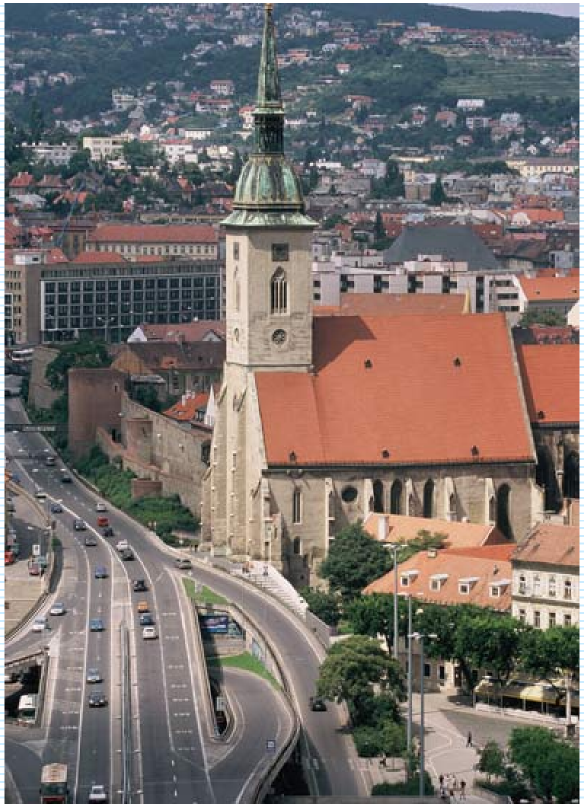
Laboratori Nazionali del Gran Sasso
Thursday 25, 2015

I. Massive Neutrinos in Nuclear Processes

Fedor Šimkovic



OUTLINE

- *Introduction*
- *Neutrino mass and single β-decay*
- *Effective Majorana neutrino mass in vacuum and nuclear matter*
- *0νββ-decay mechanisms*
- *distinguishing 0νββ-decay mechanisms*
- *0νECEC-decay*
- *Partly bosonic neutrino*
- *Conclusion*

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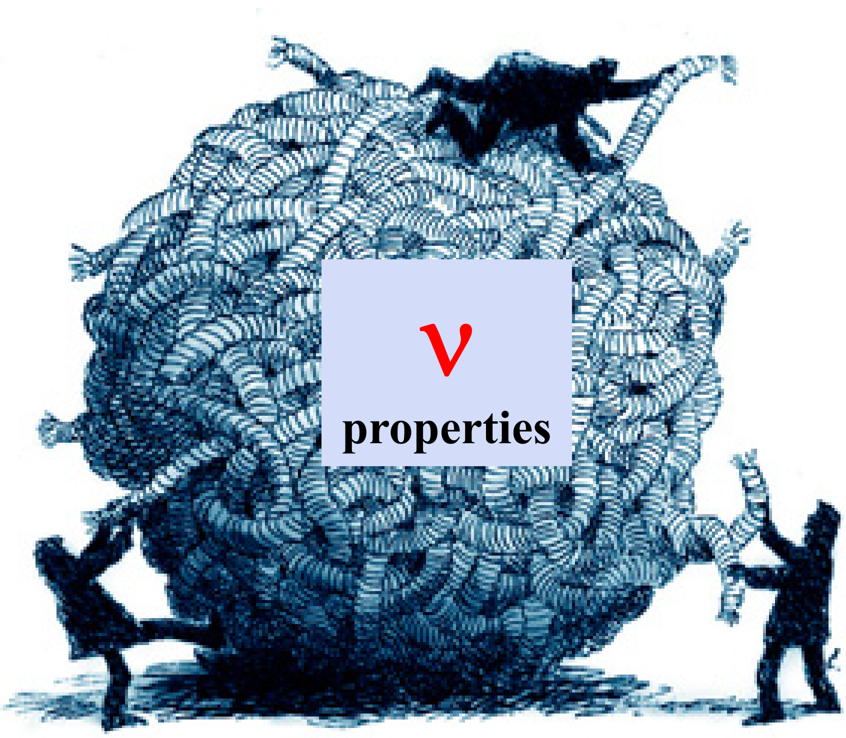
J. Engel (North Caroline U.), S. Petcov (SISSA Trieste),
A. Smirnov (ICTP Trieste), A. Dolgov (Bologna U.)…

After 59 years
we know

- 3 families of light (V-A) neutrinos: ν_e, ν_μ, ν_τ
- ν are massive: we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)



No answer yet

- Are ν Dirac or Majorana?
- Is there a CP violation in ν sector?
- Are neutrinos stable?
- What is the magnetic moment of ν?
- Sterile neutrinos?
- Statistical properties of ν? Fermionic or partly bosonic?

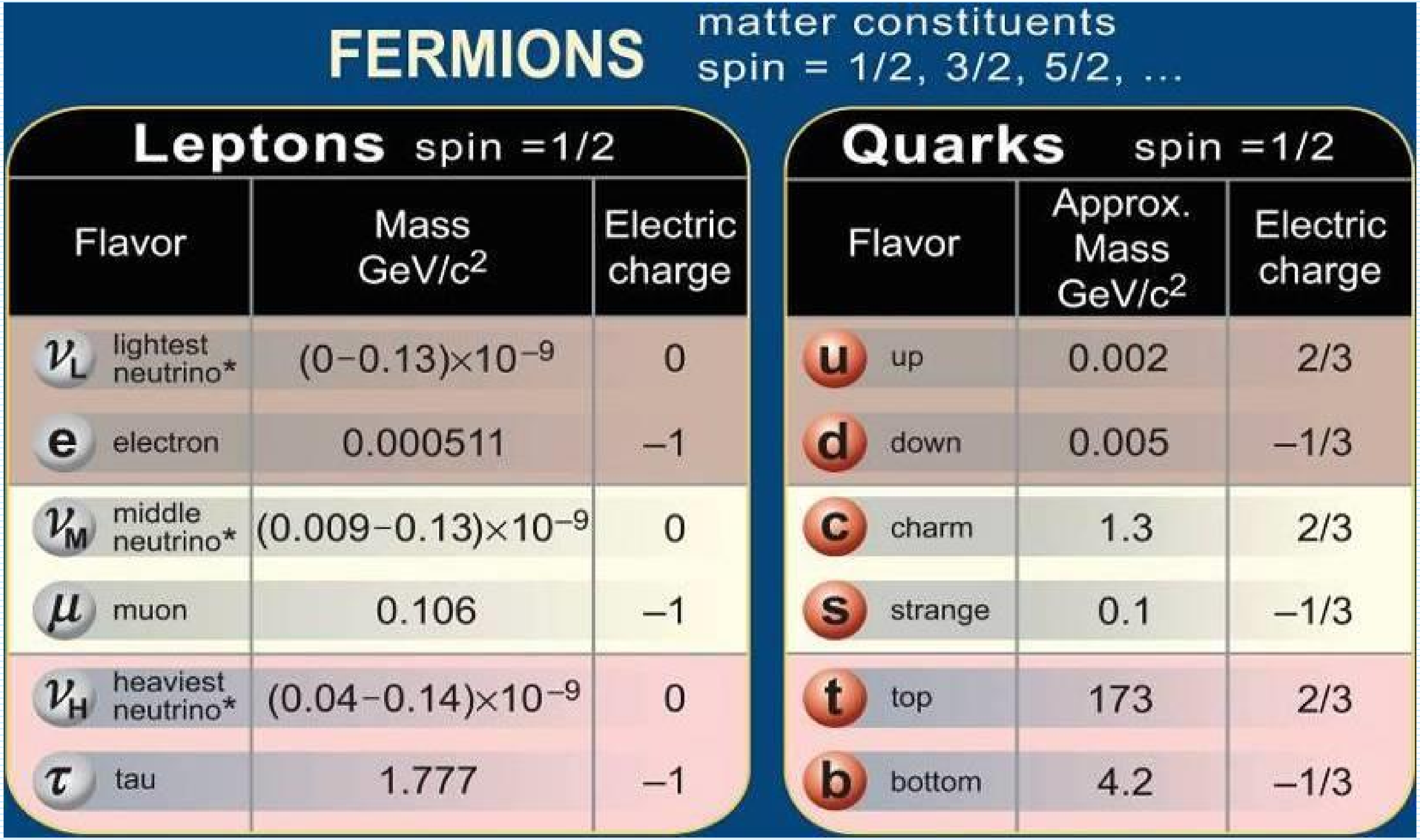
Currently main issue

Nature, Mass hierarchy, CP-properties, sterile ν

The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties

Why is neutrino mass so small?
Need right-handed neutrinos
to generate neutrino mass

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$



FERMIONS

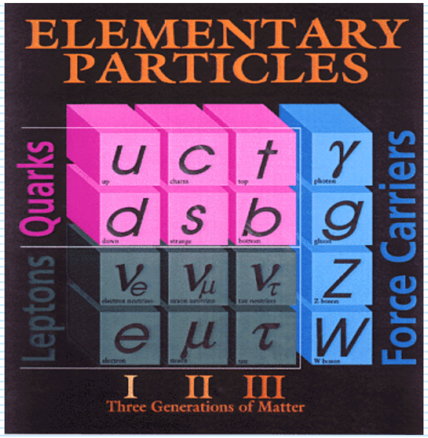
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor		Mass GeV/c^2	Electric charge
ν_L	lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e	electron	0.000511	–1
ν_M	middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ	muon	0.106	–1
ν_H	heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ	tau	1.777	–1

Quarks spin =1/2

Flavor		Approx. Mass GeV/c^2	Electric charge
u	up	0.002	2/3
d	down	0.005	–1/3
c	charm	1.3	2/3
s	strange	0.1	–1/3
t	top	173	2/3
b	bottom	4.2	–1/3



Standard Model Lepton Universality

Particle	Symbol	Anti − p.	mass [MeV]	L_e	L_μ	L_τ	life − time [s]
electron	e^-	e^+	0.511	1	0	0	stable
el.neutrino	ν_e	$\overline{\nu}_e$	$< 2.2\ 10^{-6}$	1	0	0	stable
muon	μ^-	μ^+	105.6	0	1	0	$2.2\ 10^{-6}$
muon neutr.	ν_μ	$\overline{\nu}_\mu$	< 0.19	0	1	0	stable
tau	τ^-	τ^+	1777.	0	0	1	$2.9\ 10^{-13}$
tau neutrino	ν_τ	$\overline{\nu}_\tau$	< 18.2	0	0	1	stable

Lepton Family Number Violation

NEW PHYSICS

massive neutrinos, SUSY...

Total Lepton Number Violation

$\nu_{e,\mu\tau} \leftrightarrow \nu_{e,\mu\tau}$, $\overline{\nu}_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	observed
$\mu^+ \to e^+ + \gamma$	$R \leq 1.2 \times 10^{-11}$
$\mu^+ \to e^+ + e^- + e^+$	$R \leq 1.0 \times 10^{-12}$
$K^+ \to \pi^+ + e^- + \mu^+$	$R \leq 4.7 \times 10^{-12}$
$\tau^- \to e^- + \mu^+ + \mu^-$	$R \leq 1.8 \times 10^{-6}$
$Z^0 \to e^\pm + \mu^\mp$	$R \leq 1.7 \times 10^{-6}$
$\mu_b^- + (A,Z) \to (A,Z) + e^-$	$R \leq 1.2 \times 10^{-11}$

$\nu_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	not observed
$K^+ \to \pi^- + e^+ + \mu^+$	$R \leq 5 \times 10^{-10}$
$\tau^- \to \pi^- + \pi^+ + e^+$	$R \leq 1.9 \times 10^{-6}$
$W^- + W^- \to e^- + e^-$	
$(A,Z) \to (A,Z+2) + e^- + e^-$	$T^{0\nu} \geq 1.9 \times 10^{-25}$
$\mu_b^- + (A,Z) \to (A,Z-2) + e^+$	$R \leq 3.6 \times 10^{-11}$
$e^- + e^- \to \pi^- + \pi^-$	?

The observed small neutrino masses have profound implications for our understanding of the Universe and are now a major focus in astro, particle and nuclear physics and in cosmology.

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

- **3 neutrino mixing angles are measured and non-zero**
- **Large θ_{13} opens door for searching of CP-violation in lepton sector**
- **Large θ_{13} gives good chances for measurement of mass hierarchy (MH) and CP violation in neutrino oscillations using present neutrino beams and detectors**
- **Time to start MH and δ measurements**

The size of θ_{13} → Future Program of neutrino physics

Nσ ranges for single parameters (all data included):

[F. Capozzi, G.L. Fogli, E. Lisi, D. Montanino, A. Marrone, and A. Palazzo, arXiv:1312.2878]

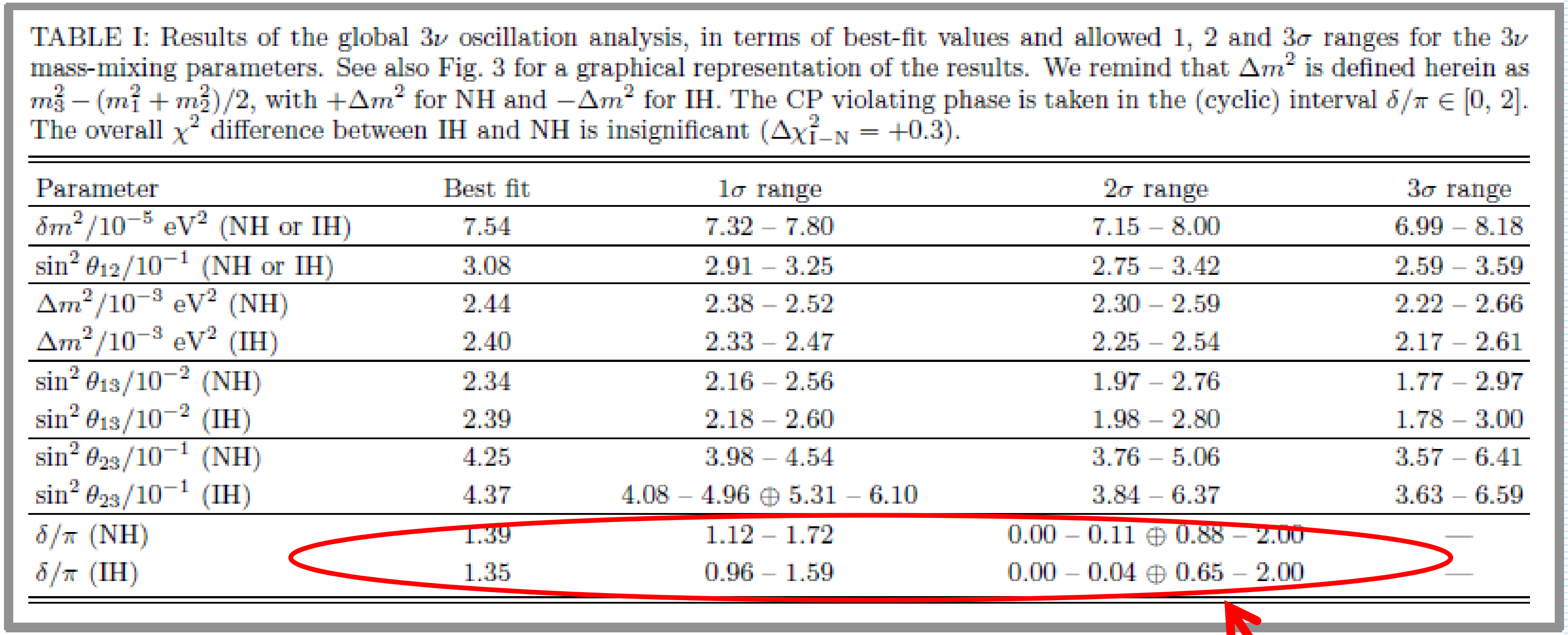


TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass-mixing parameters. See also Fig. 3 for a graphical representation of the results. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH. The CP violating phase is taken in the (cyclic) interval $\delta/\pi \in [0, 2]$. The overall χ^2 difference between IH and NH is insignificant ($\Delta\chi^2_{\mathrm{I-N}} = +0.3$).

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.08	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.44	2.38 – 2.52	2.30 – 2.59	2.22 – 2.66
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.40	2.33 – 2.47	2.25 – 2.54	2.17 – 2.61
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.34	2.16 – 2.56	1.97 – 2.76	1.77 – 2.97
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.39	2.18 – 2.60	1.98 – 2.80	1.78 – 3.00
$\sin^2\theta_{23}/10^{-1}$ (NH)	4.25	3.98 – 4.54	3.76 – 5.06	3.57 – 6.41
$\sin^2\theta_{23}/10^{-1}$ (IH)	4.37	4.08 – 4.96 $\oplus$ 5.31 – 6.10	3.84 – 6.37	3.63 – 6.59
δ/π (NH)	1.39	1.12 – 1.72	0.00 – 0.11 $\oplus$ 0.88 – 2.00	—
δ/π (IH)	1.35	0.96 – 1.59	0.00 – 0.04 $\oplus$ 0.65 – 2.00	—

Fractional uncertainties (defined as 1/6 of 3σ ranges):

δm^2 = Δm^2_{21}

$\theta_{12}, \theta_{23}, \theta_{13}, \delta$ = as in PDB

δ range = $[0, 2\pi]$ (others prefer $[-\pi,+\pi]$)

Δm^2 = $(\Delta m^2_{31} + \Delta m^2_{32})/2$

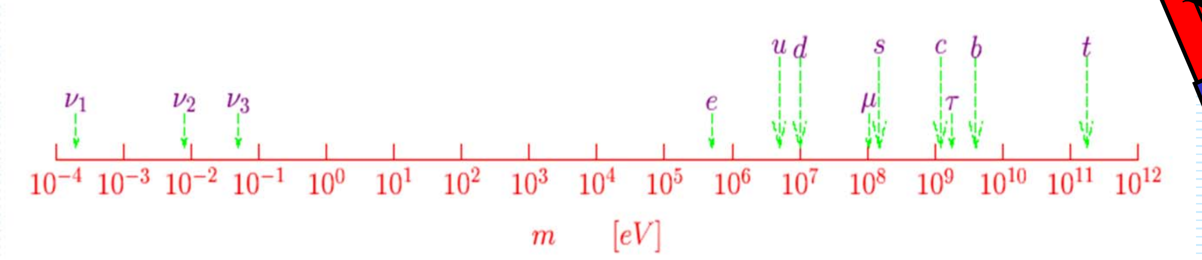
δm^2	2.6 %
Δm^2	3.0 %
$\sin^2\theta_{12}$	5.4 %
$\sin^2\theta_{13}$	8.5 %
$\sin^2\theta_{23}$	~ 11 %

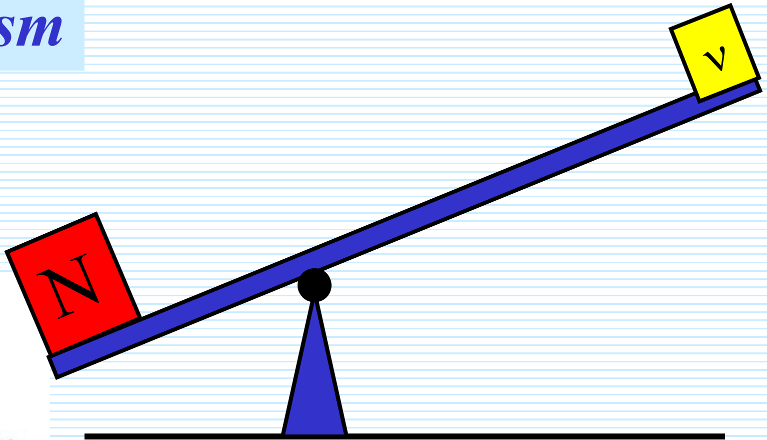
An indication of CP violation in neutrino sector

Assumption $M_R \gg m_D$

See-Saw mechanism

$$\left(\overline{\nu}_L \quad \overline{(\nu_R)^c}\right)\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}\begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix}$$





Left-right symmetric models SO(10)

$$\nu_{eL} = \sum_{i=1}^{light} U_{ei}\chi_{iL} + \sum_{i=1}^{heavy} U_{ei} N_{iL}$$

↑ large ↑ small

$$(\nu_{eR})^c = \sum_{i=1}^{light} V_{ei}\chi_{iL} + \sum_{i=1}^{heavy} V_{ei} N_{iL}$$

↑ small ↑ large

Probability of Neutrino Oscillations

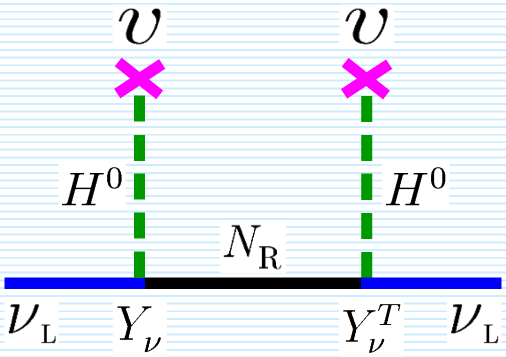
As N increases, the formalism gets rapidly more complicated!

N	Δm_{ij}^2	θ_{ij}	CP
2	1	1	0+1
3	2	3	1+2
6	5	15	10+9

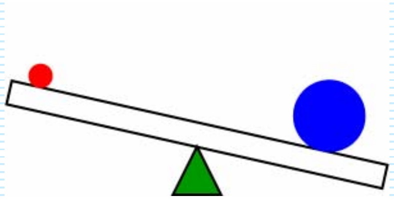
See-saws

A natural theoretical way to understand why 3 ν-masses are very small.

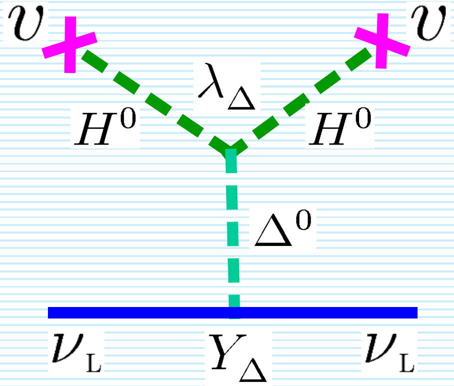
Type-I Seesaw



$$\boxed{M_\nu \approx -v^2 Y_\nu \frac{1}{M_{\mathrm{R}}} Y_\nu^T}$$



Type-II Seesaw



$$\boxed{M_\nu \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}}$$

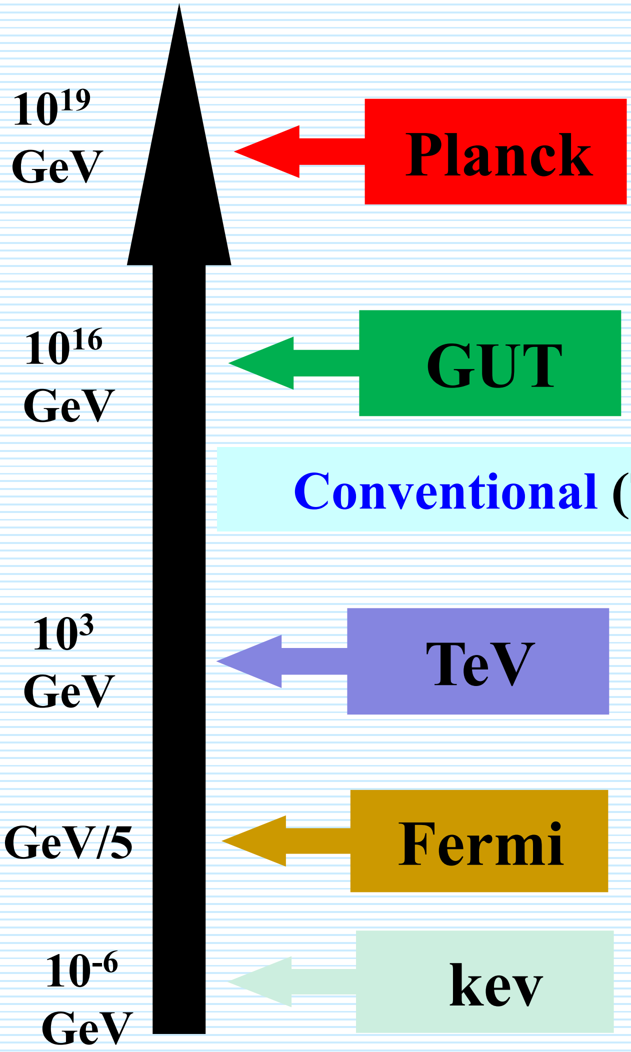
Type-I Seesaw: a right-handed Majorana neutrinos is added into the SM.

Type-II Seesaw: a few right-handed Majorana neutrinos and one Higgs triplet are both added into the SM.

Why TeV Seesaws?

Is the seesaw scale very close to a fundamental physics scale?

How heavy are the heavy Majorana neutrinos or the Higgs triplet?



10^{19} GeV — **Planck**

10^{16} GeV — **GUT** to unify strong, weak & electromagnetic forces?

Conventional (Type-one) Seesaw Picture: close to the GUT scale

10^{3} GeV — **TeV**

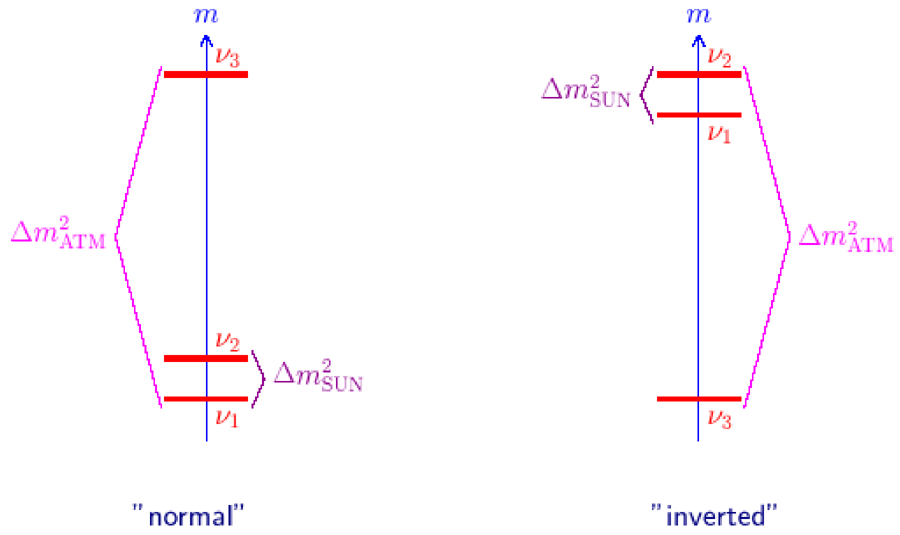
TeV Seesaw Idea: driven by testability at LHC

to solve the unnatural gauge hierarchy problem?

GeV/5 — **Fermi**

10^{-6} GeV — **kev** Hot dark mattter

Neutrinos mass spectrum

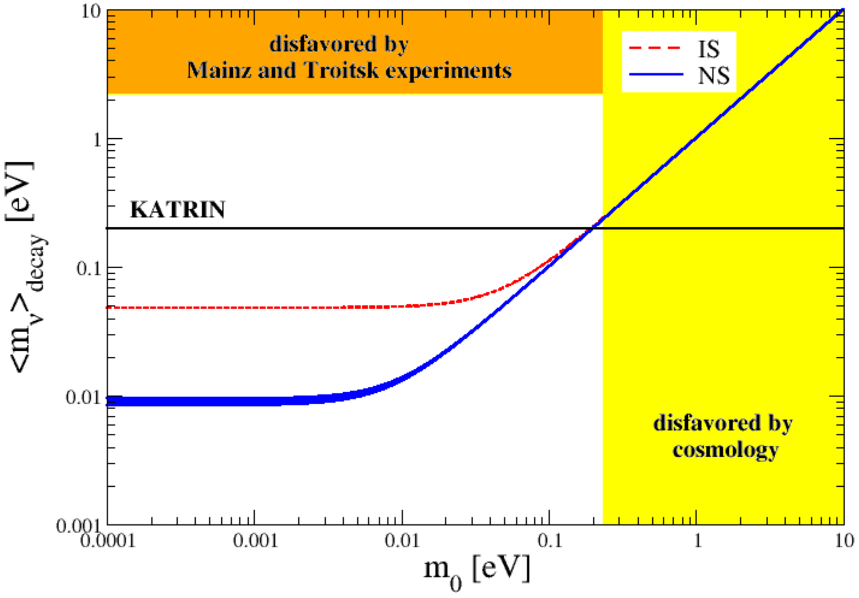


0νββ Measurements

$$\langle m_{\beta\beta}^2\rangle = \mid \sum_{i}^{n_\nu} U_{ei}^2 m_{\nu,i} \mid^2$$

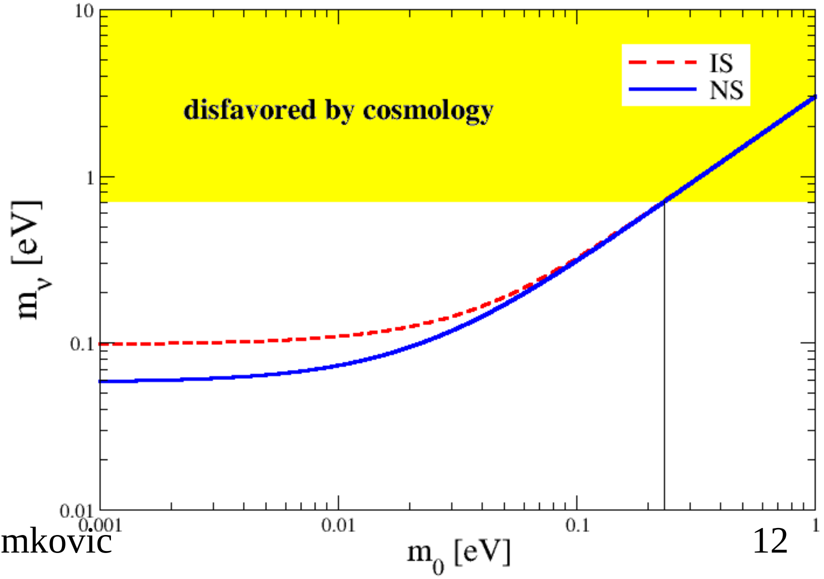
Beta Decay Measurements

$$m_\beta = \sqrt{\Sigma_{i=1}^3 |U_{ei}|^2 \; m_i^2}$$



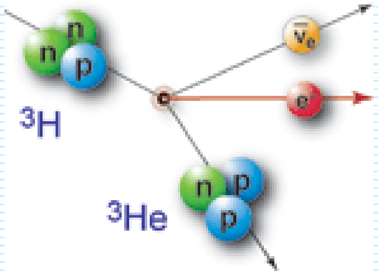
Cosmological Measurements

$$\Sigma_{i=1}^3 m_i$$



Measuring mass of neutrinos with β-decays of ^{3}H, ^{187}Re, ^{115}In and electron capture of ^{163}Ho

Tritium beta decay: $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$

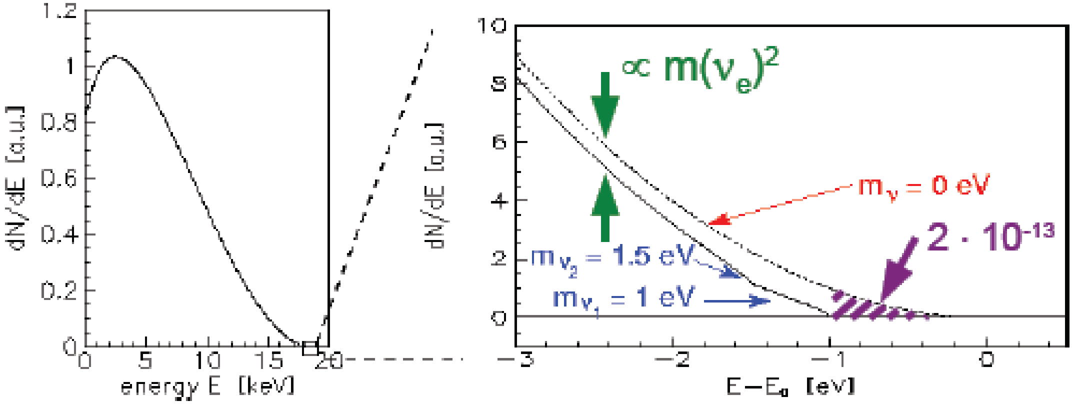


$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |M|^2 F(E) pE (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

1934 – Fermi pointed out that shape of electron spectrum in β–decay near the endpoint is sensitive to neutrino mass

First measured by Hanna and Pontecorvo with estimation $m_\nu \sim 1$ keV [Phys. Rev. 75, 983 (1940)]

**$Q = M_H - M_{He} - m_e$
= 1858 keV**



Troitsk

$m_\nu^2 = -2.3 \pm 2.5 \pm 2.0$ eV^2

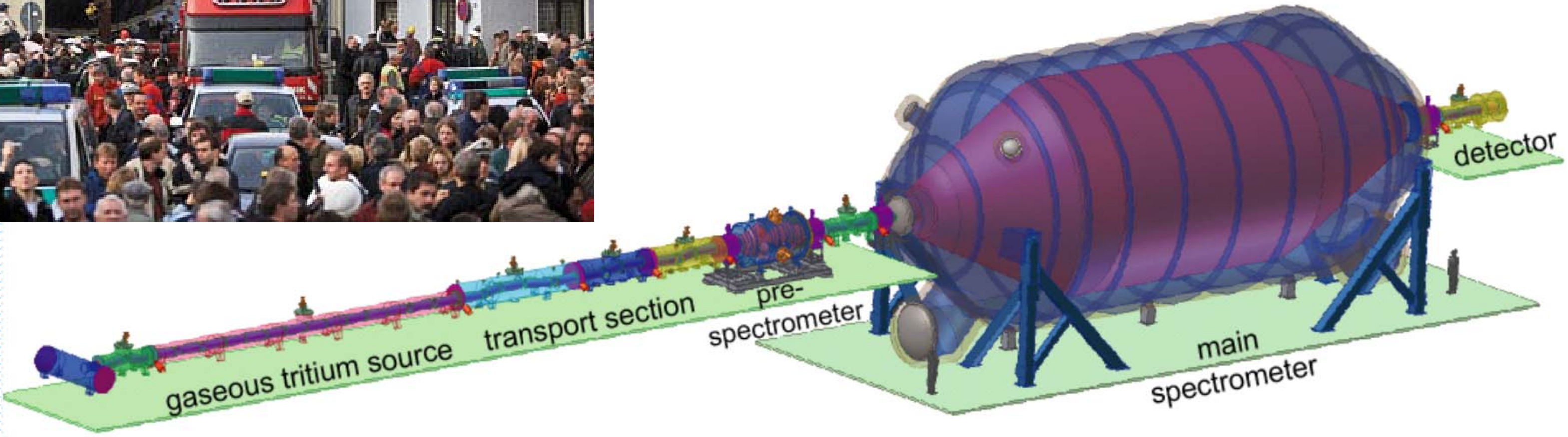
$m_\nu \leq$ 2.2 eV (95% CL.)

Mainz

$m_\nu^2 = -1.2 \pm 2.2 \pm 2.1$ eV^2

$m_\nu \leq$ 2.2 eV (95% CL.)

Karlsruhe TRItium Neutrino experiment (KATRIN)



$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 \; m_i^2}$$

Evidence for neutrino mass signal
KATRIN discovery potential:

No neutrino mass signal
KATRIN sensitivity

m_β = 0.35 eV (5σ)
m_β = 0.30 eV (3σ)

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2} < 0.2\ eV \qquad m_\beta \approx m_1$$

Relativistic approach to ^3H decay nuclear recoil (3.4 eV) taken into account

Standard approach

- **non-relativistic nuclear w.f.**
- **nuclear recoil neglected**
- **phase space analysis**

$$E_e^{\max} = M_i - M_f - m_\nu$$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\,(Q-T)\boxed{\sqrt{(Q-T)^2 - m_{\nu_e}^2}}$$

Relativistic EPT approach (Primakoff)

- **Analogy with n-decay**
  (^{3}H,^{3}He) ↔ (n,p)
- **nuclear recoil of 3.4 eV by $E_e^{\max}$**
- **relevant only phase space**

$$E_e^{\max} = \frac{1}{2M_f}\left[M_i^2 + m_e^2 - \left(M_f^2 - m_\nu^2\right)\right]$$

$$\begin{aligned}
\frac{d\Gamma}{dE_e} &= \frac{1}{(\pi)^3}(G_F\cos\theta_c)^2 F(Z,E_e)p_e \\
&\times \frac{M_i^2}{(m_{12})^2}\boxed{\sqrt{y\left(y+2m_\nu\frac{M_f}{M_i}\right)}} \\
&\times \left[(g_V+g_A)^2 y\left(y+m_\nu\frac{M_f}{M_i}\right)\frac{M_i^2(E_e^2-m_e^2)}{3(m_{12})^4}\right. \\
&\quad \underline{(g_V+g_A)^2}(y+m_\nu\frac{M_f+m_\nu}{M_i})\frac{(M_iE_e-m_e^2)}{m_{12}^2} \\
&\quad \times (y+M_f\frac{M_f+m_\nu}{M_i})\frac{(M_i^2-M_iE_e)}{m_{12}^2} \\
&\quad -(g_V^2-g_A^2)M_f\left(y+m_\nu\frac{(M_f+M_\nu)}{M_i}\right) \\
&\quad \times \frac{(M_iE_e-m_e^2)}{(m_{12})^2} \\
&\quad \left.+(g_V-g_A)^2E_e\left(y+m_\nu\frac{M_f}{M_i}\right)\right]
\end{aligned}$$

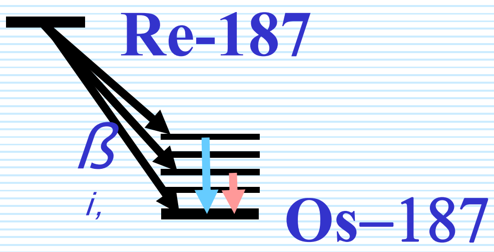
$$\begin{aligned}
y &= E_e^{max} - E_e \\
(m_{12})^2 &= M_i^2 - 2M_iE_e + m_e^2
\end{aligned}$$

Numerics:

Practically the same dependence of Kurie function on m_ν for $E_e \approx E_e^{max}$

F.Š., R. Dvornický, A. Faessler,
PRC 77 (2008) 055502

Bolometer experiments for ^{187}Re



- **Rhenium experiments (MANU, MIBETA, MARE)**

^{187}Re as ß-emitter: natural isotope content = 62.8 %

$$^{187}_{85}\mathrm{Re} \rightarrow {}^{187}_{86}\mathrm{Os} + e^- + \bar{\nu}_e$$

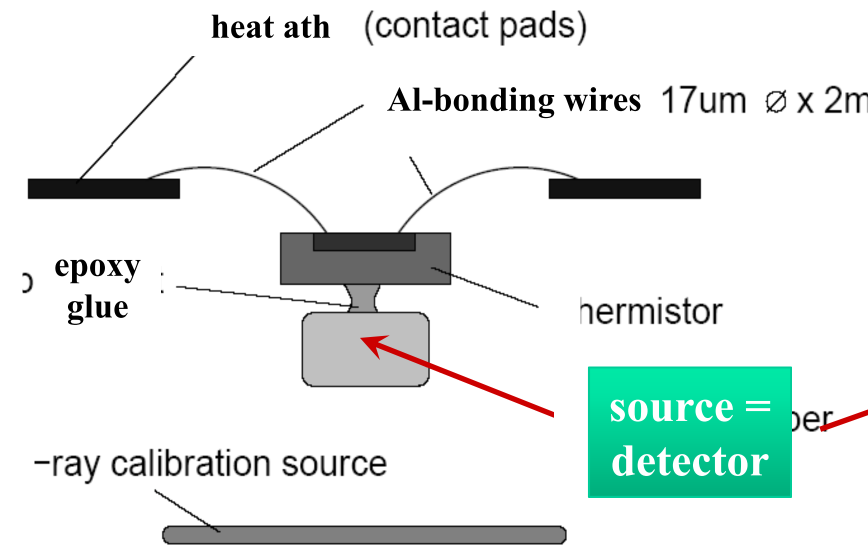
$5/2^+ \rightarrow 1/2^-$ ´unique´ 1st forbidden transition (shape factor), BEFS

^{187}Re: unique 1st	
E_0	2.47 keV
$t_{1/2}$	4.35 10^{10} y

- **previous ^{187}Re-experiments MANU, MIBETA**

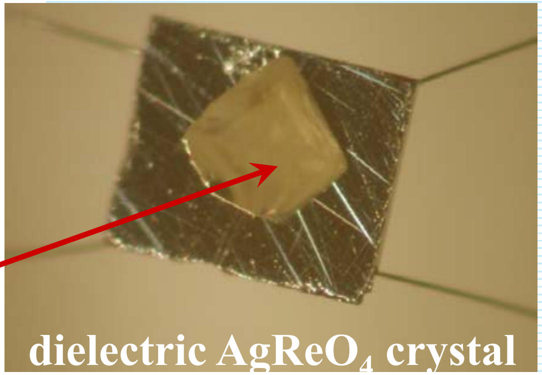
MANU: metallic Rhenium — group in Genova

MIBETA: dielectric $AgReO_4$ crystals — group in Milano



source = detector

measure entire ß-decay energy


dielectric $AgReO_4$ crystal

The entire energy is measured in the detector, except the neutrino, including the molecular & atomic excitations

MIBETA: 10 crystals

Spectrum of emitted electrons in rhenium β-decay

Dvornický, F. Š., Muto, Faessler, PPNP (2009)

$$\frac{d\Gamma}{dE}=\frac{G_F^2V_{ud}^2}{2\pi^3}\left|M\right|^2 pE(E_0-E)\sqrt{(E_0-E)^2-m_\nu^2}\,\frac{1}{3}R^2\left(p^2F_1(Z,E)+k^2F_0(Z,E)\right)$$

$$k=\sqrt{(E_0-E)^2-m_\nu^2}$$

Electron $p_{3/2}$ decay channel clearly dominates

$$\Gamma_S/\Gamma_P=1.011\times10^{-4}$$

In agreement with

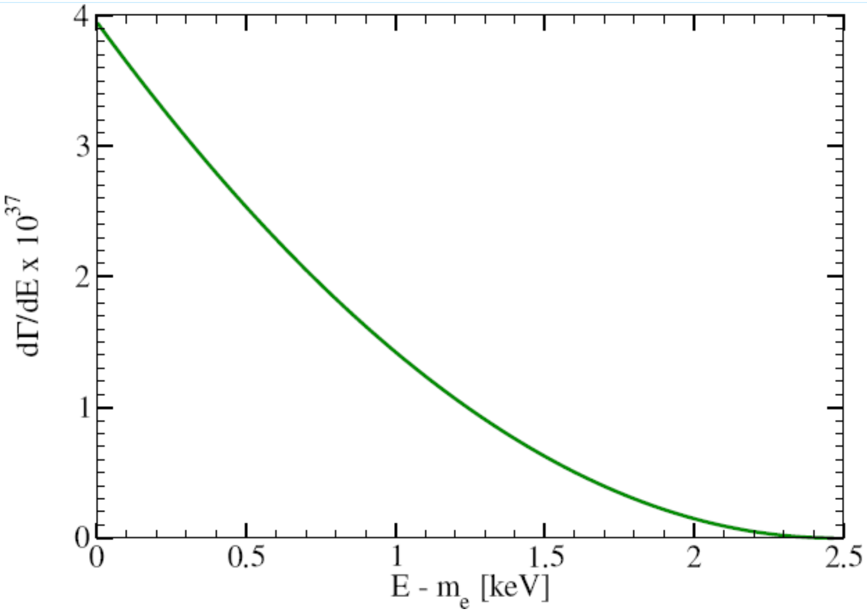
Arnaboldi et al.: PRL 96, 042503 (2006)

