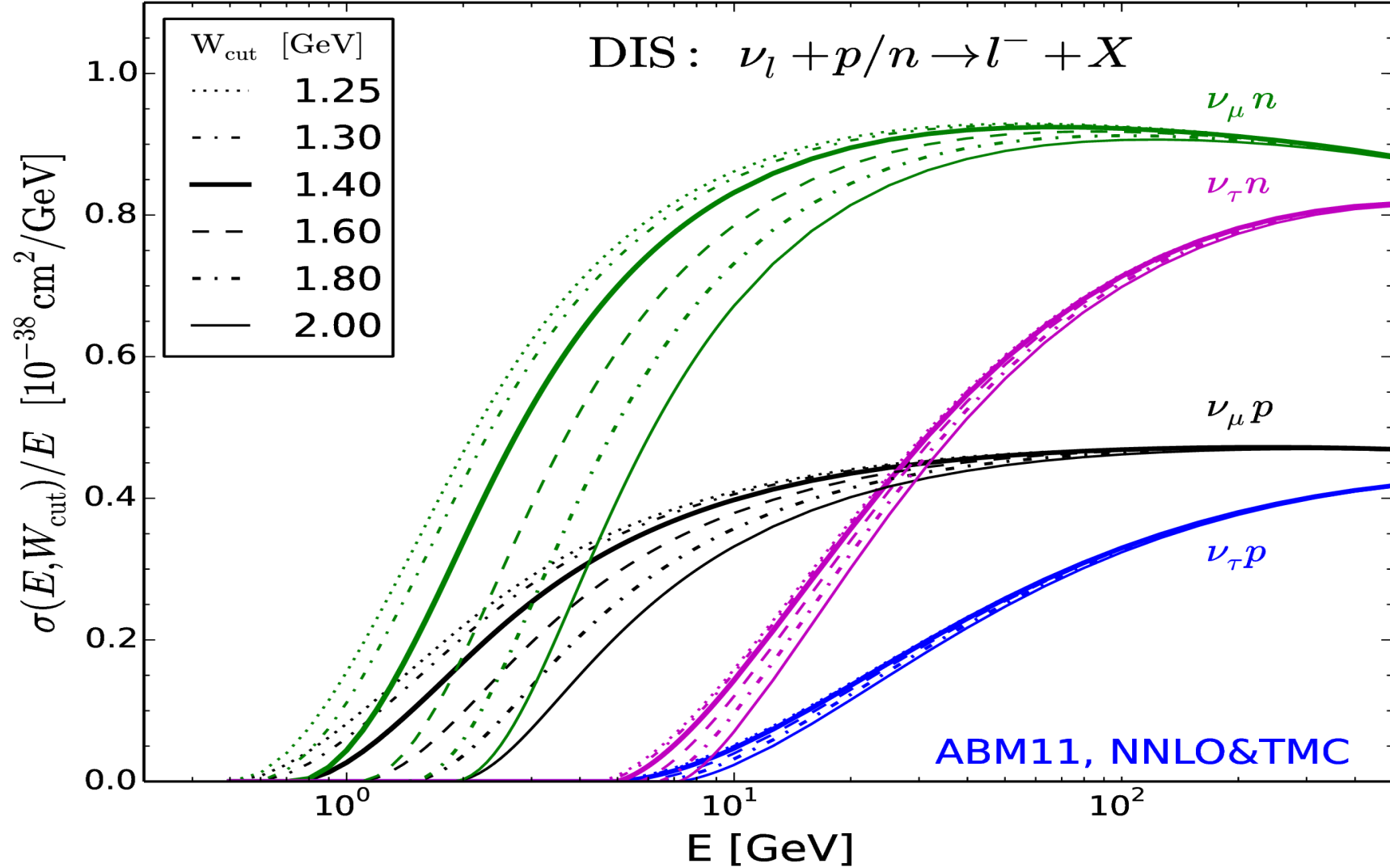


$$F_j^{\text{TMC}}(x, Q^2) = \sum_{i=1}^5 A_j^i F_i(\xi, Q^2) + B_j^i h_i(\xi, Q^2) + C_j g_2(\xi, Q^2).$$

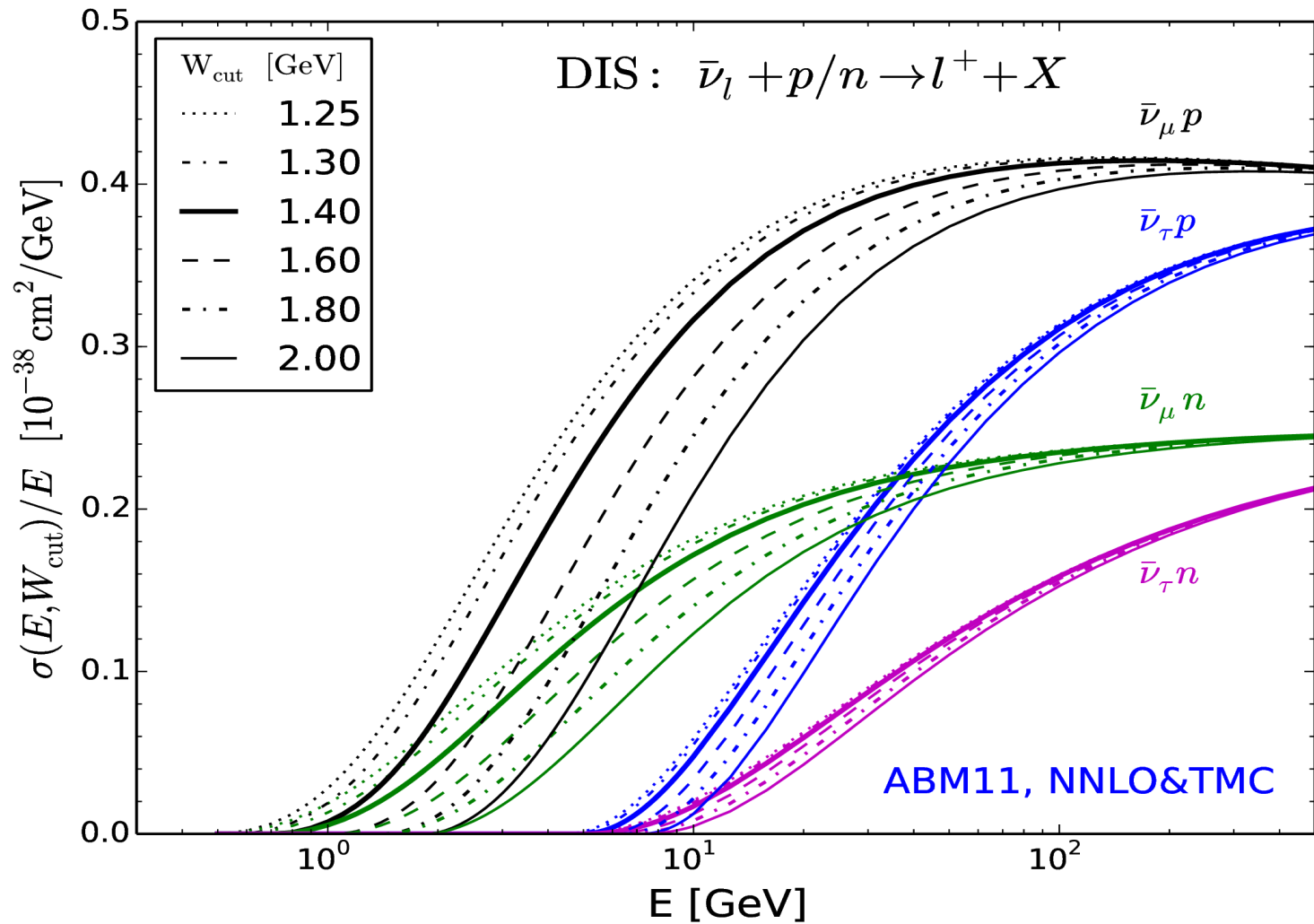
Extrapolations of SFs to small Q^2



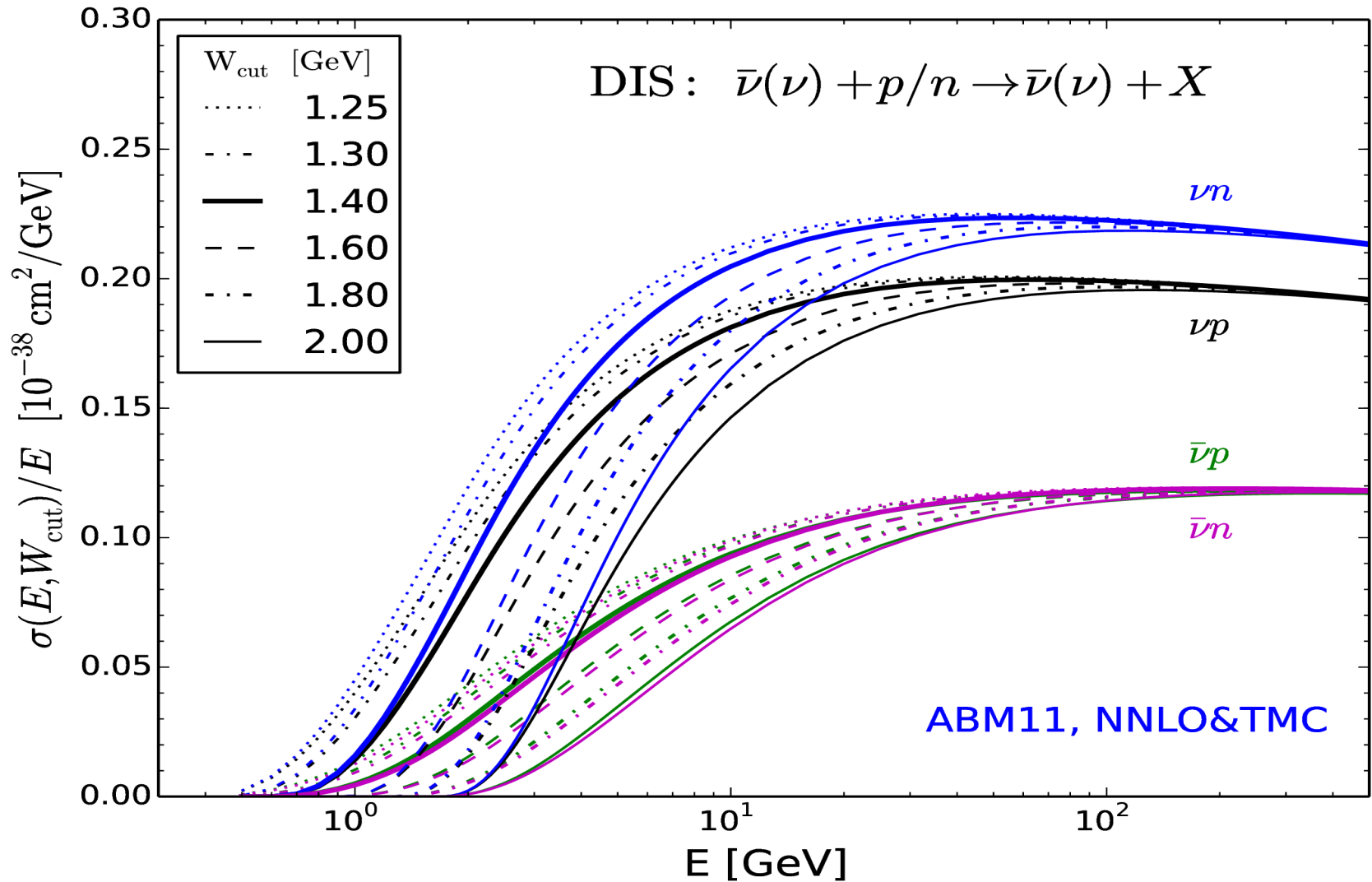
Dependence of νN CC DIS on W_{cut}



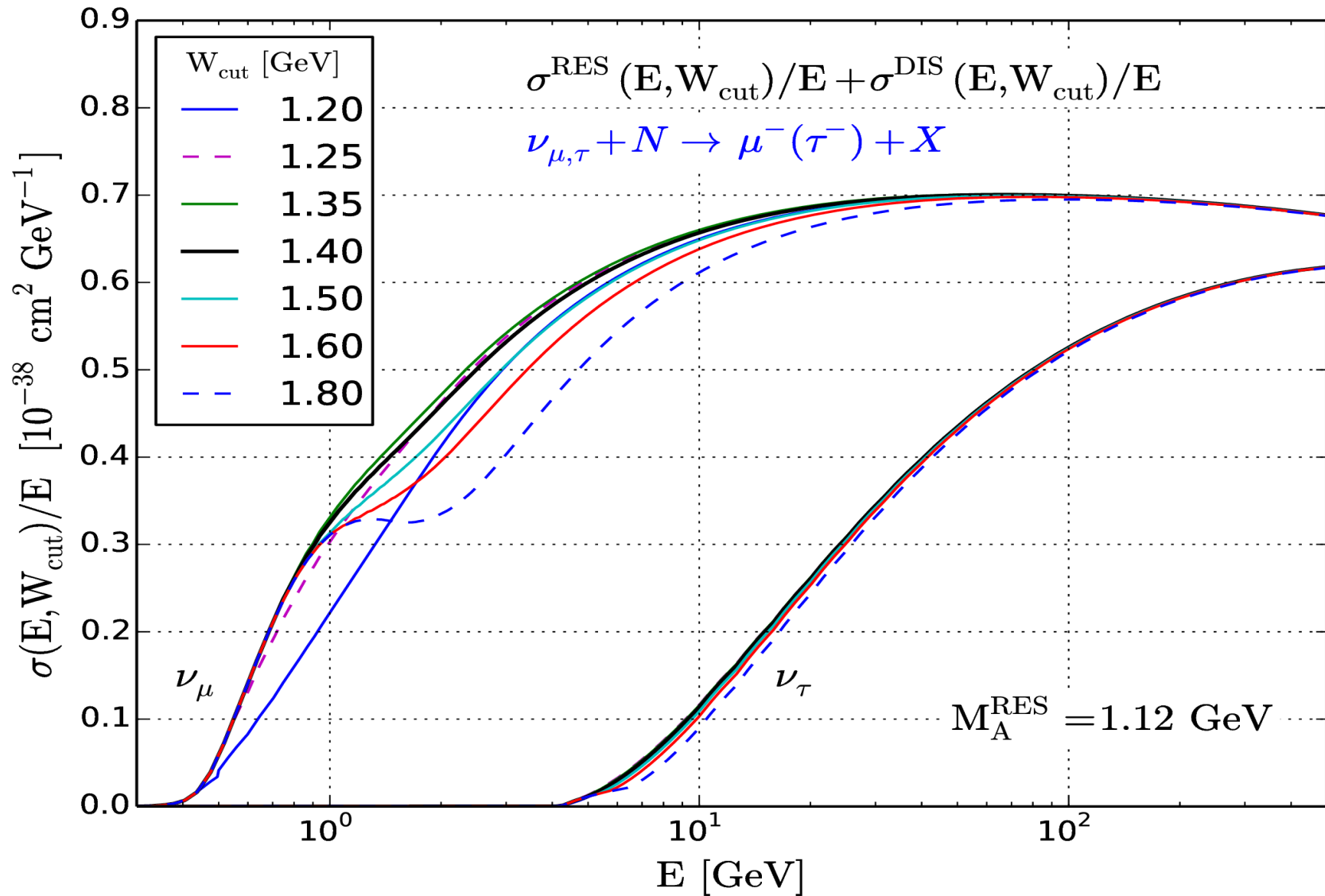
Dependence of $\bar{\nu}N$ CC DIS on W_{cut}