Electron in the $p_{3/2}$ state

$$p^{\max}\cong 50keV$$

Electron in the $s_{1/2}$ state

$$k^{\max}=2.47keV$$



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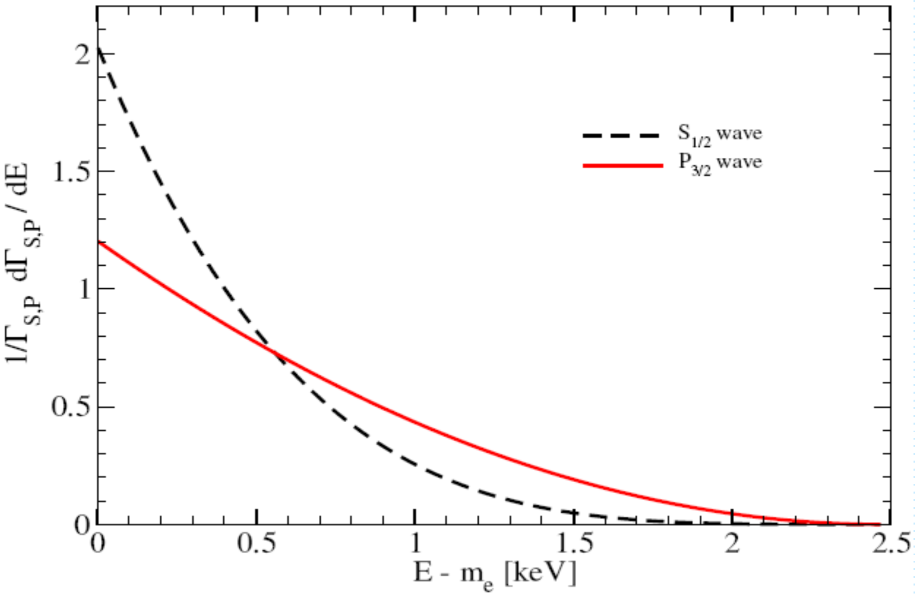
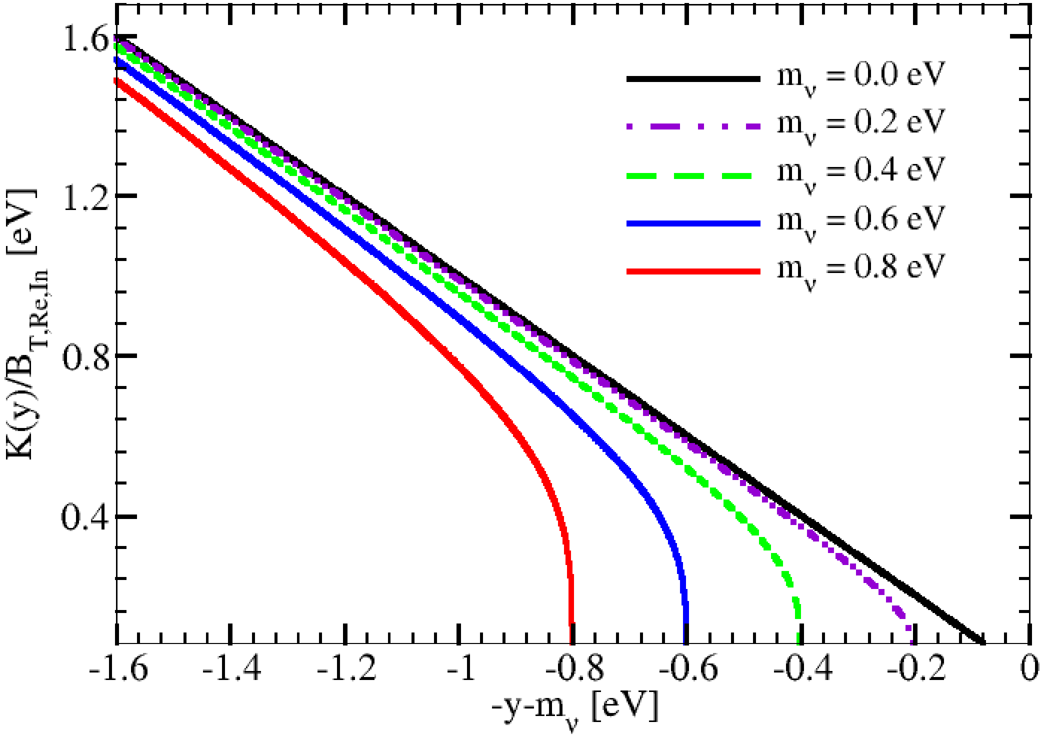


Kurie plots for tritium, rhenium and indium single β-decay

$$p^2 \frac{F_1(Z,E)}{F_0(Z,E)} \cong 1 + 2\frac{E-m_e}{m_e} \cong 1$$

$K(E)/B_{Re}$,$K(E)/B_{In} \cong K(y)/B_T$

Normalized Kurie functions become identical



ECHO exp.: Measuring electron-capture in ^{163}Ho

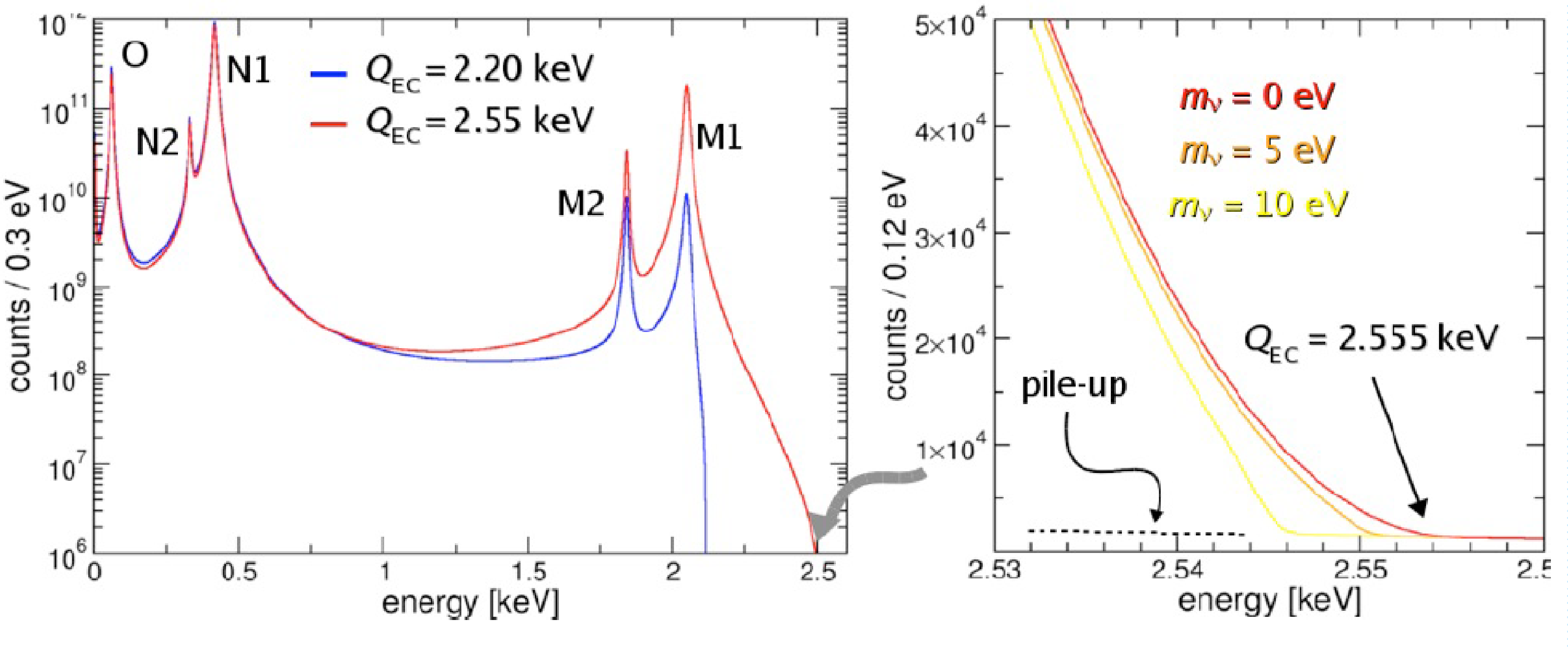
From ν phase space

$$\frac{d\Gamma}{dE_c} \propto \boxed{(Q-E_c)\sqrt{(Q-E_c)^2-m_\nu^2}}$$
$$*\sum_{\mathrm{H}} \varphi_{\mathrm{H}}^2(0)\, B_{\mathrm{H}} \frac{\Gamma_{\mathrm{H}}}{2\pi} \frac{1}{(E_c-E_{\mathrm{H}})^2+\Gamma_{\mathrm{H}}^2/4}$$
$$\Longrightarrow \mathcal{K}\,(Q-E_c)\sqrt{(Q-E_c)^2-m_\nu^2}\,,$$

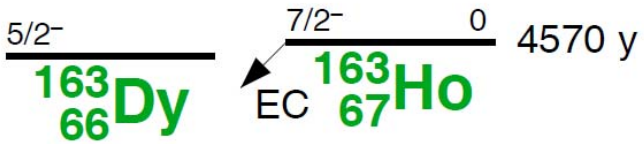
Not much progress in theory for a long period

$$E_c = Q - m_\nu$$

2 keV/Γ_M= 2keV/13 eV≈ 100



$1S_{1/2}$	$2S_{1/2}$	$2P_{1/2}$	$2P_{3/2}$	$3S_{1/2}$	$3P_{1/2}$	$3P_{3/2}$	$3D_{3/2}$	$3D_{5/2}$	...
K	L_I	L_{II}	L_{III}	M_I	M_{II}	M_{III}	M_{IV}	M_V	...



El	K	L_1	L_2	L_3	M_1	M_2	M_3	M_4	M_5	N_1	N_2	N_3	N_4	N_5
66 Dy	53.7885	9.0458	8.5806	7.7901	2.0468	1.8418	1.6756	1.3325	1.2949	0.4163	0.3318	0.2929	0.1542	0.1542
67 Ho	55.6177	9.3942	8.9178	8.0711	2.1283	1.9228	1.7412	1.3915	1.3514	0.4357	0.3435	0.3066	0.1610	0.1610

El	N_6	N_7	O_1	O_2	O_3	O_4	O_5	O_6	O_7	P_1	P_2	P_3	P_4	P_5
66 Dy	0.0042	0.0042	0.0629	0.0263	0.0263									
67 Ho	0.0037	0.0037	0.0512	0.0203	0.0203									

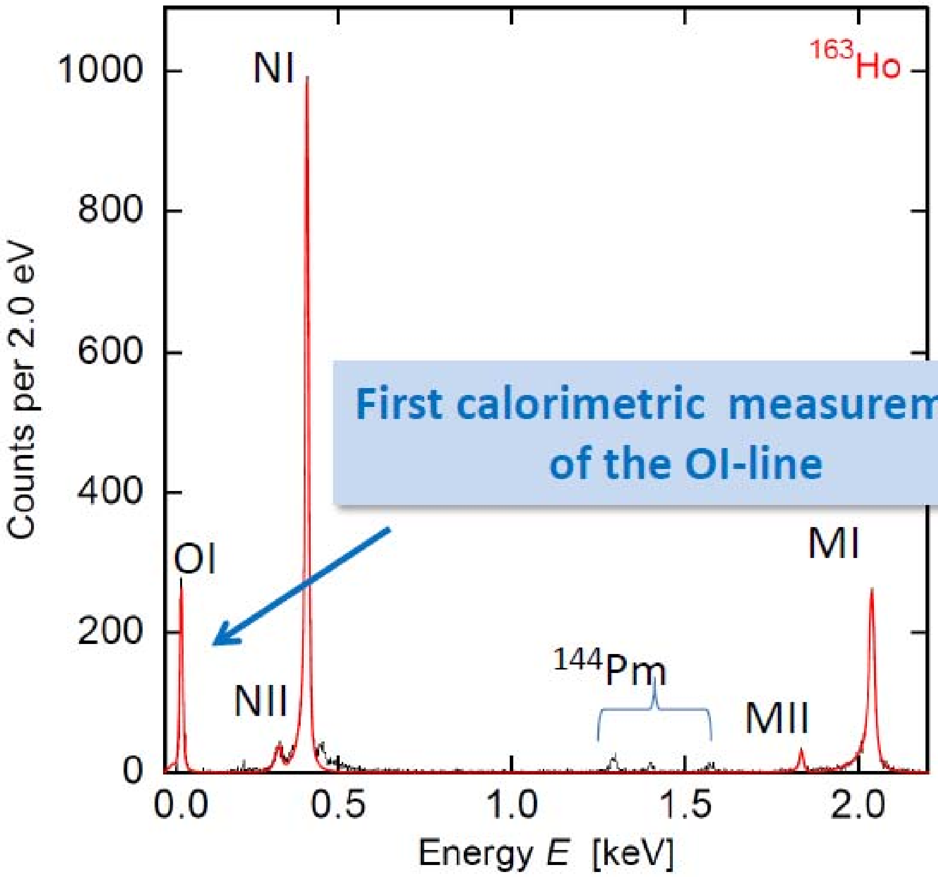
The same endpoint by capture of any tomic electron determined by the Q-value

Capture of K-electron also possible and realized through intermediate virtual state possible, but suppressed

Level	E_i [eV]	Γ_i [eV]	$\beta_i^2/\beta_{M_1}^2$
M_1	2047	13.2	1.0
M_2	1842	6.0	0.0526
N_1	414.2	5.4	0.2329
N_2	333.5	5.3	0.0119
O_1	49.9	3.0	0.0345
O_2	26.3	3.0	0.0015

Neutrino in $s_{1/2}$-wave

Presently most precise ^{163}Ho spectrum



First calorimetric measurement of the OI-line

Ratio of peaks, widths, fine structure

	E_H lit.	E_H exp.	Γ_H lit.	Γ_H exp
MI	2.047	2.040	13.2	13.7
MII	1.845	1.836	6.0	7.2
NI	0.420	0.411	5.4	5.3
NII	0.340	0.333	5.3	8.0
OI	0.050	0.048	5.0	4.3

Q_{EC} =(2.80±0.08) keV

Great progress in measurement of X/γ spectra
(many important details for theory)

Progress in theoretical description needed

^{163}Ho →^{163}Dy*

Atoms in mettallic compounds
X-ray spectrum
Auger electron emission

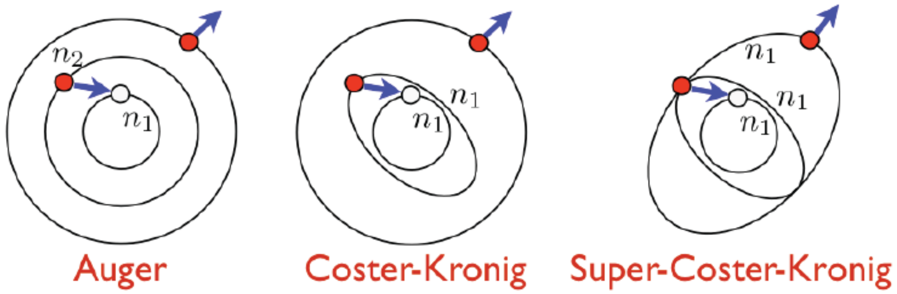
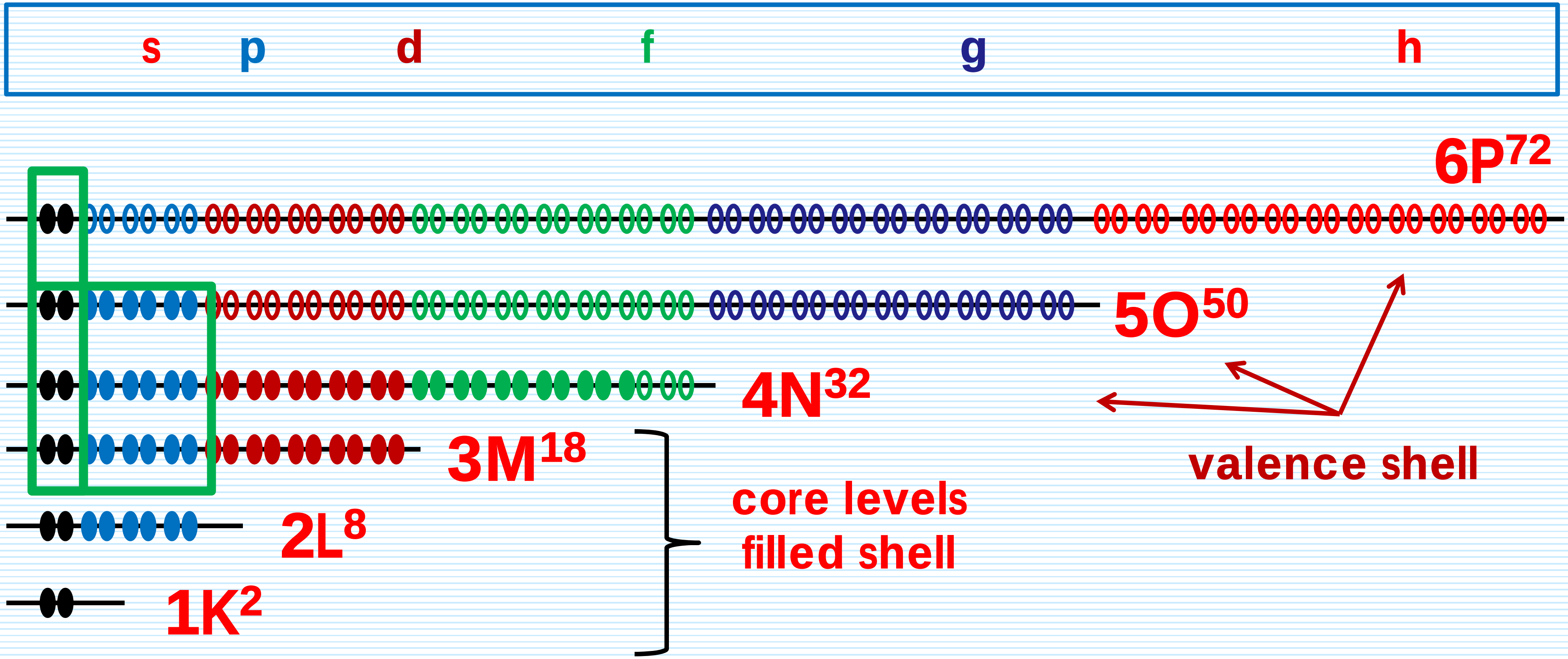


FIG. 10: The names of electron-ejection de-excitations. The holes left by EC are the hollow circles. The n_i are principal quantum numbers, which in the right-most figure all coincide.

$^{163}Ho_{67}$ electron shell



s p d f g h

$6P^{72}$

$5O^{50}$

$4N^{32}$

$3M^{18}$

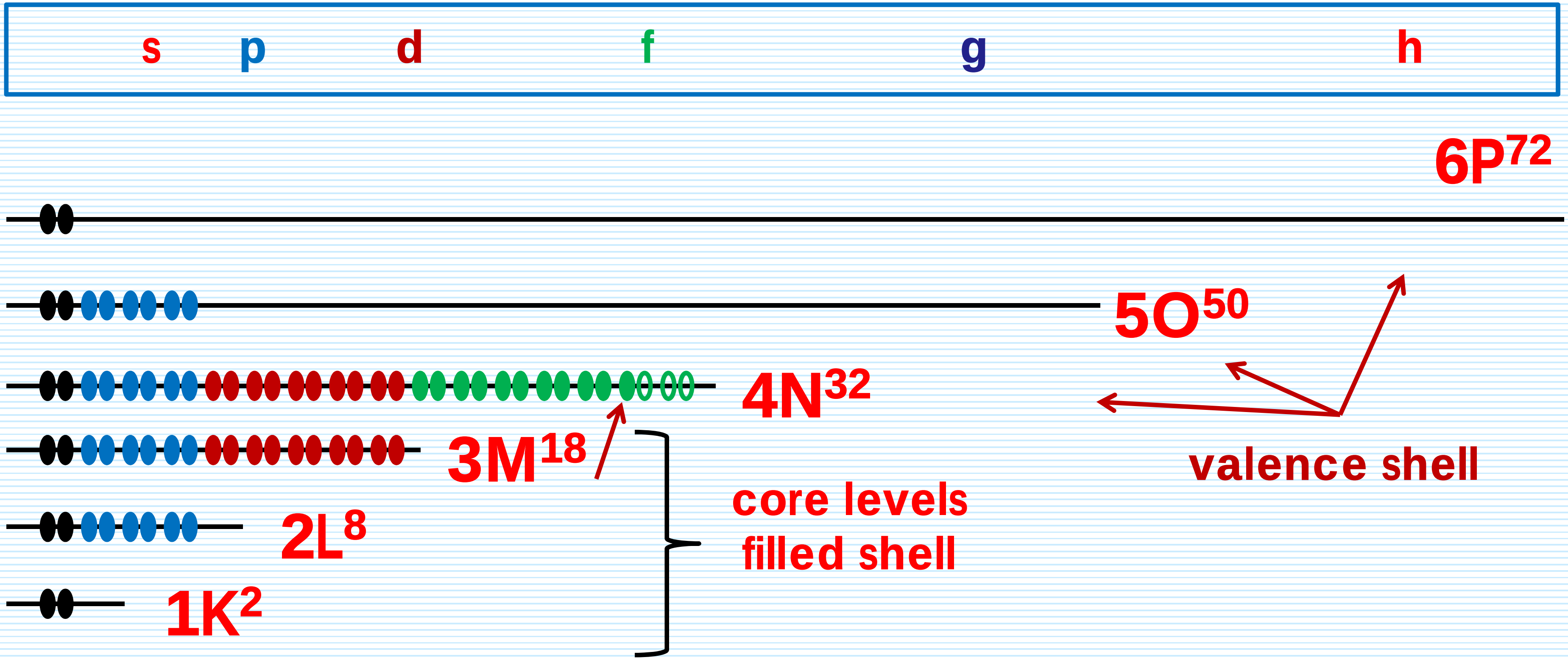
$2L^{8}$

$1K^{2}$

core levels
filled shell

valence shell

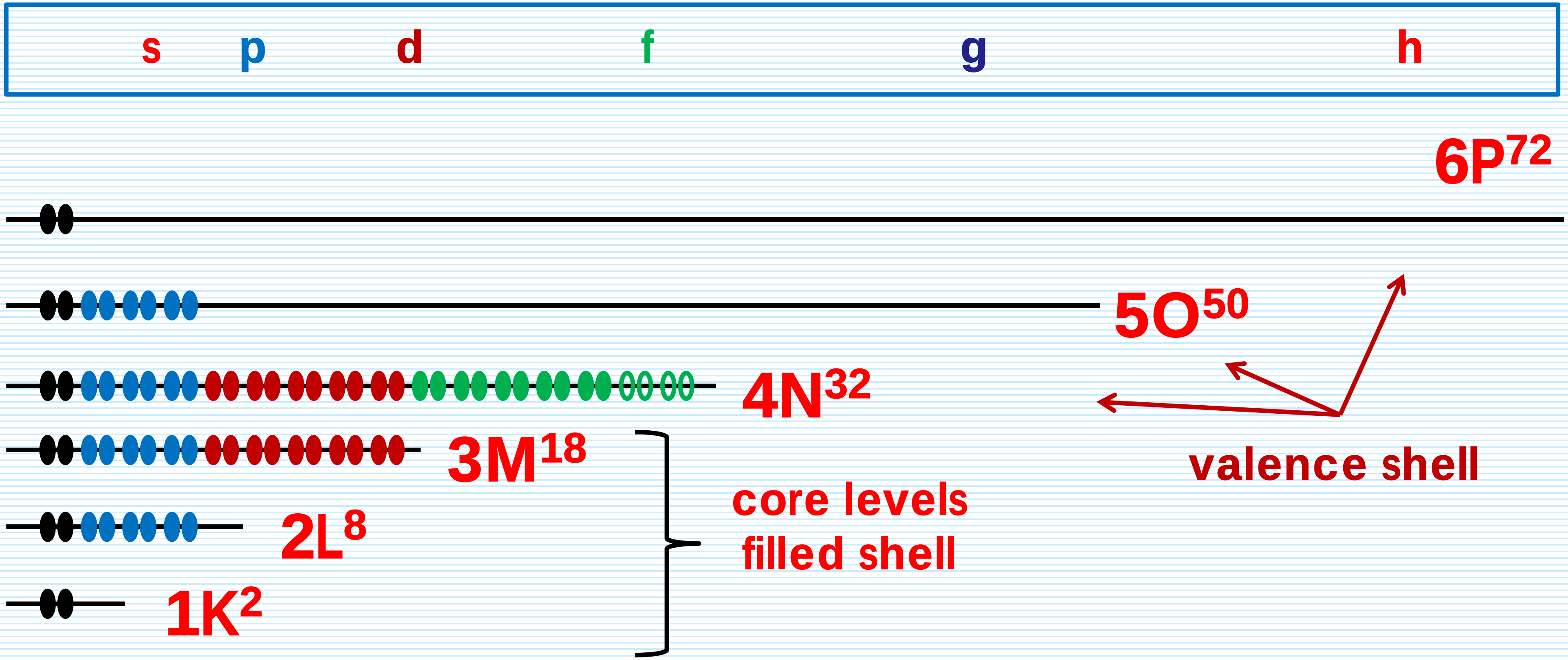
$^{163}Ho_{67}$ electron shell



valence shell

core levels
filled shell

$^{163}Dy_{66}$ electron shell



s p d f g h

$6P^{72}$

$5O^{50}$

$4N^{32}$

$3M^{18}$

$2L^{8}$

$1K^{2}$

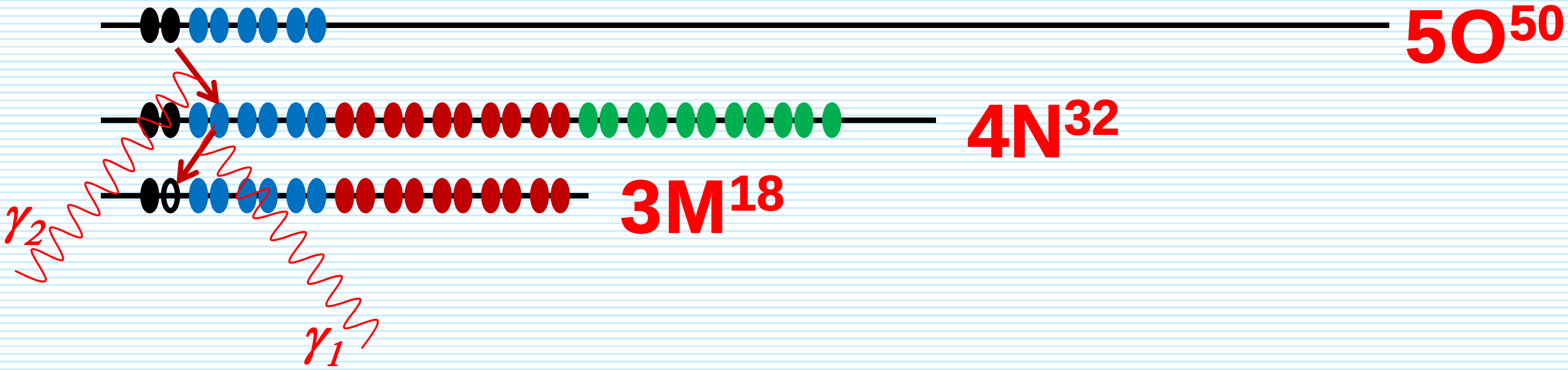
core levels
filled shell

valence shell

X-rays spectrum

In the process of de-excitation, X-ray photons are emitted with energies clustered around

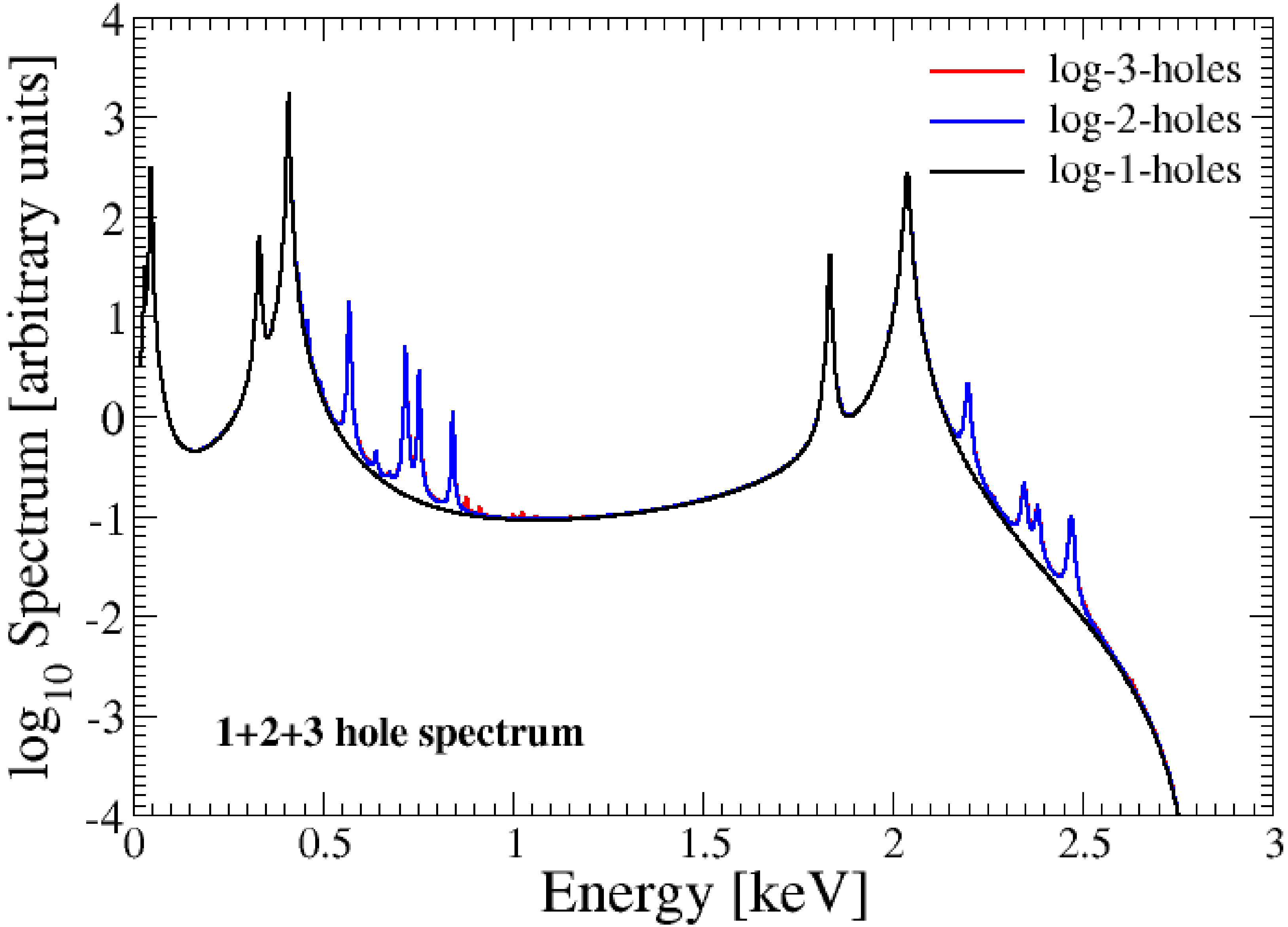
$$\omega_{\text{X-ray}} = E' - E$$

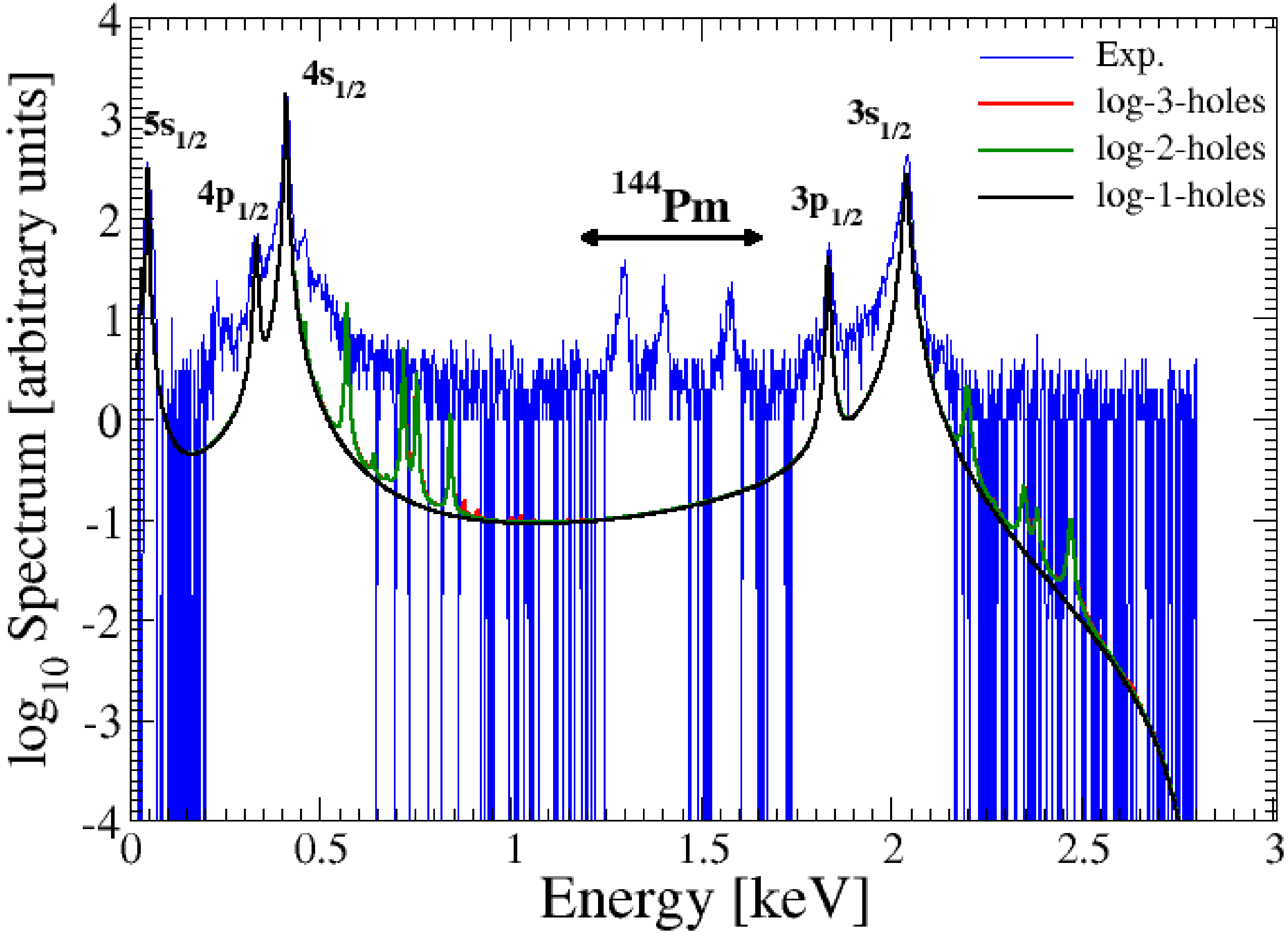


In transitions of such type, dipole transitions are dominant

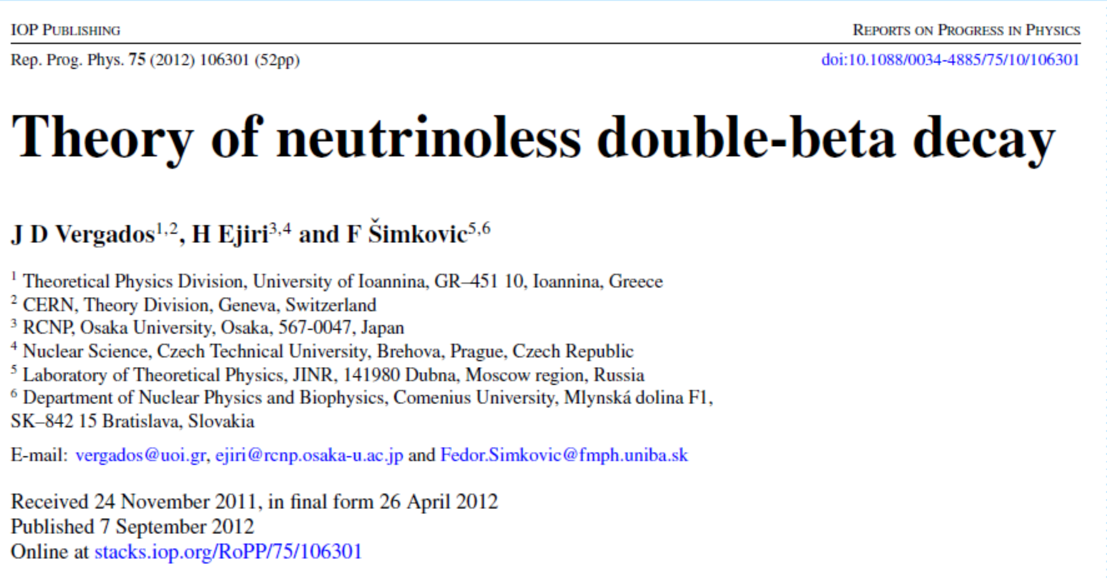
Total energy distribution?

A. Faessler, Ch. Enss, L. Gastaldo, F.Š., PRC 91, 064302 (2015) (two and 3 holes)
R.G.H. Robertson, PRC 91, 035504 (2015) (two-holes)





5s$_{1/2}$
4p$_{1/2}$
4s$_{1/2}$
^{144}Pm
3p$_{1/2}$
3s$_{1/2}$
Exp.
log-3-holes
log-2-holes
log-1-holes
log$_{10}$ Spectrum [arbitrary units]
Energy [keV]
4
3
2
1
0
-1
-2
-3
-4
0
0.5
1
1.5
2
2.5
3



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Theory of neutrinoless double-beta decay

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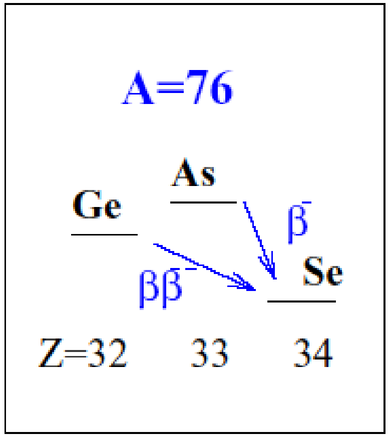
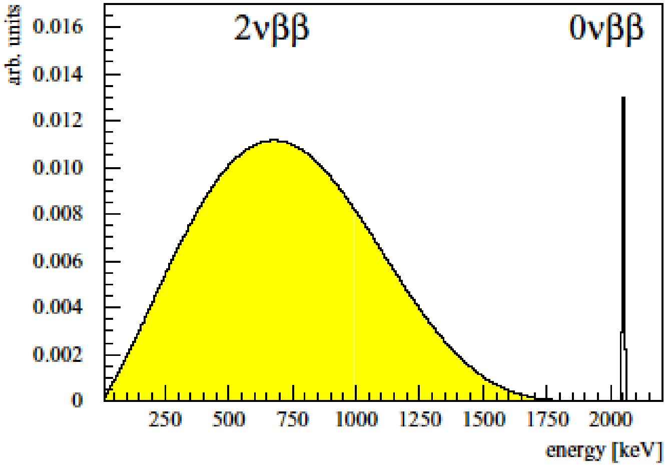
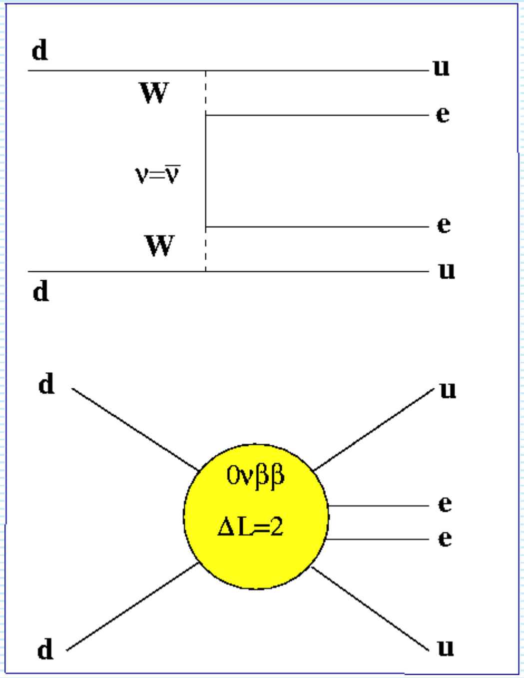
Neutrinoless Double-Beta Decay

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$







The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

What is the nature of neutrinos?

Actually, when NMEs will be needed to analyze data?

ν $\Rightarrow$

GUT's

Only the 0νββ-decay can answer this fundamental question

Analogy with kaons: K_0 and $\bar{K}_0$

Could we have both?
(light Dirac and heavy Majorana)

Analogy with π_0

1937 Beginning of Majorana neutrino physics

Ettore Majorana discoveres the possiility of existence of truly neutral fermions

Charged fermion (electron) + electromagnetic field

$$\left(i\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu} - m\right)\Psi = 0$$

$$\left(i\gamma^{\mu}\partial_{\mu} + e\gamma^{\mu}A_{\mu} - m\right)\Psi^{c} = 0$$

$\Psi^{c} = \Psi$ **forbidden**

Neutral fermion (neutrino) + electromagnetic field

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\nu = 0$$

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\nu^{c} = 0$$

$\nu^{c} = \nu$ **allowed**

Majorana condition

Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Here is the beginning of Nonstandard Neutrino Properties

Light ν-exchange 0νββ–decay mechanism

S.M. Bilenky, S. Petcov, Rev. Mod. Phys. 59, 671 (1987)

Majorana condition

$$C\,\overline{\chi_k}^T(x) \;=\; \xi_k\;\chi_k(x)$$

Majorana particle propagator

$$\begin{aligned} <\chi_\alpha(x_1)\overline{\chi}_\beta(x_2)> &= \frac{-1}{(2\pi)^4}\int\left(\frac{1}{\gamma p - im}\right)_{\alpha\beta} e^{ip(x_1-x_2)}dp \\ &= S_{\alpha\beta}(x_1-x_2) \end{aligned}$$

$$\begin{aligned} <\chi(x_1)\chi^T(x_2)> &= -\xi S(x_1-x_2)C \\ <\overline{\chi}^T(x_1)\overline{\chi}(x_2)> &= \xi C^{-1}S(x_1-x_2) \end{aligned}$$

Weak β-decay Hamiltonian

$$\mathcal{H}_W^\beta = \frac{G_F}{\sqrt{2}}\,\bar{e}\gamma_\alpha(1+\gamma_5)\nu_e\; j_\alpha + h.c.$$

Neutrino mixing

$$\nu_{eL} \;=\; \sum_k U^L_{lk}\;\chi_{kL}$$

S-matrix term

$$S^{(2)} = -\frac{(-i)^2}{2} 4 \left(\frac{G_F}{\sqrt{2}}\right)^2 \int N\left[\overline{e_L}(x_1)\gamma_\alpha < \nu_{eL}(x_1)\nu^T_{eL}(x_2) > \gamma^T_\beta \overline{e_L}^T(x_2)\right] \times$$

$$T\left(j_\alpha(x_1) j_\beta(x_2) e^{-i\int \mathcal{H}_{str}(x)dx}\right) dx_1 dx_2$$

Contraction of ν-fields

$$\begin{aligned} < \nu_{eL}(x_1)\nu_{eL}{}^T(x_2) > &= -\sum_k \left(U^L_{ek}\right)^2 \xi_k \frac{1+\gamma_5}{2} S_k(x_1 - x_2)\frac{1+\gamma_5}{2} C \\ &= \frac{i}{(2\pi)^4}\sum_k \left(U^L_{ek}\right)^2 \xi_k m_k \int \frac{e^{iq(x_1-x_2)}dq}{q^2+m_k^2}\frac{1+\gamma_5}{2} C \end{aligned}$$

Effective mass of Majorana neutrinos

$$m_{\beta\beta} = \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k$$

0νββ-decay matrix element

$$
\begin{aligned}
< f|S^{(2)}|i > \;=\; & m_{\beta\beta}\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\overline{u}(p_1)\gamma_\alpha(1+\gamma_5)\gamma_\beta C\overline{u}^T(p_2)\times \\
& \int e^{-ip_1x_1}e^{-ip_2x_2}\frac{-i}{(2\pi)^4}\int\frac{e^{iq(x_1-x_2)}dq}{q^2}\times \\
& < A'|T\left[J_\alpha(x_1)J_\beta(x_2)\right]|A > dx_1dx_2-(p_1\leftrightarrow p_2)
\end{aligned}
$$

Use of completness $1=\Sigma_n|n><n|$

$$
\begin{aligned}
< A'|J_\alpha(x_1)J_\beta(x_2)|A > \;=\; & \sum_n < A'|J_\alpha(0,\vec{x}_1)|n >< n|J_\beta(0,\vec{x}_2)|A > \times \\
& e^{-i(E'-E_n)x_{10}}e^{-i(E_n-E)x_{20}}
\end{aligned}
$$

After integration over time variables

$$
\begin{aligned}
< f|S^{(2)}|i > \;=\; & im_{\beta\beta}\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\overline{u}(p_1)\gamma_\alpha(1+\gamma_5)\gamma_\beta C\overline{u}^T(p_2) \\
& \times\int d\vec{x}_1d\vec{x}_2e^{-i\vec{p}_1\cdot\vec{x}_1}e^{-i\vec{p}_2\cdot\vec{x}_2}\frac{1}{(2\pi)^3}\int\frac{e^{i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}d\vec{q}}{\vec{q}^2}\times \\
& \sum_n\left(\frac{< A'|J_\alpha(0,\vec{x}_1)|n >< n|J_\beta(0,\vec{x}_2)|A >}{E_n+q_0+p_{20}-E}+\right. \\
& \left.\frac{< A'|J_\beta(0,\vec{x}_1)|n >< n|J_\alpha(0,\vec{x}_2)|A >}{E_n+q_0+p_{10}-E}\right) \\
& \times\; 2\pi\delta(E'+p_{10}+p_{20}-E)
\end{aligned}
$$

Approximations and simplifications

1) Non-relativistic impulse approx. for nuclear current

$$J_\alpha(0,\vec{x}) = \sum_n \tau_n^+(\delta_{\alpha 4} + i g_A (\vec{\sigma})_k \delta_{\alpha k})\delta(\vec{x}-\vec{x}_n)$$

2) Long-wave approximation for lepton wave functions

$$e^{-i\vec{p}_1\cdot\vec{x}_1 - i\vec{p}_2\cdot\vec{x}_2} \longrightarrow 1$$

3) Closure approximation

$$E_n \;\longrightarrow\; < E_n >$$

$$< f|S^{(2)}|i > = \overline{u}(p_1)\gamma_\alpha(1+\gamma_5)\gamma_\beta C\overline{u}^T(p_2)A_{\alpha\beta}, \quad A_{\alpha\beta} = A_{\beta\alpha}$$

Hadron part is symmetric

$$J_\alpha(0,\vec{x}_1)J_\beta(0,\vec{x}_2) = J_\beta(0,\vec{x}_2)J_\alpha(0,\vec{x}_1)$$

contribute

$$\gamma_\alpha\gamma_\beta = \delta_{\alpha\beta} + \frac{1}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)$$

0νββ-decay matrix element

$$\begin{aligned} < f|S^{(2)}|i > \quad &= \quad i\, m_{\beta\beta}\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\overline{u}(p_1)(1-\gamma_5)C\overline{u}^T(p_2)\frac{1}{R} \\ &\times \left(M_F - g_A^2 M_{GT}\right)\delta(p_{10}+p_{20}+M'-M) \end{aligned}$$

Nuclear matrix elements

$$M_F = < A'|\sum_{n,m}\tau_n^+\tau_m^+ h(|\vec{x}_n - \vec{x}_m|)|A >$$

$$M_F = < A'|\sum_{n,m}\tau_n^+\tau_m^+ h(|\vec{x}_n - \vec{x}_m|)\vec{\sigma}_n \cdot \vec{\sigma}_m|A >$$

Neutrino exchange potential

$$\begin{aligned} h(|\vec{x}_n - \vec{x}_m|) &= \frac{1}{2\pi^2}\int \frac{e^{i\vec{q}\cdot\vec{x}}d\vec{q}}{q_0(q_0+ < E_n > -(E+E')/2)} \\ &\approx \frac{1}{|\vec{x}|} \end{aligned}$$

Differential 0νββ-decay rate

$$\begin{aligned} d\Gamma_{0\nu} &= \frac{1}{2}\frac{G_F^4 m_e^5}{(2\pi)^5}|m_{\beta\beta}|^2\frac{1}{R^2}|M_F - g_A^2 M_{GT}|^2(1-\cos\theta) \\ &\quad F^2(Z)(\varepsilon_0 - \varepsilon + 1)^2(\varepsilon+1)d\varepsilon\sin\theta d\theta \end{aligned}$$

$$F(Z) = \frac{2\pi\alpha(Z+2)}{1 - exp[-2\pi\alpha(Z+2)]} \qquad \varepsilon_0 = \frac{1}{m_e}(M - M' - 2m_e)$$

Full 0νββ-decay rate

$$\begin{aligned} \Gamma_{0\nu} &= \frac{1}{2}\frac{G_F^4 m_e^5}{(2\pi)^5}|m_{\beta\beta}|^2\frac{1}{R^2}|M_F - g_A^2 M_{GT}|^2 F^2(Z) \\ &\quad \times \frac{1}{15}\left(\varepsilon_0^5 + 10\varepsilon_0^4 + 40\varepsilon_0^3 + 60\varepsilon_0^2 + 30\varepsilon_0\right) \end{aligned}$$

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

Absolute ν mass scale

Normal or inverted hierarchy of ν masses

CP-violating phases

Бруно Понтекорво

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.