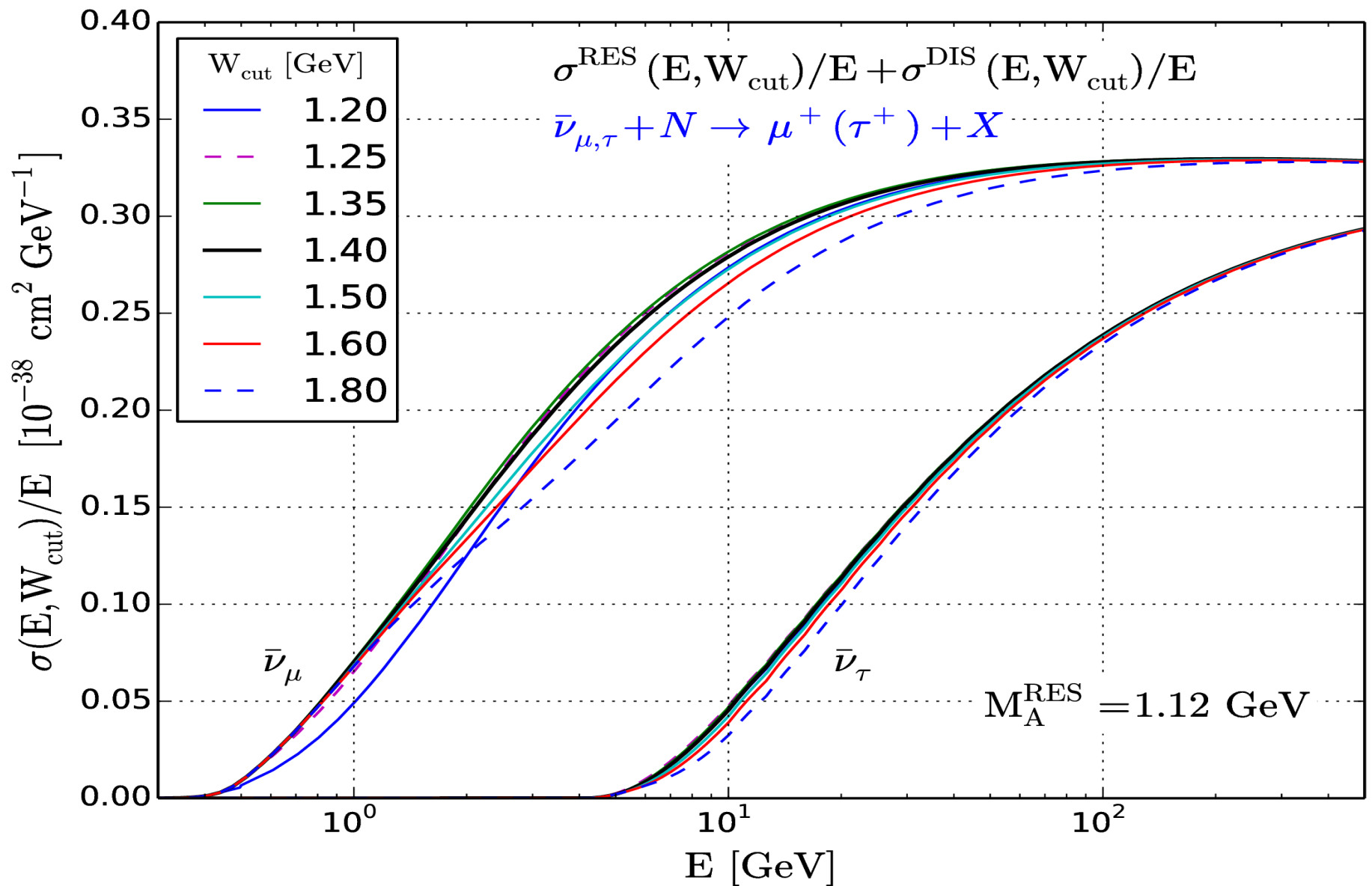
Dependence of $\nu(\bar{\nu})N$ NC DIS on W_{cut}



Optimization of W_{cut} for νN



Optimization of W_{cut} for $\bar{\nu}N$



Kinematic Limits

$$y = 1 - E_\ell/E_\nu$$

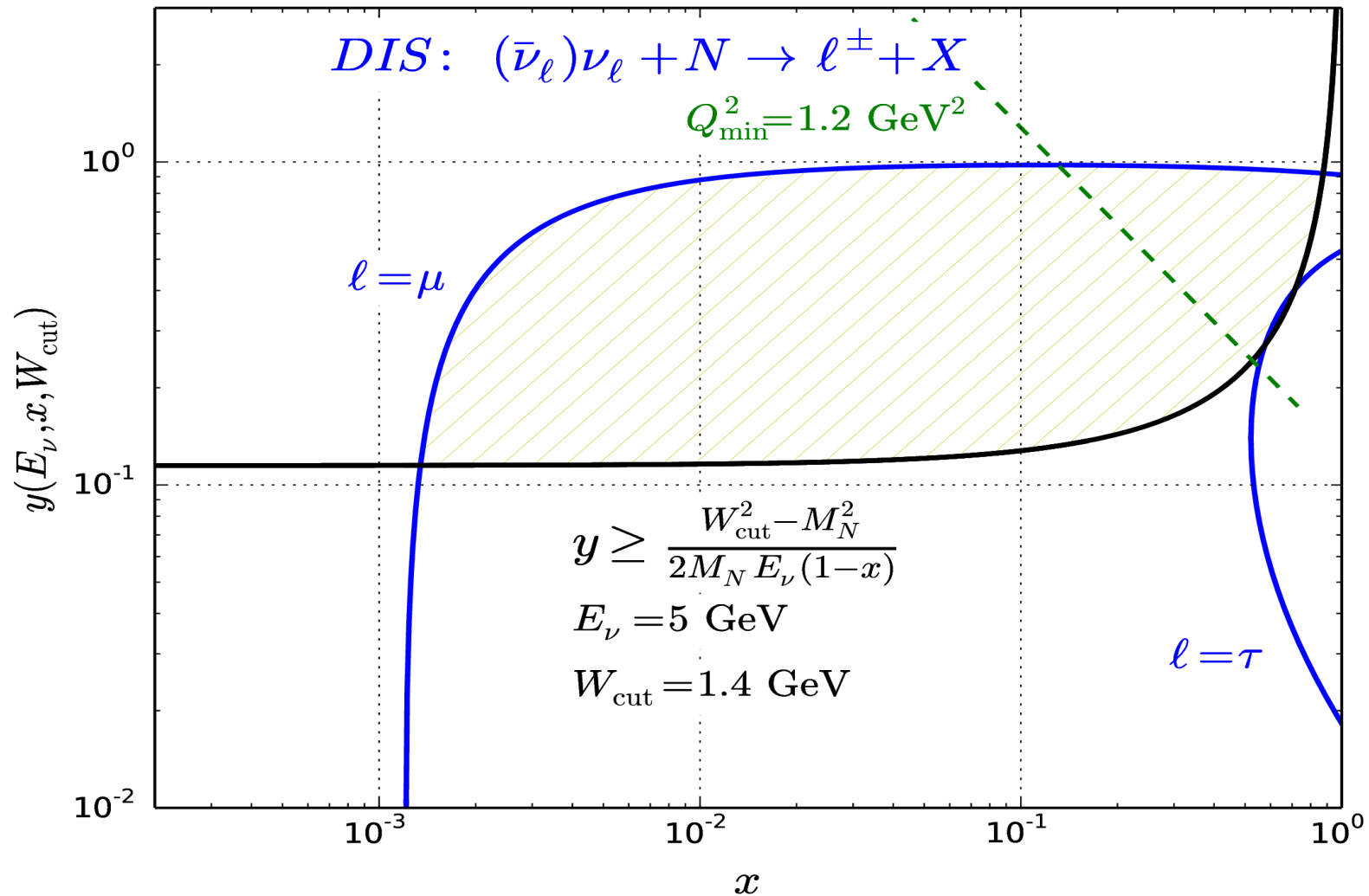
$$W^2 = M_N^2 + 2M_N E_\nu y(1 - x) \quad y \geq \frac{W_{\text{cut}}^2 - M_N^2}{2M_N E_\nu (1 - x)}$$