Effective mass of Majorana neutrinos

$$m^{\rm vac}_{\beta\beta} = \sum_i (U^L_{ei})^2 m_i$$

Majorana phases

$$\begin{aligned} P &= diag(e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, e^{-i\alpha_3/2}) \\ \alpha_3/2 &= \delta \end{aligned}$$

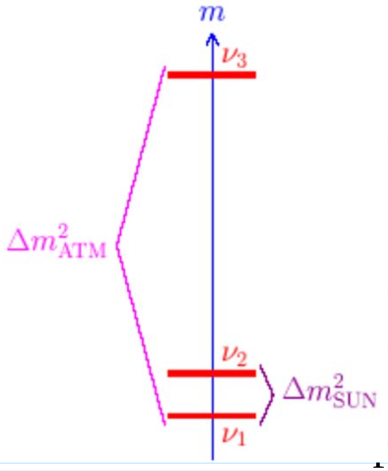
$$\begin{aligned} |m_{\beta\beta}| &= |c^2_{12}c^2_{13}e^{i\alpha_1}m_1 \\ &\quad +s^2_{12}c^2_{13}e^{i\alpha_2}m_2 + s^2_{13}m_3| \end{aligned}$$

Measured quantity

$$\begin{aligned} |m_{\beta\beta}|^2 &= c^4_{12}c^4_{13}m^2_1 + s^4_{12}c^4_{13}m^2_2 + s^4_{13}m^2_3 \\ &\quad +2c^2_{12}s^2_{12}c^4_{13}m_1 m_2 \cos{(\alpha_1 - \alpha_2)} \\ &\quad +2c^2_{12}c^2_{13}s^2_{13}m_1 m_3 \cos{\alpha_1} + 2s^2_{12}c^2_{13}s^2_{13}m_2 m_3 \cos{\alpha_2}. \end{aligned}$$

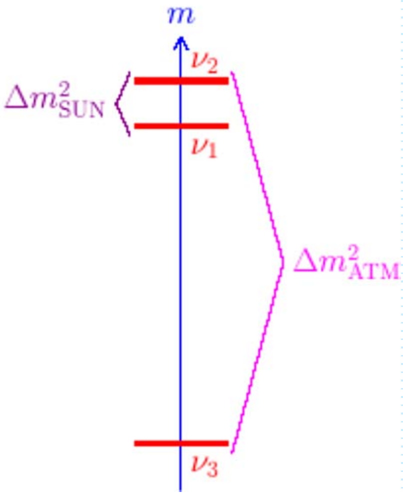
Normal hierarchy

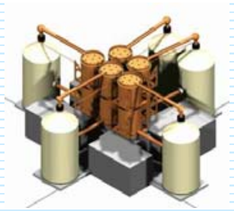
$$\begin{aligned} m_1 &\ll \sqrt{\Delta m^2_{\rm SUN}} \\ m_2 &\simeq \sqrt{\Delta m^2_{\rm SUN}} \\ m_3 &\simeq \sqrt{\Delta m^2_{\rm ATM}} \end{aligned}$$



Inverted hierarchy

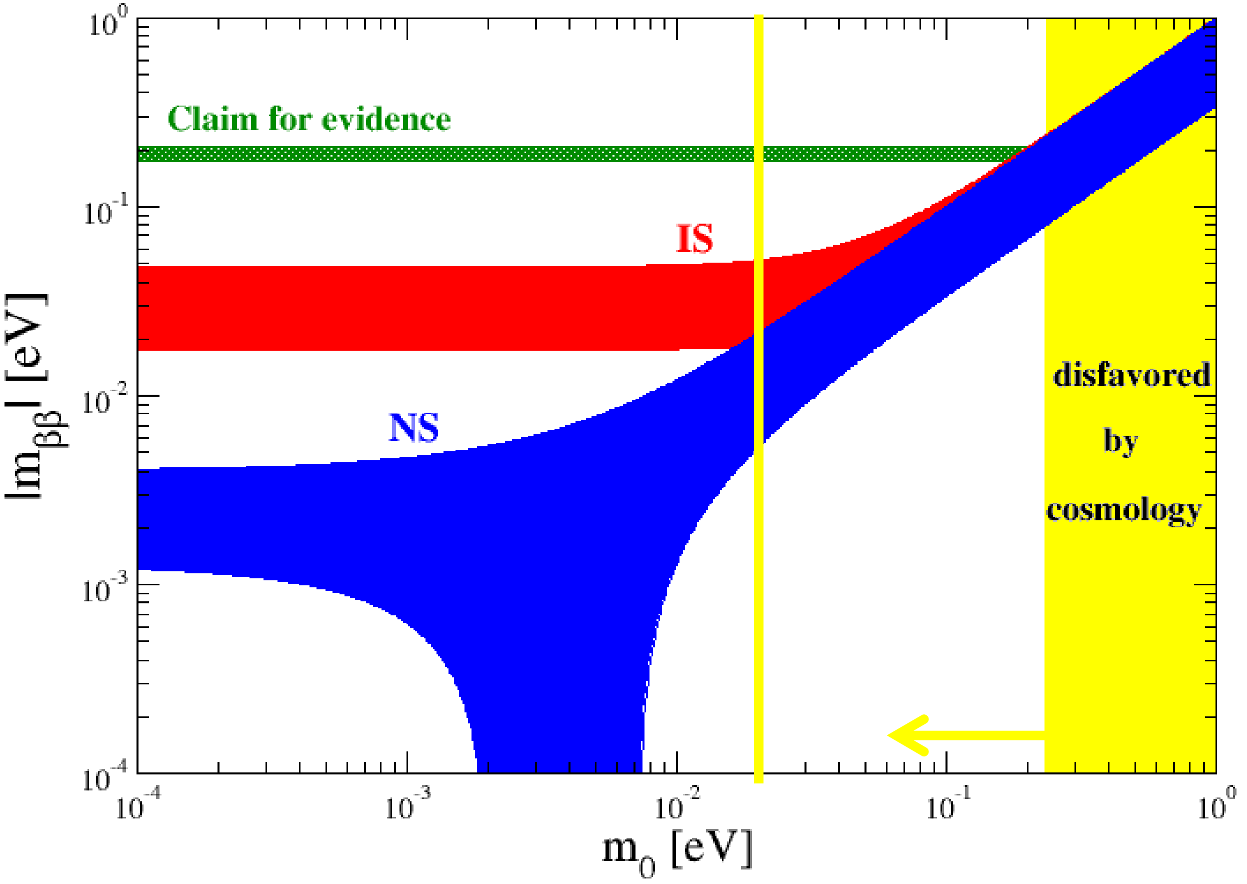
$$\begin{aligned} m_3 &\ll \sqrt{\Delta m^2_{\rm ATM}} \\ m_1 &\simeq m_2 \simeq \sqrt{\Delta m^2_{\rm ATM}} \end{aligned}$$





Complementarity of 0νββ-decay, β-decay and cosmology

Issue: Lightest neutrino mass m_0



GUT's

β-decay (Mainz, Troitsk)

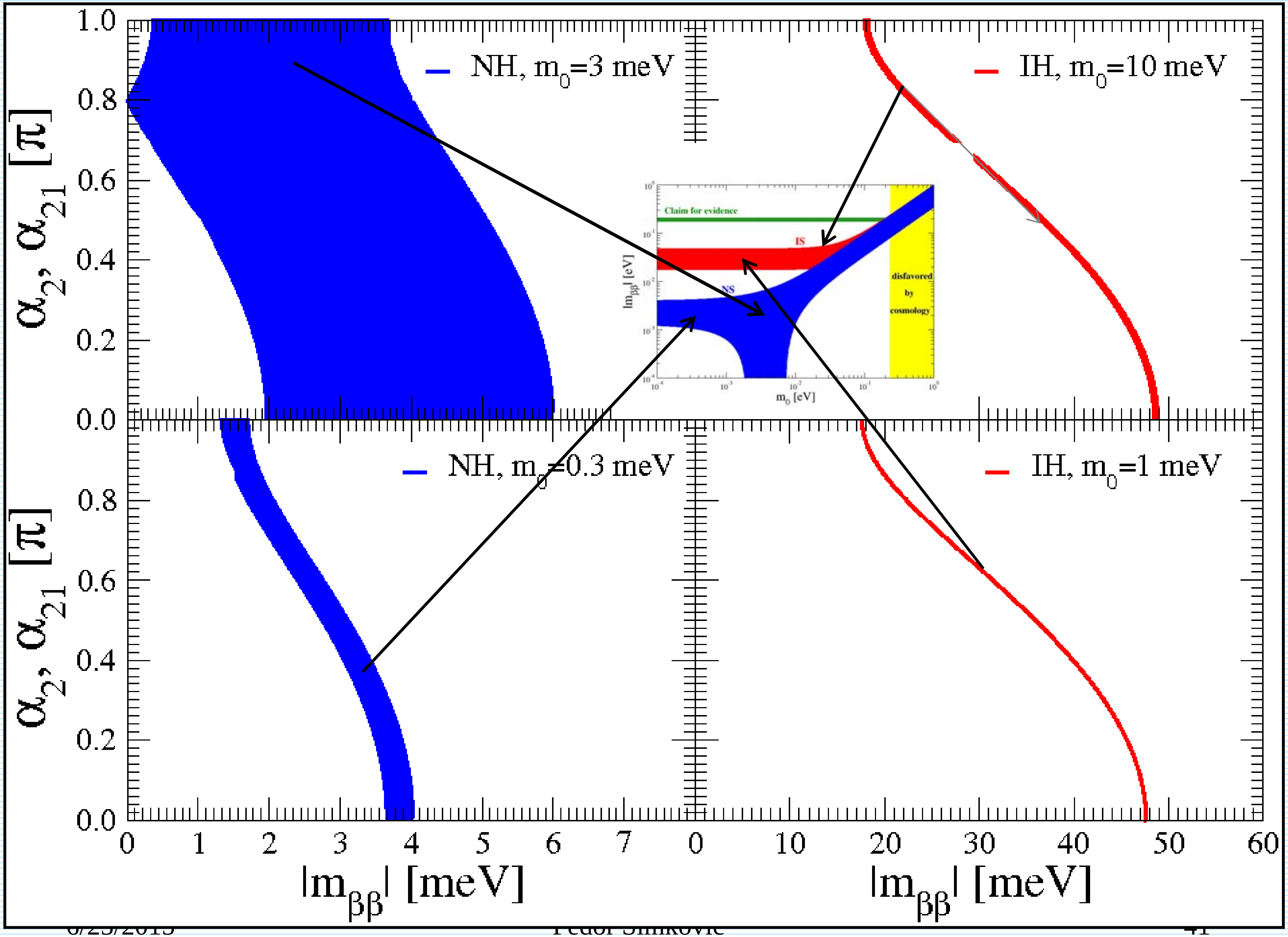
$$m_\beta^2 = \sum_i |U_{ei}^L|^2 m_i^2 \le (2.2\ \mathrm{eV})^2$$

KATRIN: $(0.2\ \mathrm{eV})^2$

Cosmology (Planck)

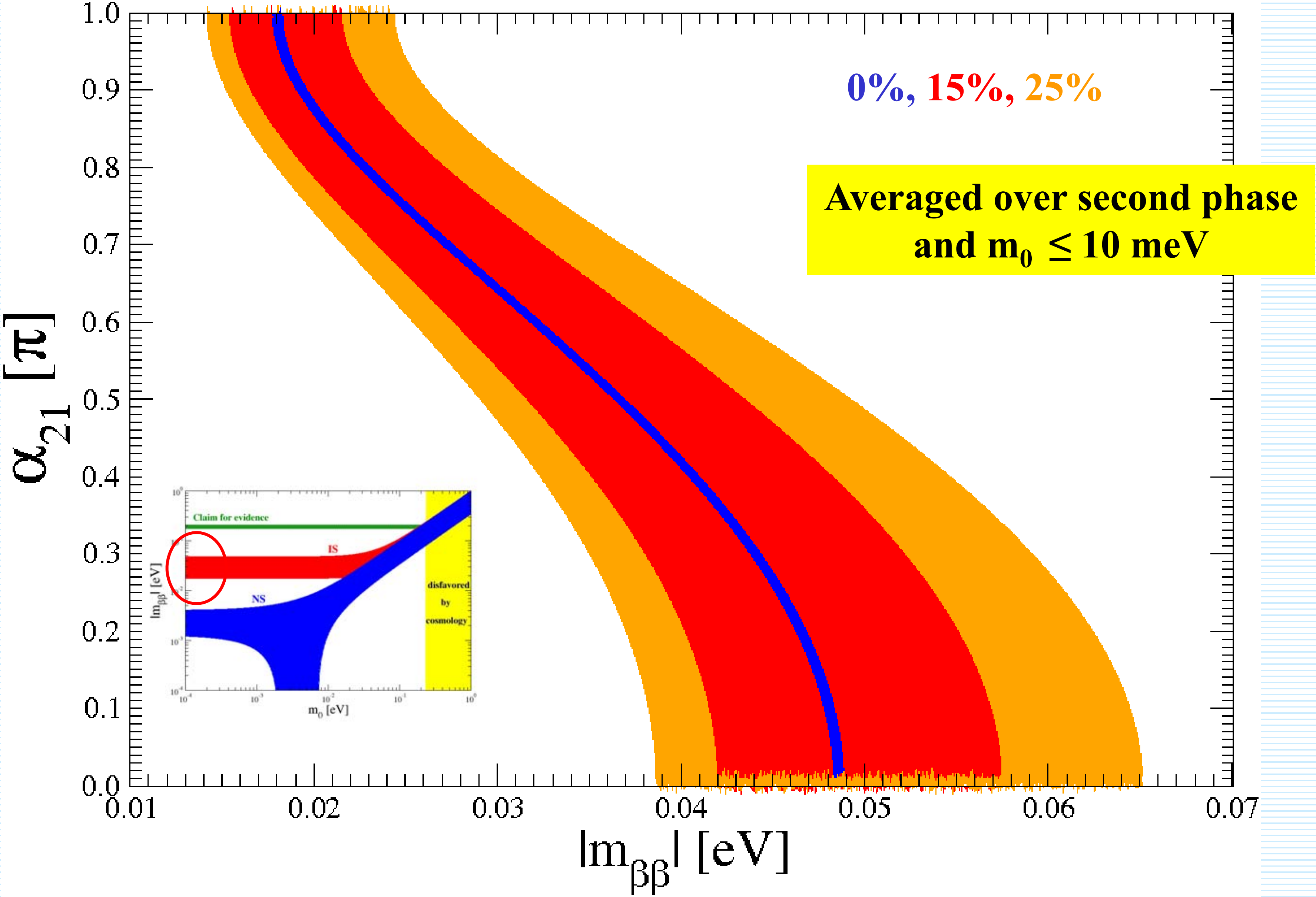
$$\sum_i m_i \le 0.23\text{–}1.08\ \mathrm{eV}$$

$$m_0 \le 0.07\ \mathrm{eV}$$


NH, m0=3 meV
IH, m0=10 meV
NH, m0=0.3 meV
IH, m0=1 meV
α2, α21 [π]
α2, α21 [π]
|mββ| [meV]
|mββ| [meV]
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
10
20
30
40
50
60
Claim for evidence
IS
NS
disfavored by cosmology
|mββ| [eV]
m0 [eV]

$$|m_{\beta\beta}| = \frac{1}{\sqrt{T^{0\nu}_{1/2} G^{0\nu}(Q_{\beta\beta}, Z)}|M'^{0\nu}|}$$

$$\frac{\sigma_{\beta\beta}}{|m_{\beta\beta}|^{obs}} = \sqrt{\frac{1}{4}\left(\frac{\sigma_{exp}}{T^{0\nu-obs}_{1/2}}\right)^2 + \left(\frac{\sigma_{th}}{|M'^{0\nu}|}\right)^2}$$



(3+1) mixing

(3+2) mixing

6 angles
3+3 =6 phases

$$\begin{aligned} U &= R_{34}\tilde{R}_{24}\tilde{R}_{14} \\ & \quad R_{23}\tilde{R}_{13} \\ & \quad R_{12}P \\ P &= diag\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, e^{i(\alpha_3/2+\delta_{13})}, e^{i\delta_{14}}\right) \end{aligned}$$

$$m_{\beta\beta}^{(3+1)} = c_{12}^2 c_{13}^2 c_{14}^2 e^{i\alpha_1} m_1 + c_{13}^2 c_{14}^2 s_{12}^2 e^{i\alpha_2} m_2 + c_{14}^2 s_{13}^2 e^{i\alpha_3} m_3 + s_{14}^2 m_4$$

4 masses
3 angles
3 phases

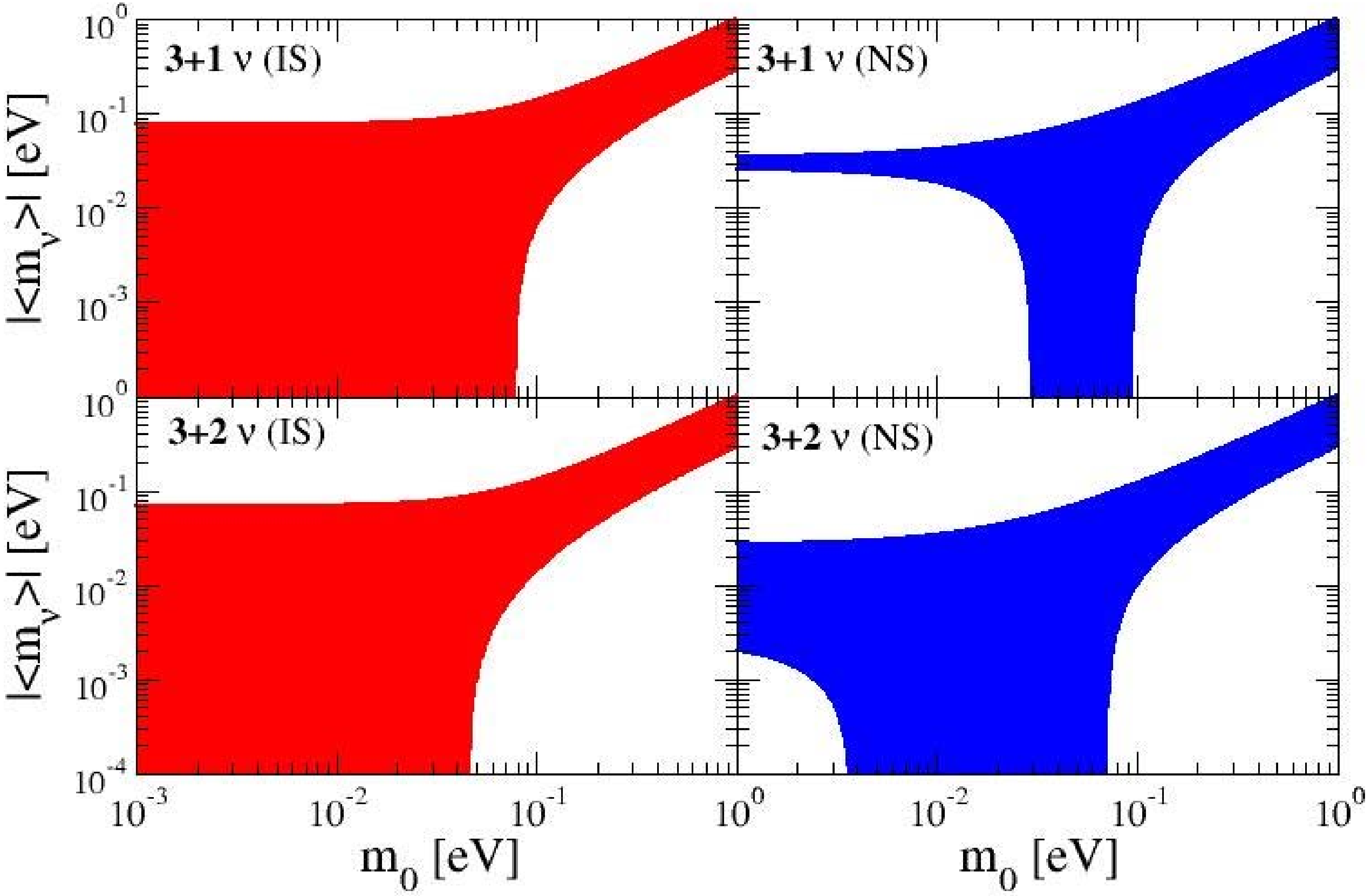
10 angles
6+4 =10 phases

$$\begin{aligned} U &= R_{45} \\ & \quad \tilde{R}_{35}R_{34} \\ & \quad \tilde{R}_{25}\tilde{R}_{24}R_{23} \\ & \quad \tilde{R}_{15}\tilde{R}_{14}\tilde{R}_{13}R_{12}P \\ P &= diag\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, e^{i(\alpha_3/2+\delta_{13})}, e^{i(\alpha_4/2+\delta_{14})}, e^{i\delta_{15}}\right) \end{aligned}$$

$$m_{\beta\beta}^{3+2} = c_{12}^2 c_{13}^2 c_{14}^2 c_{15}^2 e^{i\alpha_1} m_1 + c_{13}^2 c_{14}^2 c_{15}^2 s_{12}^2 e^{i\alpha_2} m_2 + c_{14}^2 c_{15}^2 s_{13}^2 e^{i\alpha_3} m_3 + c_{15}^2 s_{14}^2 e^{i\alpha_4} m_4 + s_{15}^2 m_5$$

5 masses
4 angles
4 phases

$$\Delta m^2_{41} = 1.78\ eV^2, \quad U_{e4} = 0.151$$
$$\Delta m^2_{51} = 0.89\ eV^2, \quad U_{e5} = 0.124$$

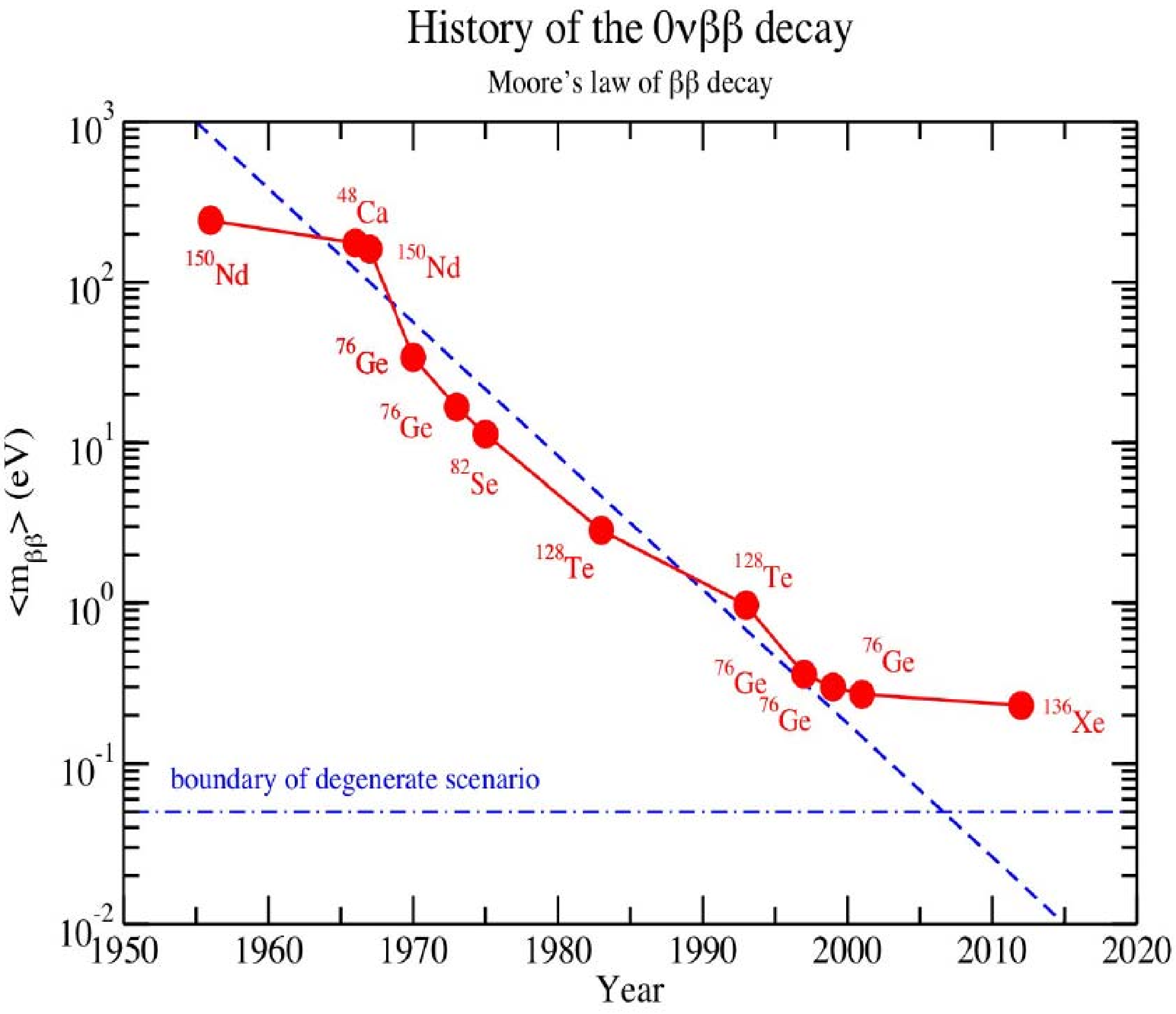


If (or when) the $0\nu\beta\beta$ decay is observed two theoretical problems must be resolved

How to relate the observed decay rate to the fundamental parameters, i.e., what is the value of the corresponding nuclear matrix elements.

What is the mechanism of the decay, i.e., what kind of virtual particle is exchanged between the affected nucleons (quarks).

Historically, there are > 100 experimental limits on $T_{1/2}$ of the $0\nu\beta\beta$ decay.



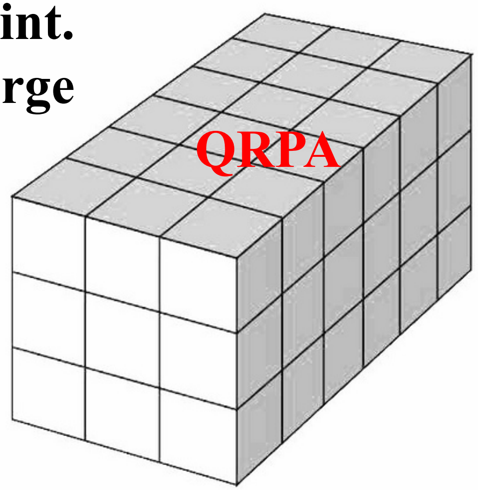
During the last decade the complexity and costs of $0\nu\beta\beta$-decay experiments increased dramatically

QRPA uncertainties and their correlations in the analysis of 0νββ decay

A. Faessler, G.L. Fogli, E. Lisi, V. Rodin, A. M. Rotunno,F.Š.,
PRD 87, 053002 (2013)

Effects of isospin restoration not included yet.

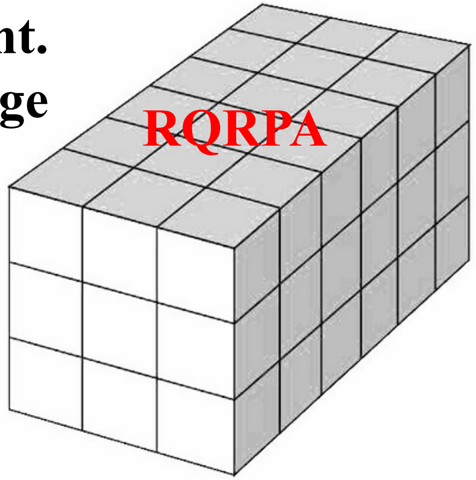
s.ms.: small
int.
large



src: Argonne
CD-Bonn

g_A=1.0,1.25

s.ms.: small
int.
large



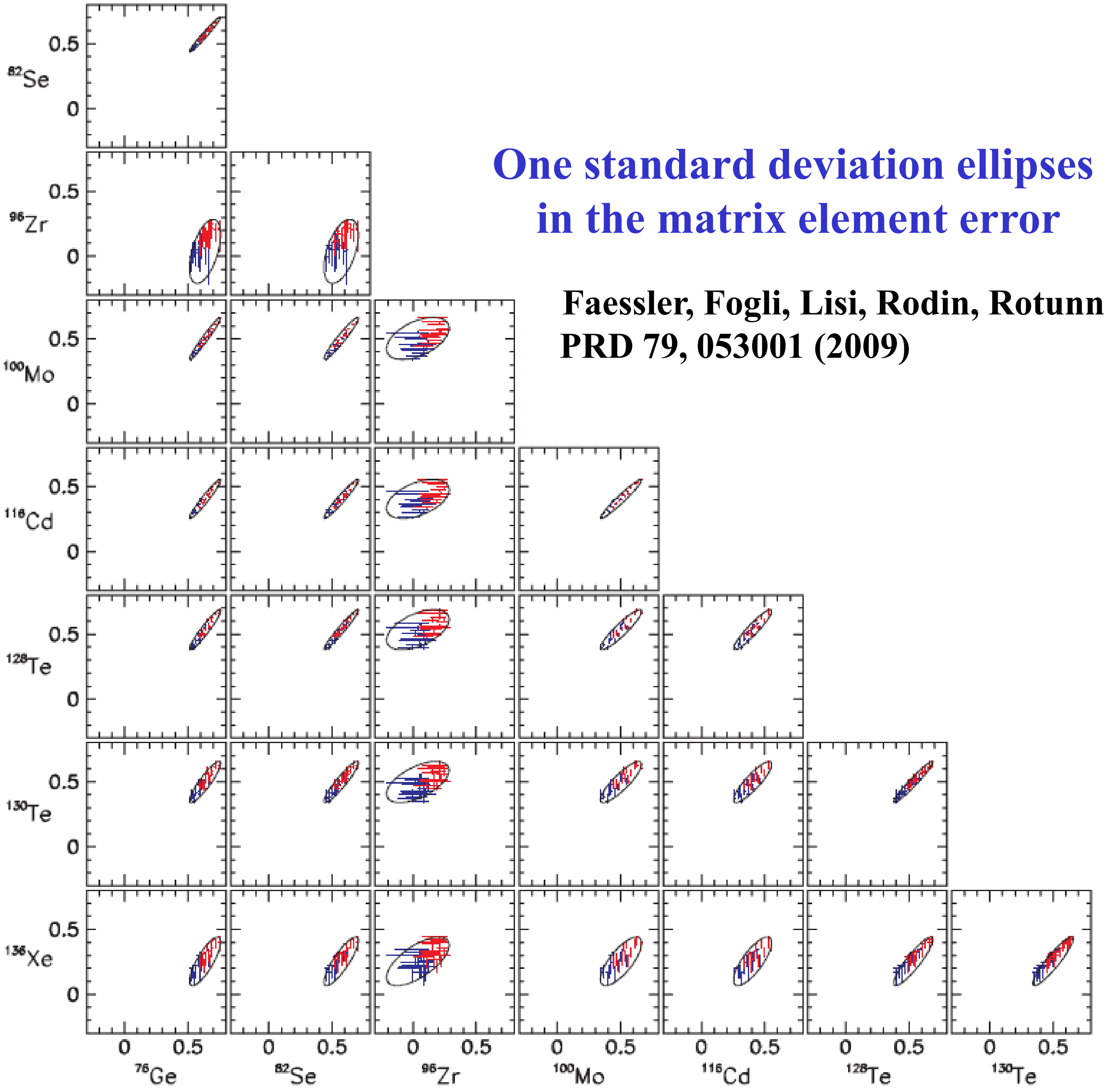
src: Argonne
CD-Bonn

g_A=1.0,1.25

For each nucleus 2x2x2x3=24 NMEs

Could multiple 0νββ measurements be helpful to extract $m_{\beta\beta}$?

Problem: Uncertainties in NME from different nuclei are highly Correlated.

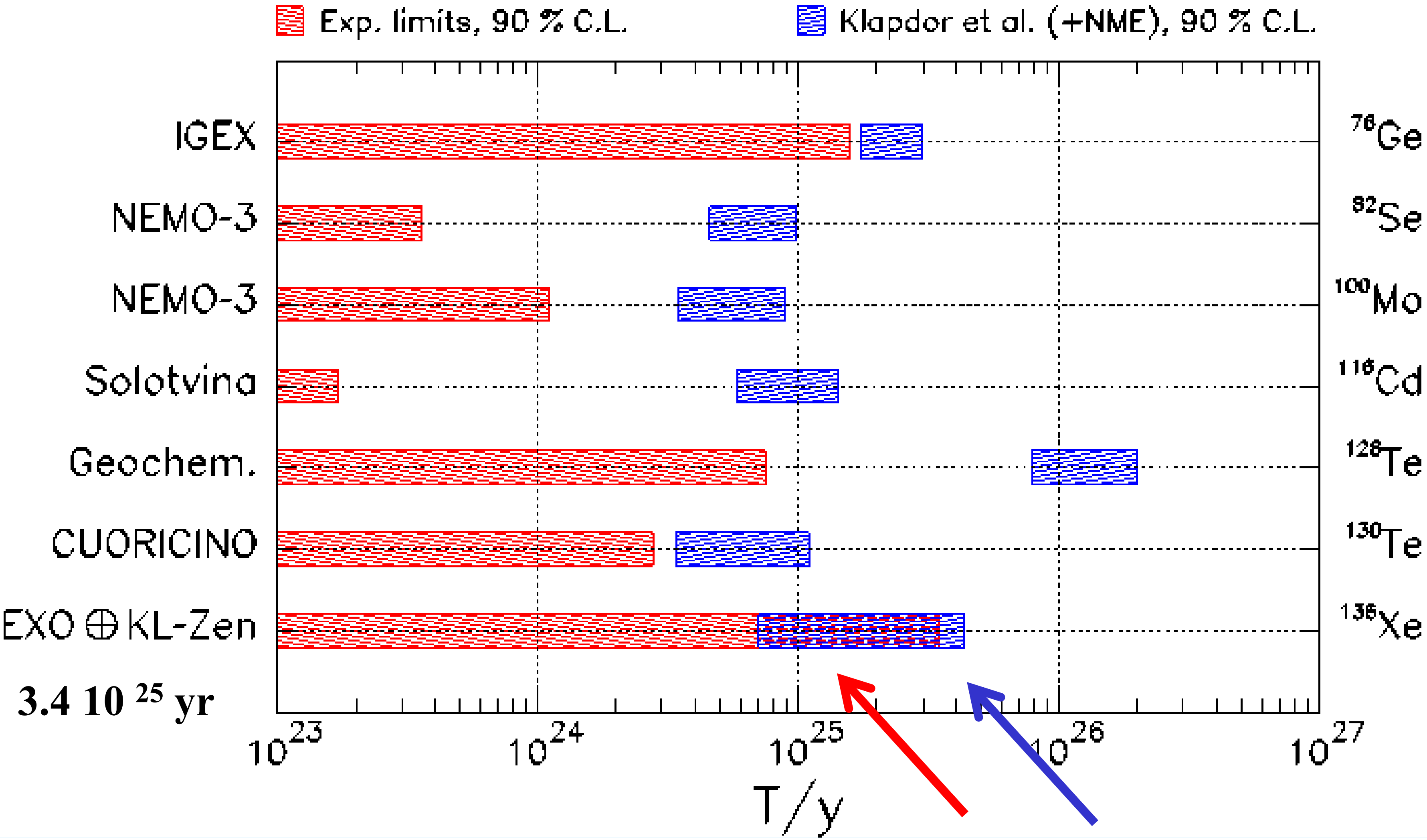


One standard deviation ellipses in the matrix element error

Faessler, Fogli, Lisi, Rodin, Rotunno, PRD 79, 053001 (2009)

Calculations:
varying method (QRPA, RQRPA),
the value g_A^{eff} (1.0 and 1.25),
the treatment of src (Jastr. and UCOM),
the size of model space (3 choices)

Range of half-lives preferred at 90% C.L. by the 0νββ claim of evidence compared with the 90% exclusion limits placed by other experiments.



3.4 10 ^{25} yr

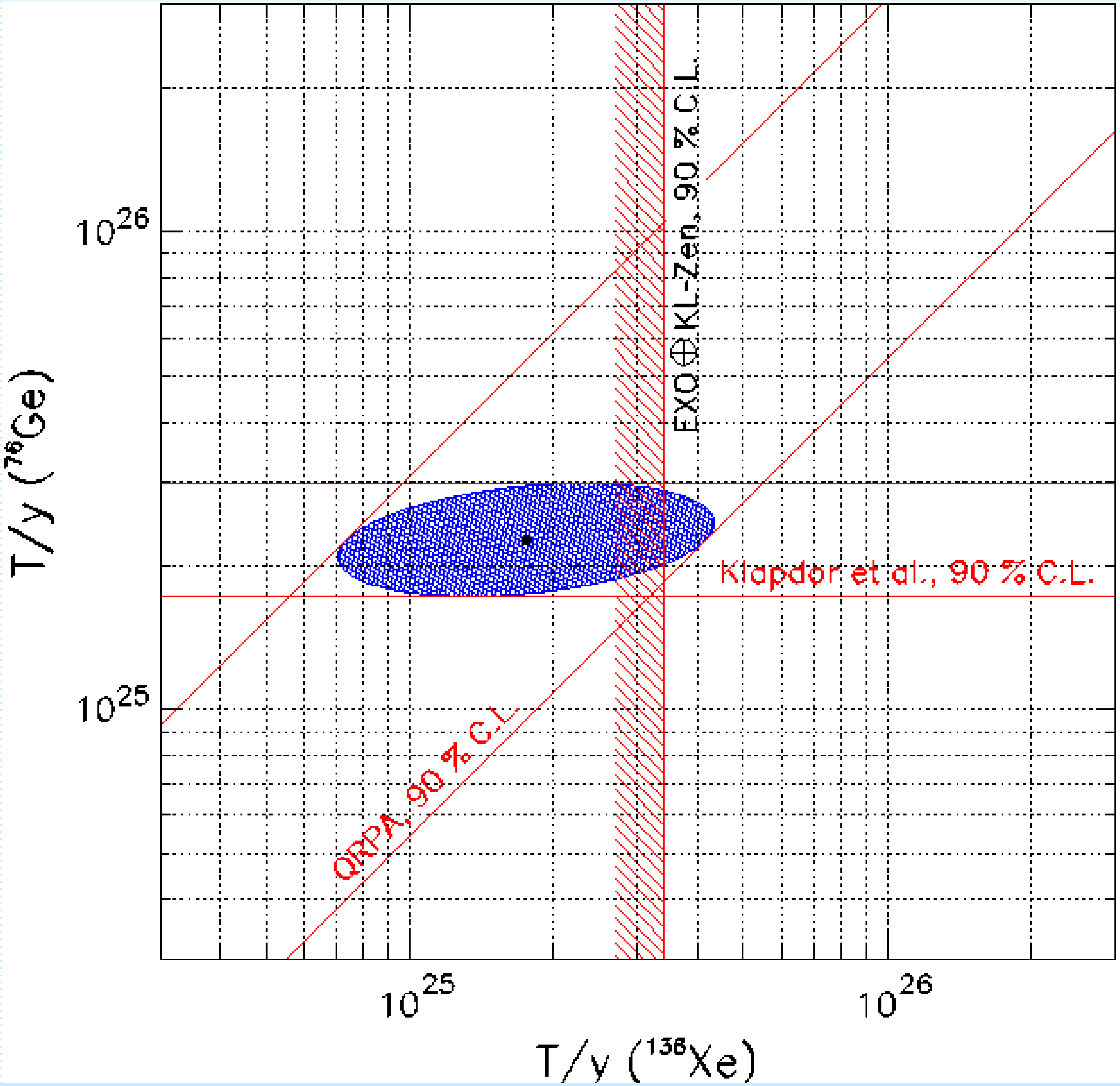
disfavored favored

The comparison involves the NME and their errors as well as their correlations

The comparison involves the NME and their errors as well as their correlations

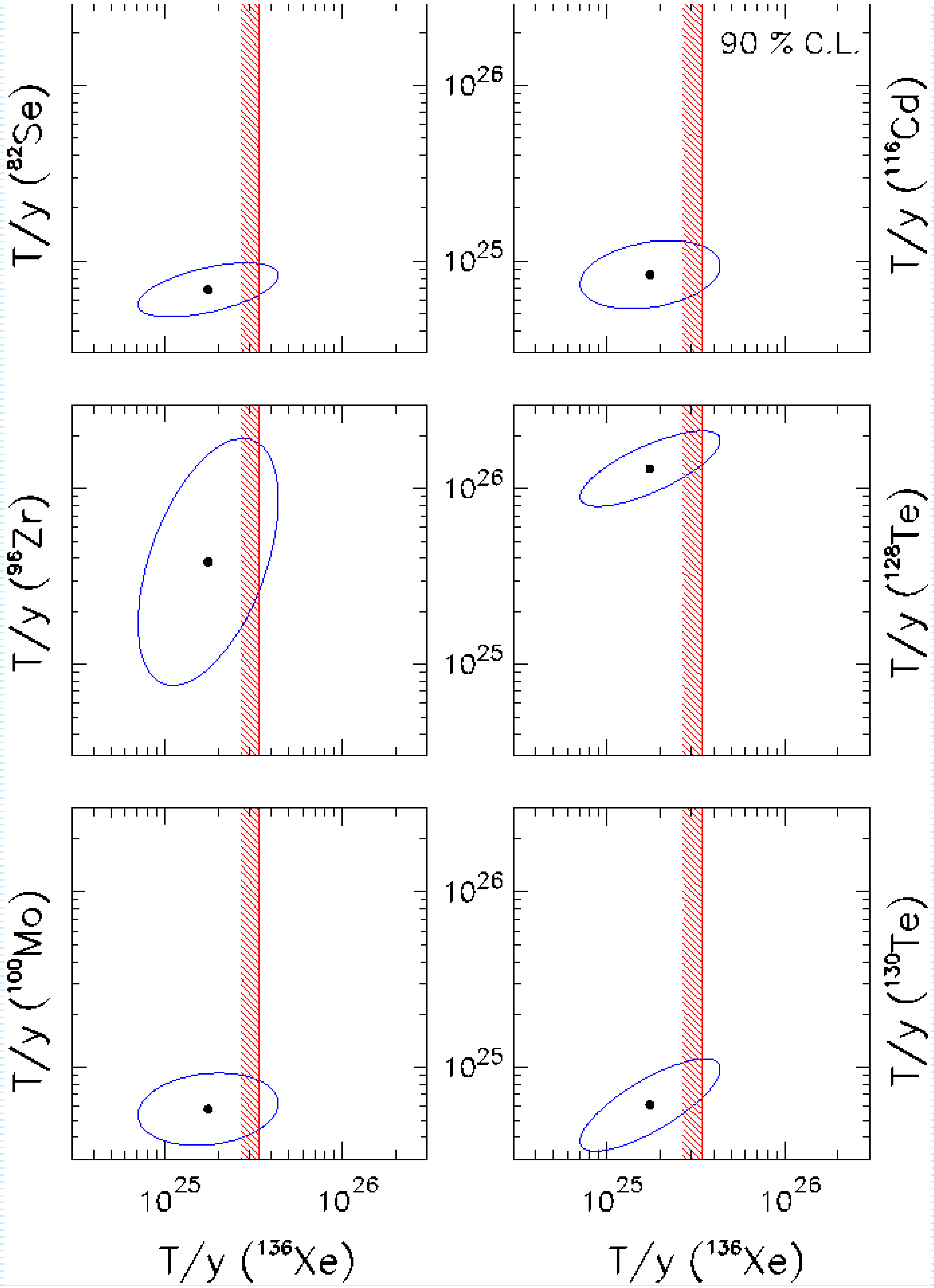
Theoretical and expperimental constraints in the plane charted by the 0νββ half-lives of ^{76}Ge and ^{136}Xe.