$$\frac{m_\ell^2}{2M_N(E_\nu - m_\ell)} \leq x \leq 1,$$

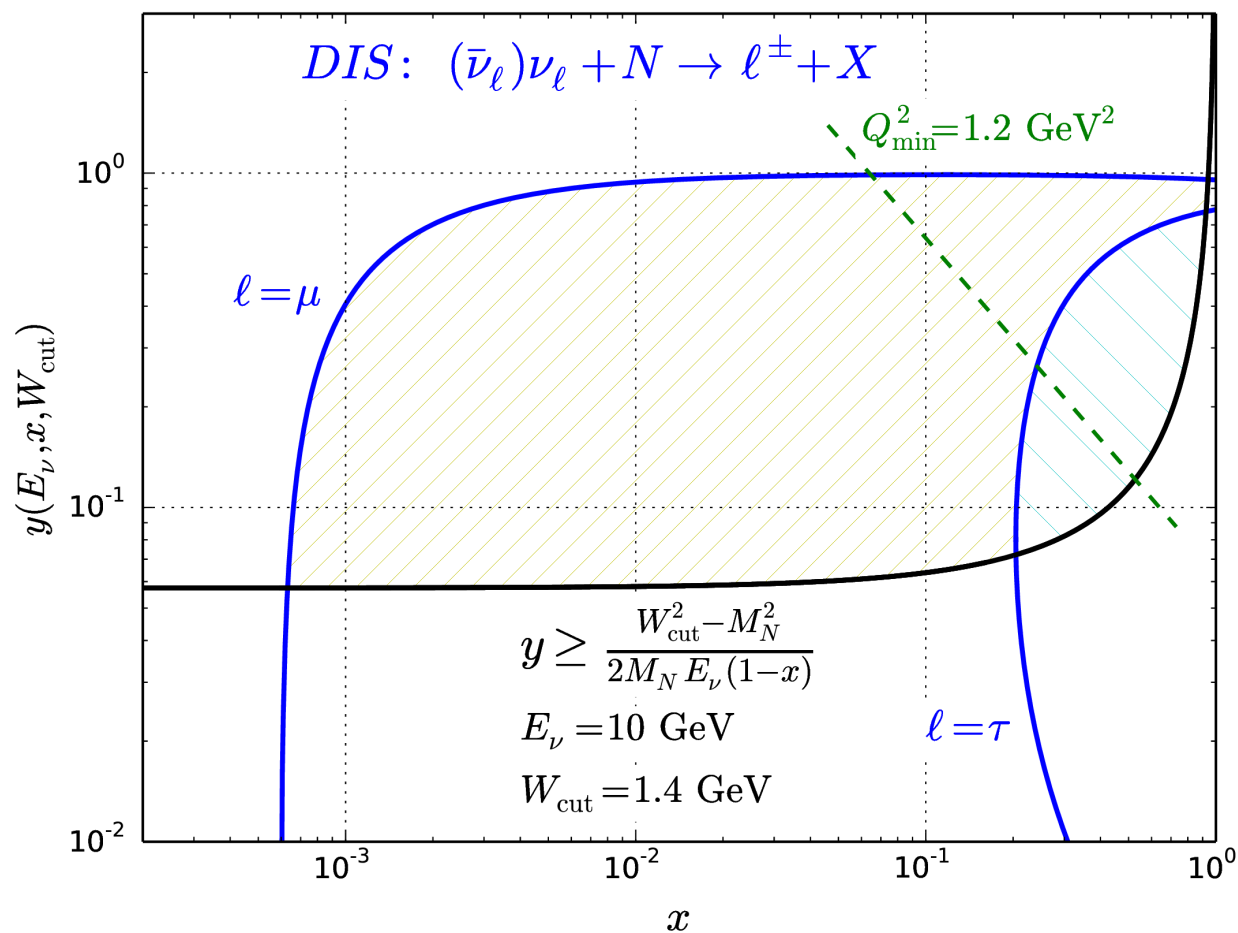
$$A - B \leq y \leq A + B$$

$$A = \frac{1 - \frac{m_\ell^2}{2M_N E_\nu x} - \frac{m_\ell^2}{2E_\nu^2}}{2 + M_N x/E_\nu}, \quad B = \frac{\sqrt{\left(1 - \frac{m_\ell^2}{2M_N E_\nu x}\right)^2 - \frac{m_\ell^2}{E_\nu^2}}}{2 + M_N x/E_\nu}$$