A.Faessler, G.L. Fogli,
E. Lisi, V. Rodin,
M. Rotunno, F.Š.,
PRD 87, 053002 (2013)



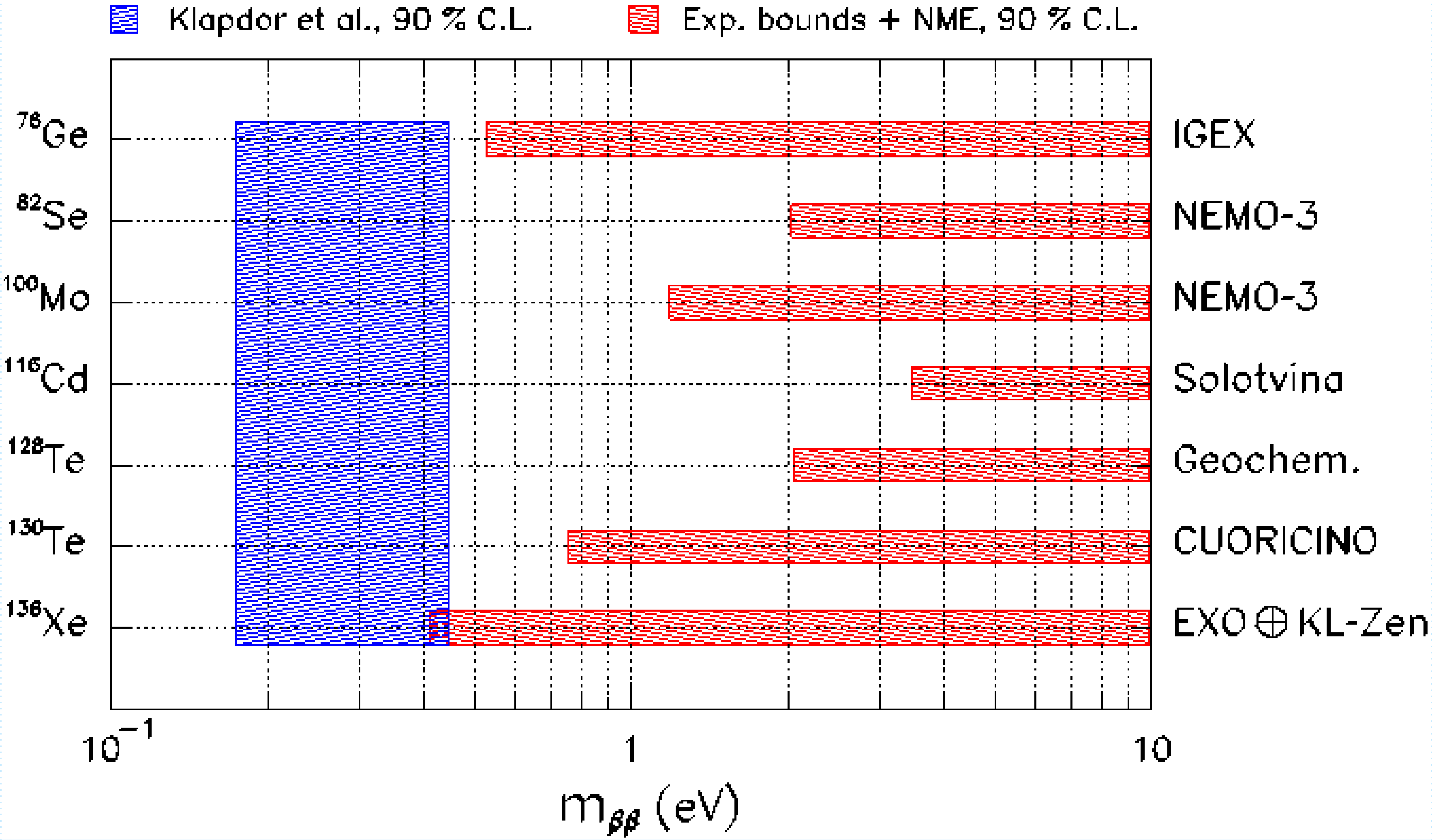
Horizontal band:range prefered by claim. Slanted band: constraint place by our QRPA estimates. The combination provides the shaded ellipse, whose projection on the abscissa gives the range preferred at 90% C.L. for the ^{136}Xe half-life.

Allowed regions (ellipses) as derived from Klapdor's claim in the plane charted by the half-lives of ^{136}Xe and each of six nuclei



A large fraction of each ellipse Is excluded by the combined EXO & KL-Zen results. (All bounds are at 90% C.L. on one variable)

Range of $m_{\beta\beta}$ allowed by the $0\nu\beta\beta$ claim of evidence compared with the limits placed by other experiments (All at 90% C.L.).



A.Faessler, G.L. Fogli, E. Lisi, V. Rodin, M. Rotunno, F.Š., PRD 87, 053002 (2013)

Nuclear medium effect on the light neutrino mass exchange mechanism of the 0νββ decay

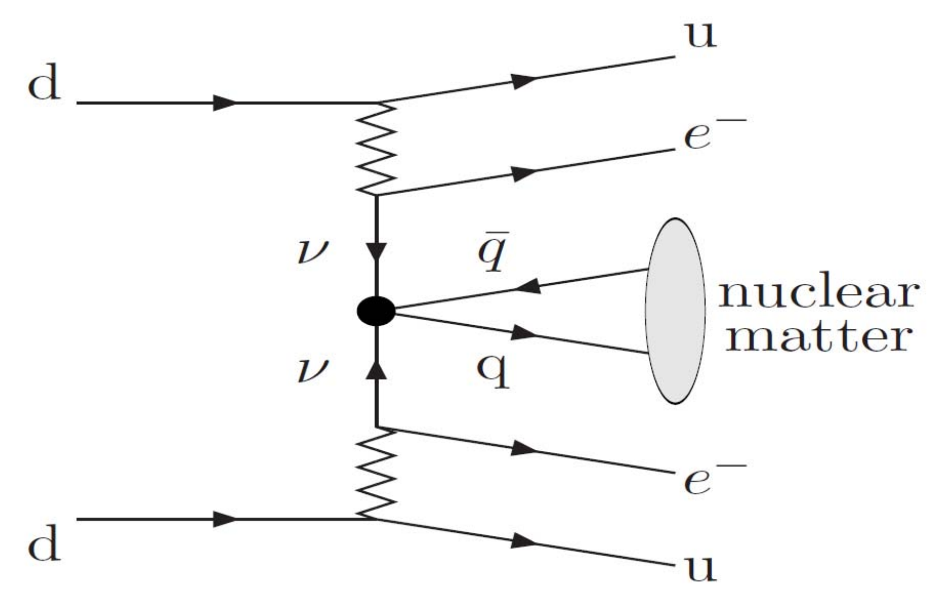
S.G. Kovalenko, M.I. Krivoruchenko, F. Š., Phys. Rev. Lett. 112 (2014) 142503

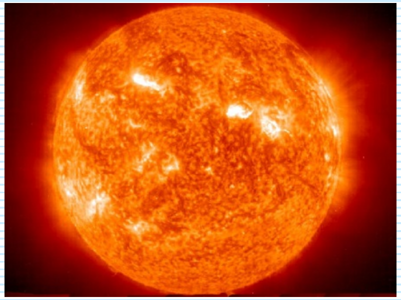
A novel effect in 0νββ decay related with the fact, that its underlying mechanisms take place in the nuclear matter environment:

+ Low energy 4-fermion ΔL $\neq$ 0 Lagrangian

+ In-medium Majorana mass of neutrino

+ 0νββ constraints on the universal scalar couplings





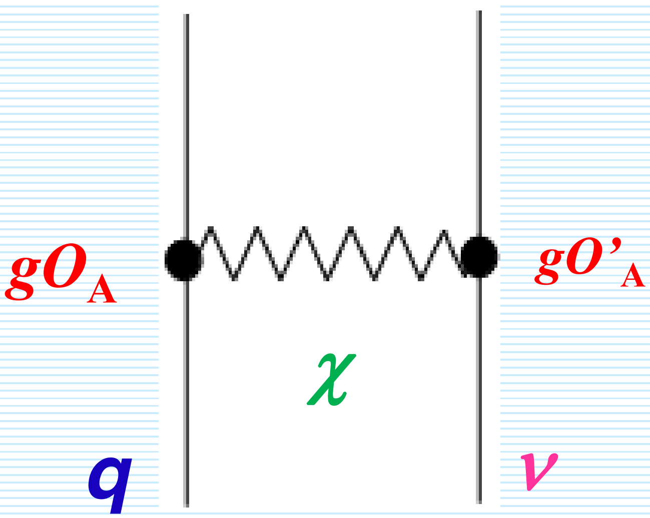
Non-standard
ν–int. discussed
e.g., in the context
of ν–osc. at Sun

$\rho_{Sun} = 1.4\ g/cm^3$

$\rho_{Earth} = 5.5\ g/cm^3$

$\rho_{nucleus} = 2.3\ 10^{14}\ g/cm^3$

Non-standard interactions might be easily detected in nucleus rather than in vacuum



Low energy 4-fermion
ΔL ≠ 0 Lagrangian

$$L_{\rm eff} = \frac{g^2}{m_\chi^2}\sum_A (\bar{q}O_A q)(\bar{\nu}O'_A \nu),$$

$$m_\chi \gtrsim M_W.$$

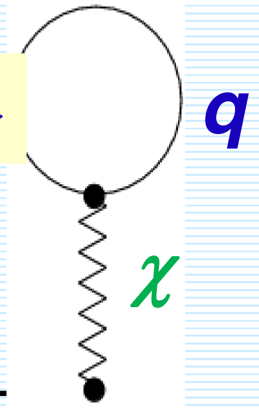
0νββ-decay

oscillation experiments
tritium β-decay, cosmology

density →

$$\Sigma_\nu^{\rm vac} = \text{-}\!\times\!\text{-},$$

$$\Sigma_\nu^{\rm medium} = \text{-}\!\times\!\text{-} +$$



Classification of the vertices gO$_A$ and gO'$_A$

$$\mathcal{L}_{\mathrm{free},\nu} = \frac{1}{4}\sum_i \bar{\nu}_i i\gamma^\mu \overleftrightarrow{\partial}_\mu \nu_i - \frac{1}{2}\sum_i m_i \bar{\nu}_i \nu_i .$$

$$\mathcal{L}_{\mathrm{eff}} = \frac{g_\chi}{m_\chi^2}\bar{q}q \sum_{a=1}^{6}\sum_{ij} g_{ij}^a J_{ij}^a$$

In nuclei, mean fields are created by scalar and vector currents (σ, ω).

Vector currents do not flip the spin of neutrinos and do not contribute to the 0νββ decay.

Symmetric and antisymmetric scalar neutrino currents J$^a_{ij}$

a	S	a	S	a	A
1	$\bar{\nu}_i^c \nu_j$	3	$\partial_\mu(\bar{\nu}_i^c \gamma_5 \gamma^\mu \nu_j)$	5	$\partial_\mu(\bar{\nu}_i^c \gamma^\mu \nu_j)$
2	$\bar{\nu}_i^c i\gamma_5 \nu_j$	4	$\bar{\nu}_i^c \gamma^\mu i \overleftrightarrow{\partial}_\mu \nu_j$	6	$\bar{\nu}_i^c \gamma_5 \gamma^\mu i \overleftrightarrow{\partial}_\mu \nu_j$

g^a_{ij} are real symmetric for a = 1,2,3,4 and imaginary antisymmetric for a = 5,6. In the limit of $R = \infty$, the currents a = 3,5 vanish.

Mean field approximation

Mean field: $\bar{q}q \rightarrow \langle \bar{q}q \rangle$ **and** $\langle \bar{q}q \rangle \approx 0.5 \langle q^\dagger q \rangle \approx 0.25\,\mathrm{fm}^{-3}$

The effect depends on the product

$$\langle \chi \rangle = -\frac{g_\chi}{m_\chi^2} \langle \bar{q}q \rangle$$

To compare with weak interaction:

$$\frac{g_\chi g_{ij}^a}{m_\chi^2} = \frac{G_F}{\sqrt{2}} \varepsilon_{ij}^a$$

Typical scale:

$$\langle \chi \rangle g_{ij}^a = -\frac{G_F}{\sqrt{2}} \langle \bar{q}q \rangle \varepsilon_{ij}^a \approx -25\,\varepsilon_{ij}^a\ \mathrm{eV}$$

We expect:

$$25\,\varepsilon_{ij}^a < 1 \rightarrow m_\chi^2 > 25 \frac{g_\chi g_{ij}^a \sqrt{2}}{G_F} \sim 1\,\mathrm{TeV}^2$$

In-medium Lagrangian of Majorana ν

$$\Delta\mathcal{L} = \frac{1}{4}\sum_{ij}\bar{\nu}_i(Z_{ij} + \gamma_5 Z'_{ij})i\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}\nu_j - \frac{1}{2}\sum_{ij}\bar{\nu}_i(M_{ij} + i\gamma_5 M'_{ij})\nu_j,$$

where

$$Z_{ij} = \delta_{ij} - \langle\chi\rangle g^4_{ij}, \qquad Z'_{ij} = -\langle\chi\rangle g^6_{ij}$$

$$M_{ij} = m_i\delta_{ij} + \langle\chi\rangle g^1_{ij}, \quad M'_{ij} = \langle\chi\rangle g^2_{ij}.$$

Universal scalar interaction

$$g^a_{ij} = \delta_{ij} g_a \quad \varepsilon^a_{ij} = \delta_{ij}\varepsilon_a$$

In medium ν mass

$$\mu_i = \frac{\sqrt{(m_i + \langle\chi\rangle g_1)^2 + (\langle\chi\rangle g_2)^2}}{\lambda_i}$$

In medium effective Majorana ν mass

$$m_{\beta\beta} = \sum_{i=1}^{n} U^2_{ei}\xi_i \frac{\sqrt{(m_i + \langle\chi\rangle g_1)^2 + (\langle\chi\rangle g_2)^2}}{(1 - \langle\chi\rangle g_4)^2}.$$

Radiative corrections

The Majorana ν mass in vaccum and the 4-fermion operators should originate from the same LNV physics at the energy scale above $\Lambda_{LNV.}$ However, mechanisms generating these two effective Lagrangian terms may be very different.

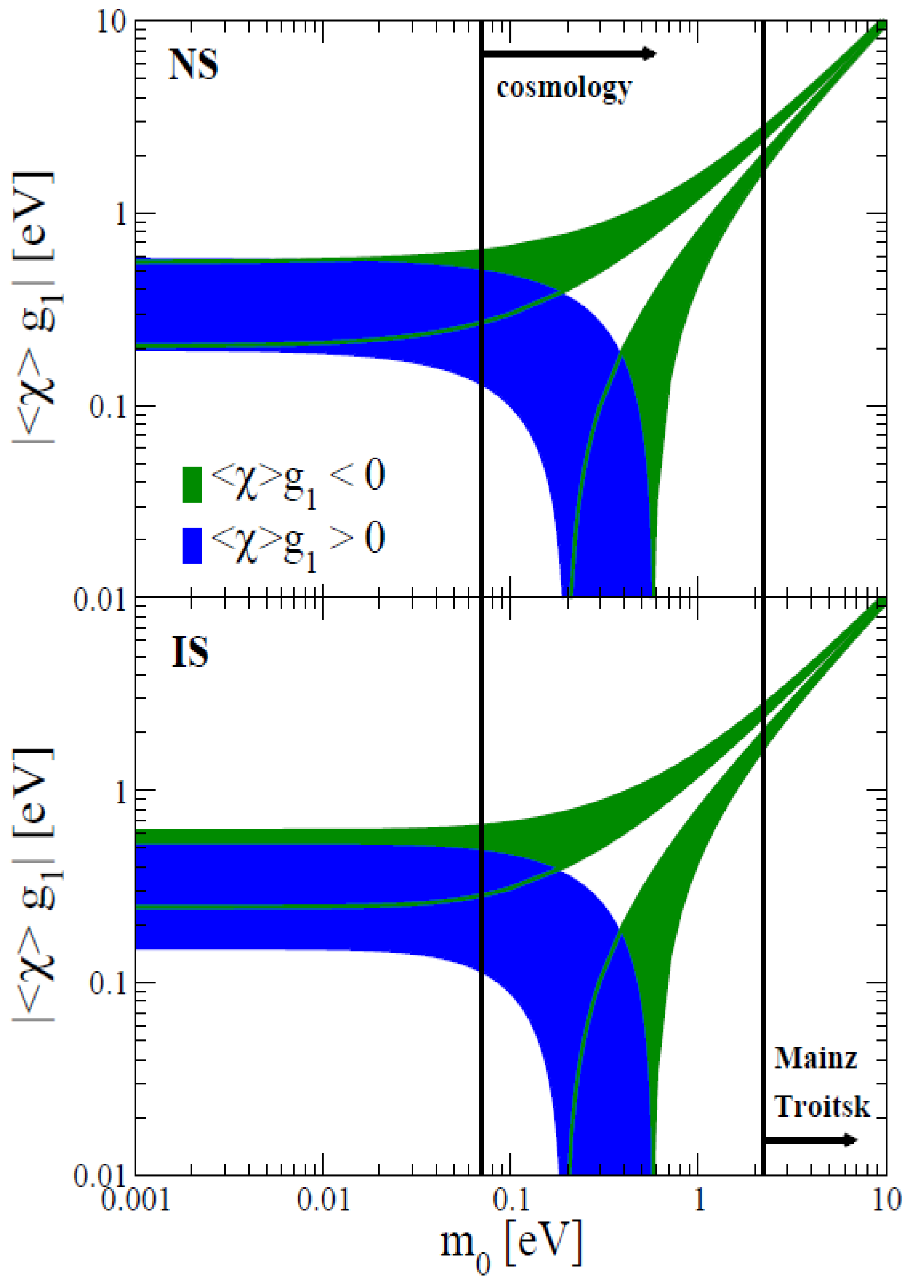
The direct contribution of the LNV operators to ν mass (quark bubble attached to neutrino line)

$$\delta m_\nu \sim g^q/(4\pi\Lambda_{LNV})^2 m_q^3 \log(\Lambda_{LNV}/m_q)$$

For Λ_{LNV}=2.4 TeV, m_d= 5 MeV and g^q=1 we find $\delta m_\nu = 10^{-6}$ eV

Thus there must be another mechanism of neutrino mass generation compatible with the neutrino oscillation data.

Regions of admissible values of $\langle\chi\rangle g_1$ and m_0 ($m_{\beta\beta}$=0.2 eV)



$$\langle\chi\rangle = 0.17\ fm^{-3} = \frac{0.17}{(5.07)^3} GeV^3$$

$$\Lambda_{LNV} \geq 2.4\,\text{TeV (Planck)}$$

$$1.1\,\text{TeV (Tritium)}$$

$$\varepsilon_{ij} \leq 0.02\ \text{(Planck)},\ 0.1\ \text{(Tritium)}$$

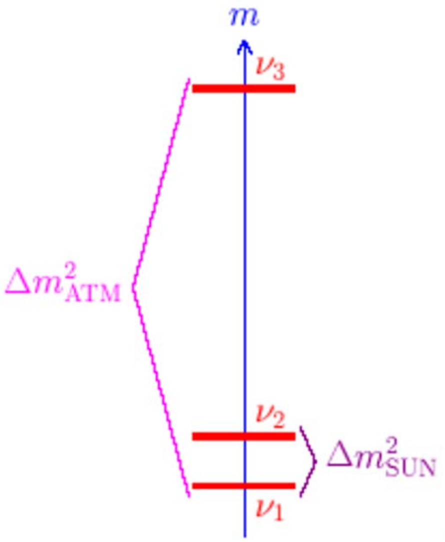
Using experimental data on the 0νββ decay in combination with β-decay and cosmological data we evaluated the characteristic scales of 4-fermion neutrino-quark operators, which is Λ_{LNV} > 2.4 TeV.

Pion decay: BR($\pi^0 \to \nu\nu$) ≤ 2.7 10^{-7}

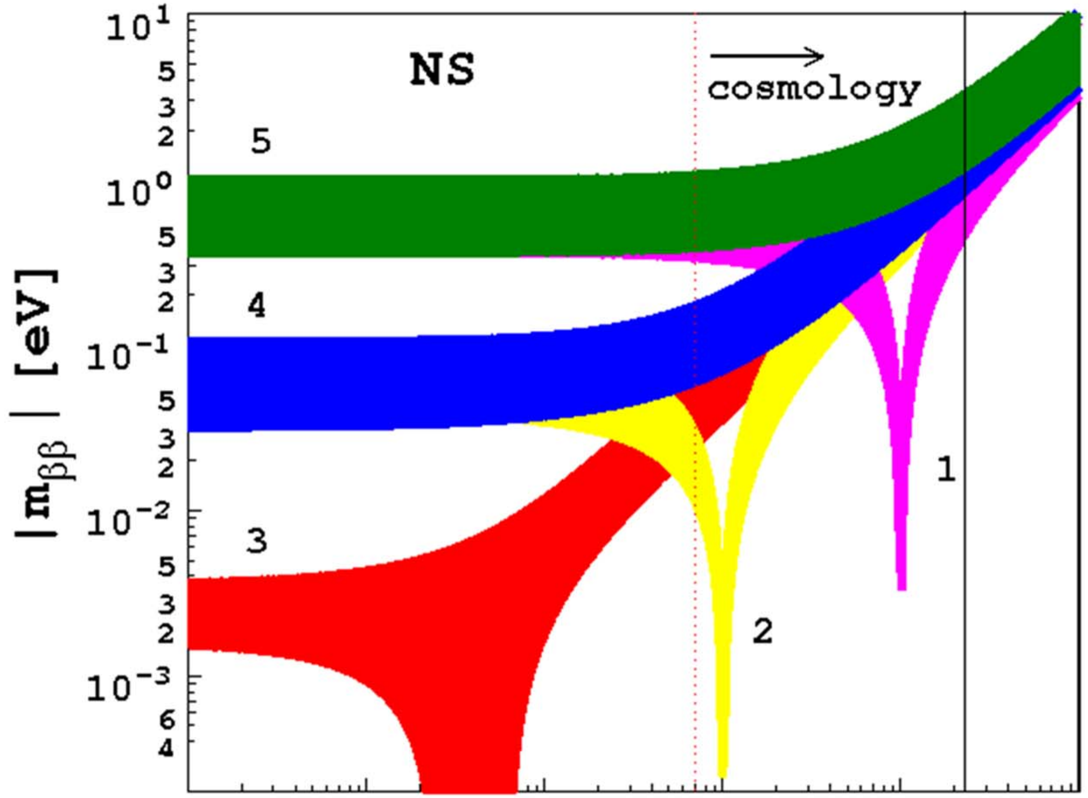
$$\Lambda_{LNV} \geq 560\ \text{GeV}$$

The normal neutrino mass spectrum (NS)

$$m_1 < m_2 < m_3$$



$|m_{\beta\beta}|=0.2$ eV

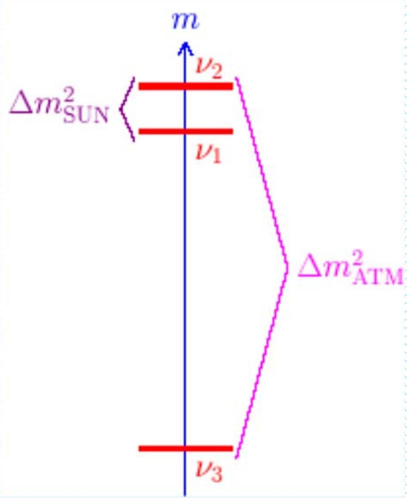


Area	$\langle\chi\rangle g_1$ [eV]
1	-1
2	-0.1
3	0
4	0.1
5	1

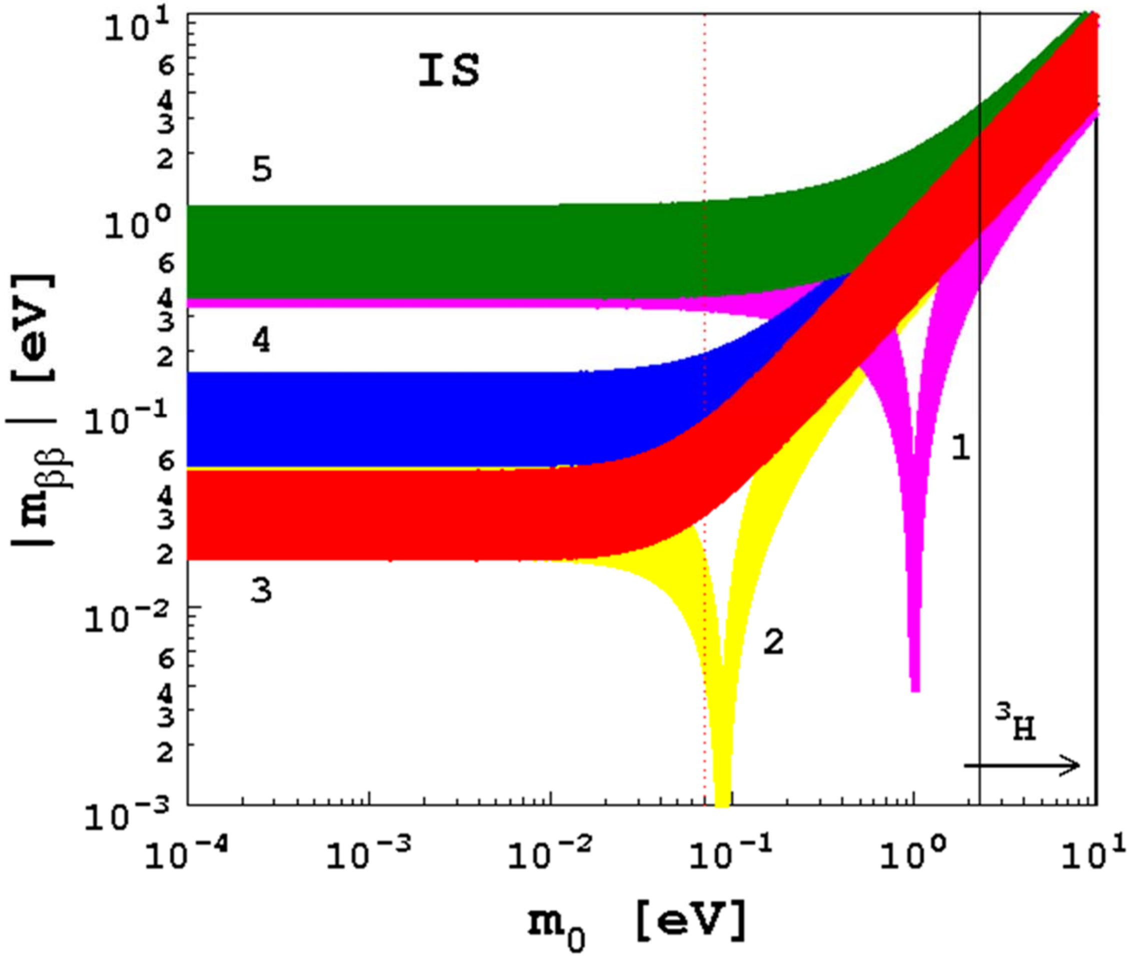
$$g_2 = g_4 = 0$$

The inverted neutrino mass spectrum (IS)

$$m_3 < m_1 < m_2$$

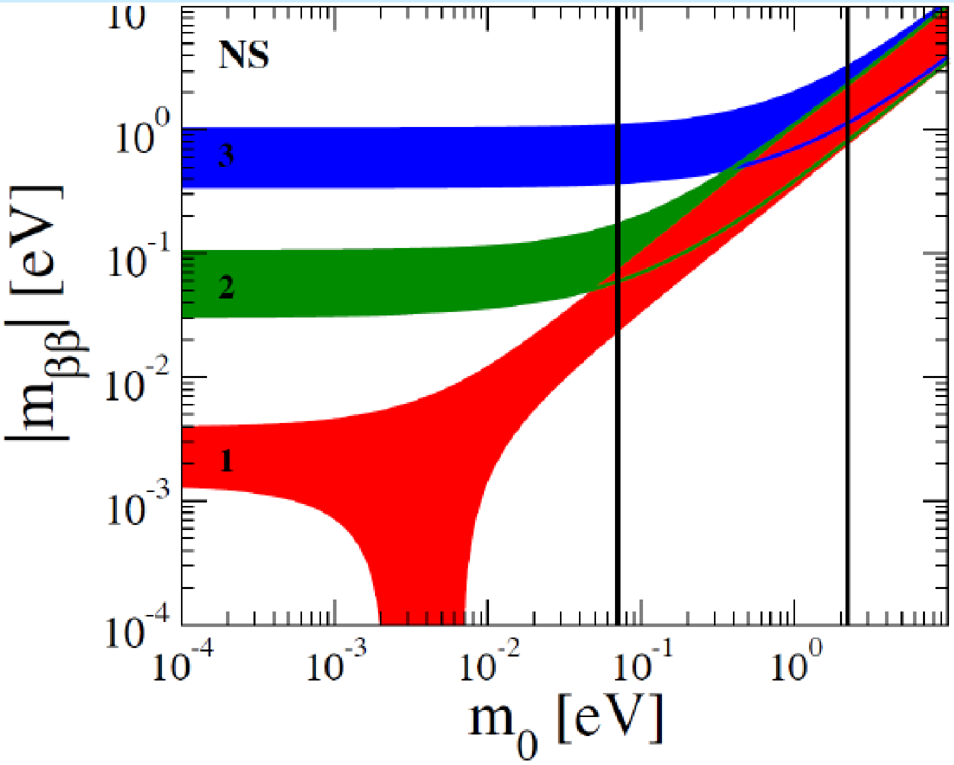


$|m_{\beta\beta}|=0.2$ eV



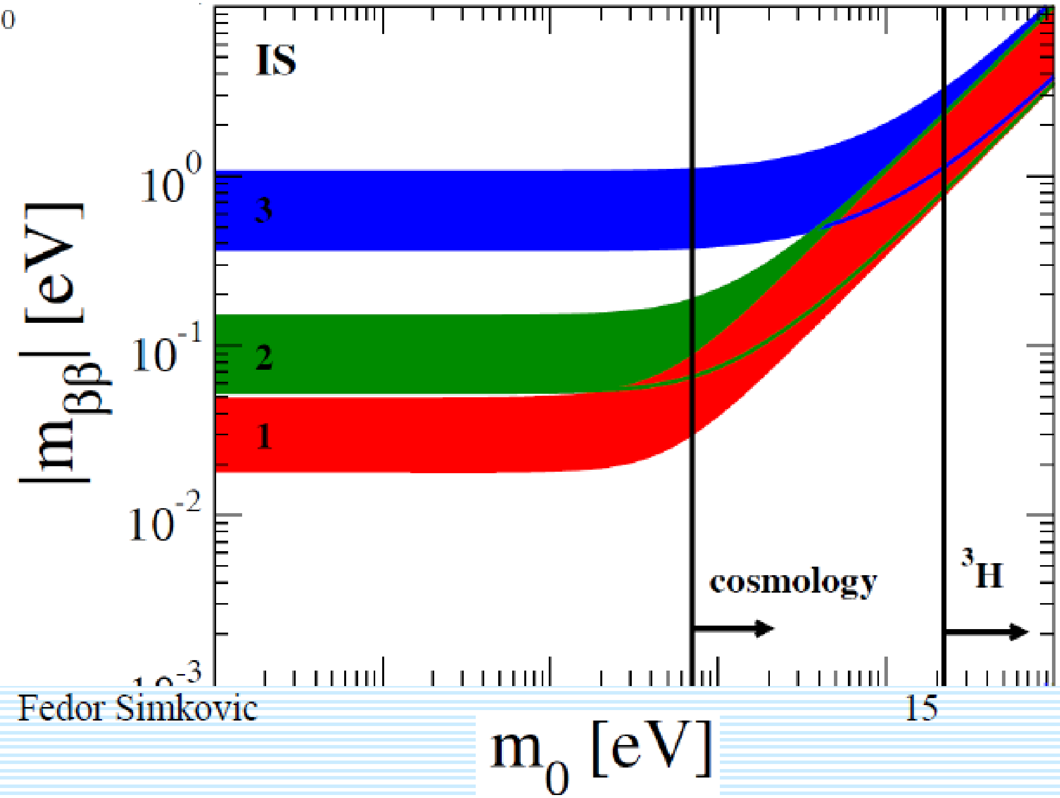
Area	$\langle\chi\rangle g_1$ **[eV]**
1	-1
2	-0.1
3	0
4	0.1
5	1

$$g_2 = g_4 = 0$$



Difference between the NS and IS disappears

For positive LNV interaction effective Majorana neutrino mass becomes larger



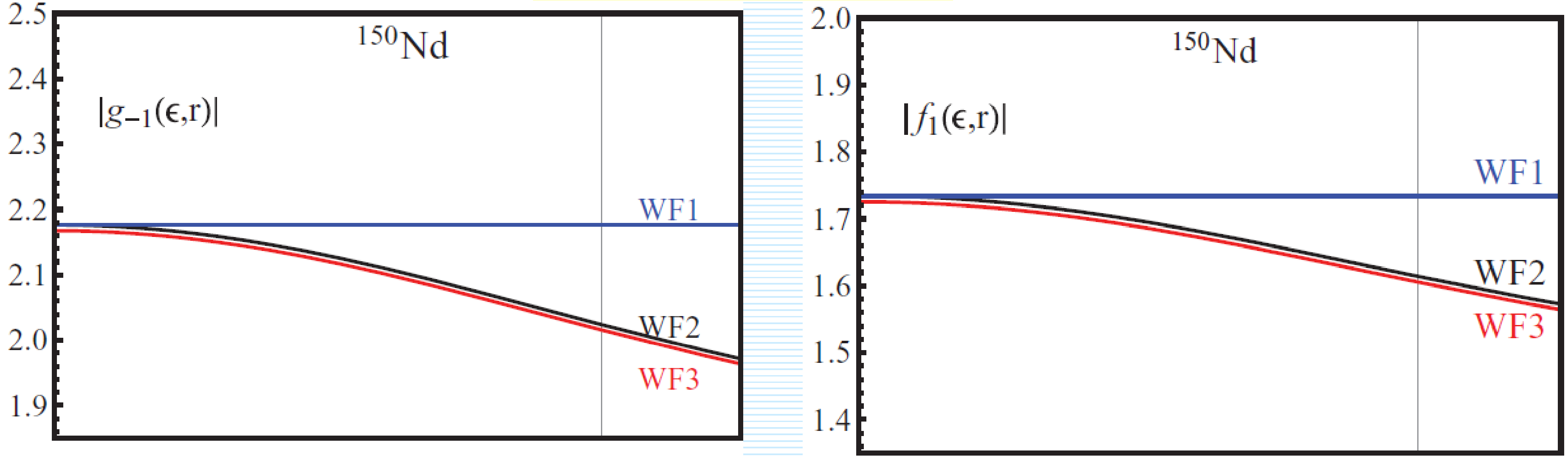
What is today exotic tomorrow is standard!!

6/25/2015

Phase-space factor

$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

Electron w.f. at r=R



Dirac equations:

$$\psi_{\epsilon\kappa\mu}(\mathbf{r}) = \begin{pmatrix} g_\kappa(\epsilon,r)\chi^\mu_\kappa \\ i f_\kappa(\epsilon,r)\chi^\mu_{-\kappa}, \end{pmatrix}$$

$$\frac{dg_\kappa(\epsilon,r)}{dr} = -\frac{\kappa}{r}g_\kappa(\epsilon,r) + \frac{\epsilon - V + m_e c^2}{c\hbar}f_\kappa(\epsilon,r)$$

$$\frac{df_\kappa(\epsilon,r)}{dr} = -\frac{\epsilon - V - m_e c^2}{c\hbar}g_\kappa(\epsilon,r) + \frac{\kappa}{r}f_\kappa(\epsilon,r)$$

$s_{1/2}$ electron wave state

Finite nuclear size

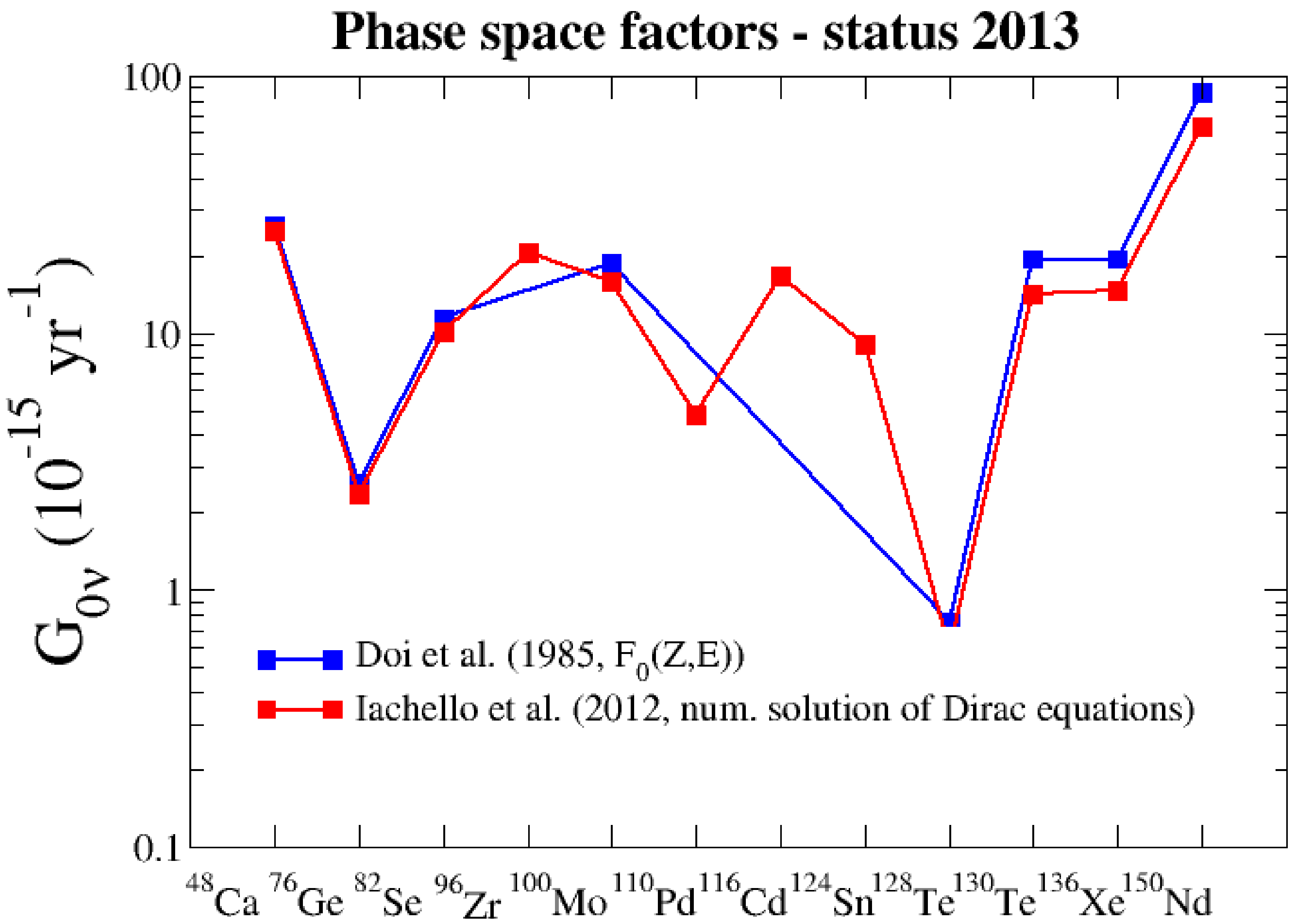
$$e_s^{S_{1/2}}(\epsilon,\mathbf{r}) = \begin{pmatrix} g_{-1}(\epsilon,r)\chi_s \\ f_1(\epsilon,r)(\hat{\mathbf{p}}\cdot\vec{\sigma})\chi_s \end{pmatrix}$$

WF1: approx. sol. inside nucl. (M. Rose approach)

WF2: numerical solution of Dirac eq.

WF3: numerical solution of Dirac eq. with consideration of electron screening

Nucleus is considered to be spherical



Nuclear Matrix Elements (NMEs)

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M^{0\nu}_{\nu}\right|^2 G^{0\nu}$$

The 0νββ-decay Nuclear Matrix Elements

Systematic and statistical errors

In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus

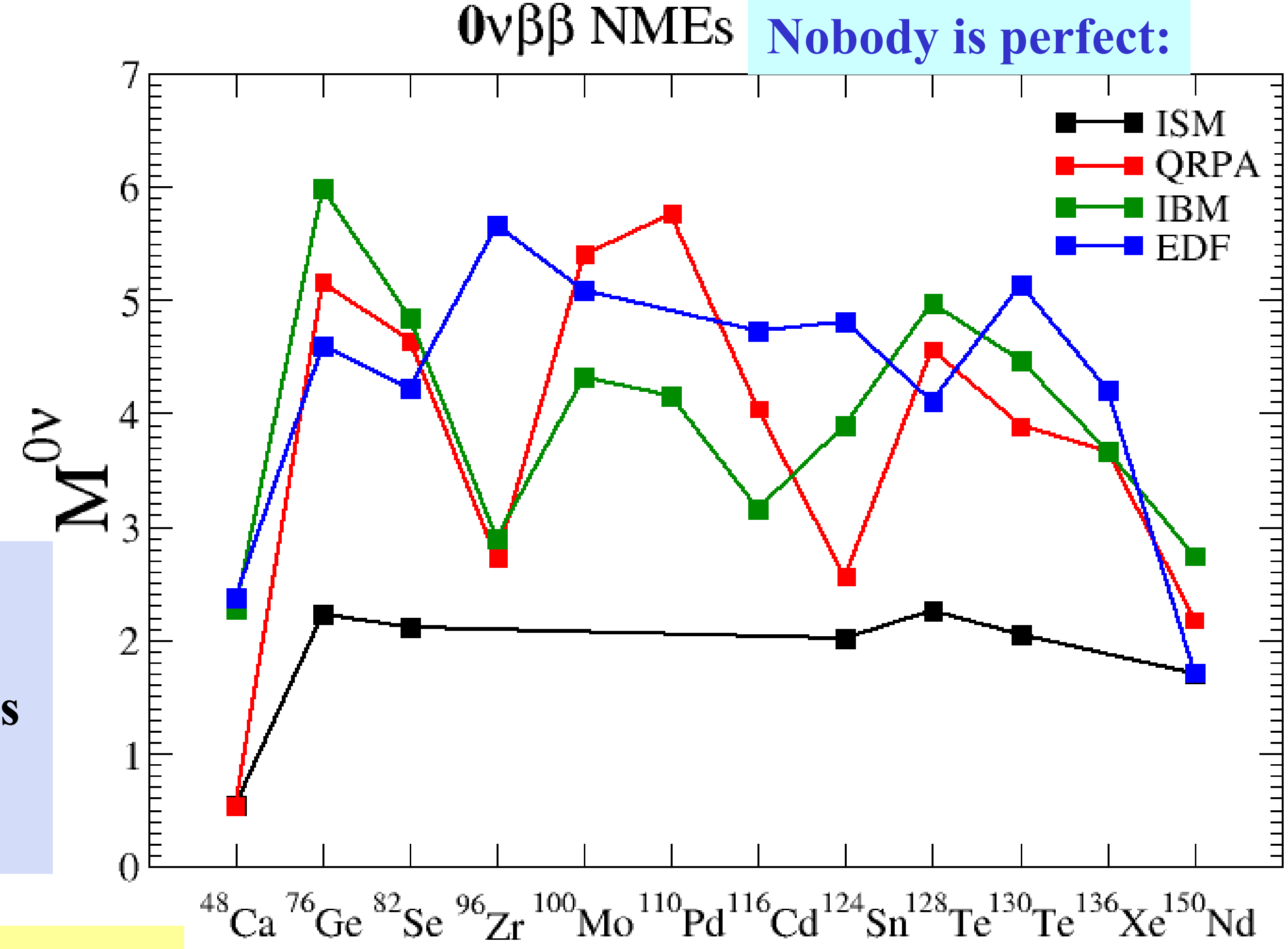
It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the 0νββ-decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge directly the quality of the result.