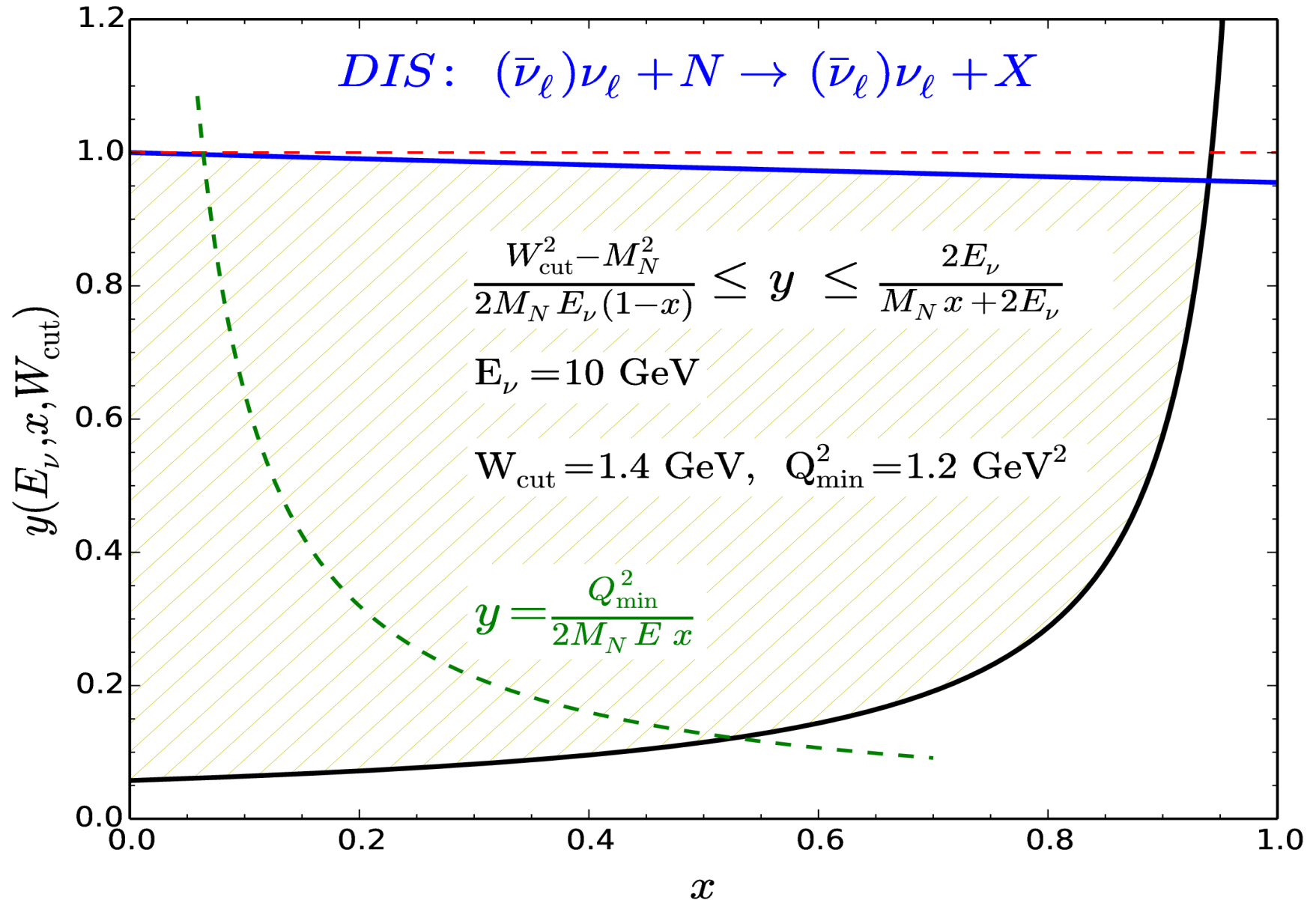
Kinematic Limits for CC DIS at E=5 GeV



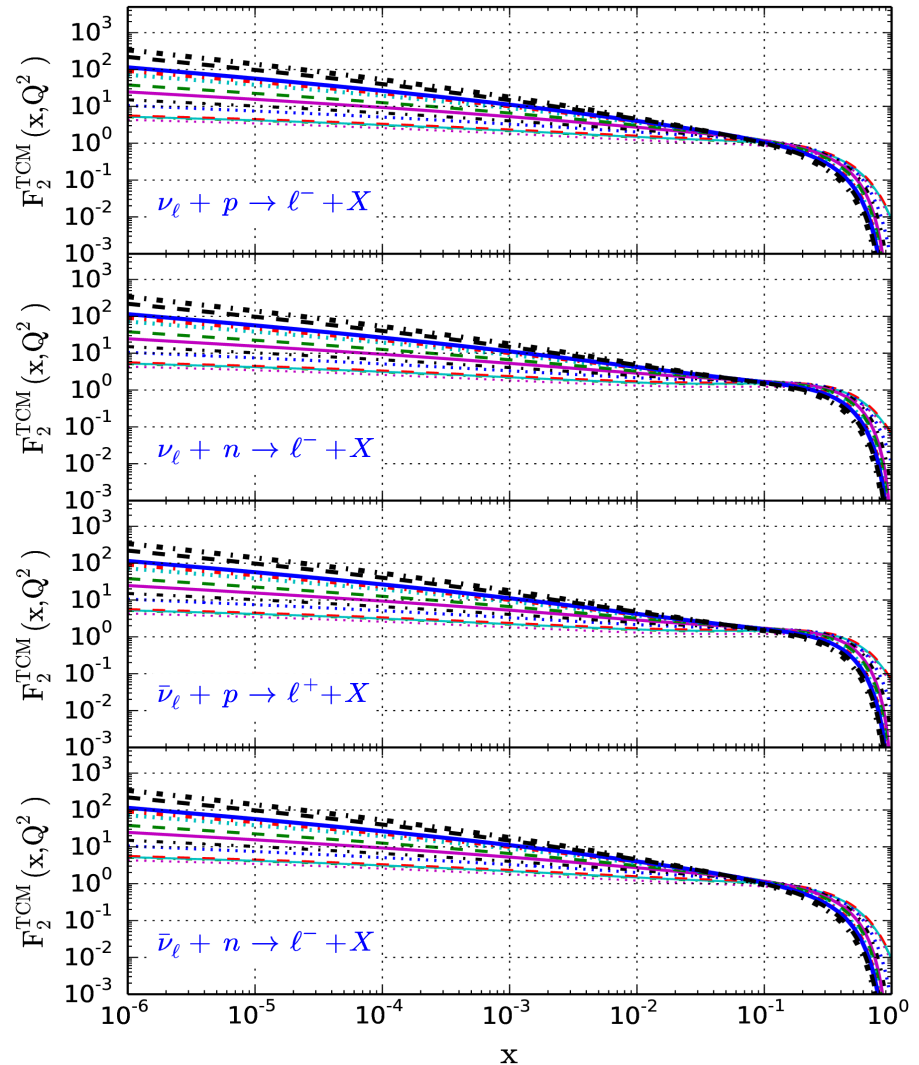
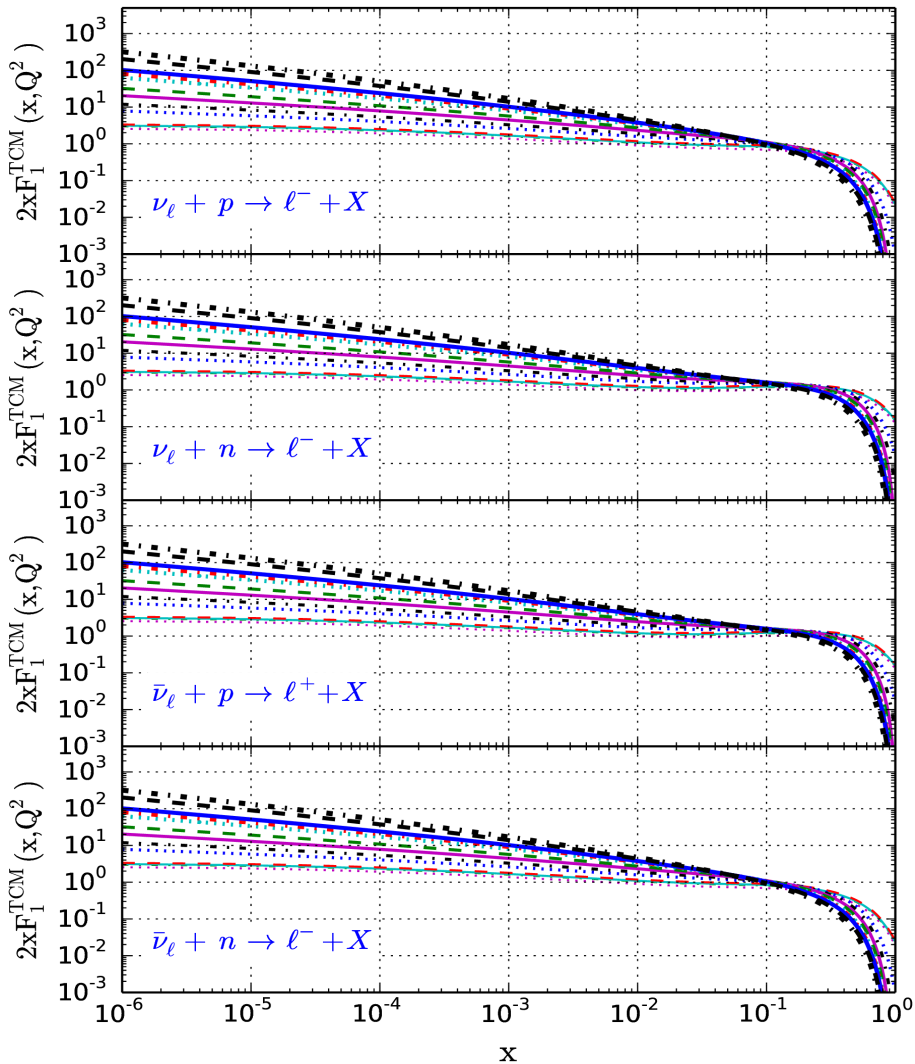
Kinematic Limits for CC DIS at E=10 GeV