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M^{0\nu}_{\nu}\right|^2 G^{0\nu}$$

The 0νββ-decay NMEs (Status:2015)

Nobody is perfect:



g_A=1.25(7), CCm or UCOM s.r.c., r_0=1.20 fm

Systematic errors – Calculations can be improved

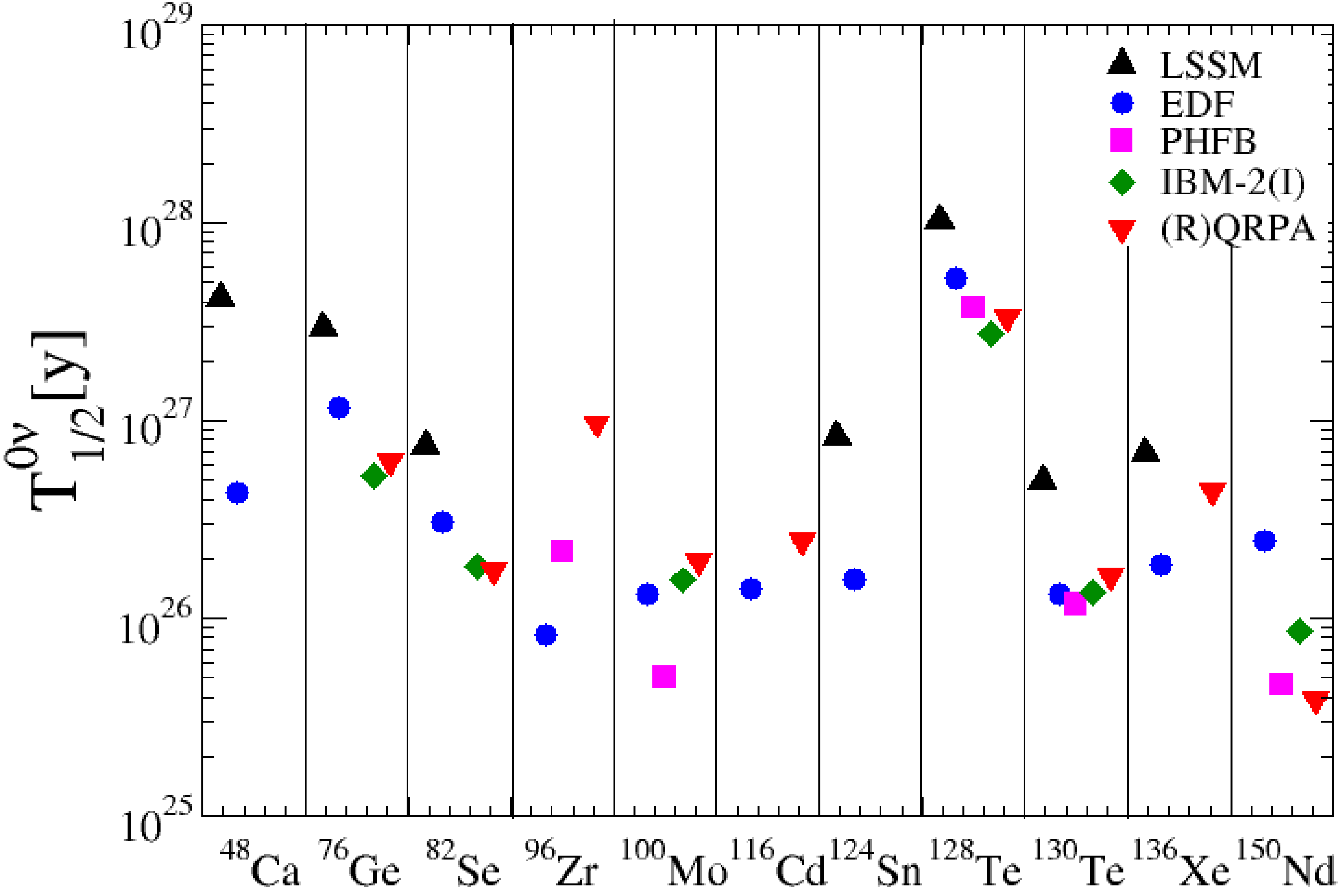
Differences:

i) mean field;
ii) residual int.;
iii) size of the m.s.
iv) many-body appr.

LSSM (small m.s., negative parity states)
PHFB (GT force neglected)
IBM (Hamiltonian truncated)
(R)QRPA (g.s. correlations not accurate enough)

$0\nu\beta\beta$-decay half-life

$m_{\beta\beta}$=50 meV



Quenching of g_A (systematic error)

$g_A{}^4 = (1.269)^4 = 2.6$

$(g^{eff}{}_A)^4 {}_= 1.0$

Strength of GT trans. has to be quenched to reproduce experiment

$g_{A=}1.269 \Rightarrow g^{eff}{}_{A=}0.75\, g_A \approx 1$

$(g^{eff}{}_A)^4 {}_= (0.8)^4 = 0.41$

$(g^{eff}{}_A)^4 {}_= (0.7)^4 = 0.24$

In QRPA $g^{eff}{}_A$ and isoscalar force were fitted to reproduce the $2\nu\beta\beta$-decay half-life, β^- decay rate and β^+/EC rate
=> $g^{eff}{}_A$ is smaller than unity.

Faessler, Fogli, Lisi, Rodin, Rotunno, F. Š, J. Phys. G 35, 075104 (2008).

$g_A{}^{eff\text{-}ISM} = 0.57\text{-}0.90$

$g_A{}^{eff\text{-}IBM} = 0.35\text{-}0.71$

$g_A{}^{eff}$ is highly dependent on the model calculations and assumptions made

Barea, Kotila, Iachello, PRC 87, 014315 (2013)

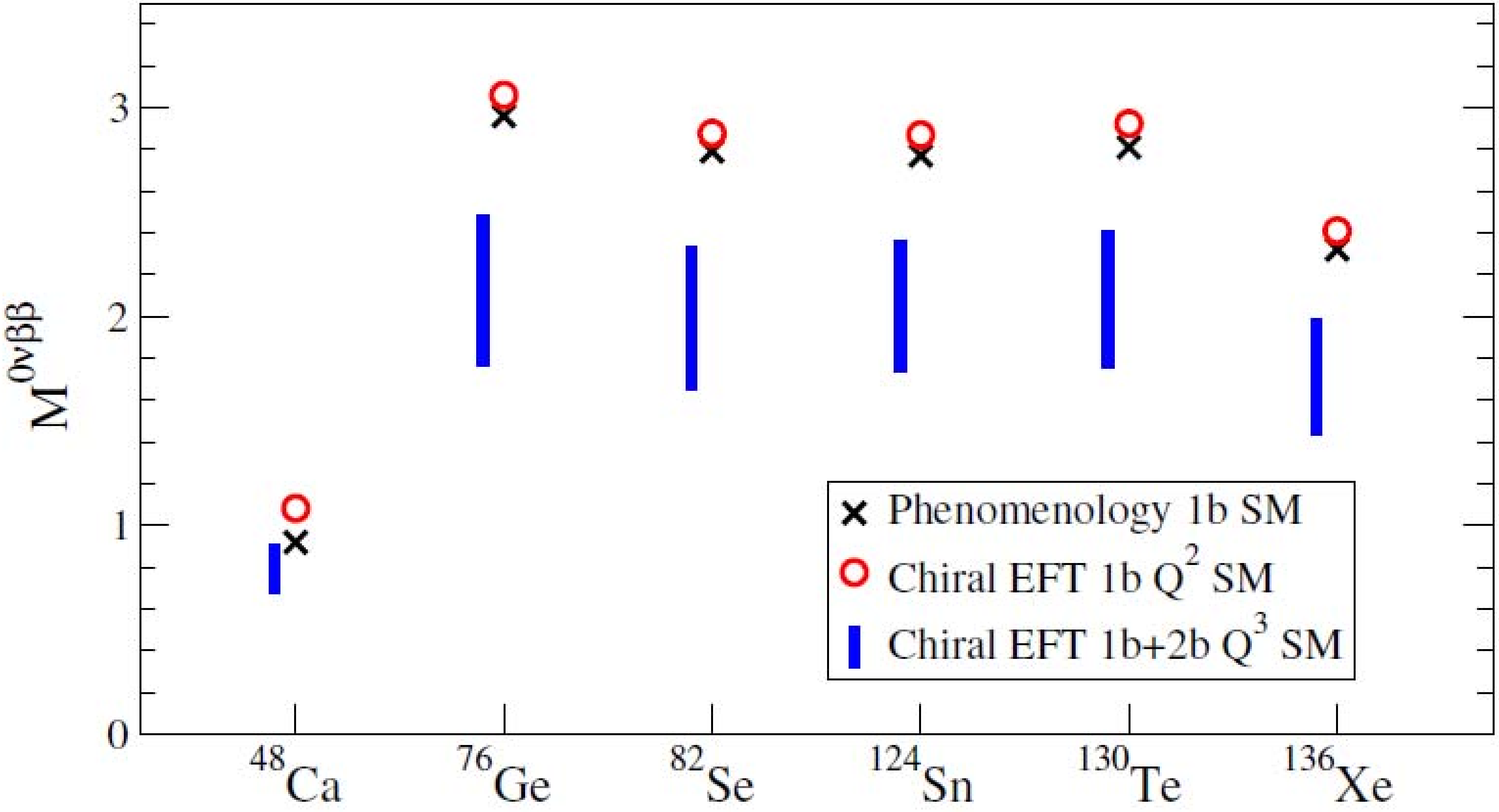
Is $g_A{}^{eff}$ different for different (A,Z) and different spin-dependent transition operators?

Quenching of g_A and two-body currents

Menendez, Gazit, Schwenk, PRL 107 (2011) 062501; MEDEX13 contribution

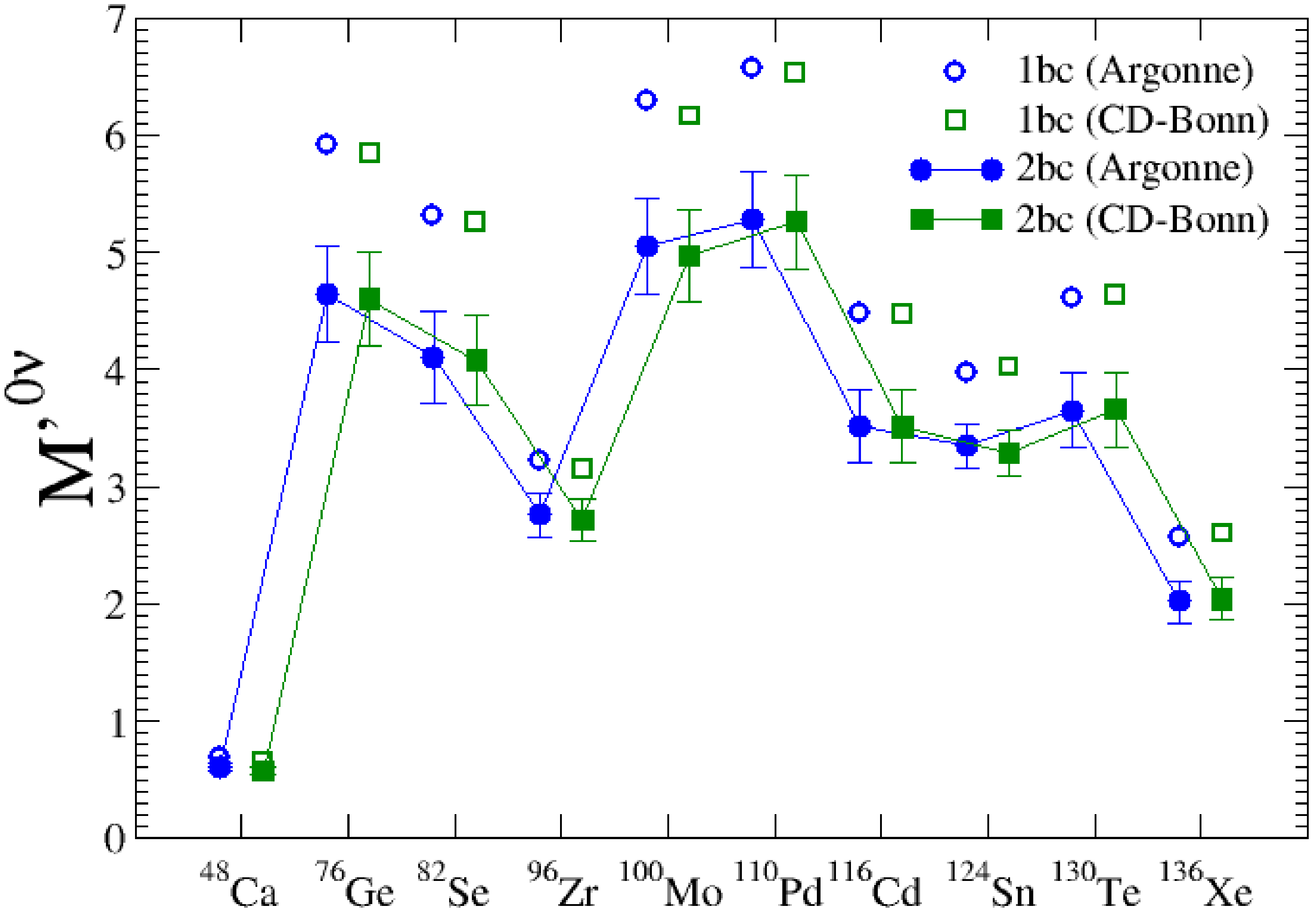
$$\mathbf{J}^{\text{eff}}_{i,2b} = -g_A \boldsymbol{\sigma}_i \tau_i^- \frac{\rho}{F_\pi^2}\left[\frac{2}{3}c_3\frac{p^2}{4m_\pi^2+p^2}+I(\rho,P)\left(\frac{1}{3}(2c_4-c_3)+\frac{1}{6m}\right)\right] = -g_A \boxed{\delta(p)} \boldsymbol{\sigma}_i \tau_i^-$$

The 0νββ operator calculated within effective field theory. Corrections appear as 2-body current predicted by EFT. The 2-body current contributions are related to the quenching of Gamow-Teller transitions found in nuclear structure calc.



Quenching of g_A , two-body currents and QRPA

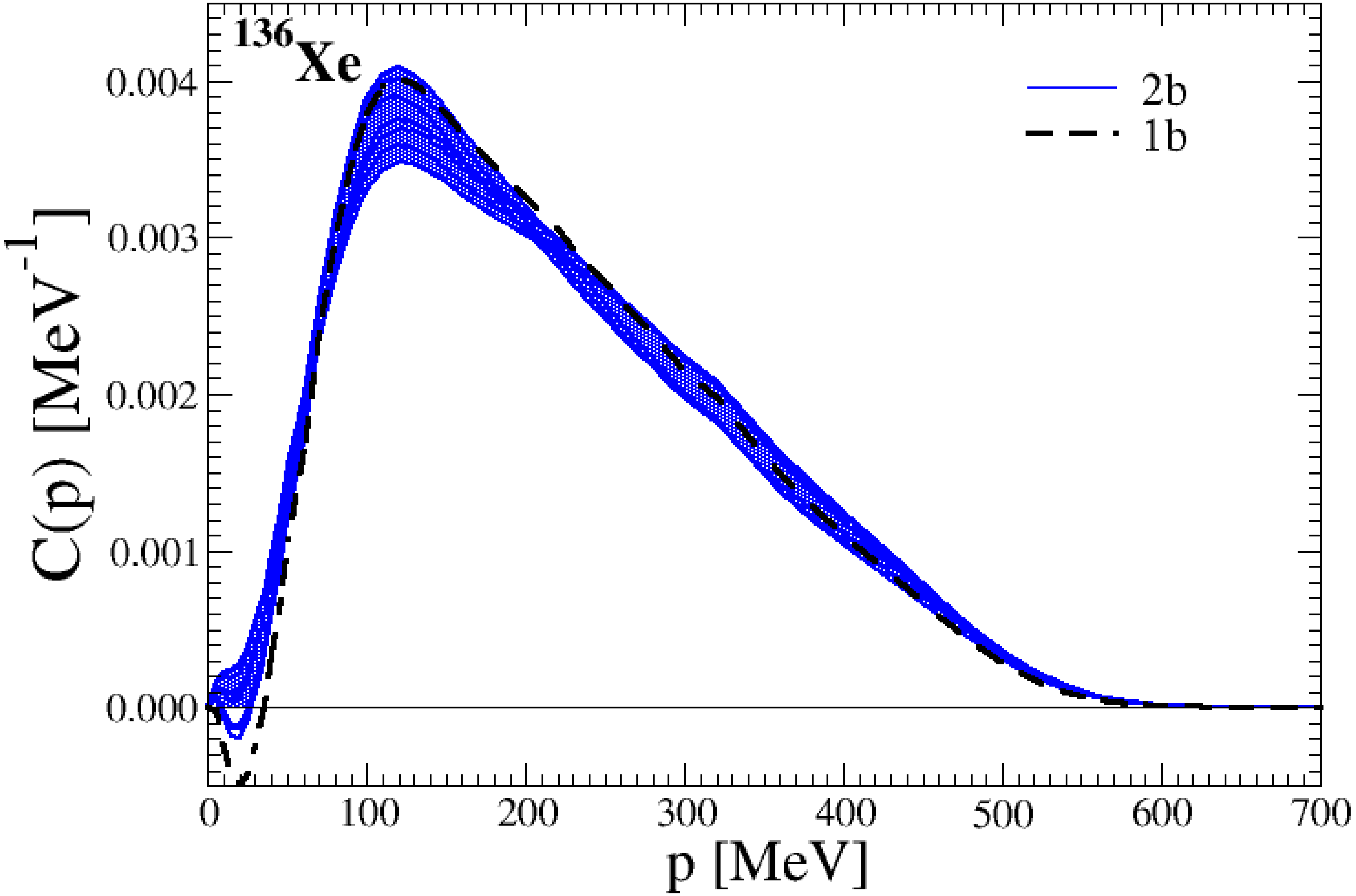
(Suppression of about 20%)



Engel, Vogel, Faessler, F.Š., PRC 89 (2014) 064308

Momentum distribution of NME normalized to unity

<p> ≈ 230 MeV, sqrt(<p²>) ≈ 250 MeV



The 0νββ-decay with emission of electrons in $p_{1/2}$ wave state

D. Štefánik, R. Dvornický, F.Š., Nuclear Theory 33 (2014) 115 and to be submitted

$$\psi(\mathbf{r},p,s) \simeq \psi_{s_{1/2}}(\mathbf{r},p,s) + \psi_{p_{1/2}}(\mathbf{r},p,s) =$$

$$\begin{pmatrix} g_{-1}(\varepsilon,r)\chi_s \\ f_{+1}(\varepsilon,r)\,(\vec{\sigma}\cdot\hat{\mathbf{p}})\,\chi_s \end{pmatrix} + \begin{pmatrix} i g_{+1}(\varepsilon,r)\,\boxed{(\vec{\sigma}\cdot\hat{\mathbf{r}})}\,(\vec{\sigma}\cdot\hat{\mathbf{p}})\,\chi_s \\ -i f_{-1}(\varepsilon,r)\,\boxed{(\vec{\sigma}\cdot\hat{\mathbf{r}})}\,\chi_s \end{pmatrix}$$

Exact relativ. electron w.f.

$$J^{\rho}(\mathbf{x}) = \sum_n \tau_n^+ \delta(\mathbf{x}-\mathbf{r}_n)\left[(g_V - g_A C_n)\, g^{\rho 0} + g^{\rho k}\right.$$
$$\left.\times \left(g_A \sigma_n^k - g_V D_n^k - g_P \left(p_n^k - p_n^{'k}\right) \frac{\vec{\sigma}_n \cdot \left(\mathbf{p}_n - \mathbf{p}_n^{'}\right)}{2m_N}\right)\right]$$

Higher order terms of nucleon current with nucleon recoil

$$C_n = \frac{\vec{\sigma}_n \cdot \boxed{\left(\mathbf{p}_n + \mathbf{p}_n^{'}\right)}}{2m_N} - \frac{g_P}{g_A}\left(E_n - E_n^{'}\right)\frac{\vec{\sigma}_n \cdot \boxed{\left(\mathbf{p}_n - \mathbf{p}_n^{'}\right)}}{2m_N}$$

$$\mathbf{D}_n = \frac{\boxed{\left(\mathbf{p}_n + \mathbf{p}_n^{'}\right)}}{2m_N} - i\left(1 + \frac{g_M}{g_V}\right)\frac{\vec{\sigma}_n \times \boxed{\left(\mathbf{p}_n - \mathbf{p}_n^{'}\right)}}{2m_N}.$$

0νββ-decay rate with $p_{1/2}$ electrons (2 additional NMEs and 5 phase-space factors)

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = \frac{|m_{\beta\beta}|^2}{m_e^2} g_A^4 \left(2Re\left\{M_s M_r^*\right\} G_{sr} + 2Re\left\{M_s M_p^*\right\} G_{sp} + 2Re\left\{M_r M_p^*\right\} G_{rp} + \boxed{G_{ss}\left|M_s\right|^2} + G_{rr}\left|M_r\right|^2 + G_{pp}\left|M_p\right|^2\right),$$

$$\boxed{M_s} = -\frac{M_F}{g_A^2} + M_{GT} + M_T$$

$$M_{F,GT,T} = \sum_{r,s} \langle 0|\, h_{F,GT,T}(r_-) \mathcal{O}_{\mathcal{F},\mathcal{GT},\mathcal{T}} \,|0\rangle$$

$$\boxed{M_p} = -\frac{M_F'}{g_A^2} + M_{GT}' + M_T' + M_V + M_A + M_A'$$

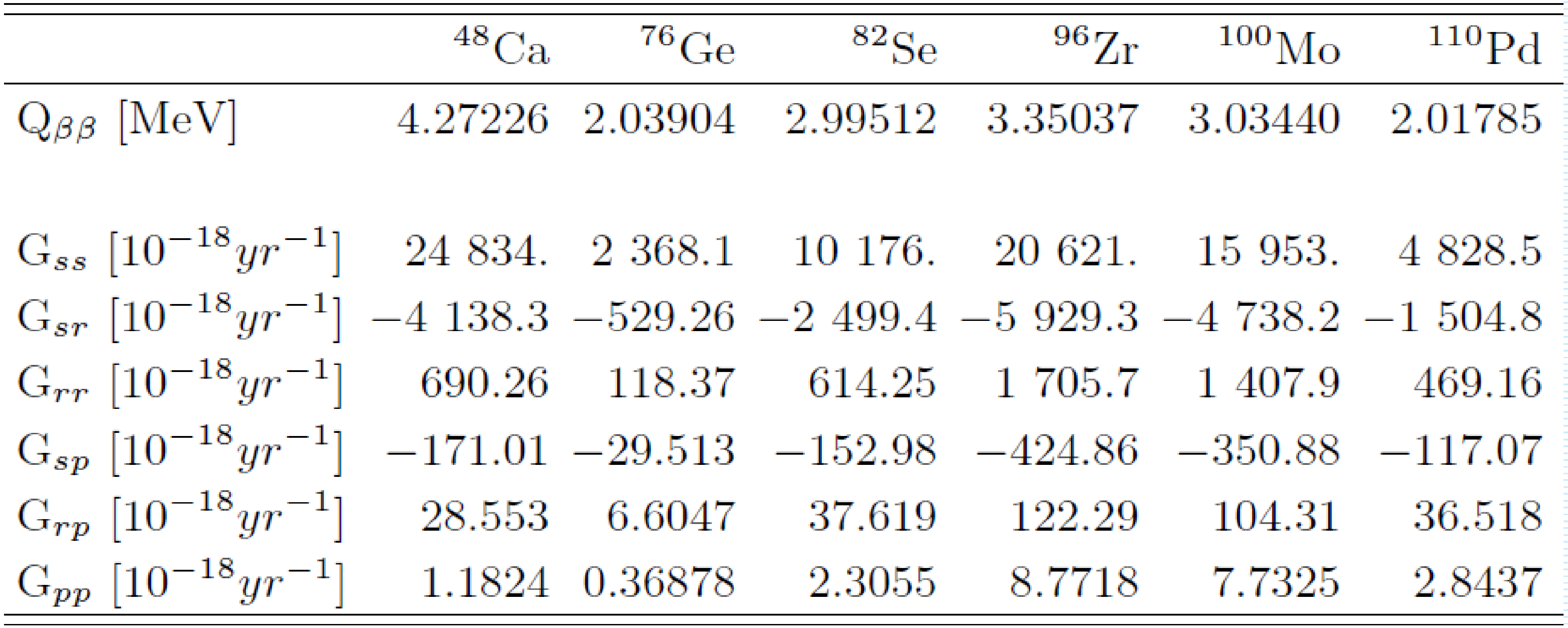
$$M_V = i \sum_{r.s} \langle 0|\, \frac{h_{AV}(r_-) + h_{VP}(r_-)}{2R^2} \tau_r^+ \tau_s^+ (\mathbf{r}_- \times \mathbf{r}_+) \cdot \vec{\sigma}_r \,|0\rangle$$

$$M_{F,GT,T}' = \sum_{r.s} \langle 0|\, h_{F,GT,T}(r_-) \mathcal{O}_{\mathcal{F},\mathcal{GT},\mathcal{T}} \left(\frac{|\mathbf{r}_-|^2 - |\mathbf{r}_+|^2}{4R^2}\right)$$

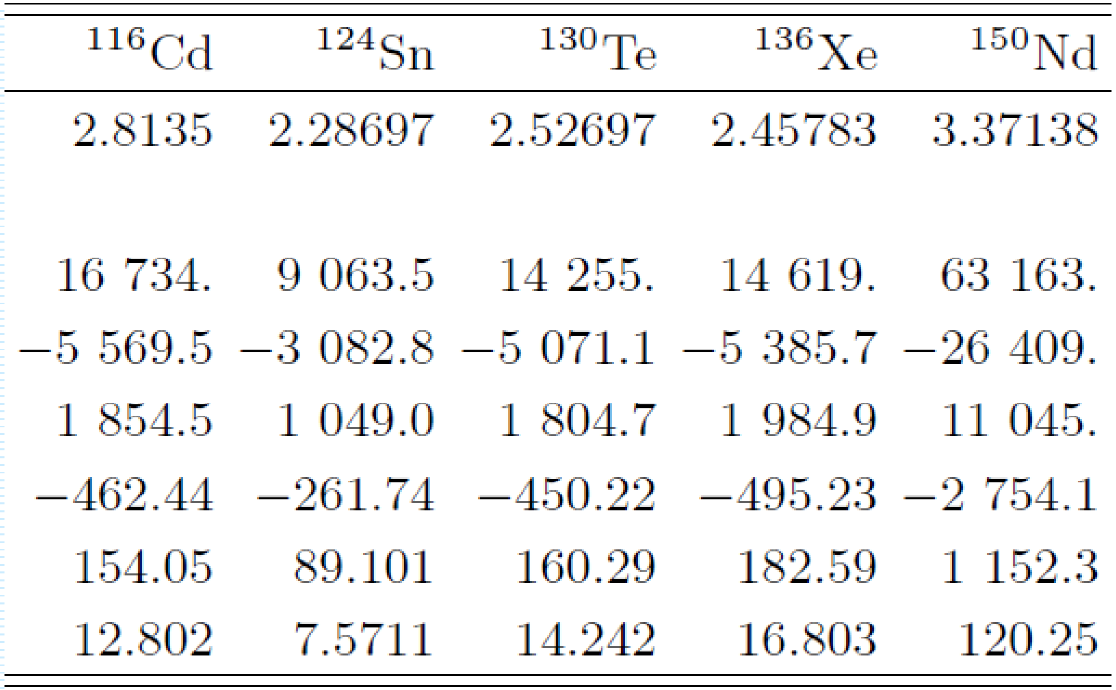
$$M_A = \sum_{r,s} \langle 0|\, \frac{h_{AP}(r_-) + h_{AA}(r_-) + h_{MM}(r_-)}{2R^2} \times \tau_r^+ \tau_s^+ (\vec{\sigma}_r \cdot \mathbf{r}_-)(\vec{\sigma}_s \cdot \mathbf{r}_+) \,|0\rangle$$

$$M_A' = \sum_{r,s} \langle 0|\, h_A(r_-) \frac{(\mathcal{O}_\mathcal{T} + \mathcal{O}_{\mathcal{GT}})\, \mathbf{r}_- \cdot \mathbf{r}_+}{2R^2} \,|0\rangle$$

$$\boxed{M_r} = \sum_{r.s} \langle 0|\, \left(h_R(r_-) + h_R'(r_-)\right) \mathcal{O}_\mathcal{T} - 2h_R(r_-) \mathcal{O}_{\mathcal{GT}} \,|0\rangle$$



	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd
$Q_{\beta\beta}$ [MeV]	4.27226	2.03904	2.99512	3.35037	3.03440	2.01785
G_{ss} $[10^{-18} yr^{-1}]$	24 834.	2 368.1	10 176.	20 621.	15 953.	4 828.5
G_{sr} $[10^{-18} yr^{-1}]$	−4 138.3	−529.26	−2 499.4	−5 929.3	−4 738.2	−1 504.8
G_{rr} $[10^{-18} yr^{-1}]$	690.26	118.37	614.25	1 705.7	1 407.9	469.16
G_{sp} $[10^{-18} yr^{-1}]$	−171.01	−29.513	−152.98	−424.86	−350.88	−117.07
G_{rp} $[10^{-18} yr^{-1}]$	28.553	6.6047	37.619	122.29	104.31	36.518
G_{pp} $[10^{-18} yr^{-1}]$	1.1824	0.36878	2.3055	8.7718	7.7325	2.8437

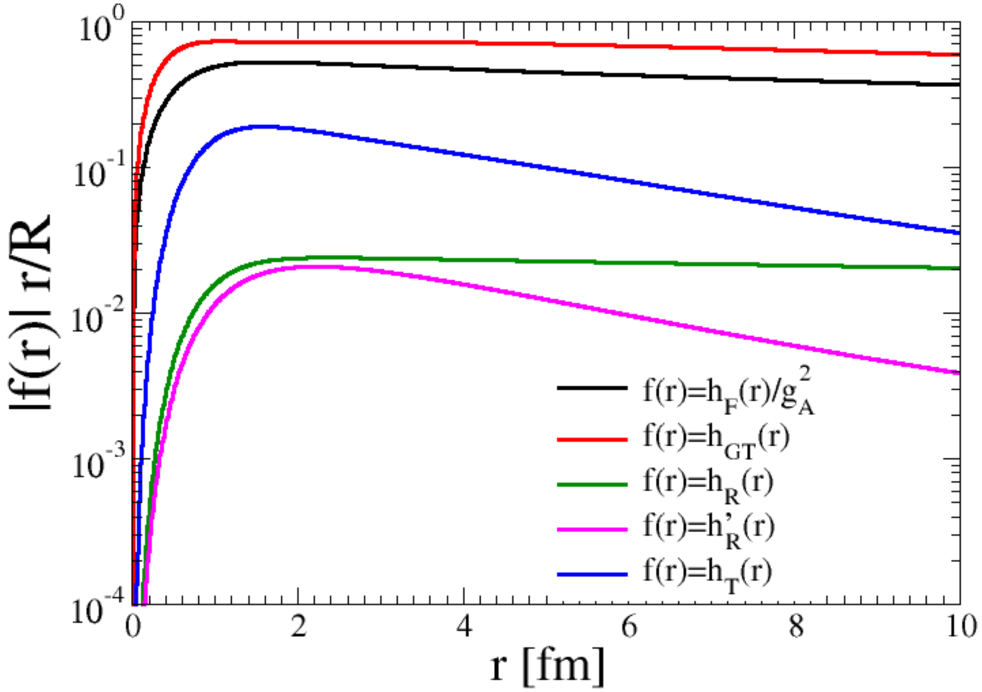


	^{116}Cd	^{124}Sn	^{130}Te	^{136}Xe	^{150}Nd
	2.8135	2.28697	2.52697	2.45783	3.37138
	16 734.	9 063.5	14 255.	14 619.	63 163.
	−5 569.5	−3 082.8	−5 071.1	−5 385.7	−26 409.
	1 854.5	1 049.0	1 804.7	1 984.9	11 045.
	−462.44	−261.74	−450.22	−495.23	−2 754.1
	154.05	89.101	160.29	182.59	1 152.3
	12.802	7.5711	14.242	16.803	120.25

Calculated phase-space factor for 0νββ-decay with emission of $s_{1/2}$ and $p_{1/2}$ electrons

($m_{\beta\beta}$ mechanism)

	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd	^{116}Cd	^{124}Sn
				r = 1 fm				
$2h_R(r)/h_{GT}(r)$	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04
$\Delta[\%]$	0.31	0.55	0.93	1.81	2.03	2.44	4.22	3.24
				r = 2 fm				
$2h_R(r)/h_{GT}(r)$	0.71	0.07	0.06	0.06	0.06	0.06	0.06	0.06
$\Delta[\%]$	1.19	0.46	0.20	0.60	0.82	1.21	3.60	1.95



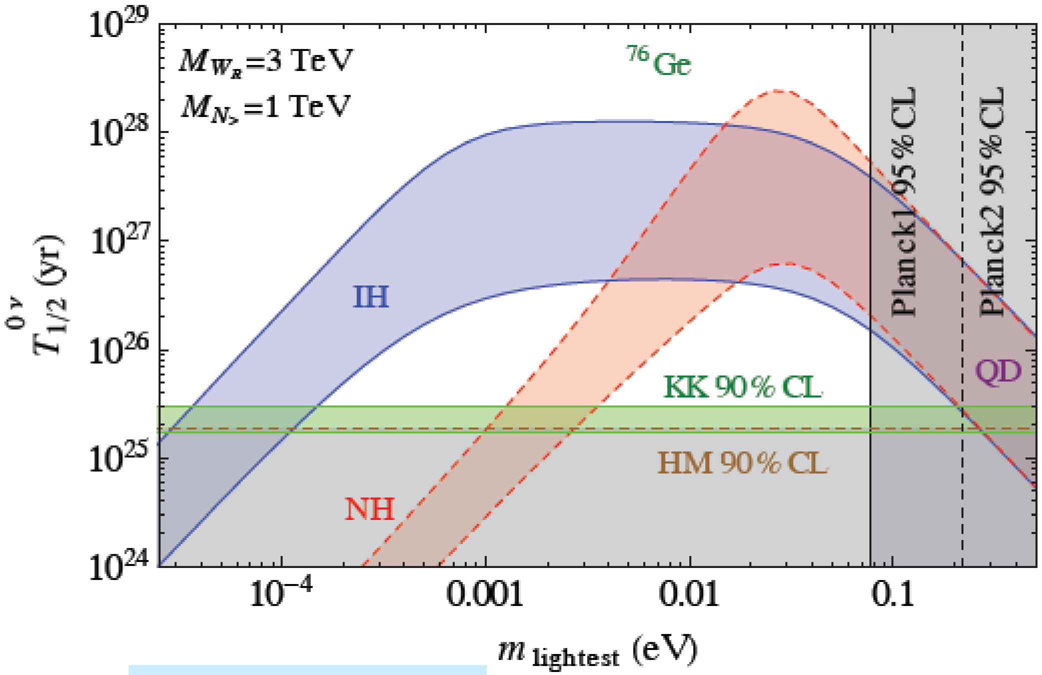
^{130}Te	^{136}Xe	^{150}Nd
0.04	0.04	0.03
3.70	4.10	5.74
0.06	0.05	0.05
2.37	2.75	4.27

Effect of $p_{1/2}$ wave is below 10%.

Heavy/sterile ν's and $0\nu\beta\beta$-decay

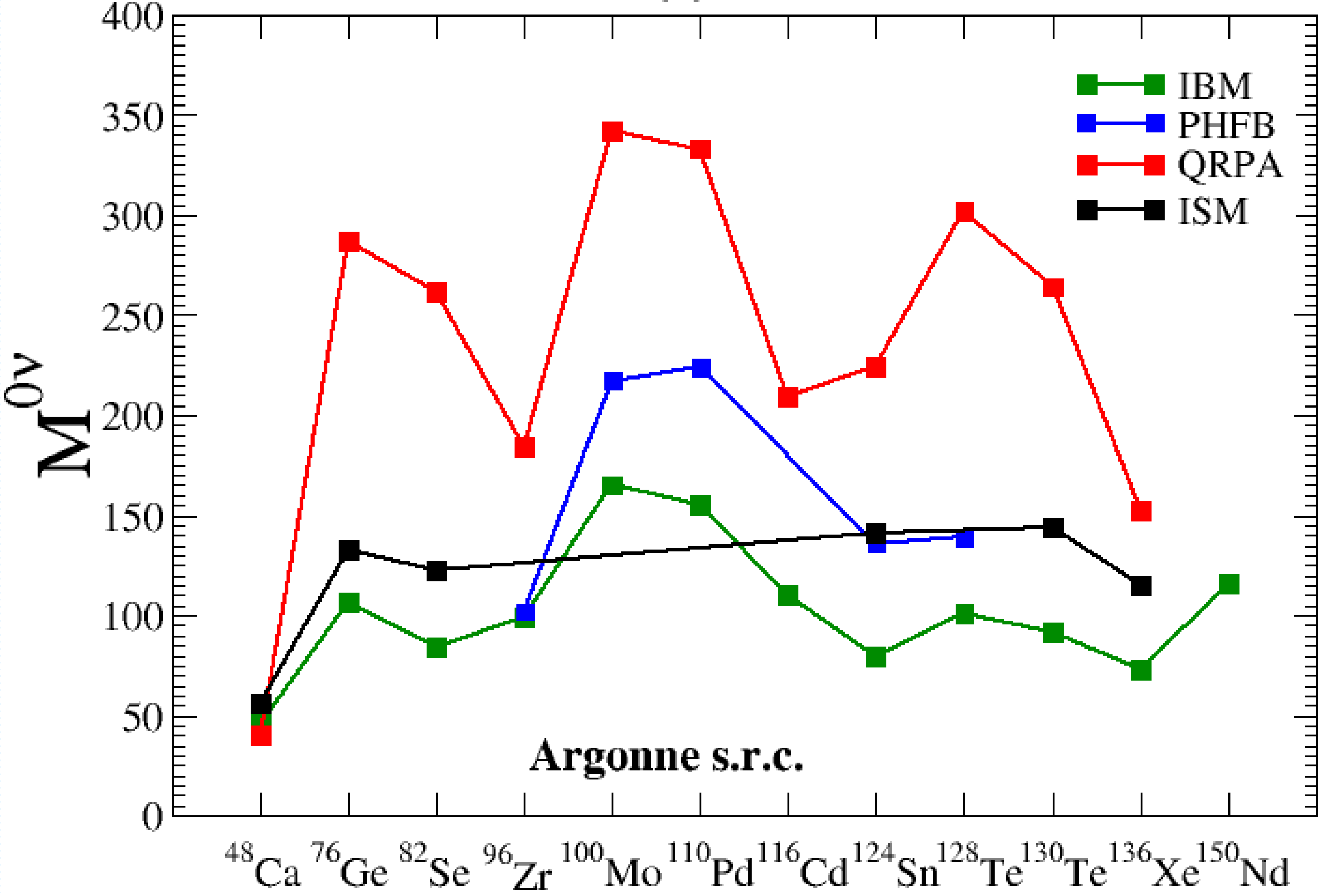
LHC (scale!?)
and L-R symmetric models

Discrete LR symmetry to parity (U=V)



W. Rodejohan

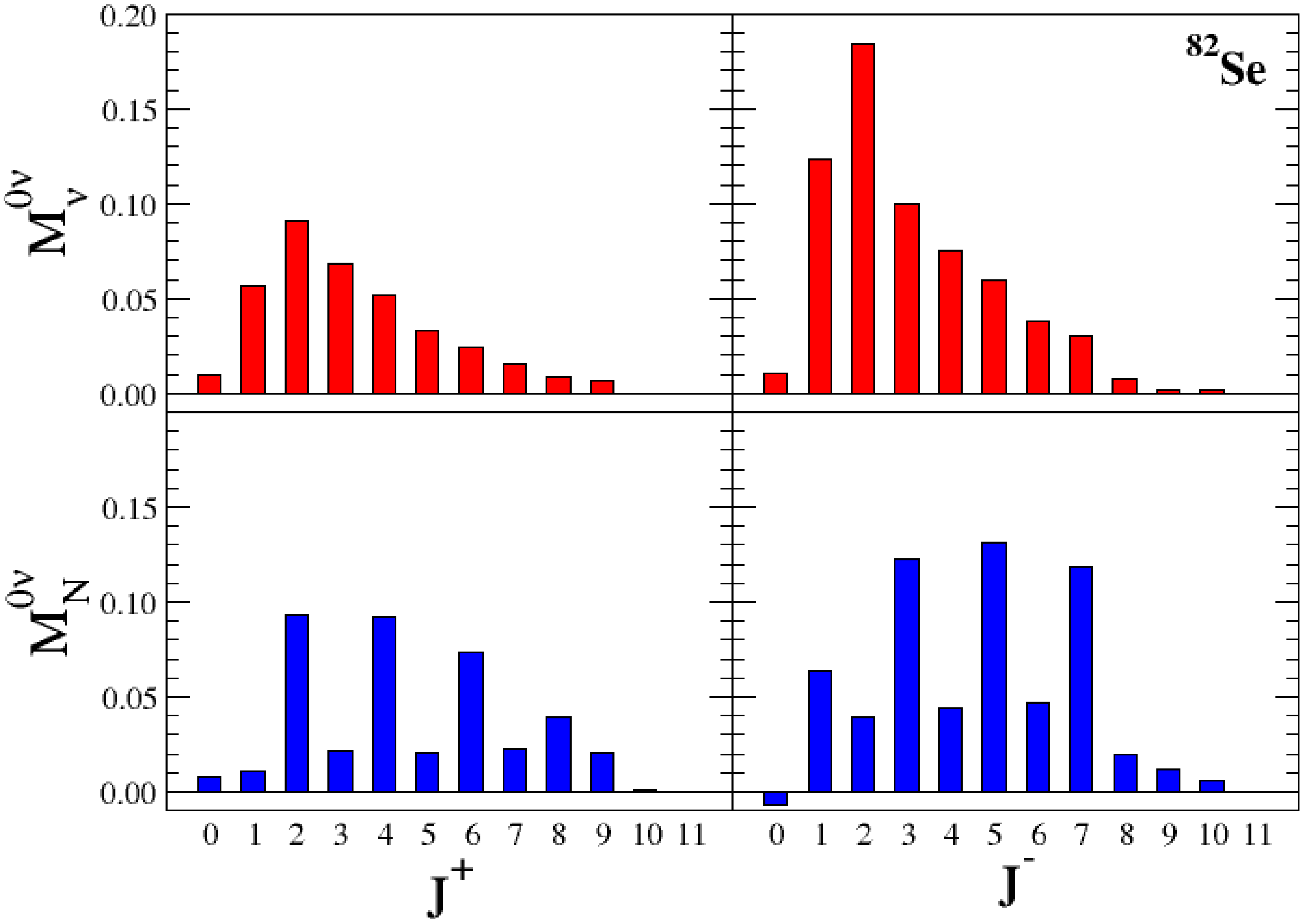
Heavy ν: 0νββ NMEs -status 2014



PHFB: K. Rath et al., PRC 85 (2012) 014308
IBM: Barea, Kotila, Iachello, PRC (2013) 014315

SQRPA: Vergados, Ejiri, F. Š., RPP 75 (2012) 106301
ISM: Menendez, privite communications

Multipole decomposition of NMEs normalized to unity



Majorana neutrino mass eigenstate N with arbitrary mass m_N

$$N = \sum_{\alpha=s,e,\mu,\tau} U_{N\alpha}\, \nu_\alpha$$

Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_{\mathrm{A}}^4 \left| \sum_{\mathrm{N}} \left(U^2_{\mathrm{eN}} m_{\mathrm{N}}\right) m_{\mathrm{p}}\, M'^{0\nu}(m_{\mathrm{N}}, g_{\mathrm{A}}^{\mathrm{eff}}) \right|^2$$

General case

$$M'^{0\nu}(m_{\mathrm{N}}, g_{\mathrm{A}}^{\mathrm{eff}}) = \frac{1}{m_{\mathrm{p}} m_{\mathrm{e}}} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x\, d^3y\, d^3p \times e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \frac{\langle 0_F^+|J^{\mu\dagger}(\mathbf{x})|n\rangle\langle n|J_\mu^\dagger(\mathbf{y})|0_I^+\rangle}{\sqrt{p^2+m_N{}^2}\left(\sqrt{p^2+m_N{}^2}+E_n-\frac{E_I-E_F}{2}\right)}$$

$$M'^{0\nu}(m_{\mathrm{N}} \to 0, g_{\mathrm{A}}^{\mathrm{eff}}) = \frac{1}{m_{\mathrm{p}} m_{\mathrm{e}}} M'^{0\nu}_{\nu}(g_{\mathrm{A}}^{\mathrm{eff}})$$

$$M'^{0\nu}(m_{\mathrm{N}} \to \infty, g_{\mathrm{A}}^{\mathrm{eff}}) = \frac{1}{m_{\mathrm{N}}^2} M'^{0\nu}_{\mathrm{N}}(g_{\mathrm{A}}^{\mathrm{eff}})$$

Particular cases

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_{\mathrm{A}}^4 \times$$
$$\times \begin{cases} \left|\frac{\langle m_\nu\rangle}{m_e}\right|^2 \left|M'^{0\nu}_{\nu}(g_{\mathrm{A}}^{\mathrm{eff}})\right|^2 & \text{for } m_{\mathrm{N}} \ll p_{\mathrm{F}} \\ \left|\langle\frac{1}{m_{\mathrm{N}}}\rangle m_{\mathrm{p}}\right|^2 \left|M'^{0\nu}_{\mathrm{N}}(g_{\mathrm{A}}^{\mathrm{eff}})\right|^2 & \text{for } m_{\mathrm{N}} \gg p_{\mathrm{F}} \end{cases}$$