Kinematic Limits for NC DIS at E=10 GeV



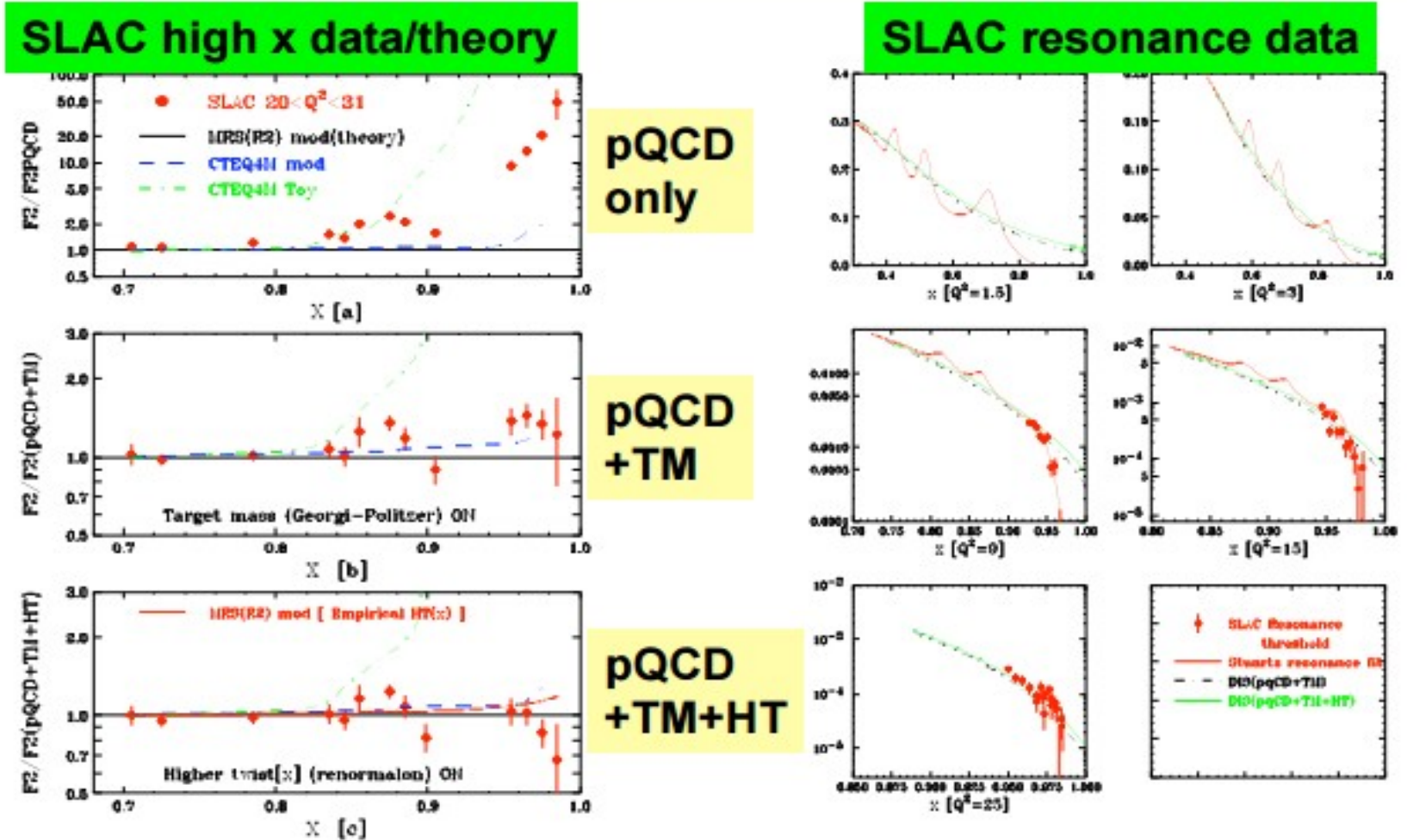
TMC SFs Dependence on x



PDF are from **ABM-11** and **OPENQCDRAD** by Alekhin, Bluemlein, Moch.

Approach of Bodek&Yang

Very high x F_2 proton data (DIS + resonance)

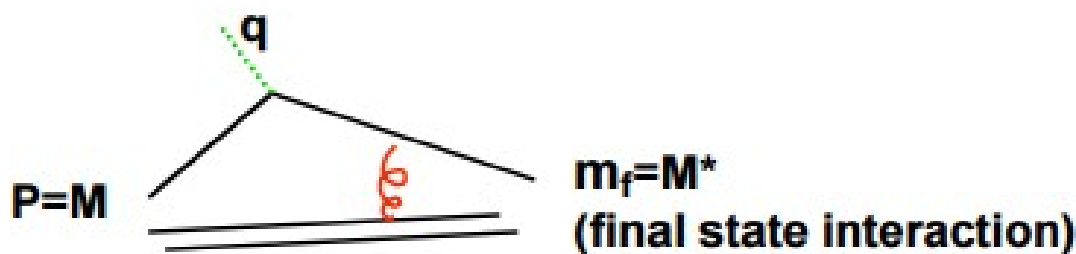


The data are well described by the predictions (duality works!)

B&Y Pseudo NLO

Original approach (NNLO pQCD+TM) was to explain the non-perturbative QCD effects at low Q^2 , but now we reverse the approach:

- Use LO PDFs and “effective target mass and final state masses” to account for initial target mass, final target mass, and even missing higher orders