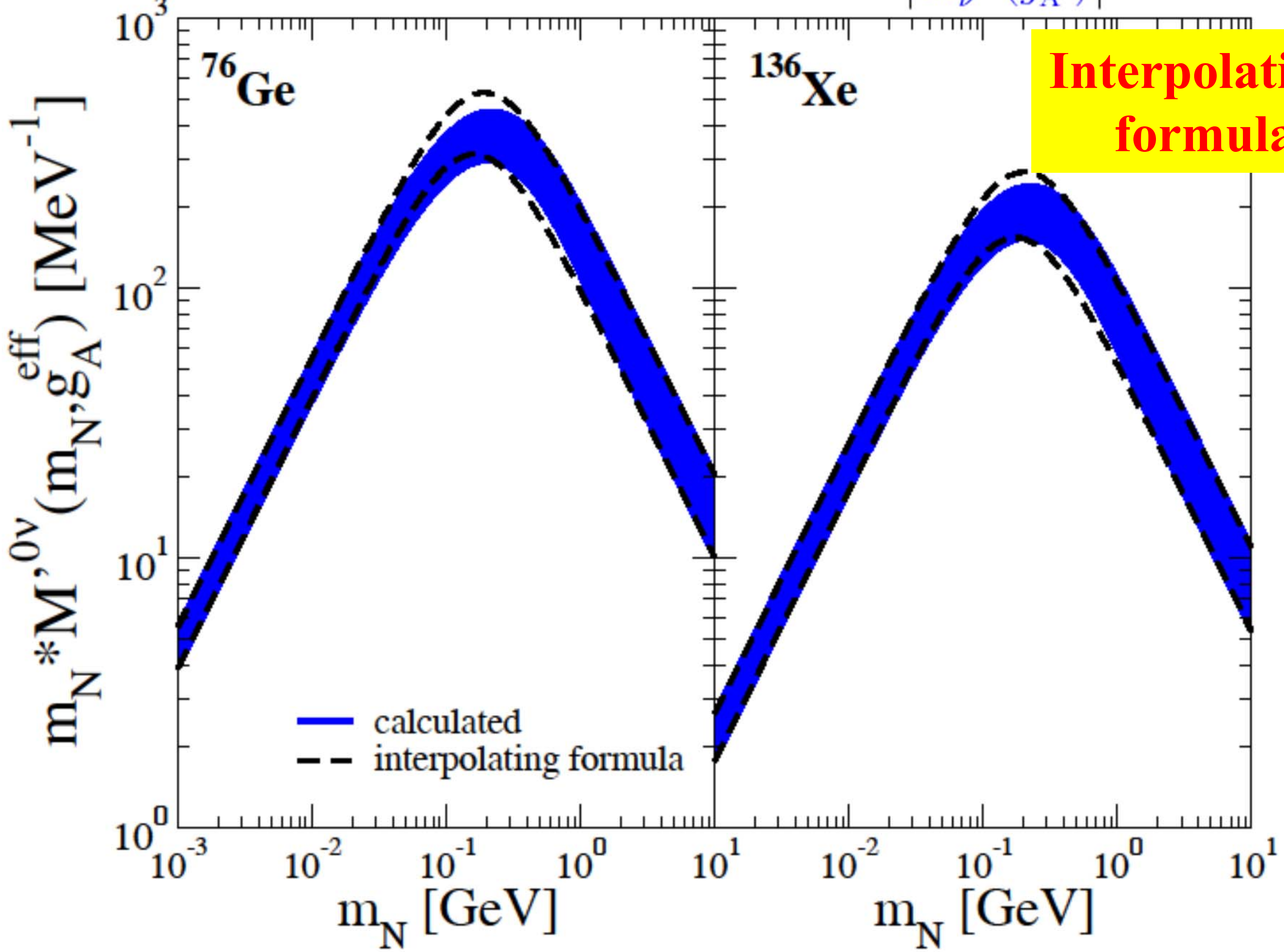
$$\langle m_\nu\rangle = \sum_{\mathrm{N}} U^2_{\mathrm{eN}} m_{\mathrm{N}}$$

$$\left\langle \frac{1}{m_{\mathrm{N}}} \right\rangle = \sum_{\mathrm{N}} \frac{U^2_{\mathrm{eN}}}{m_{\mathrm{N}}}$$

$$[T^{0\nu}_{1/2}]^{-1} = \mathcal{A}\cdot\left|m_{\mathrm{p}} \sum_{\mathrm{N}} U^2_{e\mathrm{N}}\frac{m_{\mathrm{N}}}{\langle p^2\rangle + m_{\mathrm{N}}^2}\right|^2,$$

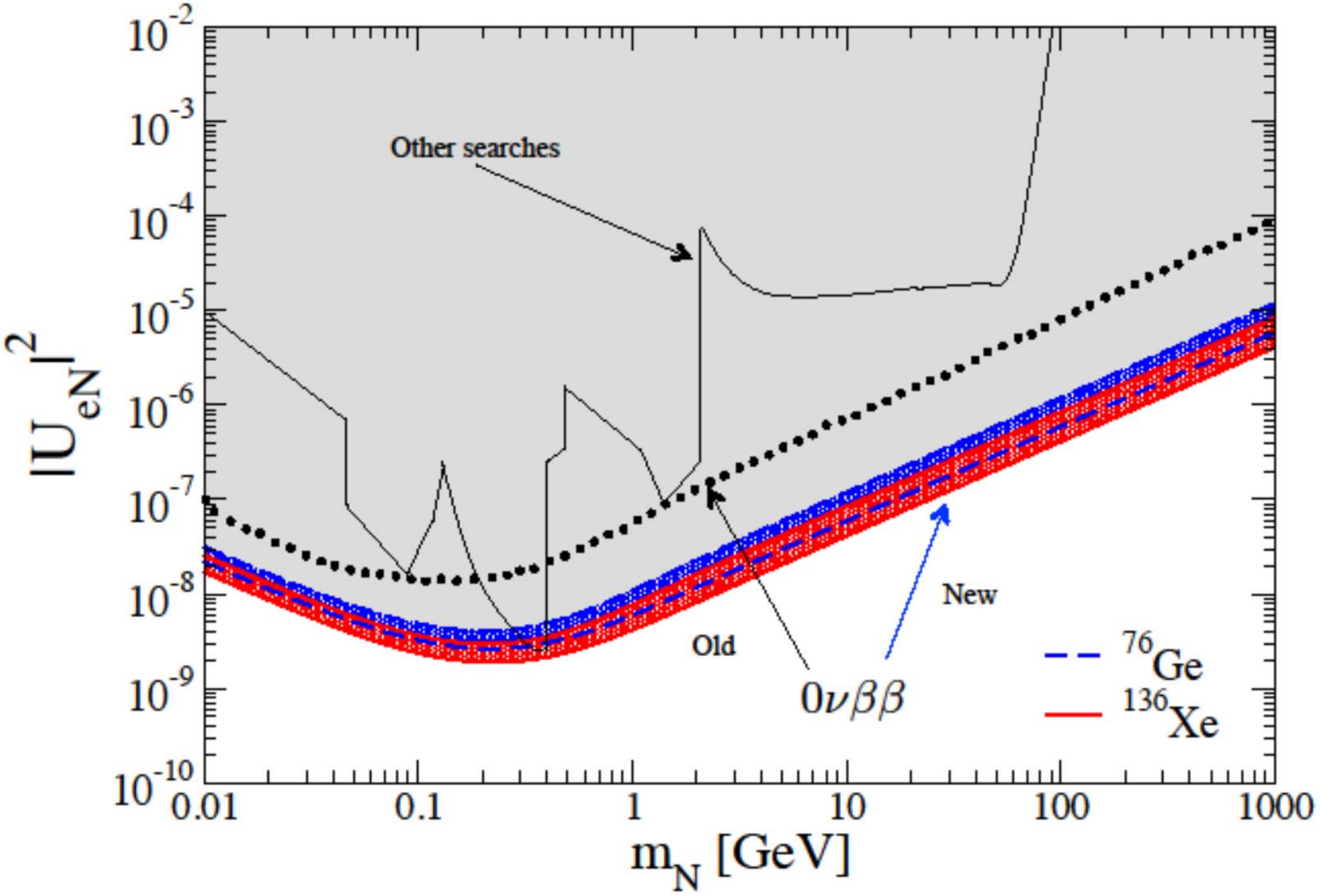
$$\mathcal{A} = G^{0\nu} g_{\mathrm{A}}^4 \left|M'^{0\nu}_{\mathrm{N}}(g_{\mathrm{A}}^{\mathrm{eff}})\right|^2,$$

$$\langle p^2\rangle = m_{\mathrm{p}} m_{\mathrm{e}} \left|\frac{M'^{0\nu}_{\mathrm{N}}(g_{\mathrm{A}}^{\mathrm{eff}})}{M'^{0\nu}_{\nu}(g_{\mathrm{A}}^{\mathrm{eff}})}\right|^{\cancel{2}} \approx 200\ MeV$$



Interpolating formula

Exclusion plot in $|U_{eN}|^2 - m_N$ plane



Improvements: i) QRPA (constrained Hamiltonian by 2νββ half-life, self-consistent treatment of src, restoration of isospin symmetry …), ii) More stringent limits on the 0νββ half-life

The $0\nu\beta\beta$-decay with right-handed currents revisited

D. Štefánik, R. Dvornický, F.Š., P. Vogel, to be submitted

Assumption $M_R \gg m_D$

$$\begin{pmatrix} \overline{\nu}_L & \overline{(\nu_R)^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix}$$

Eigenvalues and eigenvectors

$m_1 = m_D^2/M_R \ll m_D \quad m_2 \approx M_R$

$\nu_1 = \nu_L - m_D/M_R\,(\nu_R)^c \quad \nu_2 = \nu_R + m_D/M_R\,(\nu_L)^c$

Left-right symmetric models SO(10)

Two-charged vector bosons

$W_1^\pm = \cos\zeta\ W_L^\pm + \sin\zeta\ W_R^\pm$

$W_2^\pm = -\sin\zeta\ W_L^\pm + \cos\zeta\ W_R^\pm$

Parameters

$\zeta \leq 2.5\ 10^{-3},\ 1.3\ 10^{-2}$

$M_1 = 81$ GeV, $M_2 > 2.9$ TeV, $(M_1/M_2)^2 < 10^{-3}$ LHC

Currents

$$j_L^\rho = \bar{e}\gamma_\rho(1-\gamma_5)\nu_{eL} \qquad j_R^\rho = \bar{e}\gamma_\rho(1+\gamma_5)\nu_{eR}$$

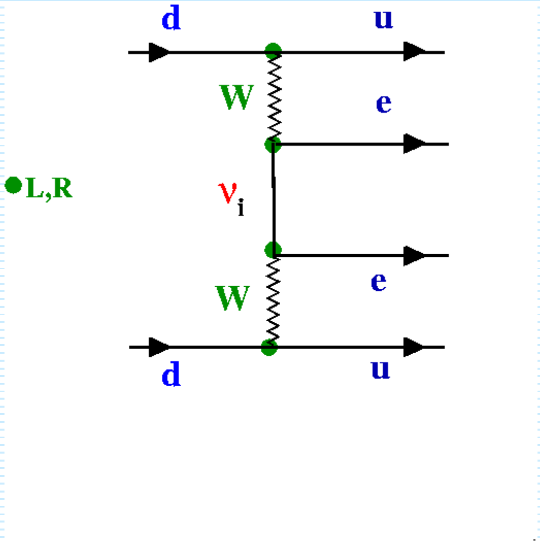
See-saw scenario

$$\nu_{eL} = \sum_{i=1}^{light} \underset{\uparrow\ \text{large}}{U_{ei}}\chi_{iL} + \sum_{i=1}^{heavy} \underset{\uparrow\ \text{small}}{U_{ei}} N_{iL} \qquad (\nu_{eR})^c = \sum_{i=1}^{light} \underset{\uparrow\ \text{small}}{V_{ei}}\chi_{iL} + \sum_{i=1}^{heavy} \underset{\uparrow\ \text{large}}{V_{ei}} N_{iL}$$

$$H^{\beta} = \frac{G_{\beta}}{\sqrt{2}} \left[j_L^{\rho} J_{L\rho}^{\dagger} + \chi j_L^{\rho} J_{R\rho}^{\dagger} + \eta j_R^{\rho} J_{L\rho}^{\dagger} + \lambda j_R^{\rho} J_{R\rho}^{\dagger} + h.c. \right]$$

$$\eta \simeq -\tan\zeta \qquad \chi = \eta$$

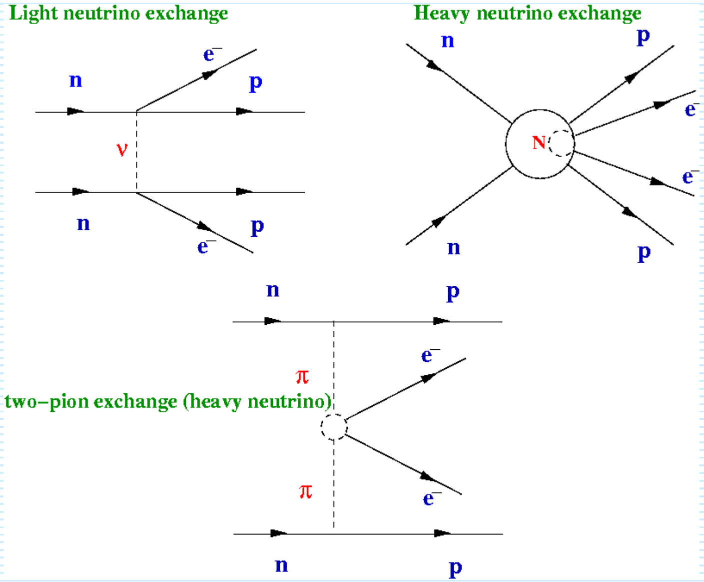
$$\lambda \simeq (M_{W_1}/M_{W_2})^2$$



$$P_{L}\frac{\hat{q}+im}{q^2+m^2}P_{L} \Rightarrow \frac{im}{q^2}$$

$$P_{L}\frac{\hat{q}+im}{q^2+m^2}P_{R} \Rightarrow \frac{i\hat{q}}{q^2}$$

$$P_{L,R}\frac{\hat{q}+iM}{q^2+M^2}P_{L,R} \Rightarrow \frac{i}{M}$$



quark level

Mechanism

nucleon level

neutrino	lept.v.	quarkv.	hadr.m.	supp.f.	LNVp.	limit
light	LL	LL	$2n$		$\sum^{light} UUm$	$m_{\beta\beta} \lesssim 0.5\ eV$
	LR	LR	$2n$	$(M_1/M_2)^2$	$\sum^{light} UV$	$<\lambda> \lesssim 7\ 10^{-7}$
	LR	LL	$2n$	$\tan\zeta$	$\sum^{light} UV$	$<\eta> \lesssim 4\ 10^{-9}$
heavy	LL	LL	$2n$	$-$	$\sum^{heavy} U\ U\ m_p/M$	$\eta_N \lesssim 8\ 10^{-8}$
	RR	RR	$2n$	$(M_1/M_2)^4$	$\sum^{heavy} V\ V\ m_p/M$	
	RR	LL	$2n$	$(\tan\zeta)^4$	$\sum^{heavy} V\ V\ m_p/M$	
	RR	RL	2π	$\tan\zeta\ (M_1/M_2)^2$	$\sum^{heavy} V\ V\ m_p/M$	

3x3 block matrices
U, S, T, V are
generalization of PMNS matrix

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

Basis

$$(\nu_L, (N_R)^{\bar{C}})^T$$

6x6 neutrino mass matrix

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

Decomposition

$$\mathcal{U} = \begin{pmatrix} 1 & 0 \\ 0 & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & 0 \\ 0 & 1 \end{pmatrix}$$

Type seesaw I

$$A \approx 1, B \approx 1, R \approx \frac{m_D}{m_{LNV}} 1, S \approx -\frac{m_D}{m_{LNV}} 1$$

Approximation

$$U_0 \simeq V_0$$

LNV parameters

$$\langle\lambda\rangle \approx (M_{W_1}/M_{W_2})^2 \frac{m_D}{m_{LNV}} |\xi|$$

$$\langle\eta\rangle \approx -\tan\zeta \frac{m_D}{m_{LNV}} |\xi| ,$$

$$\begin{aligned} |\xi| &= |c_{23} c_{12}^2 c_{13} s_{13}^2 - c_{12}^3 c_{13}^3 - c_{13} c_{23} c_{12}^2 s_{13}^2 \\ &\quad - c_{12} c_{13} \left(c_{13}^2 s_{12}^2 + s_{13}^2 \right) | \\ &\simeq 0.82 \end{aligned}$$

The 0νββ-decay rate with right-handed currents

$$\left[T^{0\nu\beta\beta}_{1/2}\right]^{-1} = g_A^4\, |M_{GT}|^2 \left\{ C_1 \left(\frac{|m_{\beta\beta}|}{m_e}\right)^2 + C_4 \langle\lambda\rangle^2 + C_2 \frac{|m_{\beta\beta}|}{m_e}\langle\lambda\rangle \cos\psi_1 + C_3 \frac{|m_{\beta\beta}|}{m_e}\langle\eta\rangle \cos\psi_2 + C_5 \langle\eta\rangle^2 + C_6 \langle\lambda\rangle\langle\eta\rangle \cos(\psi_1 - \psi_2) \right\}$$

Two additioanal phase-space factor G_{010} and G_{011}
(For w.f. A $G_{010}=G_{03}$, $G_{011}=G_{04}$)

The induced pseudoscalar term included

$$\langle\lambda\rangle = \lambda |\sum_j U_{ej} V_{ej} (g'_V/g_V)|,$$

$$\langle\eta\rangle = \eta |\sum_j{}' U_{ej} V'_{ej}|,$$

$$\psi_1 = \arg[\{\sum_j{}' m_j U^2_{ej}\}\{\sum_j{}' U_{ej} V_{ej} (g'_V/g_V)\}],$$

$$\psi_2 = \arg[\{\sum_j{}' m_j U^2_{ej}\}\{\sum_j{}' U_{ej} V'_{ej}\}^*].$$

$$\begin{aligned}
C_1 &= (1-\chi_F+\chi_T)^2 G_{01},\\
C_2 &= -(1-\chi_F+\chi_T)\left[\chi_{2-}G_{03} - \chi_{1+}G_{04}\right],\\
C_3 &= (1-\chi_F+\chi_T)\\
&\times \left[\chi_{2+}G_{03} - \chi_{1-}G_{04} - \chi_P G_{05} + \chi_R G_{06}\right],
\end{aligned}$$

$$\begin{aligned}
C_4 &= \chi^2_{2-}G_{02} + \frac{1}{9}\chi^2_{1+}G_{011} - \frac{2}{9}\chi_{1+}\chi_{2-}G_{010},\\
C_5 &= \chi^2_{2+}G_{02} + \frac{1}{9}\chi^2_{1-}G_{011} - \frac{2}{9}\chi_{1-}\chi_{2+}G_{010} + \chi^2_P G_{08}\\
&- \chi_P\chi_R G_{07} + \chi^2_R G_{09},\\
C_6 &= -2[\chi_{2-}\chi_{2+}G_{02} - \frac{1}{9}(\chi_{1+}\chi_{2+} + \chi_{2-}\chi_{1-})G_{010}\\
&+ \frac{1}{9}\chi_{1+}\chi_{1-}G_{011}]
\end{aligned}$$

Different types of electron wave functions

$$\Psi(\varepsilon,\boldsymbol{r}) \;=\; \Psi^{(s_{1/2})}(\varepsilon,\boldsymbol{r}) + \Psi^{(p_{1/2})}(\varepsilon,\boldsymbol{r})$$

w.f. A (Doi et al.), uniform charge distribution, only the lowest term in expansion r/R

$$\begin{pmatrix} g_{-1}(\varepsilon,r) \\ f_{+1}(\varepsilon,r) \end{pmatrix} \approx \sqrt{F_0(Z_f,\varepsilon)} \begin{pmatrix} \sqrt{\frac{\varepsilon+m_e}{2\varepsilon}} \\ \sqrt{\frac{\varepsilon-m_e}{2\varepsilon}} \end{pmatrix} \qquad \begin{pmatrix} g_{+1}(\varepsilon,r) \\ f_{-1}(\varepsilon,r) \end{pmatrix} \approx \sqrt{F_0(Z_f,\varepsilon)} \begin{pmatrix} \sqrt{\frac{\varepsilon-m_e}{2\varepsilon}}[\alpha Z_f/2+(\varepsilon+m_e)r/3] \\ -\sqrt{\frac{\varepsilon+m_e}{2\varepsilon}}[\alpha Z_f/2+(\varepsilon-m_e)r/3] \end{pmatrix}$$

w.f. B, the analytical solution of the Dirac equation for a point-like nucleus

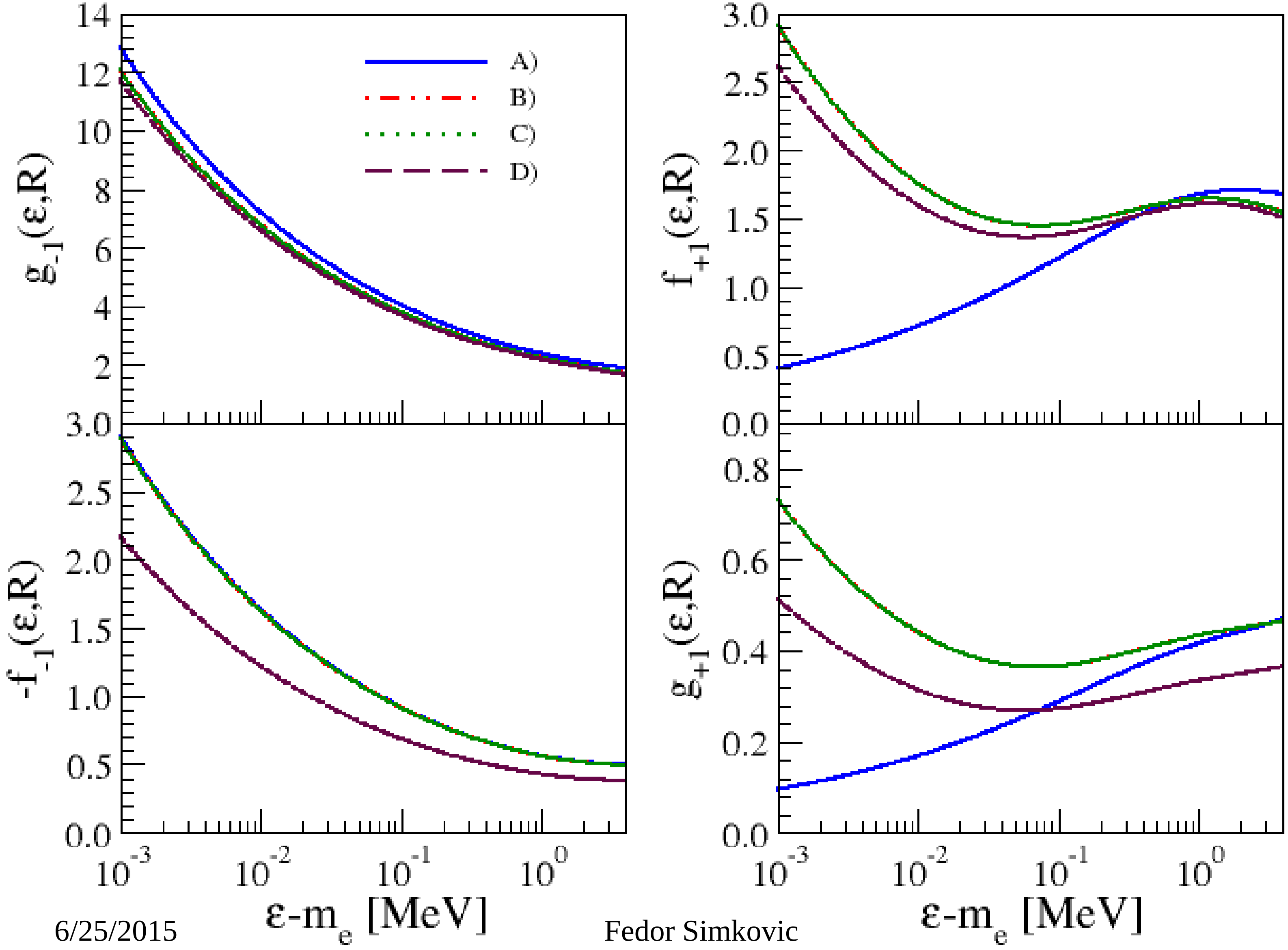
$$g_\kappa(\varepsilon,r) \;=\; \frac{1}{pr}\sqrt{\frac{\varepsilon+m_e}{2\varepsilon}}\frac{|\Gamma(1+\gamma_k+iy)|}{\Gamma(1+2\gamma_k)}(2pr)^{\gamma_k} \Im\left\{e^{i(pr+\xi)}\;{}_1F_1(\gamma_k-iy,1+2\gamma_k,-2ipr)\right\}$$

$$f_\kappa(\varepsilon,r) \;=\; \frac{1}{pr}\sqrt{\frac{\varepsilon-m_e}{2\varepsilon}}\frac{|\Gamma(1+\gamma_k+iy)|}{\Gamma(1+2\gamma_k)}(2pr)^{\gamma_k} \Re\left\{e^{i(pr+\xi)}\;{}_1F_1(\gamma_k-iy,1+2\gamma_k,-2ipr)\right\}$$

w.f. C, the exact Dirac wave functions with finite nuclear size corrections, which are taken into account in by a uniform charge distribution in a sphere of nucleus

w.f. D , the same as w.f. C but the screening of atomic electrons included

Radial components of electron wave functions at nuclear surface



Effect of Coulomb corrections more important as of finite nuclear size

w.f. A (Doi et al.), uniform charge distribution, only the lowest term in expansion r/R

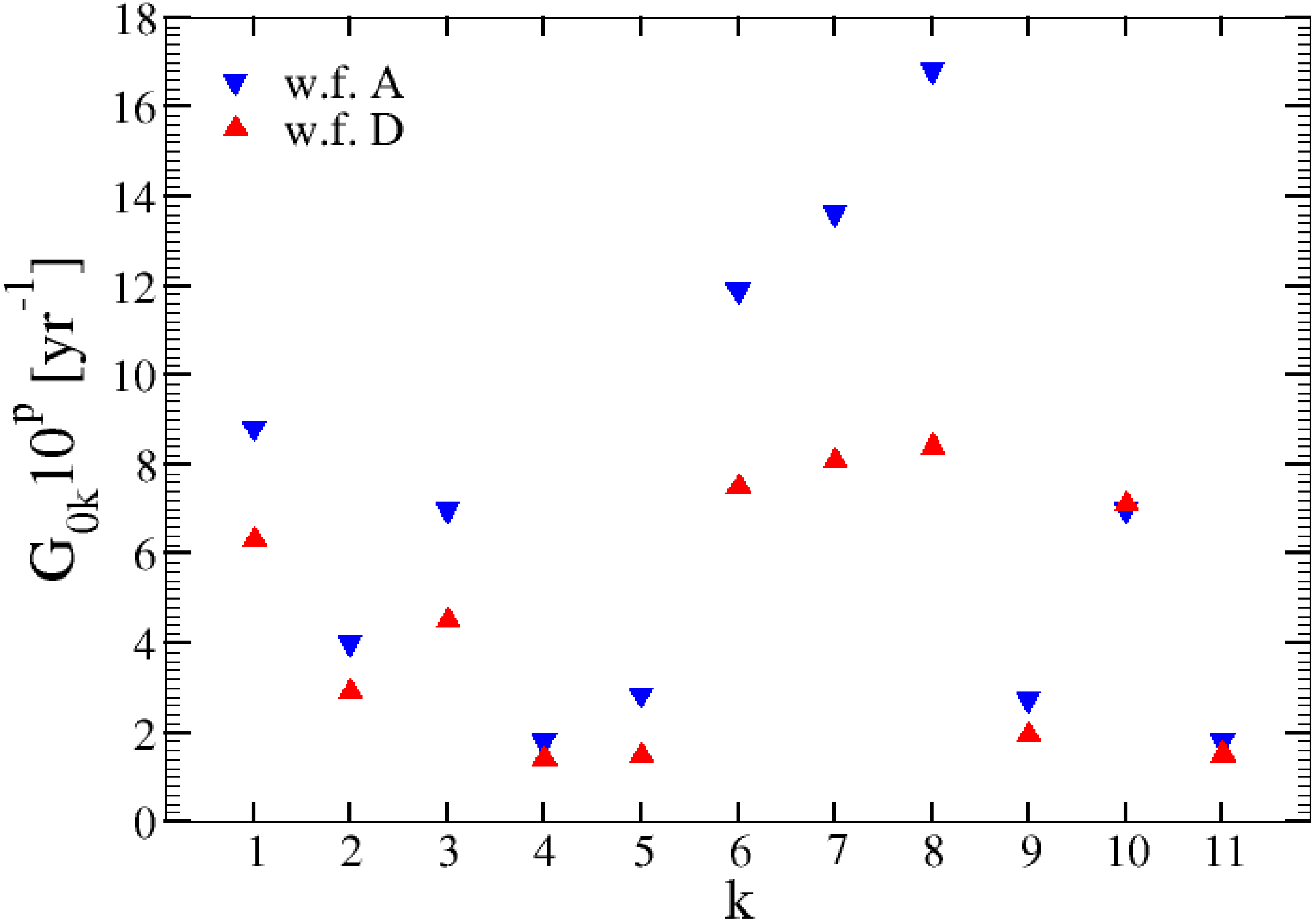
w.f. B, the analytical solution of the Dirac equation for a point-like nucleus

w.f. C, exact Dirac wave functions with finite nuclear size

w.f. C, exact Dirac wave functions with finite nuclear size and electron screening

	^{76}Ge				^{130}Te				^{150}Nd			
w.f.	A	B	C	D	A	B	C	D	A	B	C	D
$G_{01}.10^{14}$	0.261	0.244	0.240	0.237	1.807	1.535	1.453	1.425	8.827	6.986	6.432	6.316
$G_{02}.10^{14}$	0.428	0.404	0.397	0.391	4.683	4.064	3.851	3.761	40.190	32.401	29.869	29.187
$G_{03}.10^{15}$	1.478	1.340	1.316	1.305	12.237	9.566	9.065	8.967	70.032	49.465	45.593	45.130
$G_{04}.10^{15}$	0.501	0.489	0.477	0.470	3.625	3.315	3.086	3.021	18.343	16.000	14.348	14.066
$G_{05}.10^{13}$	0.791	0.727	0.572	0.566	6.390	5.185	3.842	3.790	28.537	21.183	15.061	14.873
$G_{06}.10^{12}$	0.605	0.547	0.536	0.531	3.091	2.398	2.258	2.227	11.922	8.323	7.591	7.497
$G_{07}.10^{10}$	0.365	0.345	0.274	0.270	2.713	2.383	1.788	1.755	13.625	11.362	8.233	8.085
$G_{08}.10^{11}$	0.245	0.236	0.151	0.149	2.877	2.653	1.579	1.549	16.833	14.996	8.564	8.405
$G_{09}.10^{10}$	1.360	1.263	1.238	1.223	6.398	5.354	5.063	4.972	27.582	21.530	19.799	19.454
$G_{010}.10^{15}$	1.478	1.531	1.423	1.410	12.237	14.602	11.616	11.455	70.032	105.415	72.249	71.154
$G_{011}.10^{15}$	0.501	0.500	0.484	0.476	3.625	3.564	3.220	3.148	18.343	18.334	15.376	15.055

Phase-space factors for ^{150}Nd



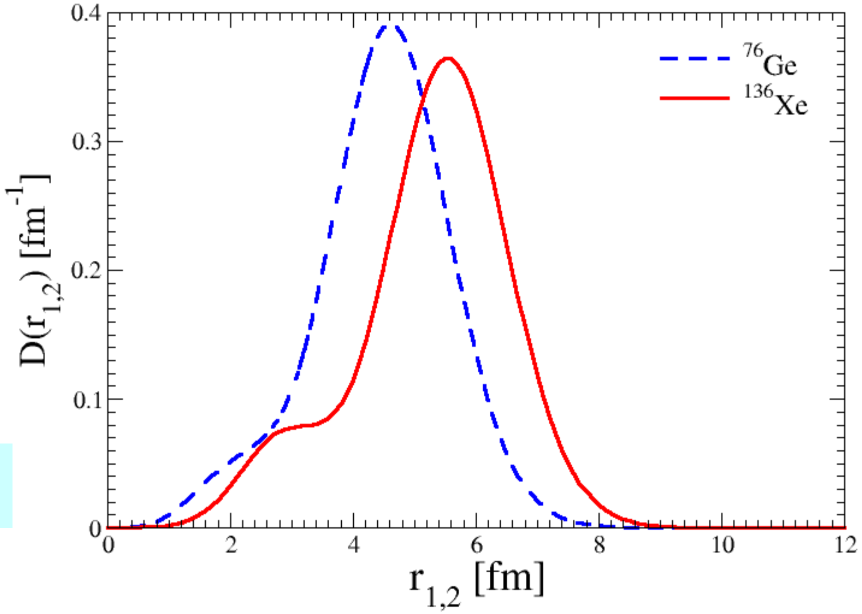
Phase space factors for a given distribution of beta decays in a nucleus

$$g_{\pm 1}(\varepsilon, R) = \int_0^\infty g_{\pm 1}(\varepsilon, r_1) D(r_1) dr_1,$$
$$f_{\pm 1}(\varepsilon, R) = \int_0^\infty f_{\pm 1}(\varepsilon, r_1) D(r_1) dr_1$$

i) Just exact w.f. (standard approach)

$$D(r_1) = \delta(r_1 - R)$$

ii) D(r_1) deduced from the QRPA calcul.



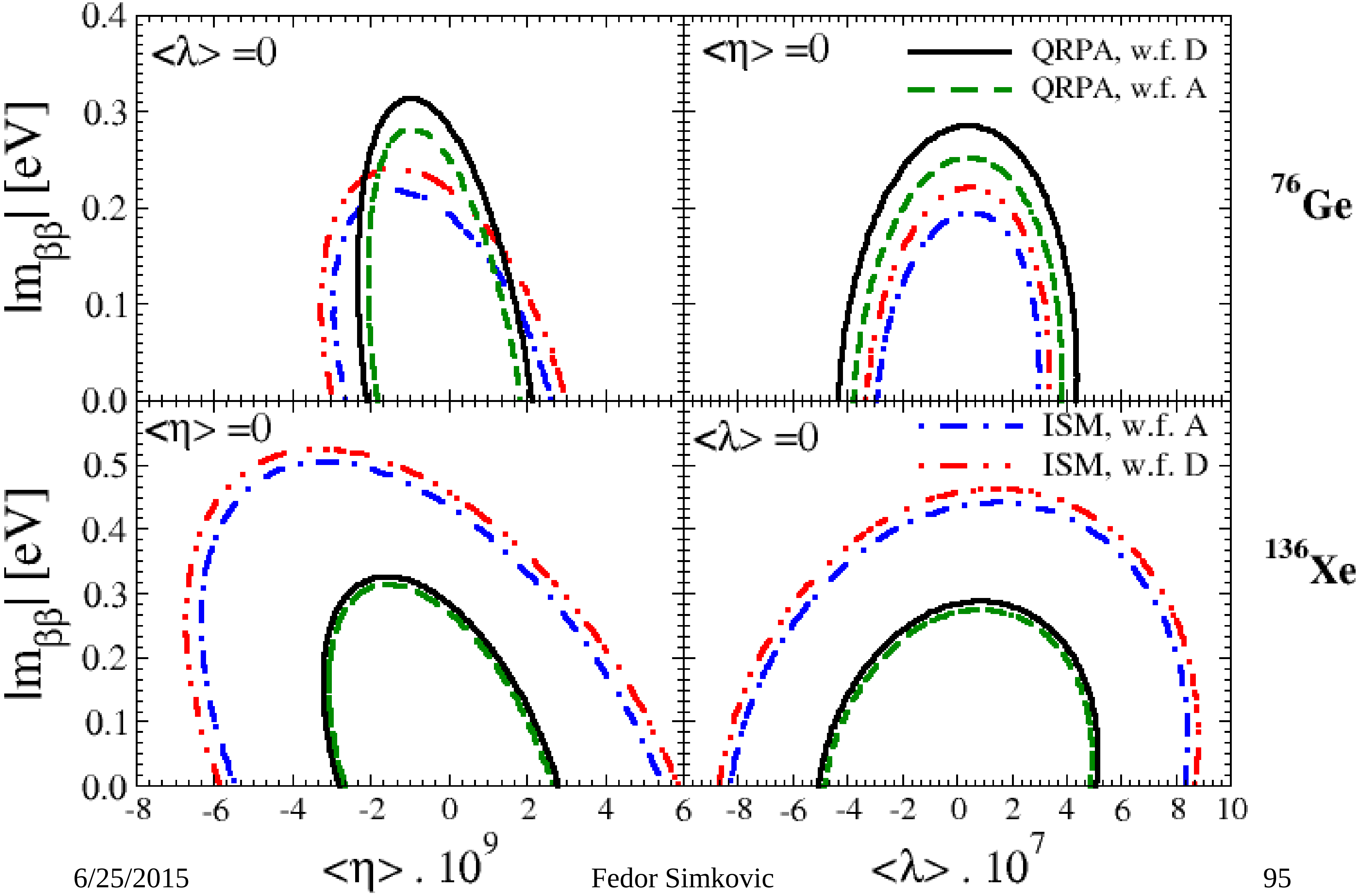
	$G_{01}.10^{14}$		
	^{76}Ge	^{130}Te	^{136}Xe
Exact	0.23681	1.42547	1.46187
Exact and averaged	0.23987	1.47396	1.52851

10% effect for ^{136}Xe

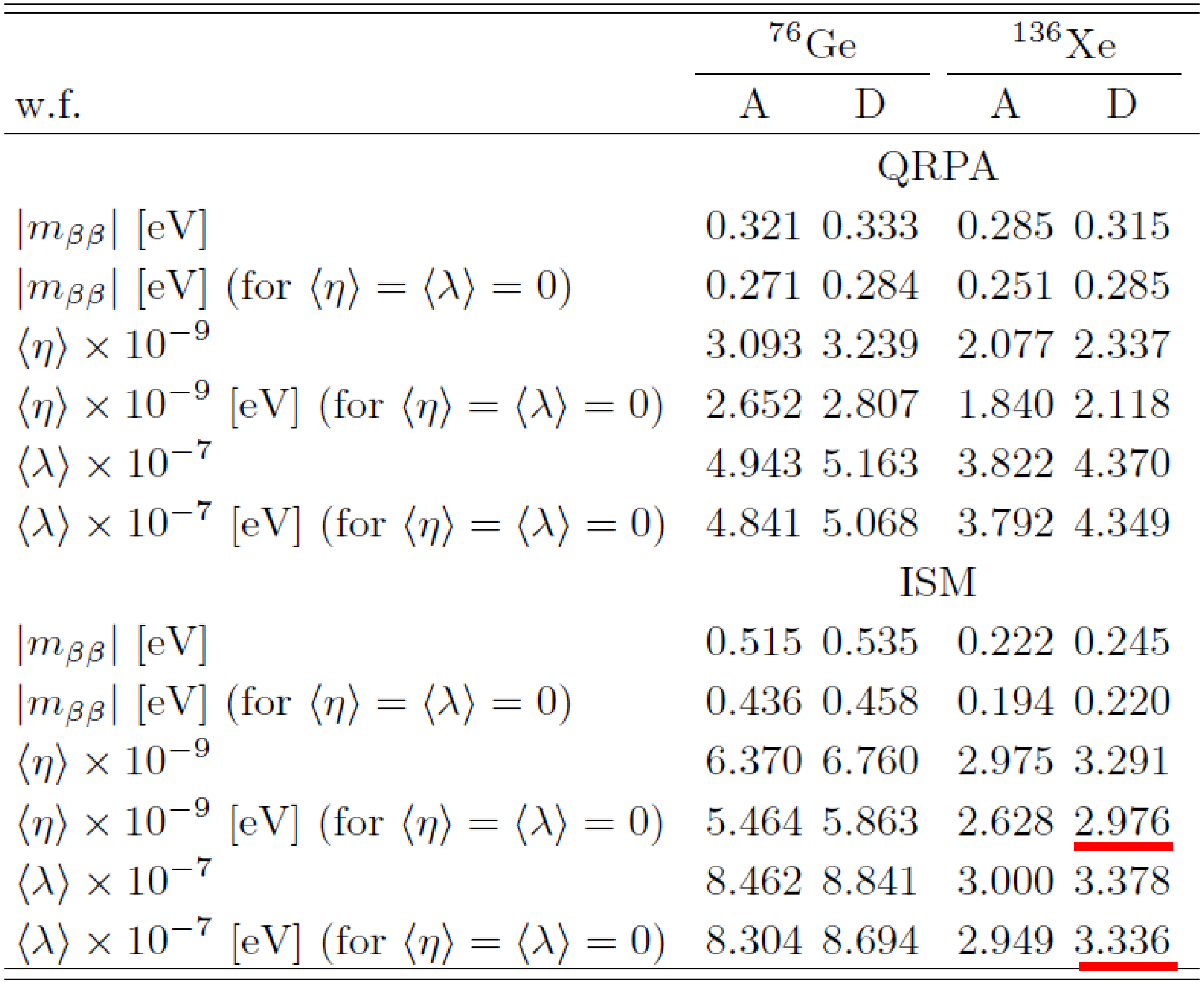
Phase-space factors for nuclei of experimental interest in the case of left and right-handed current mechanisms of the 0νββ-decay (light neutrino exchange)

	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd	^{116}Cd	^{124}Sn	^{130}Te	^{136}Xe	^{150}Nd
$Q_{\beta\beta}$ [MeV]	4.27226	2.03904	2.99512	3.35037	3.03440	2.01785	2.8135	2.28697	2.52697	2.45783	3.37138
$G_{01}.10^{14}$	2.483	0.237	1.018	2.062	1.595	0.483	1.673	0.906	1.425	1.462	6.316
$G_{02}.10^{14}$	16.229	0.391	3.529	8.959	5.787	0.814	5.349	1.967	3.761	3.679	29.187
$G_{03}.10^{15}$	18.907	1.305	6.913	14.777	10.974	2.672	11.128	5.403	8.967	9.047	45.130
$G_{04}.10^{15}$	5.327	0.470	2.141	4.429	3.400	0.978	3.569	1.886	3.021	3.099	14.066
$G_{05}.10^{13}$	3.007	0.566	2.004	4.120	3.484	1.400	4.060	2.517	3.790	4.015	14.873
$G_{06}.10^{12}$	3.984	0.531	1.733	3.043	2.478	0.934	2.563	1.543	2.227	2.275	7.497
$G_{07}.10^{10}$	2.682	0.270	1.163	2.459	1.927	0.599	2.062	1.113	1.755	1.812	8.085
$G_{08}.10^{11}$	1.109	0.149	0.708	1.755	1.420	0.462	1.703	0.939	1.549	1.657	8.405
$G_{09}.10^{10}$	16.246	1.223	4.779	8.619	6.540	1.939	6.243	3.301	4.972	4.956	19.454
$G_{010}.10^{14}$	2.116	0.141	0.801	1.855	1.359	0.309	1.418	0.660	1.146	1.165	7.115
$G_{011}.10^{15}$	5.376	0.476	2.183	4.557	3.502	1.010	3.704	1.955	3.148	3.238	15.055

Constraints on LNV mechanisms from the GERDA and EXO+KamLAND-Zen half-life limits



Current constraints on the effective neutrino mass and effective right-handed current parameters



w.f.	^{76}Ge A	^{76}Ge D	^{136}Xe A	^{136}Xe D
	QRPA			
$\lvert m_{\beta\beta}\rvert$ [eV]	0.321	0.333	0.285	0.315
$\lvert m_{\beta\beta}\rvert$ [eV] (for $\langle\eta\rangle = \langle\lambda\rangle = 0$)	0.271	0.284	0.251	0.285
$\langle\eta\rangle \times 10^{-9}$	3.093	3.239	2.077	2.337
$\langle\eta\rangle \times 10^{-9}$ [eV] (for $\langle\eta\rangle = \langle\lambda\rangle = 0$)	2.652	2.807	1.840	2.118
$\langle\lambda\rangle \times 10^{-7}$	4.943	5.163	3.822	4.370
$\langle\lambda\rangle \times 10^{-7}$ [eV] (for $\langle\eta\rangle = \langle\lambda\rangle = 0$)	4.841	5.068	3.792	4.349
	ISM			
$\lvert m_{\beta\beta}\rvert$ [eV]	0.515	0.535	0.222	0.245
$\lvert m_{\beta\beta}\rvert$ [eV] (for $\langle\eta\rangle = \langle\lambda\rangle = 0$)	0.436	0.458	0.194	0.220
$\langle\eta\rangle \times 10^{-9}$	6.370	6.760	2.975	3.291
$\langle\eta\rangle \times 10^{-9}$ [eV] (for $\langle\eta\rangle = \langle\lambda\rangle = 0$)	5.464	5.863	2.628	2.976
$\langle\lambda\rangle \times 10^{-7}$	8.462	8.841	3.000	3.378
$\langle\lambda\rangle \times 10^{-7}$ [eV] (for $\langle\eta\rangle = \langle\lambda\rangle = 0$)	8.304	8.694	2.949	3.336