resonance, higher twist, and TM

$$\xi_W = \frac{Q^2}{Mv[1 + \sqrt{(1+Q^2/v^2)}]},$$

where $2Q'^2 = (Q^2 + m_f^2 - m_i^2)$

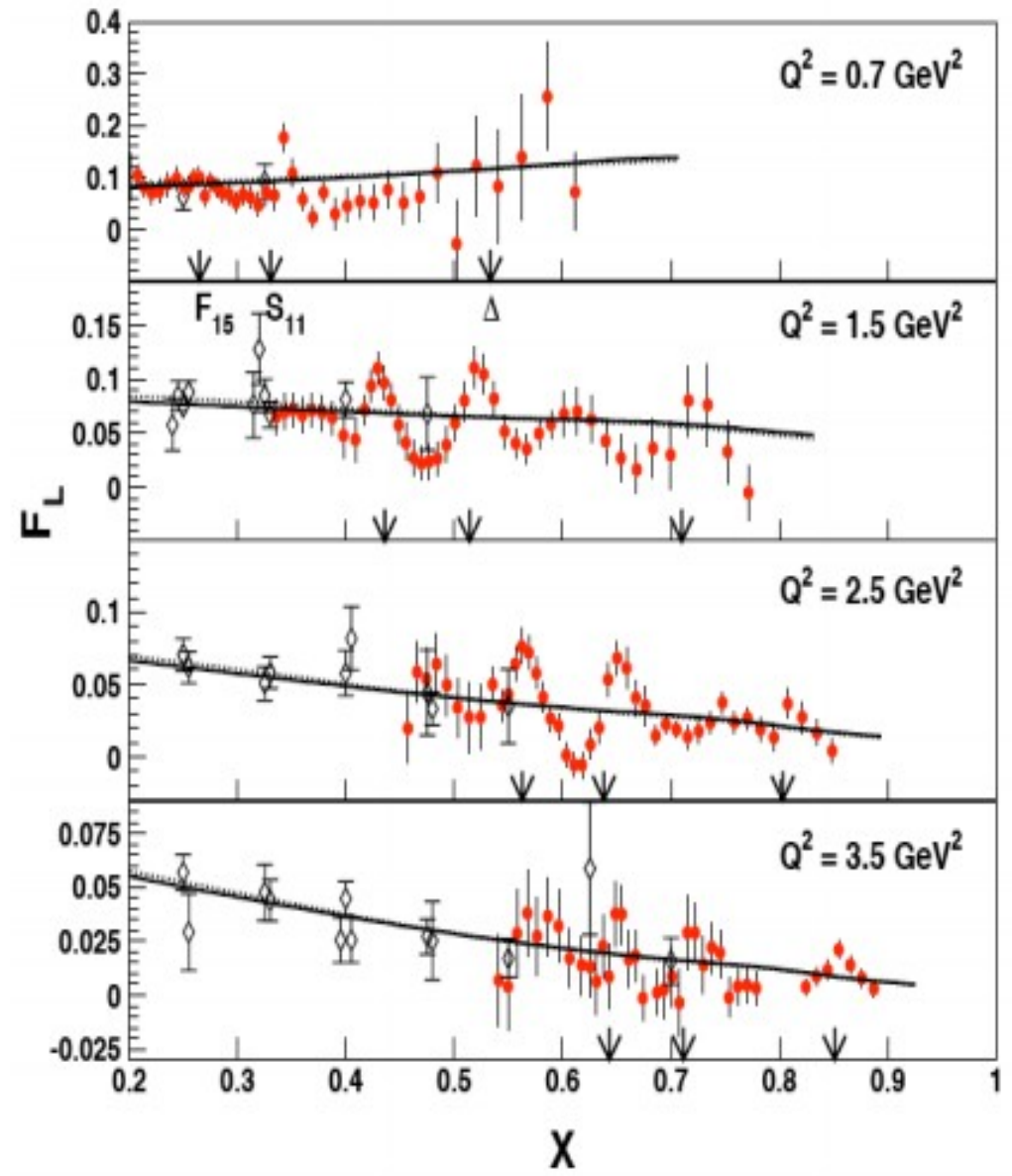
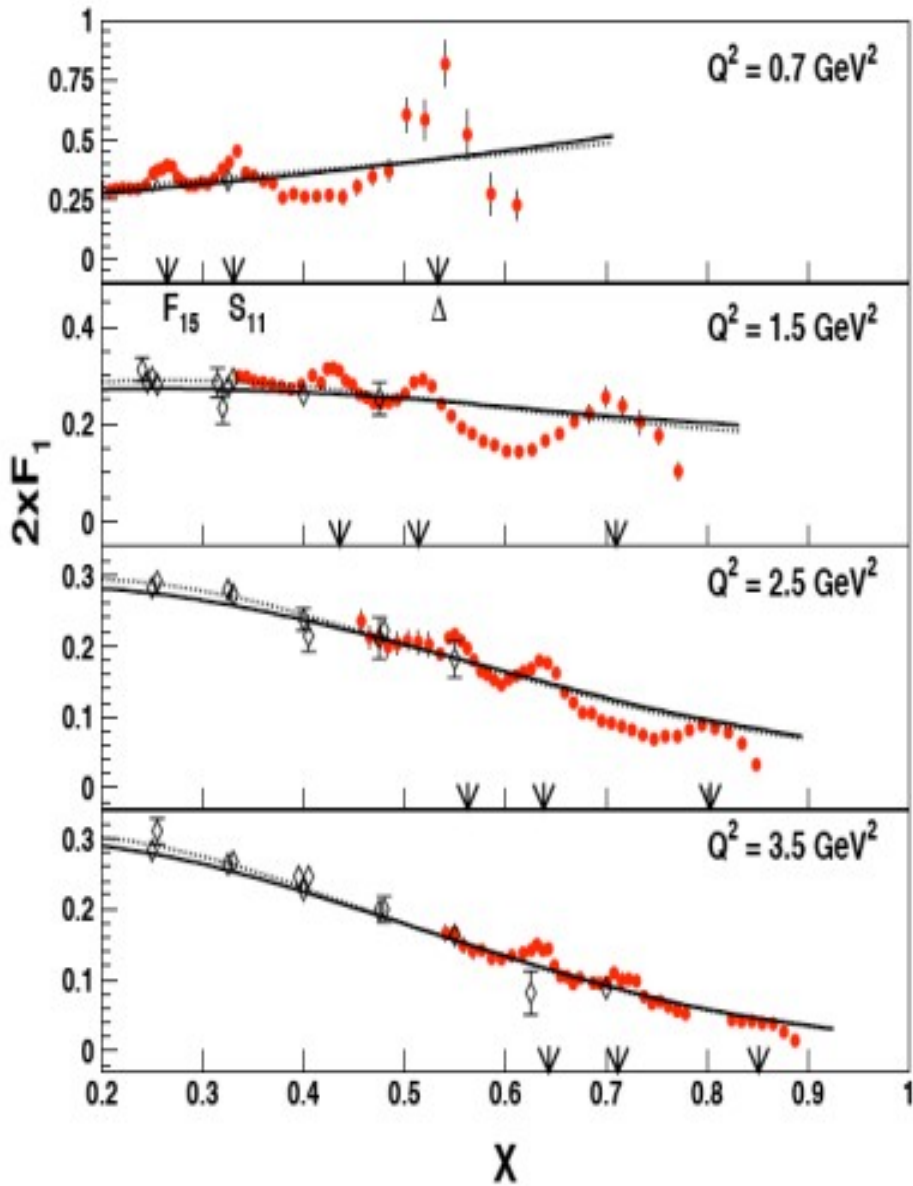
$$+ \sqrt{(Q^2 + m_f^2 - m_i^2)^2 + 4Q^2(m_i^2 + P_i^2)}$$

$$\xi_W^{\text{tr}} = \frac{Q^2 + B}{\{Mv[1 + \sqrt{(1+Q^2/v^2)}] + A\}}$$

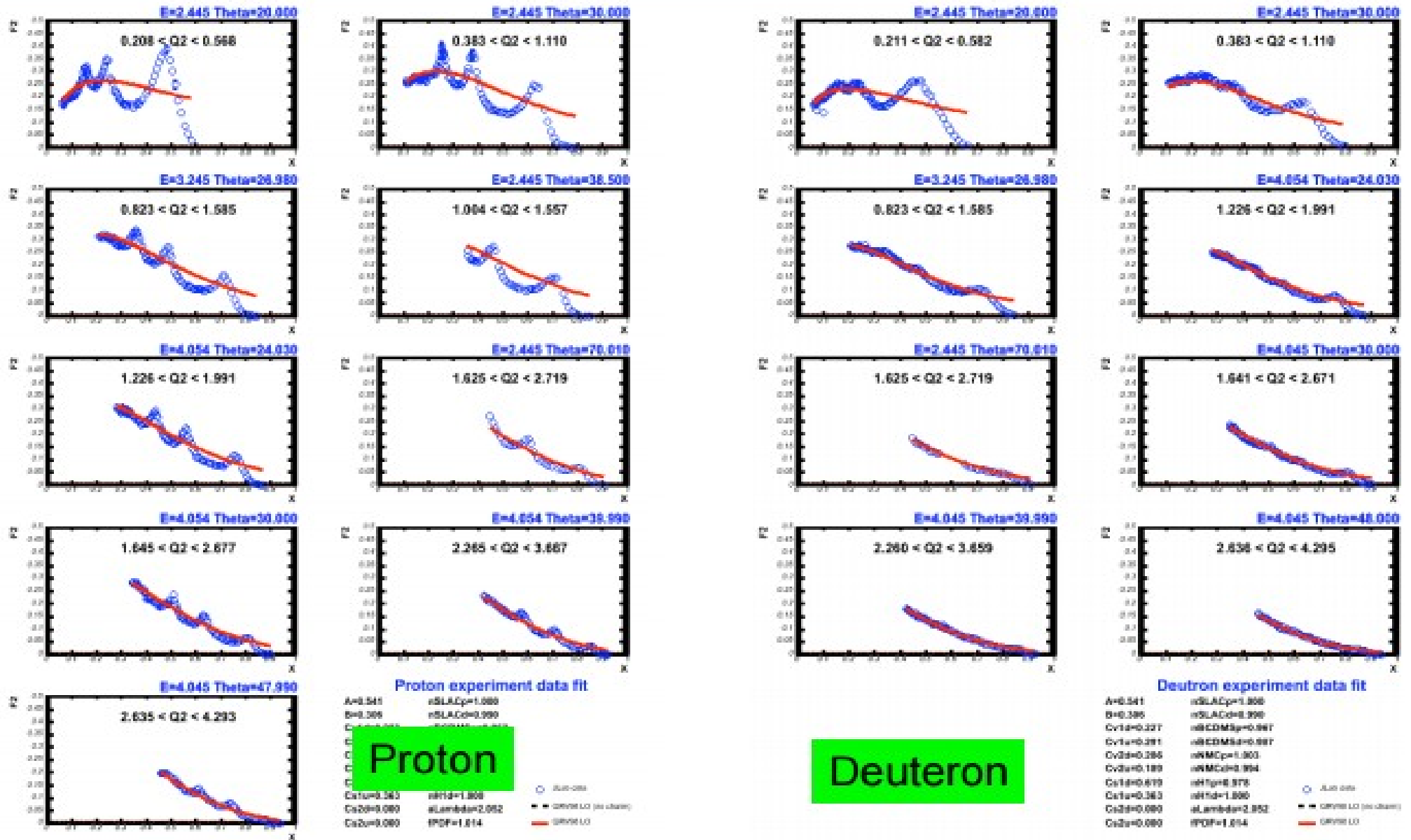
A : initial binding/target mass effect
plus higher order term

B: final state mass m_f^2 , Δm^2 and photo-
production

$2xF_1$ and F_L in B&Y Approach



Comparison of B&Y Model with Resonances



MC Generators

Neutrino event generators available in the market.

- **NEUT** – developed for Kamiokande and used by Super-Kamiokande, K2K, SciBooNE, T2K.
- **NUANCE** – developed for the IMB experiment and used by MiniBooNE.
- **NEUGEN** - developed for the SOUDAN and updated for MINOS.
- **ANIS** – developed for AMANDA and used by IceCube.

General purpose MC generators

- **FLUKA**
- **GENIE** – ArgoNeut, MicroBooNE, MINOS, MINERvA, T2K. Continuously updated.

MC Generators, cont.

- **GIBUU** – theorist group approach. It provides an unified transport framework at MeV - GeV energies. Numerous nuclear effects included.
- **NuWro** - developed by the Wroclaw group. Accounts for the impact of nuclear effects on directly observable quantities with all the final state interactions included.

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- An account for TMC, lepton mass, pQCD corrections are important.
- New precision data may change the set of parameters.

Thank you!

QES: from Strumia&Vissani

$$\mathcal{M} = \bar{v}_\nu \gamma^a (1 - \gamma_5) v_e \cdot \bar{u}_n \left(f_1 \gamma_a + g_1 \gamma_a \gamma_5 + i f_2 \sigma_{ab} \frac{q^b}{2M} + g_2 \frac{q_a}{M} \gamma_5 \right) u_p$$

where $q = p_\nu - p_e = p_n - p_p$. A straightforward calculation yields²:

$$|\mathcal{M}^2| = A(t) - (s - u)B(t) + (s - u)^2 C(t)$$

where $s = (p_\nu + p_p)^2$, $t = (p_\nu - p_e)^2$, $u = (p_\nu - p_n)^2$ are the usual Mandelstam variables and

$$16 A = (t - m_e^2) \left[4|f_1^2|(4M^2 + t + m_e^2) + 4|g_1^2|(-4M^2 + t + m_e^2) + |f_2^2|(t^2/M^2 + 4t + 4m_e^2) + 4m_e^2 t |g_2^2|/M^2 + 8\text{Re}[f_1^* f_2](2t + m_e^2) + 16m_e^2 \text{Re}[g_1^* g_2] \right]$$

$$- \Delta^2 \left[(4|f_1^2| + t|f_2^2|/M^2)(4M^2 + t - m_e^2) + 4|g_1^2|(4M^2 - t + m_e^2) + 4m_e^2 |g_2^2|(t - m_e^2)/M^2 + 8\text{Re}[f_1^* f_2](2t - m_e^2) + 16m_e^2 \text{Re}[g_1^* g_2] \right] - 32m_e^2 M \Delta \text{Re}[g_1^*(f_1 + f_2)]$$

$$16 B = 16t \text{Re}[g_1^*(f_1 + f_2)] + 4m_e^2 \Delta (|f_2^2| + \text{Re}[f_1^* f_2 + 2g_1^* g_2]) / M$$

$$16 C = 4(|f_1^2| + |g_1^2|) - t|f_2^2|/M^2.$$

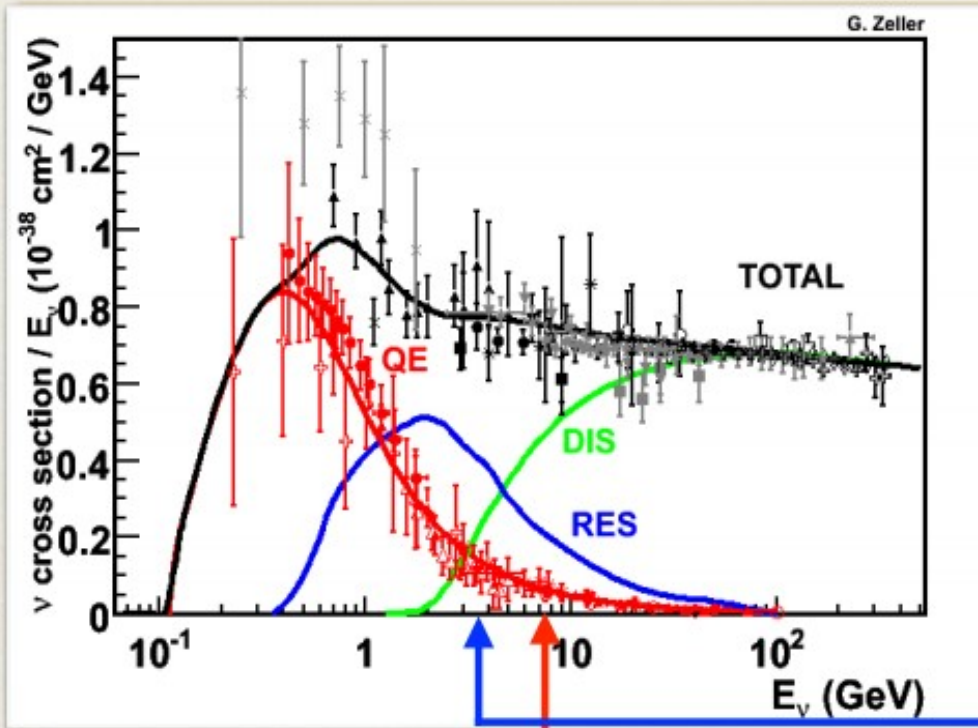
f_i and g_i are real function of $t = -Q^2$. Approximation:

$$\{f_1, f_2\} = \frac{\{1 - (1 + \xi)t/4M^2, \xi\}}{(1 - t/4M^2)(1 - t/M_V^2)^2}, \quad g_1 = \frac{g_1(0)}{(1 - t/M_A^2)^2}, \quad g_2 = \frac{2M^2 g_1}{m_\pi^2 - t}$$

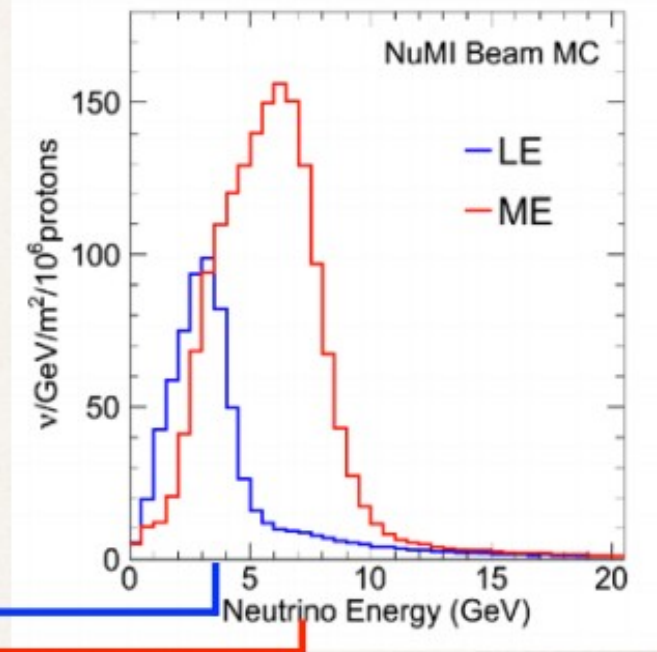
$$\xi = k_p - k_n = 3.706$$

where k_p and k_n – anomalous magnetic moments in units of nuclear magneton $\mu_N = e \hbar / 2 m_p$

The MINERvA energy range



J.A. Formaggio and G.P. Zeller,
Rev. Mod. Phys. 84, 1307-1341,
 2012



BooNE experiments, MINERvA, DUNE,
 T2K NOvA, MINOS
 PINGU

Low-energy run,
 2010-2012
 3.98×10^{20} POT (ν)
 1.7×10^{20} POT ($\bar{\nu}$)

Medium-energy run, 2013-
 Already exceeded low-energy
 POT in ν mode