The basic scale of the LRSM is TeV scale

$$\langle\eta\rangle \leq 2.98 \times 10^{-9}$$

$$m_D/m_{LNV} = 2.8 \times 10^{-7}$$

$$\langle\lambda\rangle \leq 3.34 \times 10^{-7}$$

$$m_D/m_{LNV} = 5.0 \times 10^{-6}$$

$$m_{LNV}/\mathrm{TeV}$$

$$= 0.3 \text{ - } 2\ m_D/\mathrm{MeV}$$

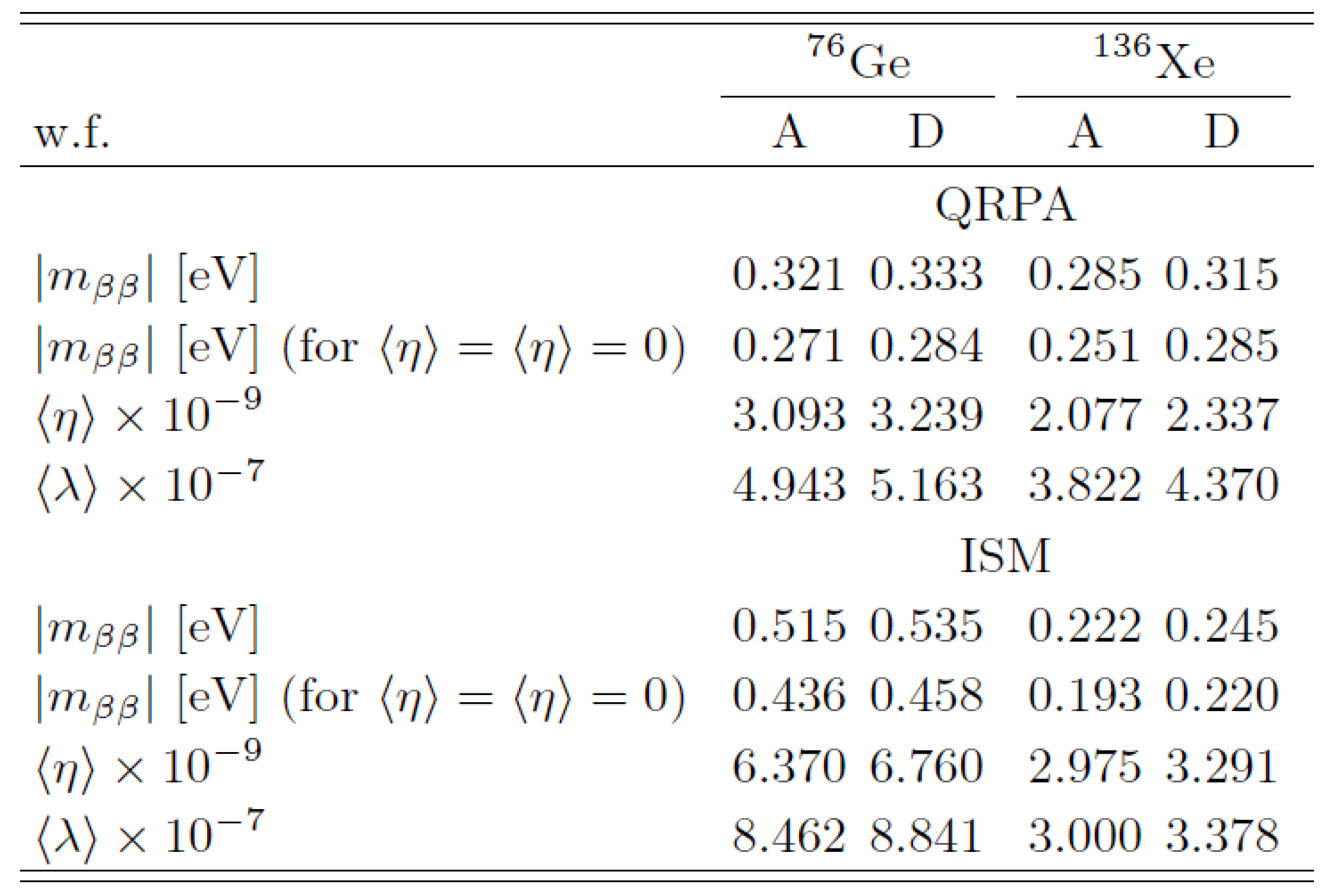
$^{76}Ge\ T^{0\nu}_{1/2} \geq 3.0\times10^{25}$ ISM

$^{136}Xe\ T^{0\nu}_{1/2} \geq 3.4 \times 10^{25}$ QRPA

E. Caurier, F. Nowacki, A. Poves and J. Retamosa, Phys. Rev. Lett. **77**, 1954 (1996)

K. Muto, E. Bender and H.V. Klapdor, Z. Phys. A **334**, 187 (1989)

Current constraints on the effective neutrino mass and effective right-handed current parameters



	^{76}Ge		^{136}Xe	
w.f.	A	D	A	D
		QRPA		
$\lvert m_{\beta\beta}\rvert$ [eV]	0.321	0.333	0.285	0.315
$\lvert m_{\beta\beta}\rvert$ [eV] (for $\langle\eta\rangle = \langle\eta\rangle = 0$)	0.271	0.284	0.251	0.285
$\langle\eta\rangle \times 10^{-9}$	3.093	3.239	2.077	2.337
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		ISM		
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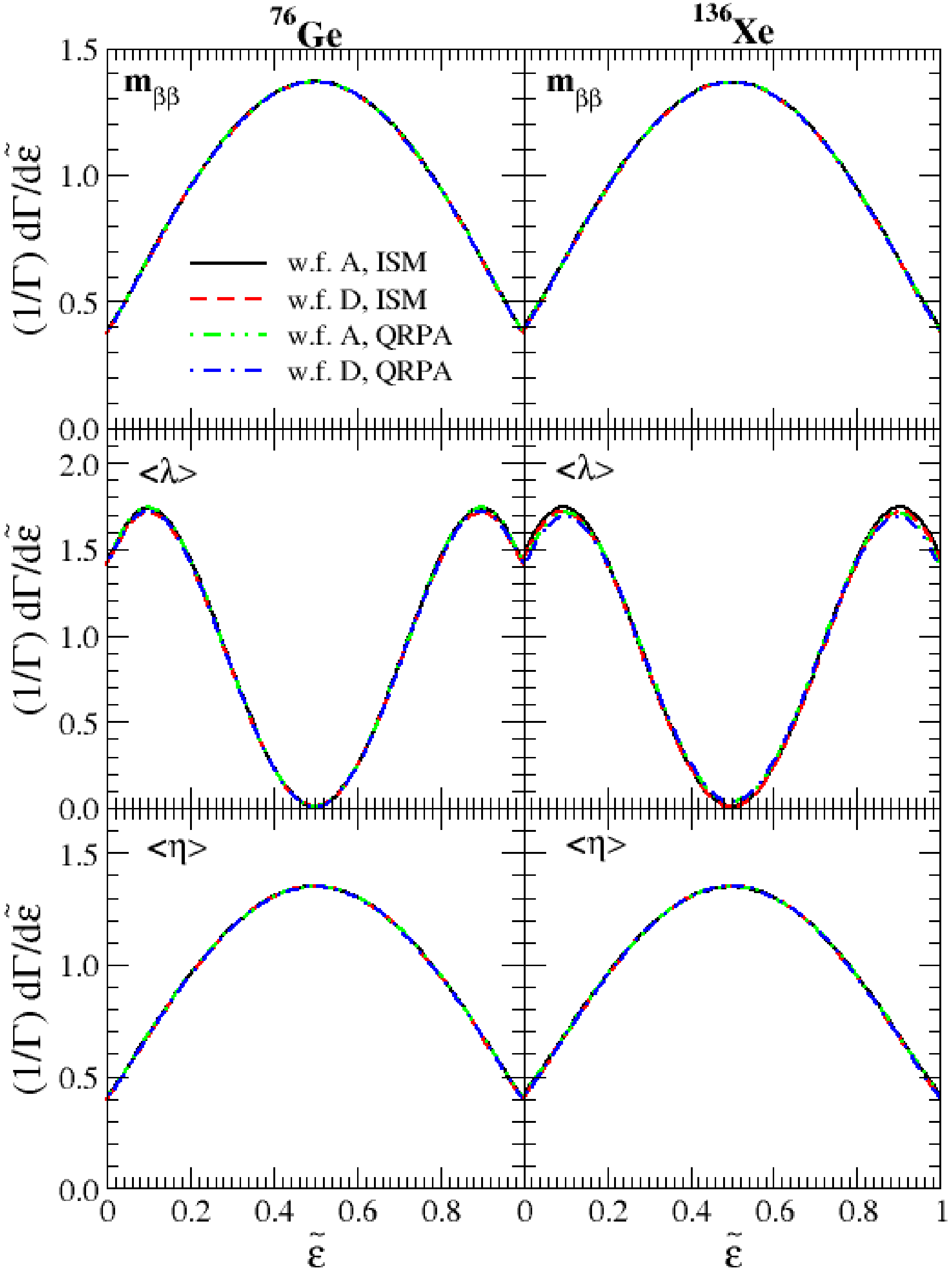
$\langle\eta\rangle \times 10^{-9}$	6.370	6.760	2.975	3.291
$\langle\lambda\rangle \times 10^{-7}$	8.462	8.841	3.000	3.378

$^{76}Ge\ T^{0\nu}_{1/2} \geq 3.0\times 10^{25}$

ISM E. Caurier, F. Nowacki, A. Poves and J. Retamosa, Phys. Rev. Lett. **77**, 1954 (1996)

$^{136}Xe\ T^{0\nu}_{1/2} \geq 3.4 \times 10^{25}$

QRP K. Muto, E. Bender and H.V. Klapdor, Z. Phys. A **334**, 187 (1989)



The single differential decay rate normalized to the total decay rate as function of electron energy for 3 limiting cases:

Results do not depend on isotope, NME and type of w.f.

i) Case $m_{\beta\beta} \neq 0$

$(\langle\lambda\rangle = 0 \text{ and } \langle\eta\rangle = 0)$

ii) Case $\langle\lambda\rangle \neq 0$

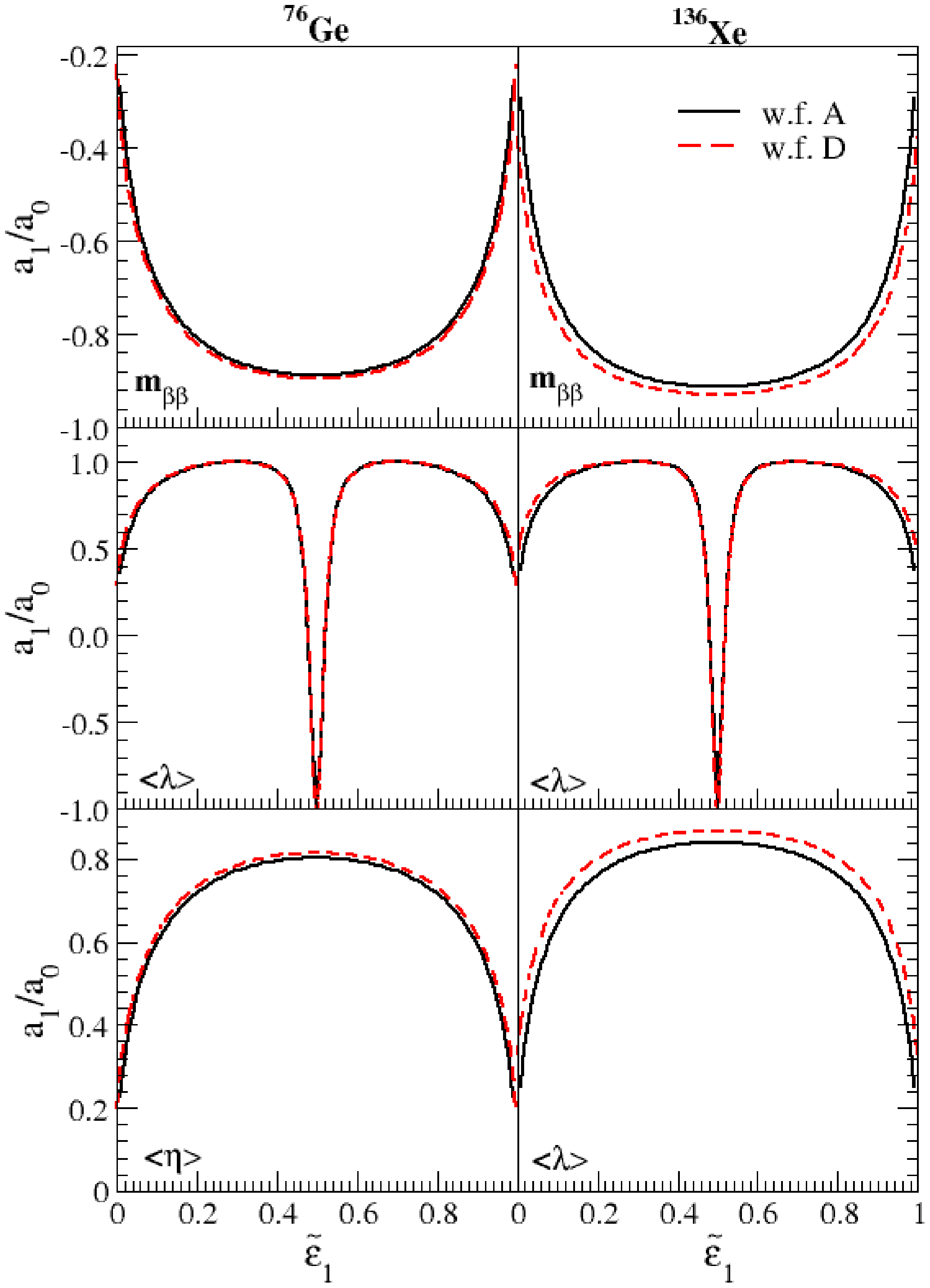
$(m_{\beta\beta} = 0 \text{ and } \langle\eta\rangle = 0)$

iii) Case $\langle\eta\rangle \neq 0$

$(m_{\beta\beta} = 0 \text{ and } \langle\lambda\rangle = 0)$

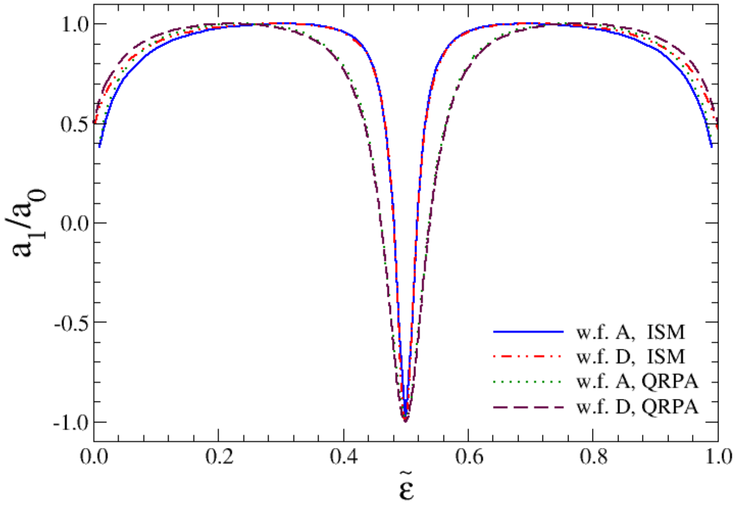
$$\varepsilon_1 \;=\; \tilde{\varepsilon}_1 Q_{\beta\beta} + m_e$$

$$\varepsilon_2 = Q_{\beta\beta} + 2m_e - \varepsilon_1)$$



Angular correlation factor as function of electron energy

$$\frac{d\Gamma}{d\cos\theta d\tilde{\varepsilon}_1} = a_0\left(1 + a_1/a_0 \cos\theta\right)$$



Minimal Supersymmetric Standard Model

Normal particles / fields		***Supersymmetric particles / fields***			
		Interaction eigenstates		**Mass eigenstates**	
Symbol	**Name**	**Symbol**	**Name**	**Symbol**	**Name**
$q = d,c,b,u,s,t$	quark	$\tilde{q}_L, \tilde{q}_R$	squark	$\tilde{q}_1, \tilde{q}_2$	squark
$l = e, \mu, \tau$	lepton	$\tilde{l}_L, \tilde{l}_R$	slepton	$\tilde{l}_1, \tilde{l}_2$	slepton
$\nu = \nu_e, \nu_\mu, \nu_\tau$	neutrino	$\tilde{\nu}$	sneutrino	$\tilde{\nu}$	sneutrino
g	gluon	$\tilde{g}$	gluino	$\tilde{g}$	gluino
$W^{\pm}$	W-boson	$\tilde{W}^{\pm}$	wino	$\left.\right\}\tilde{\chi}^{\pm}_{1,2}$	chargino
$H^{\mp}$	Higgs boson	$\tilde{H}^{\mp}_{1/2}$	Higgsino		
B	B-field	$\tilde{B}$	bino	$\left.\right\}\tilde{\chi}^{0}_{1,2,3,}$	neutralino
W^3	W^3-field	$\tilde{W}^3$	wino		
H_1^0	Higgs boson	$\tilde{H}_1^0$	Higgsino		
H_2^0	Higgs boson	$\tilde{H}_2^0$	Higgsino		
H_{31}^0	Higgs boson				

R=+1

R=–1

R-parity: R=$(-1)^{3B+L+2S}$

R-parity Breaking MSSM
(neutralino is not dark matter candidate)

$\lambda_{ij<k}$ LLE $+ \lambda'_{ijk}$ LQD$+ \lambda''_{ij<k}$ UDD

$9 + 27 + 9 = 45$ coupling constants

R-parity breaking terms In superpotential

$\lambda'_{11k} * \lambda''_{11k} < 10^{-22}$	proton decay
$\lambda < 10^{-3}$ to 10^{-1}	with $\lambda_{133} < 0.003$ limit on ν_e mass
$\lambda' < 10^{-2}$ to 10^{-1}	with $\lambda'_{111} < 4.10^{-4}$ neutrinoless beta decay

Neutrino-Neutralino mixing matrix (see-saw structure)

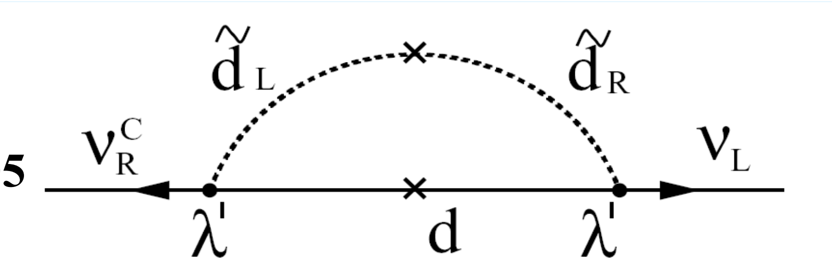
$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m^T & M_\chi \end{pmatrix}$$

$$\Psi'^T_{(0)} = (\nu_e, \nu_\mu, \nu_\tau, -i\lambda', -i\lambda_3, \tilde{H}^0_1, \tilde{H}^0_2),$$

Radiative corrections to neutrino mass

$$\mathcal{M}_\nu = \mathcal{M}^{tree} + \mathcal{M}^l + \mathcal{M}^q$$

Gozdz, Kaminski, Šimkovic, PRD 70 (2004) 095005



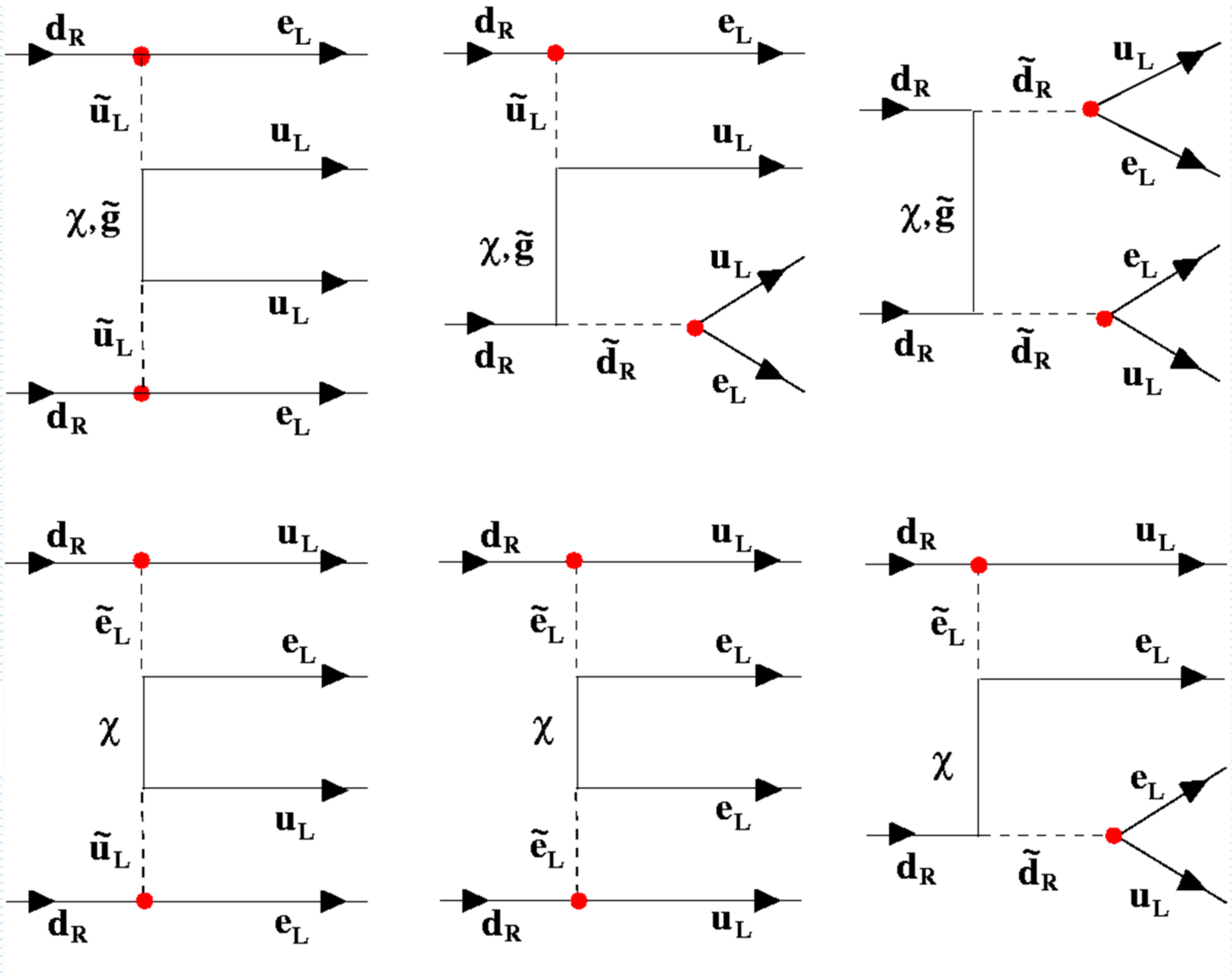
gluino/neutralino exchange R-parity breaking SUSY mechanism of the 0νββ–decay

$d+d \to u + u + e^- + e^-$

exchange of squarks, neutralinos and gluinos

$(\lambda'_{111})^2$ **mechanism**

quark-level diagrams

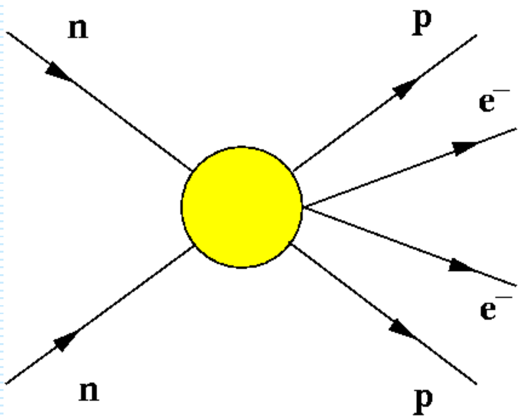


● R–parity violation

1968 Pontecorvo proposed $\pi^- \to \pi^+ + 2e^-$, superweak int.
We identified with R-parity breaking SUSYmechanism

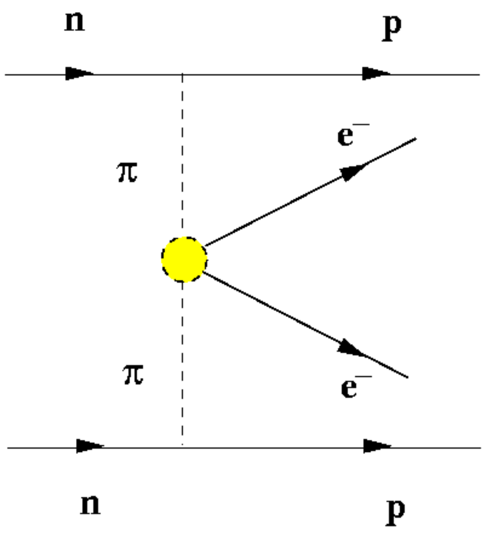
$$\mathcal{L}_{qe} = \frac{G_F^2}{2m_p}\,\bar{e}(1+\gamma_5)e^c\left[\eta^{PS}\ J_{PS}J_{PS} - \frac{1}{4}\eta^{T}\ J_T^{\mu\nu}J_{T\mu\nu}\right].$$

Two–nucleon mechanism



Can be neglected

Pion–exchange mechanism



The dominant contribution

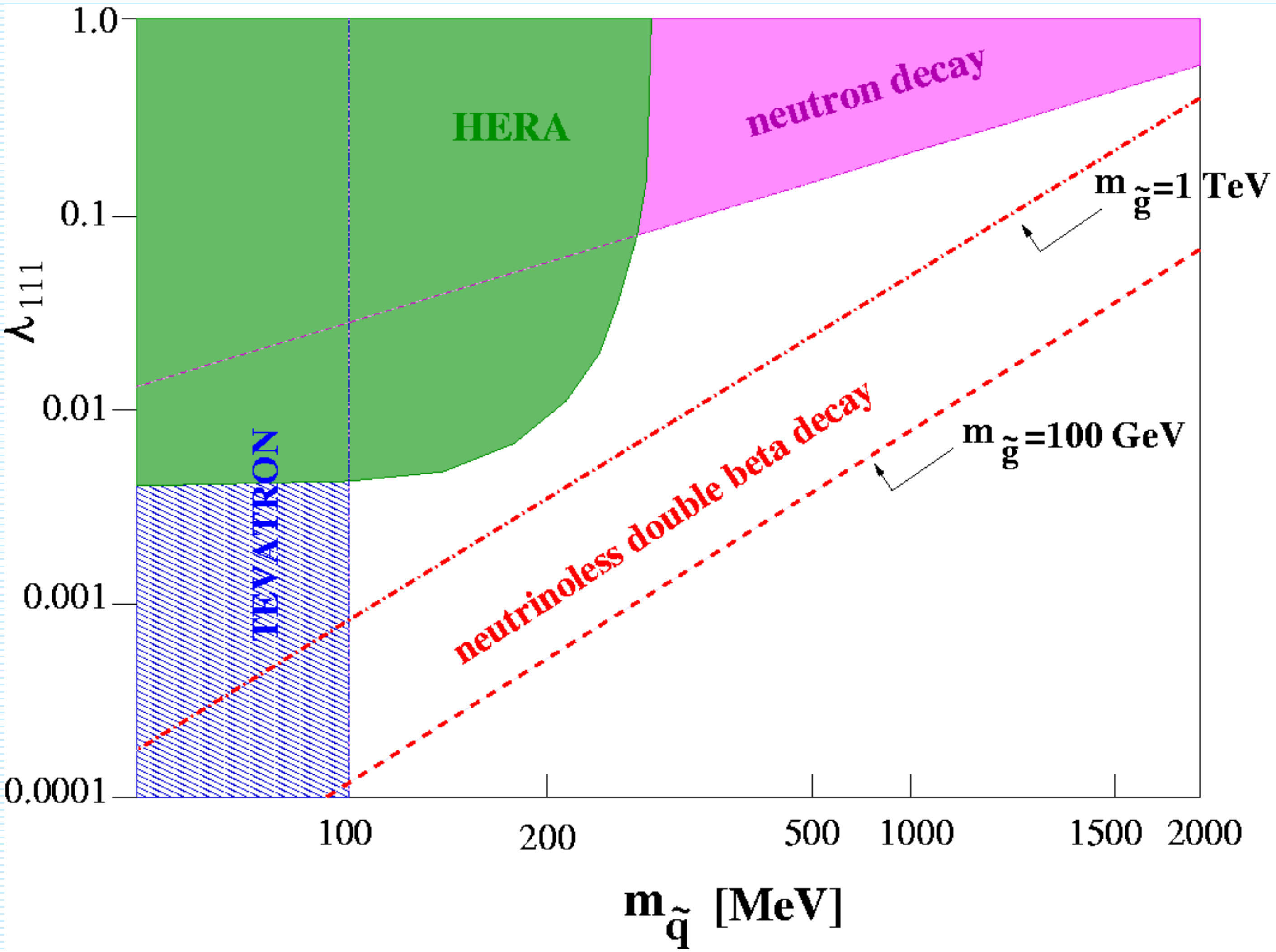
Hadron-level diagrams

Faessler, Kovalenko, Šimkovic
PRL 78 (1998) 183
Wodecki, Kaminski, Šimkovic,
PRD 60 (1999) 11507

$$\langle 0|\bar{u}\gamma_5 d|\pi^-\rangle = i\sqrt{2}f_\pi\frac{m_\pi^2}{m_u+m_d}, \quad (m_\pi/(m_u+m_d)\approx 13)$$

$$\langle 0|\bar{u}\gamma_\alpha\gamma_5 d|\pi^-\rangle = i\sqrt{2}f_\pi k_\alpha$$

Limit on R-parity breaking parameter λ'_{111}



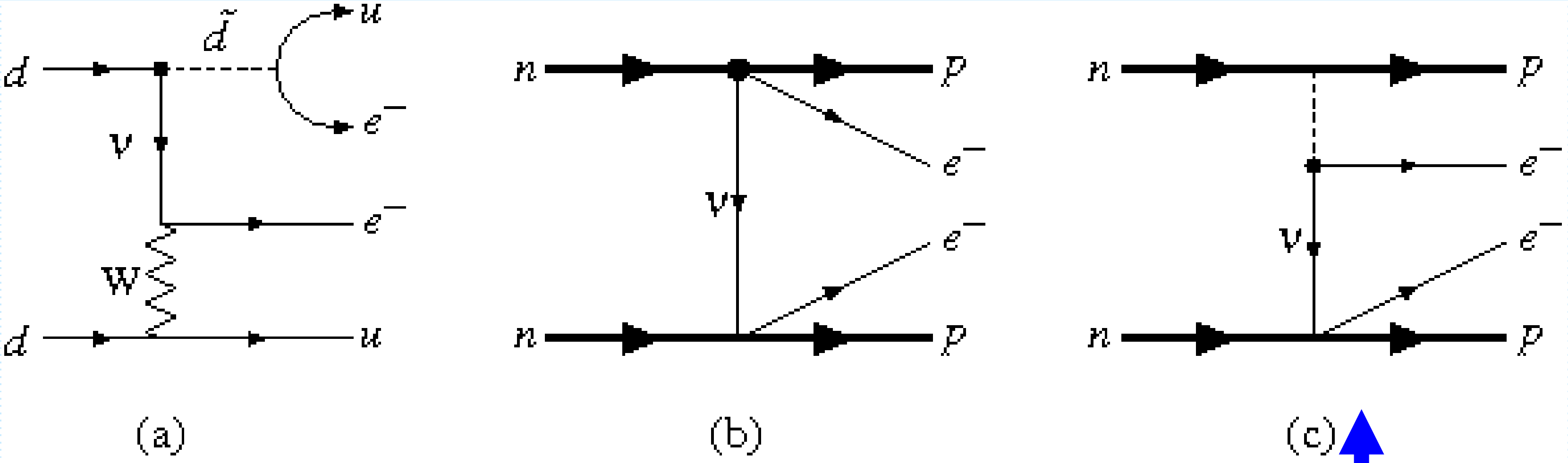
Faessler, F.Š, Kovalenko, PRD 58 (1998) 115004

$$\acute{\lambda}_{111} = 1.3\ 10^{-4} \left(\frac{m_{\tilde{q}}}{100\ GeV}\right)^{2} \left(\frac{m_{\tilde{g}}}{100\ GeV}\right)^{1/2}$$

Squark mixing SUSY mechanism

Mixing between scalar superpartners of the left- and right-handed fermions

$$M^2_{\tilde{d}^k} = \begin{pmatrix} m^2_{\tilde{d}^k_L} + m^2_{d^k} - \frac{1}{6}(2m^2_W + m^2_Z)\cos 2\beta & -m_{d^k}((\mathbf{A}_D)_{kk} + \mu \tan\beta) \\ -m_{d^k}((\mathbf{A}_D)_{kk} + \mu \tan\beta) & m^2_{\tilde{d}^k_R} + m^2_{d^k} + \frac{1}{3}(m^2_W - m^2_Z)\cos 2\beta \end{pmatrix}$$



(a) (b) (c)

Hirsch,
Klapdor-Kleingrothaus,
Kovalenko
PLB 372 (1996) 181

A. Faessler,
Th. Gutsche,
S. Kovalenko,
F.Š.,
PRD 77, 113012 (2008)

Effective SUSY ν-e Lagrangian

Neutrino vertex

$$\mathcal{L}^{LH} = \frac{G_F}{\sqrt{2}} \sum_i U_{ei} \; (\bar{e}\gamma_\alpha(1-\gamma_5)\nu) \; (\bar{u}\gamma^\alpha(1-\gamma_5)d) + h.c. \quad (V-A)$$

R-parity violating SUSY vertex

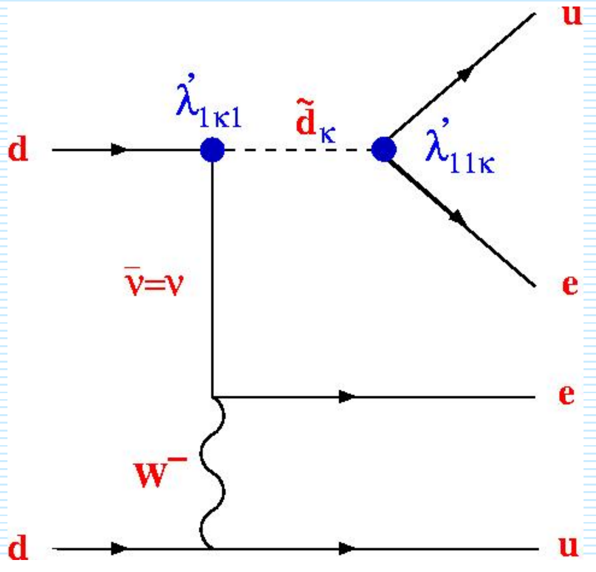
Hirsch,Klapdor-Kleingrothaus, Kovalenko
PLB 372 (1996) 181

$$\mathcal{L}^{eff}_{SUSY} = \frac{G_F}{\sqrt{2}} \left(\frac{1}{4}\eta_{(q)LR} \; \sum_i U^*_{ei} \; (\overline{\nu}(1+\gamma_5)e) \; (\overline{u}(1+\gamma_5)d) \qquad (S,P) \right.$$
$$\left. + \frac{1}{8}\eta_{(q)LR} \; \sum_i U^*_{ei} \; (\overline{\nu}\sigma_{\alpha\beta}(1+\gamma_5)e) \; (\overline{u}\sigma^{\alpha\beta}(1+\gamma_5)d) + h.c. \right) \quad (Tensor)$$

Paes, Hirsch, Klapdor-Kleingrothaus,
PLB 459 (1999) 450

LN-violating parameter

$$\eta_{(q)LR} = \sum_k \frac{\lambda'_{11k}\lambda'_{1k1}}{8\sqrt{2}G_F} \sin 2\theta^d_{(k)} \left(\frac{1}{m^2_{\tilde{d}_1(k)}} - \frac{1}{m^2_{\tilde{d}_2(k)}} \right)$$



Limits on R-breaking parameters

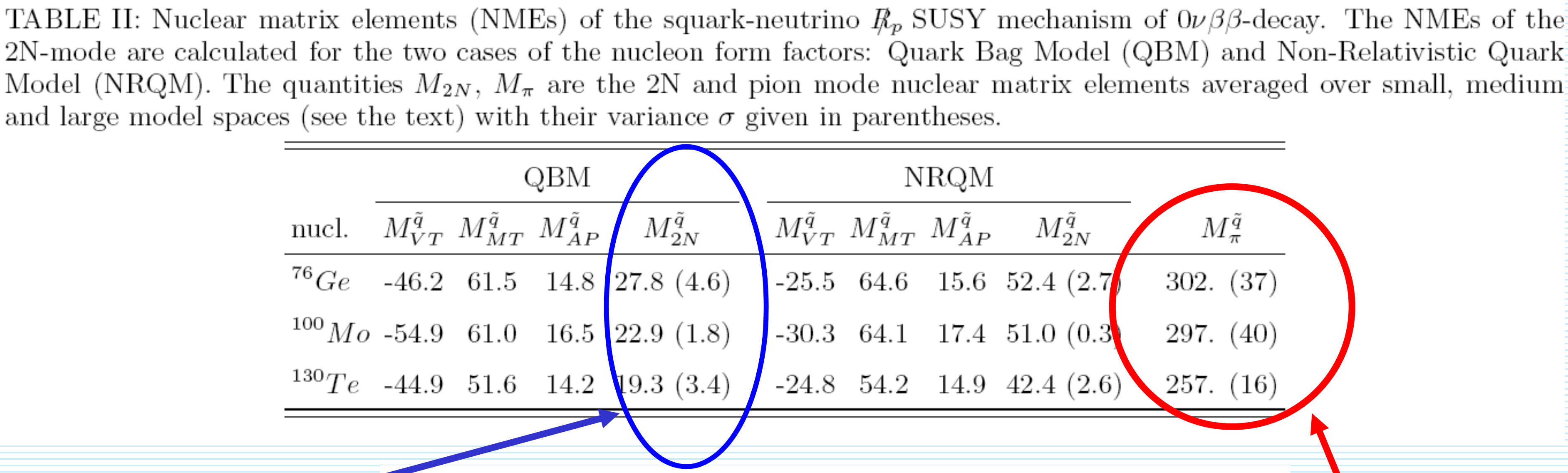


TABLE II: Nuclear matrix elements (NMEs) of the squark-neutrino $\not R_p$ SUSY mechanism of $0\nu\beta\beta$-decay. The NMEs of the 2N-mode are calculated for the two cases of the nucleon form factors: Quark Bag Model (QBM) and Non-Relativistic Quark Model (NRQM). The quantities M_{2N}, M_π are the 2N and pion mode nuclear matrix elements averaged over small, medium and large model spaces (see the text) with their variance σ given in parentheses.

	QBM				NRQM				
nucl.	$M^{\tilde q}_{VT}$	$M^{\tilde q}_{MT}$	$M^{\tilde q}_{AP}$	$M^{\tilde q}_{2N}$	$M^{\tilde q}_{VT}$	$M^{\tilde q}_{MT}$	$M^{\tilde q}_{AP}$	$M^{\tilde q}_{2N}$	$M^{\tilde q}_{\pi}$
^{76}Ge	-46.2	61.5	14.8	27.8 (4.6)	-25.5	64.6	15.6	52.4 (2.7)	302. (37)
^{100}Mo	-54.9	61.0	16.5	22.9 (1.8)	-30.3	64.1	17.4	51.0 (0.3)	297. (40)
^{130}Te	-44.9	51.6	14.2	19.3 (3.4)	-24.8	54.2	14.9	42.4 (2.6)	257. (16)

2n mode

Pion mode

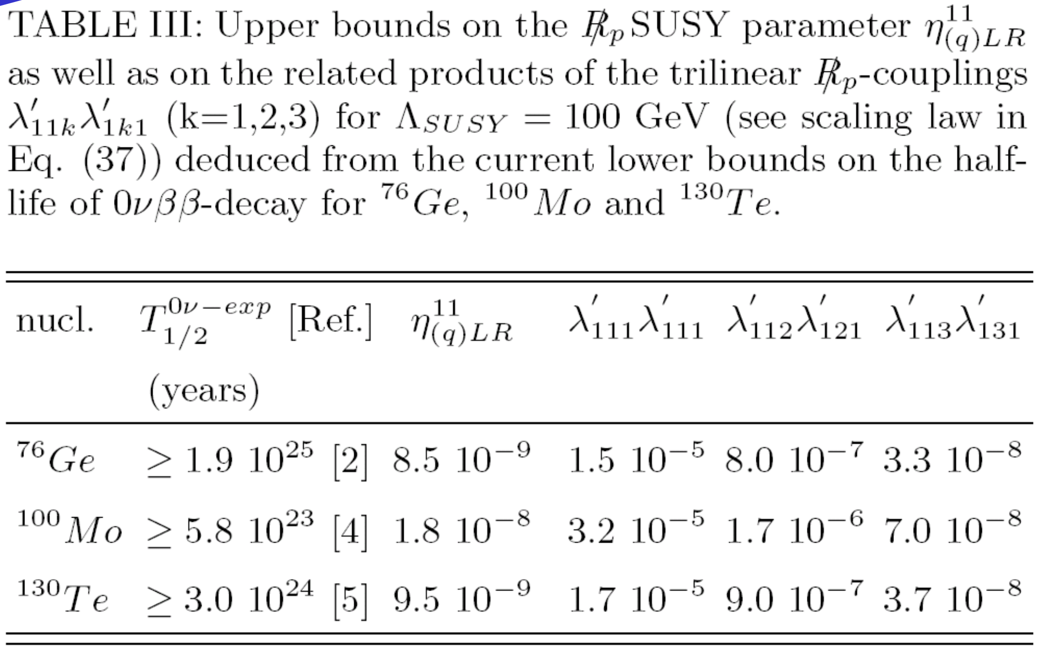


TABLE III: Upper bounds on the $\not R_p$ SUSY parameter $\eta^{11}_{(q)LR}$ as well as on the related products of the trilinear $\not R_p$-couplings $\lambda'_{11k}\lambda'_{1k1}$ (k=1,2,3) for $\Lambda_{SUSY} = 100$ GeV (see scaling law in Eq. (37)) deduced from the current lower bounds on the half-life of $0\nu\beta\beta$-decay for ^{76}Ge, ^{100}Mo and ^{130}Te.

nucl.	$T^{0\nu-exp}_{1/2}$ (years)	[Ref.]	$\eta^{11}_{(q)LR}$	$\lambda^{'}_{111}\lambda^{'}_{111}$	$\lambda^{'}_{112}\lambda^{'}_{121}$	$\lambda^{'}_{113}\lambda^{'}_{131}$
^{76}Ge	$\geq 1.9\ 10^{25}$	[2]	$8.5\ 10^{-9}$	$1.5\ 10^{-5}$	$8.0\ 10^{-7}$	$3.3\ 10^{-8}$
^{100}Mo	$\geq 5.8\ 10^{23}$	[4]	$1.8\ 10^{-8}$	$3.2\ 10^{-5}$	$1.7\ 10^{-6}$	$7.0\ 10^{-8}$
^{130}Te	$\geq 3.0\ 10^{24}$	[5]	$9.5\ 10^{-9}$	$1.7\ 10^{-5}$	$9.0\ 10^{-7}$	$3.7\ 10^{-8}$

A. Faessler,
Th. Gutsche,
S. Kovalenko,
F.Š.,
PRD 77, 113012 (2008)

Co-existence of few mechanisms of the 0νββ-decay

It may happen that in year 201? (or 2???) the 0νββ-decay will be detected for 2-3 or more isotopes …

(If there will be enough money for enrichment of isotopes!?)

Probing the see-saw mechanism

Bilenky, Faessler, Potzel, F.Š, Eur. Phys. J. C 71 (2011) 1754

There exist heavy Majorana neutral leptons N_i (singlet of SU(2)xU(1) group)

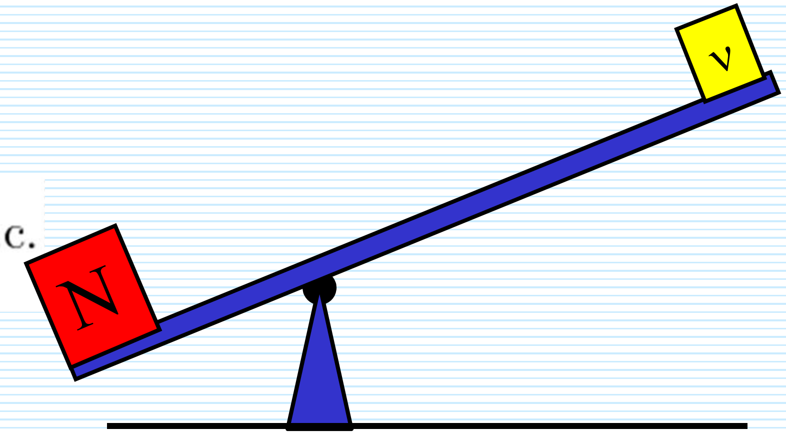
$$\mathcal{L} = -\sqrt{2}\sum_{i,l} Y_{li}\bar{L}_{lL}N_{iR}\tilde{H} + \text{h.c.}.$$

$$L_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix}$$

$$N_i = N_i^c = C\bar{N}_i^T$$

Effective interaction for processes with virtual N_i at electroweak scale

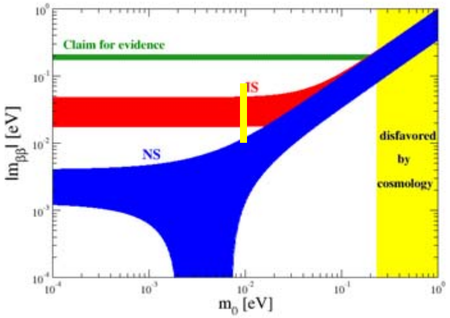
$$\mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda}\sum_{l',l,i}\overline{L_{l'L}}\tilde{H}\sum_i (Y_{l'i}\frac{\Lambda}{M_i}Y_{li})C\tilde{H}^T(\overline{L_{lL}})^T + \text{h.c.}$$



After spontaneous violation of the electroweak symmetry the left-handed Majorana mass term is generated

$$M^L = Y\frac{v^2}{M}Y^T = U\ m\ U^T$$

$$\begin{aligned}\mathcal{L}^{\text{M}} &= -\frac{1}{2}\sum_{l',l}\bar{\nu}_{l'L}\ M^L_{l'l}\ (\nu_{lL})^c + \text{h.c.} \\ &= -\frac{1}{2}\sum_i m_i\bar{\nu}_i\nu_i,\end{aligned}$$

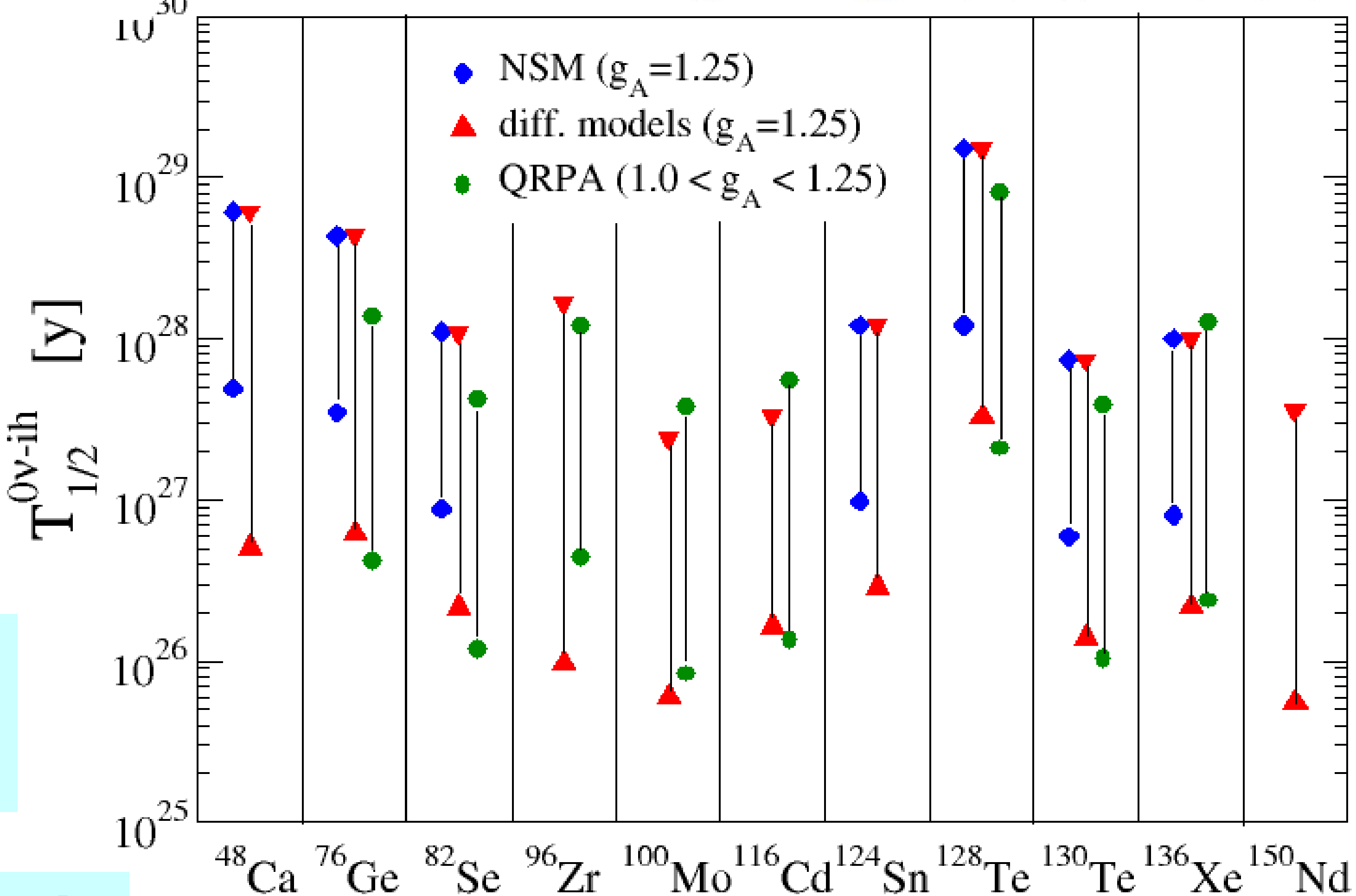


$$T^{min}_{1/2}(A,Z) \le T^{0\nu}_{1/2}(A,Z) \le T^{max}_{1/2}(A,Z)$$

$$T^{min}_{1/2}(A,Z) = \frac{1}{\Delta m_A^2 |M(A,Z)|^2 G^{0\nu}(E_0,Z)},$$

$$T^{max}_{1/2}(A,Z) = \frac{1}{\Delta m_A^2 \cos^2 2\,\theta_{12} |M(A,Z)|^2 G^{0\nu}(E_0,Z)}$$

Probing standard see-saw mech.



Heavy ν Mass at GUT scale

Inverted hierarchy

Co-existence of 2, 3 or more interferring mechanisms of $0\nu\beta\beta$-decay

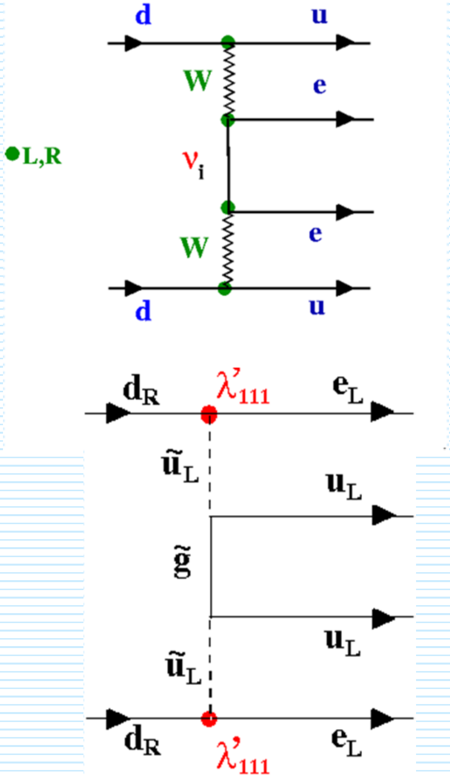
It is well-known that there exist many mechanisms that may contribute to the $0\nu\beta\beta$. Let consider 3 mechanisms: i) light ν-mass mechanism, ii) heavy ν-mass mechanism, iii) R-parity breaking SUSY mechanism with gluino exchange and CP conservation

$$\frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}(E_0, Z)|\frac{m_{\beta\beta}}{m_e} M^{0\nu}_{\nu} + \eta^L_N M^{0\nu}_N + \eta_{\lambda'_{111}} M^{0\nu}_{\lambda'_{111}} \ldots|^2$$

$$m_{\beta\beta} = \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k$$

$$\eta^L_N = \sum_k^{heavy} U^2_{ek} \frac{m_p}{M_k}$$

$$\eta_{\lambda'_{111}} = \frac{\pi\alpha_s}{6} \frac{\lambda'^2_{111}}{G^2_F m^4_{\tilde{d}_R}} \frac{m_p}{m_{\tilde{g}}} \left[1 + \left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\right]^2 .$$



Claim of evidence:

Klapdor-Kleingrothaus, Krivosheina,
Mod. Phys. A 21, 1547 (2009)

$$T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{100}\mathrm{Mo}) \geq 5.8 \times 10^{23}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{130}\mathrm{Te}) \geq 3.0 \times 10^{24}\ \mathrm{y}$$

$\xi_{Te} < 1.2$
$\xi_{Mo} < 2.6$

We introduce

$$\xi = \frac{|M^{\nu}_1|\sqrt{T_1\ G_1}}{|M^{\nu}_2|\sqrt{T_2\ G_2}}$$ r S

ξ=0, non-observation ($T_2 \rightarrow \infty$)
ξ=1, solution for single active mech. is reproduced

4 sets of two linear eq.

$$\frac{\pm 1}{\sqrt{T_1\;G_1}} = \frac{m_{\beta\beta}}{m_e} M_1^{\nu} + \eta M_1^{\eta}$$

$$\frac{\pm 1}{\sqrt{T_2\;G_2}} = \frac{m_{\beta\beta}}{m_e} M_2^{\nu} + \eta M_2^{\eta}$$

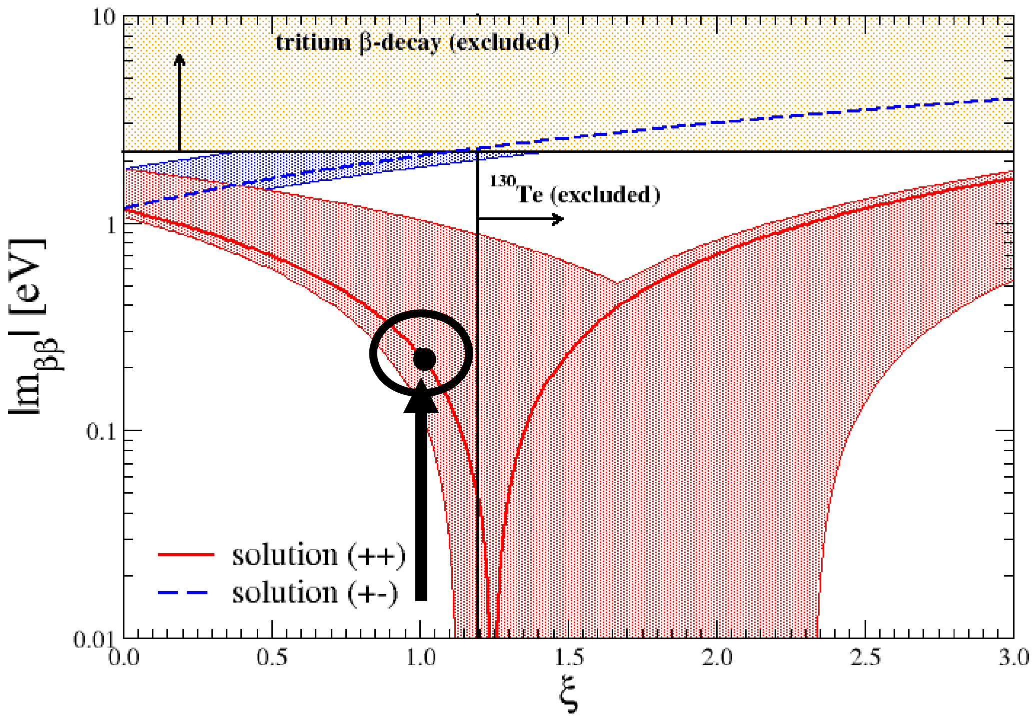
2 different solutions ***CP-conservation assumed***

$$|m_{\beta\beta}| = \left| \frac{m_e}{M_1^{\nu}\sqrt{T_1\;G_1}} \frac{M_1^{\nu}\;M_2^{\eta}}{(M_1^{\nu} M_2^{\eta} - M_2^{\nu} M_1^{\eta})} \pm \frac{m_e}{M_2^{\nu}\sqrt{T_2\;G_2}} \frac{M_2^{\nu}\;M_1^{\eta}}{(M_1^{\nu} M_2^{\eta} - M_2^{\nu} M_1^{\eta})} \right|$$

2 active mechanisms
of the 0νββ-decay:
Light and heavy
ν-mass mechanism

Non-observation of
the 0νββ-decay for some
isotopes might be
in agreement with
non- zero $m_{\beta\beta}$

F.Š., J.D. Vergados, A. Faessler, PRD 82, 113015 (2010)



● *Single solution*
for ***light ν-mass mech.***

Non-observation
for ^{130}Te

Two non-interfering mechanisms of the 0νββ-decay (light LH and heavy RH neutrino exchange)

$$T^{0\nu}_{1/2}(^{76}Ge) \geq 1.9 \times 10^{25} y, \qquad T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25} y$$

$$5.8 \times 10^{23} y \leq T^{0\nu}_{1/2}(^{100}Mo) \leq 5.8 \times 10^{24} y, \qquad 3.0 \times 10^{24} y \leq T^{0\nu}_{1/2}(^{130}Te) \leq 3.0 \times 10^{25} y$$

Half-life:

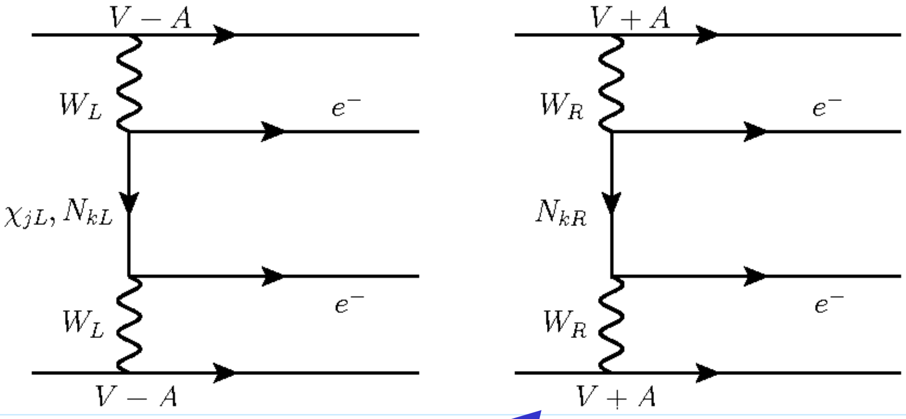
$$\frac{1}{T^{0\nu}_{1/2,i} G^{0\nu}_i(E,Z)} \cong |\eta_\nu|^2 |M'^{0\nu}_{i,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{i,N}|^2$$

$$\eta_\nu = \frac{m_{\beta\beta}}{m_e}$$

Set of equations:

$$\frac{1}{T_1 G_1} = |\eta_\nu|^2 |M'^{0\nu}_{1,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{1,N}|^2$$

$$\frac{1}{T_2 G_2} = |\eta_\nu|^2 |M'^{0\nu}_{2,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{2,N}|^2$$



Solutions:

$$|\eta_\nu|^2 = \frac{|M'^{0\nu}_{2,N}|^2/T_1G_1 - |M'^{0\nu}_{1,N}|^2/T_2G_2}{|M'^{0\nu}_{1,\nu}|^2|M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2|M'^{0\nu}_{2,\nu}|^2}$$

$$|\eta_R|^2 = \frac{|M'^{0\nu}_{1,\nu}|^2/T_2G_2 - |M'^{0\nu}_{2,\nu}|^2/T_1G_1}{|M'^{0\nu}_{1,\nu}|^2|M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2|M'^{0\nu}_{2,\nu}|^2}$$

$$\eta^R_N = \left(\frac{M_W}{M_{WR}}\right)^4 \sum_k^{heavy} V^2_{ek} \frac{m_p}{M_k}$$

A. Faessler, A. Meroni, S.T. Petcov, F. Š., J.D. Vergados,
B. Phys. Rev. D 83, 113003 (2011); JHEP 1302, 025 (2013)

Two non-interfering mechanisms of the 0νββ-decay
(light LH and heavy RH neutrino exchange)

The positivity condition:

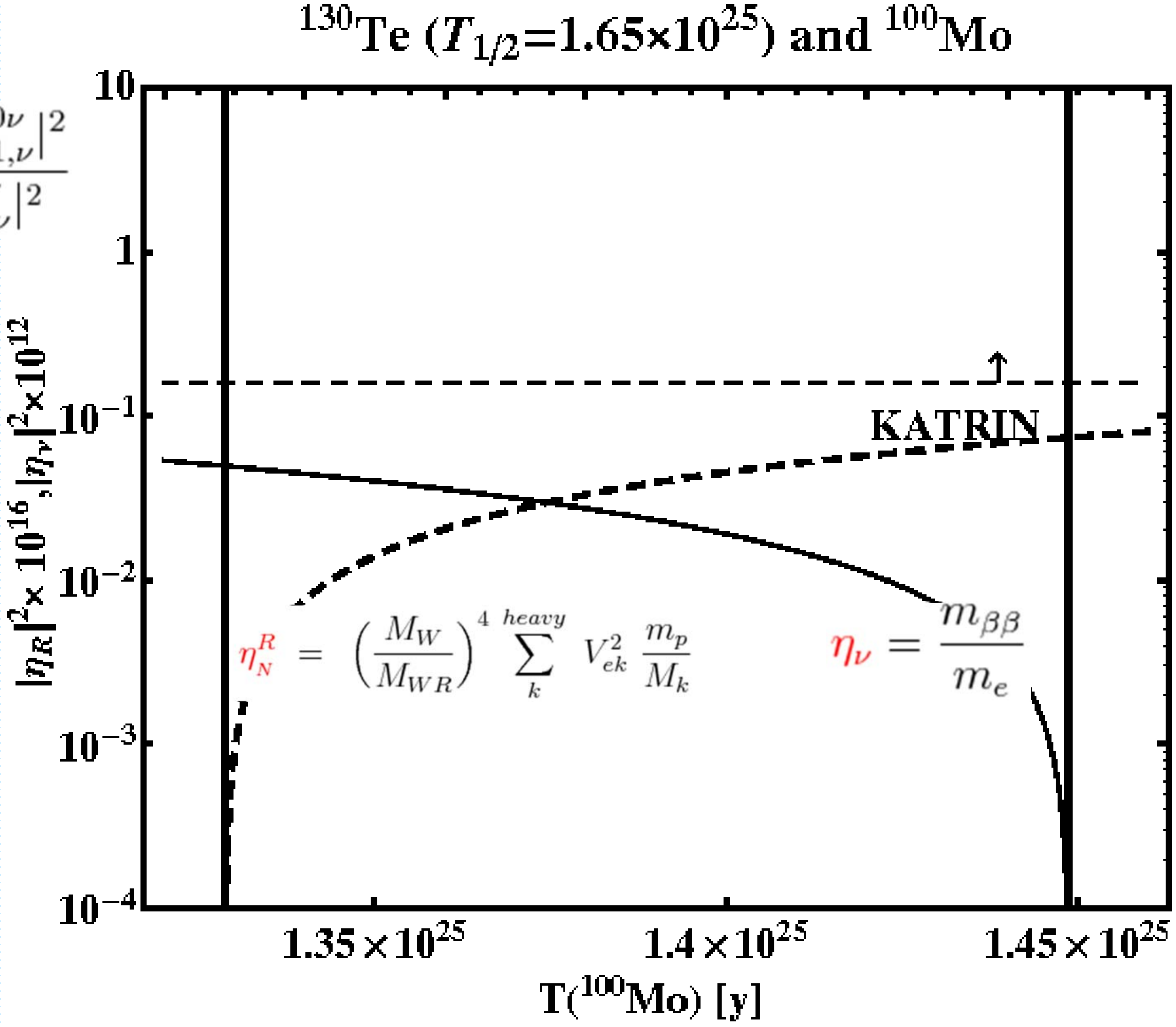
$$\frac{T_1 G_1 |M'^{0\nu}_{1,N}|^2}{G_2 |M'^{0\nu}_{2,N}|^2} \le T_2 \le \frac{T_1 G_1 |M'^{0\nu}_{1,\nu}|^2}{G_2 |M'^{0\nu}_{2,\nu}|^2}$$

Very narrow ranges!

$$0.15 \le \frac{T^{0\nu}_{1/2}(^{100}Mo)}{T^{0\nu}_{1/2}(^{76}Ge)} \le 0.18$$

$$0.17 \le \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{76}Ge)} \le 0.22$$

$$1.14 \le \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{100}Mo)} \le 1.24$$

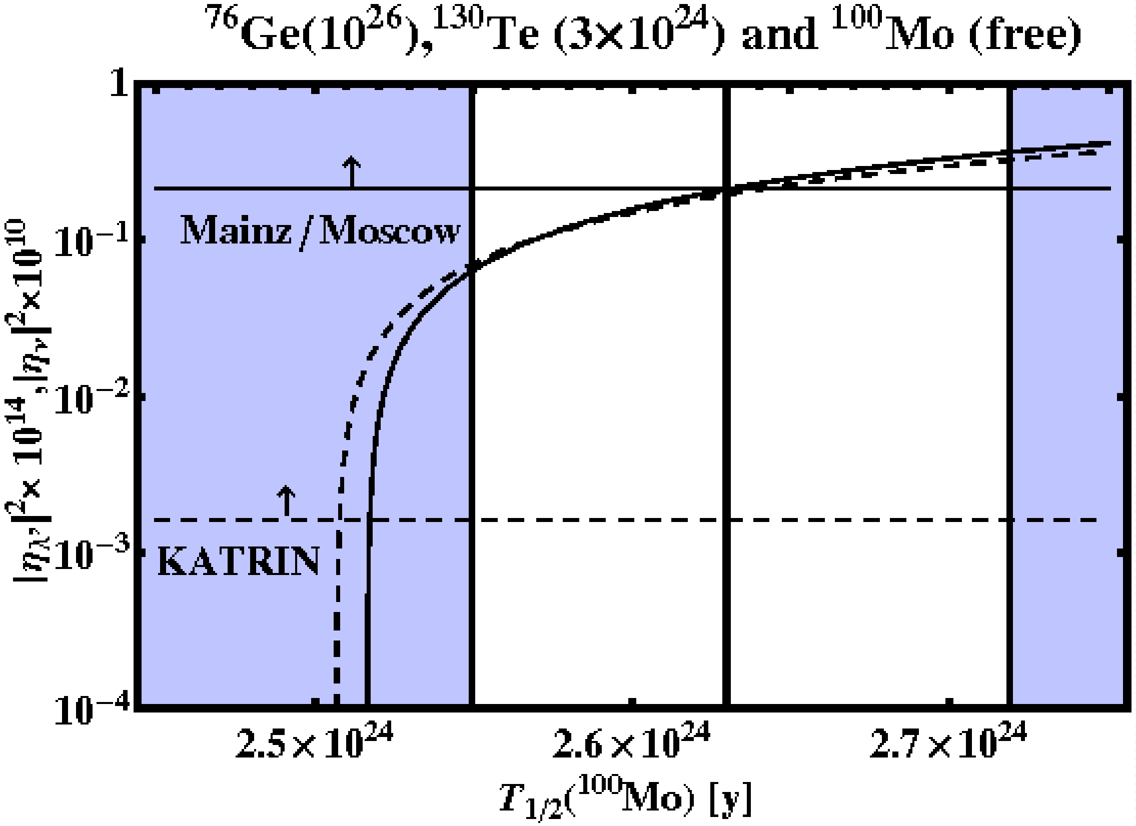


Two interfering mechanisms of the $0\nu\beta\beta$-decay
(Light neutrino and gluino exchange)

$$\frac{1}{T^{0\nu}_{1/2,i} G^{0\nu}_i(E,Z)} = |\eta_\nu|^2 |M'^{0\nu}_{i,\nu}|^2 + |\eta_{\lambda'}|^2 |M'^{0\nu}_{i,\lambda'}|^2 + 2\cos\alpha |M'^{0\nu}_{i,\lambda'}||M'^{0\nu}_{i,\nu}||\eta_\nu||\eta_{\lambda'}|$$

$$|\eta_\nu|^2 = \frac{D_1}{D}, \quad |\eta_{\lambda'}|^2 = \frac{D_2}{D}$$

$$z \equiv 2\cos\alpha|\eta_\nu||\eta_{\lambda'}| = \frac{D_3}{D}$$



Given (i.e. having data on) T_1, T_2 + using the condition on the interference term $z = 2\cos\alpha|\eta_\nu||\eta_{\lambda'}|$, Determines an interval of allowed values of T_3.

Neutrinoless Double-Electron Capture

$(A,Z) \rightarrow (A,Z-2)^{**}$

Additional
modes of the 0νECEC-decay:

$$
\begin{aligned}
e_b + e_b + (A,Z) \rightarrow (A,Z-2) &+ \gamma \\
&+ 2\gamma \\
&+ e^+e^- \\
&+ M
\end{aligned}
$$

Neutrinoless double electron capture
(resonance transitions)
$(A,Z) \rightarrow (A,Z-2)^{*HH'}$

J. Bernabeu, A. DeRujula, C. Jarlskog,
Nucl. Phys. B 223, 15 (1983)

Atom mixing amplitude
ΔM

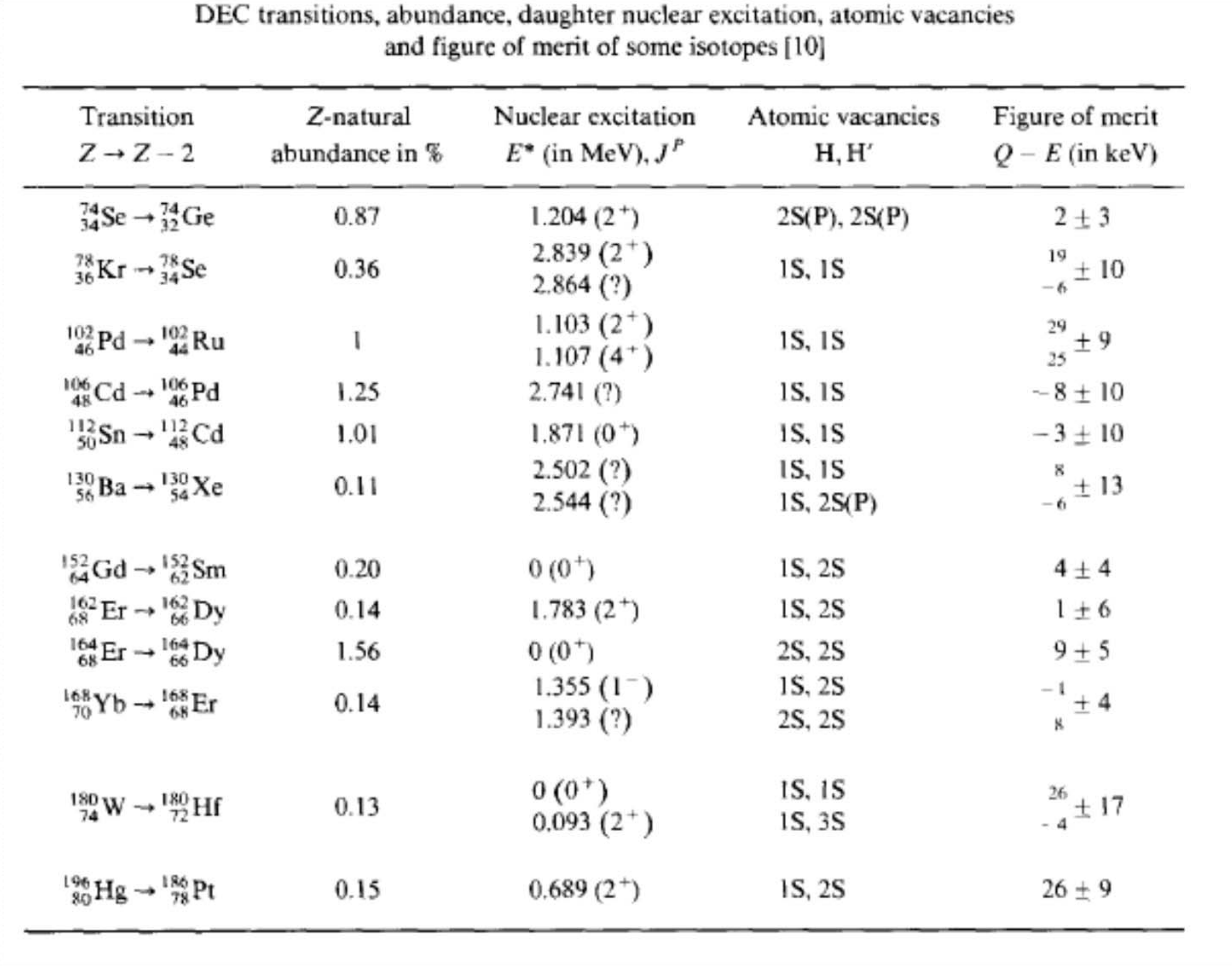
$$E \simeq E^* + E_H + E_{H'},$$

$$\Gamma \simeq \Gamma^* + \Gamma_H + \Gamma_{H'}.$$

Decay rate

$$\frac{1}{\tau} \simeq \frac{(\Delta M)^2}{(Q-E)^2 + \frac{1}{4}\Gamma^2}\Gamma,$$

2νECEC-background depends strongly on Q-value



DEC transitions, abundance, daughter nuclear excitation, atomic vacancies and figure of merit of some isotopes [10]

Transition $Z \rightarrow Z-2$	Z-natural abundance in %	Nuclear excitation E^* (in MeV), J^P	Atomic vacancies H, H'	Figure of merit $Q-E$ (in keV)
${}^{74}_{34}\mathrm{Se} \rightarrow {}^{74}_{32}\mathrm{Ge}$	0.87	1.204 (2^+)	2S(P), 2S(P)	2 ± 3
${}^{78}_{36}\mathrm{Kr} \rightarrow {}^{78}_{34}\mathrm{Se}$	0.36	2.839 (2^+) 2.864 (?)	1S, 1S	${}^{19}_{-6} \pm 10$
${}^{102}_{46}\mathrm{Pd} \rightarrow {}^{102}_{44}\mathrm{Ru}$	1	1.103 (2^+) 1.107 (4^+)	1S, 1S	${}^{29}_{25} \pm 9$
${}^{106}_{48}\mathrm{Cd} \rightarrow {}^{106}_{46}\mathrm{Pd}$	1.25	2.741 (?)	1S, 1S	-8 ± 10
${}^{112}_{50}\mathrm{Sn} \rightarrow {}^{112}_{48}\mathrm{Cd}$	1.01	1.871 (0^+)	1S, 1S	-3 ± 10
${}^{130}_{56}\mathrm{Ba} \rightarrow {}^{130}_{54}\mathrm{Xe}$	0.11	2.502 (?) 2.544 (?)	1S, 1S 1S, 2S(P)	${}^{8}_{-6} \pm 13$
${}^{152}_{64}\mathrm{Gd} \rightarrow {}^{152}_{62}\mathrm{Sm}$	0.20	0 (0^+)	1S, 2S	4 ± 4
${}^{162}_{68}\mathrm{Er} \rightarrow {}^{162}_{66}\mathrm{Dy}$	0.14	1.783 (2^+)	1S, 2S	1 ± 6
${}^{164}_{68}\mathrm{Er} \rightarrow {}^{164}_{66}\mathrm{Dy}$	1.56	0 (0^+)	2S, 2S	9 ± 5
${}^{168}_{70}\mathrm{Yb} \rightarrow {}^{168}_{68}\mathrm{Er}$	0.14	1.355 (1^-) 1.393 (?)	1S, 2S 2S, 2S	${}^{-1}_{8} \pm 4$
${}^{180}_{74}\mathrm{W} \rightarrow {}^{180}_{72}\mathrm{Hf}$	0.13	0 (0^+) 0,093 (2^+)	1S, 1S 1S, 3S	${}^{26}_{-4} \pm 17$
${}^{196}_{80}\mathrm{Hg} \rightarrow {}^{186}_{78}\mathrm{Pt}$	0.15	0.689 (2^+)	1S, 2S	26 ± 9

Resonance enhancement of neutrinoless double electron capture

M.I. Krivoruchenko, F. Šimkovic, D. Frekers, and A. Faessler,
Nucl. Phys. A 859, 140-171 (2011)

- **New physical phenomenon, oscillations of atoms, was proposed. A connection to process of resonant neutrinoless double electron capture (0νεε) estsblished.**
- **The process of the 0νεε has been revisited for those cases where the two participating atoms are nearly degenerate in mass. New 0νεε transitions with parity violation to ground and excited states of final atom/nucleus were found. Selection rules for the 0νεε transitions were established. The explicit form of corresponding NMEs was derived.**
- **Available data of atomic masses, as well as nuclear and atomic excitations were used to select the most likely candidates for resonant 0νεε transitions. Assuming an effective Majorana neutrino mass of 1 eV, some half-lives has been predicted to be as low as 10^{22} years in the unitary limit. According to obtained estimates, in the case of ^{152}Gd the sensitivity can be comparable to the favored 0νββ decays of nuclei.**
- **More accurate atomic mass measurements in the context of the 0νεε were initialized, which have been partially accomplished using the modern high-precision ion traps. In addition, new 0νεε experiments were initialized (TGV, R. Bernabei group at Gran Sasso, Muenster-Bratislava)**

Oscillations of atoms

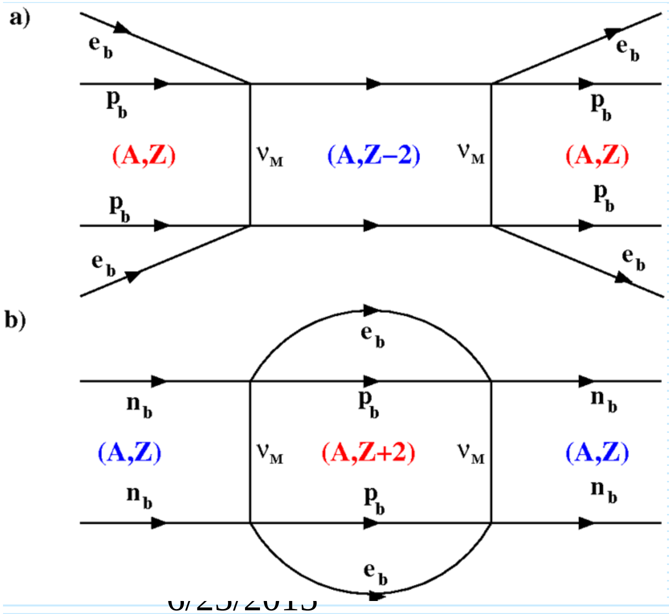
$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of atoms
(lepton number violation)

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485.

In analogy with oscillations of
n-anti{n} (baryon number violation)

$$H_{eff}^{n\bar{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$



Oscillations of stable atoms **(Γ=0)**

$$| < f|e^{-iH_{eff}t}|i > |^2 = \frac{4V^2}{(M_i - M_f)^2} \sin^2 [t\ (M_i - M_f)/2]$$

$$^{164}_{68}Er \rightarrow {}^{164}_{66}Dy$$

$$(M_i - M_f) = 24.1\ keV$$

$$| < f|e^{-iH_{eff}t}|i > |^2 \leq 3\ 10^{-55}$$

Oscillations of unstable atoms **(Γ≠0)**

Double electron capture
(resonant enhancement)

Different types of Oscillations (Effective Hamiltonian)

$$H_{eff}^{K_0\overline{K_0}} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \Gamma_{12} \\ M_{12}^* - \Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillations of ν_l-$\nu_{l'}$
(lepton flavor)

Oscillation of K_0-anti$\{K_0\}$
(strangeness)

$$H_{eff}^{n\bar{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of n-anti{n}
(baryon number)

$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of Atoms (OoA)
(total lepton number)

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485.

Full width of unstable atom/nucleus

Eigenvalues

$$\lambda_+ = M_i + \Delta M - \frac{i}{2}\Gamma_1,$$
$$\lambda_- = M_f - \frac{i}{2}\Gamma - \Delta M + \frac{i}{2}\Gamma_1$$

$$\Delta M = \frac{V^2(M_i - M_f)}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2},$$
$$\Gamma_1 = \frac{V^2\Gamma}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}.$$

Light ν-exchange potential for the 0νEEC

$$\Gamma^{0\nu ECEC}(J^\pi) = \frac{|V_{\alpha\beta}(J^\pi)|^2}{(M_i - M_f)^2 + \Gamma^2_{\alpha\beta}/4}\,\Gamma_{\alpha\beta}$$

β-decay Hamiltonian

$$\mathcal{H}^\beta(x) = \frac{G_\beta}{\sqrt{2}}\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)j_\mu(x) + \mathrm{h.c.}$$

ν-mixing decay

$$\nu_{eL}(x) = \sum_{i=1}^{3} U_{ek}\chi_{kL}(x)$$

Potential

$$\begin{aligned}
V_{\alpha\beta} &= i m_{\beta\beta}\left(\frac{G_\beta}{\sqrt{2}}\right)^2 \frac{1}{\sqrt{1+\delta_{\alpha\beta}}} \sum_{m_\alpha m_\beta} C^{JM}_{j_\alpha m_\alpha j_\beta m_\beta} \int d\vec{x}_1 d\vec{x}_2 \\
&\times \Psi_{\alpha m_\alpha}{}^T(\vec{x}_1) C\gamma^\mu\gamma^\nu(1-\gamma_5)\Psi_{\beta m_\beta}(\vec{x}_2) \int \frac{e^{-i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}}{2q_0}\frac{d\vec{q}}{(2\pi)^3} \\
&\times \sum_n \left[\frac{< A, Z-2|J_\mu(\vec{x}_1)|n >< n|J_\nu(\vec{x}_2)|A, Z >}{q_0 + E_n - M_i - \varepsilon_\beta}\right. \\
&\quad \left. + \frac{< A, Z-2|J_\nu(\vec{x}_2)|n >< n|J_\mu(\vec{x}_1)|A, Z >}{q_0 + E_n - M_i - \varepsilon_\alpha}\right] - (\alpha \leftrightarrow \beta)
\end{aligned} \qquad (2)$$

0νEEC potential - approximations

Non-relativistic impulse approximation for nucleon current

$$J^\mu(0,\vec{x}) = \sum_{n=1}^{A} \tau_n^- [g_V g^{\mu 0} + g_A (\sigma_k)_n g^{\mu k}] \delta(\vec{x} - \vec{x}_n)$$

Closure approximation

$$E_n - M_i \Rightarrow < E > \approx 8\ MeV$$

$$\sum_n |n >< n| = 1$$

$$V^{\alpha\beta}(J_f^\pi) = \frac{1}{4\pi}\ G_\beta^2 m_{\beta\beta} \frac{g_A^2}{R} \sqrt{2J_f + 1} \mathcal{M}_{\alpha\beta}(J_f^\pi)$$

Factorization of atomic and nuclear part

$$\mathcal{M}_{\alpha\beta}(J_f^\pi) \approx \mathcal{A}_{\alpha\beta}\ M^{0\nu}(J_f^\pi)$$

Similar form as for 0νββ-decay

$$M^{0\nu}(0_f^+) \quad = \quad < 0_f^+ \parallel \sum_{nm} \tau_n^- \tau_m^- h(r_{nm}) [-\frac{g_V^2}{g_A^2} + (\vec{\sigma}_n \cdot \vec{\sigma}_m)] \parallel 0_i^+ >,$$

$$M^{0\nu}(0_f^-) \quad = \quad < 0_f^- \parallel \sum_{nm} \tau_n^- \tau_m^- h(r_{nm}) (\hat{r}_n - \hat{r}_m) \cdot [\frac{g_V}{g_A}(\vec{\sigma}_n - \vec{\sigma}_m) - i(\vec{\sigma}_n \times \vec{\sigma}_m)] \parallel 0_i^+ >$$

$$h(r_{nm}) = \frac{2}{\pi} R \int_0^\infty j_0(q r_{nm}) \frac{q_0}{q_0 + < E > - m} dq.$$

Capture of $s_{1/2}$ and $p_{1/2}$ atomic electrons is prefered

$$\Psi_{\alpha m_\alpha}(\vec{x}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} f_\alpha(r)\ \chi_{m_\alpha} \\ -ig_\alpha(r)\ (\vec{\sigma}\cdot\hat{r})\ \chi_{m_\alpha} \end{pmatrix} \qquad (\alpha = n_\alpha s_{1/2})$$

$$\Psi_{\alpha m_\alpha}(\vec{x}) = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} -if_\alpha(r)\ (\vec{\sigma}\cdot\hat{r})\ \chi_{m_\alpha} \\ -g_\alpha(r)\ \chi_{m_\alpha} \end{pmatrix} \qquad (\alpha = n_\alpha p_{1/2})$$

$J^{\pi}=0^+,0^-,1^+,1^-$

Shell		^{78}Se	^{112}Cd	^{124}Te	^{130}Xe	^{156}Gd
$1s_{1/2}$	$< \mathbf{f} >$	3.45×10^3	6.80×10^3	8.83×10^3	1.09×10^4	1.33×10^4
	$< g >$	-4.34×10^2	-1.23×10^3	-1.81×10^3	-2.47×10^3	-3.30×10^3
$2s_{1/2}$	$< f >$	1.25×10^3	2.54×10^3	3.35×10^3	4.19×10^3	5.20×10^3
	$< g >$	-1.58×10^2	-4.59×10^2	-6.87×10^2	-9.48×10^2	-1.29×10^3
$3s_{1/2}$	$< f >$	6.83×10^2	1.39×10^3	1.83×10^3	2.29×10^3	2.85×10^3
	$< g >$	-8.60×10^1	-2.51×10^2	-3.76×10^2	-5.18×10^2	-7.05×10^2
$4s_{1/2}$	$< f >$	4.43×10^2	8.99×10^2	1.19×10^3	1.48×10^3	1.84×10^3
	$< g >$	-5.58×10^1	-1.63×10^2	-2.43×10^2	-3.36×10^2	-4.57×10^2
$2p_{1/2}$	$< f >$	-1.72×10^1	-7.22×10^1	-1.23×10^2	-1.87×10^2	-2.78×10^2
	$< g >$	-1.37×10^2	-3.99×10^2	-5.97×10^2	-8.25×10^2	-1.12×10^3
$2p_{3/2}$	$< f >$	8.06×10^{-1}	2.38×10^0	3.48×10^0	4.62×10^0	6.31×10^0
	$< g >$	-5.02×10^{-2}	-2.10×10^{-1}	-3.46×10^{-1}	-5.03×10^{-1}	-7.47×10^{-1}

Normalized 0vECEC half-lives

For comparison
0νββ-half-life

$$\tilde{T}_{1/2} = T_{1/2} \left|\frac{m_{\beta\beta}}{1\ \mathrm{eV}}\right|^2 \left|\frac{M^{0\nu}(J_f^\pi)}{M^{0\nu}(0_f^+)}\right|^2 .$$

$= 3$

$$\begin{aligned} \tilde{T}^{0\nu\beta\beta}_{1/2} &= (1.4, 9.7) \times 10^{24}\ y \quad (^{76}Ge) \\ &= (0.14, 1.8) \times 10^{24}\ y \quad (^{100}Mo) \\ &= (1.8, 15.6) \times 10^{24}\ y \quad (^{130}Te) \end{aligned}$$

$T^{\min}_{1/2}$: $M_i = M_f$ (full degeneracy)

$$\begin{aligned} \Delta M^2 &= (M^{**}_{A,Z-2} - M_{A,Z})^2 + \Delta M^2_{\mathrm{expt}} \\ \Delta M^2_{\mathrm{expt}} &= \delta M^2_{A,Z-2} + \delta M^2_{A,Z} + \delta R^2_{A,Z-2}. \end{aligned}$$

$^{184}_{76}$Os→ $^{184}_{74}$W* (0.02%)

J_f^π	$M_f^* - M_f$	$M_f^{**} - M_i$	$(n2jl)_\alpha$	$(n2jl)_\beta$	ϵ_α^*	ϵ_β^*	ϵ_C	$\Gamma_{\alpha\beta}$	$\tilde{T}_{1/2}^{\min}$	$\tilde{T}_{1/2}^{\max}$
(0^+)	1322.152±0.022	-11.3± 1.3± 0.9	110	110	69.53	69.53	1.31	6.7×10^{-2}	**2×10^{22}**	**3×10^{27}**

$^{180}_{74}$W→ $^{184}_{72}$Hf (0.13%)

Half-lives in years

J_f^π	$M_f^* - M_f$	$M_f^{**} - M_i$	$(n2jl)_\alpha$	$(n2jl)_\beta$	ϵ_α^*	ϵ_β^*	ϵ_C	$\Gamma_{\alpha\beta}$	$\tilde{T}_{1/2}^{\min}$	$\tilde{T}_{1/2}^{\max}$
0^+	0	12.0± 3.9± 2.1	110	110	65.35	65.35	1.26	5.9×10^{-2}	**3×10^{22}**	**5×10^{27}**

$^{106}_{48}$Cd→ $^{106}_{46}$Pd (1.25%)

All masses/energies in keV

J_f^π	$M_f^* - M_f$	$M_f^{**} - M_i$	$(n2jl)_\alpha$	$(n2jl)_\beta$	ϵ_α^*	ϵ_β^*	ϵ_C	$\Gamma_{\alpha\beta}$	$\tilde{T}_{1/2}^{\min}$	$\tilde{T}_{1/2}^{\max}$
	2717.59±0.21	3.0± 5.9± 4.1	110	110	24.35	24.35	0.74	$7.1\ 10^{-3}$	**$2\ 10^{23}$**	$8\ 10^{29}$
	2737±1	-16.5± 5.9± 4.1	110	110	24.35	24.35	0.74	$7.1\ 10^{-3}$	**$2\ 10^{23}$**	$4\ 10^{30}$
		4.8± 5.9± 4.1	110	210	24.35	3.60	0.23	$3.6\ 10^{-3}$	$3\ 10^{23}$	$7\ 10^{30}$
		5.1± 5.9± 4.1	110	211	24.35	3.33	0.21	$3.1\ 10^{-3}$	$5\ 10^{25}$	$2\ 10^{33}$
		7.9± 5.9± 4.1	110	310	24.35	0.67	0.07	$3.6\ 10^{-3}$	$1\ 10^{24}$	$4\ 10^{31}$
		8.0± 5.9± 4.1	110	311	24.35	0.56	0.06	$3.5\ 10^{-3}$	$1\ 10^{26}$	$6\ 10^{33}$

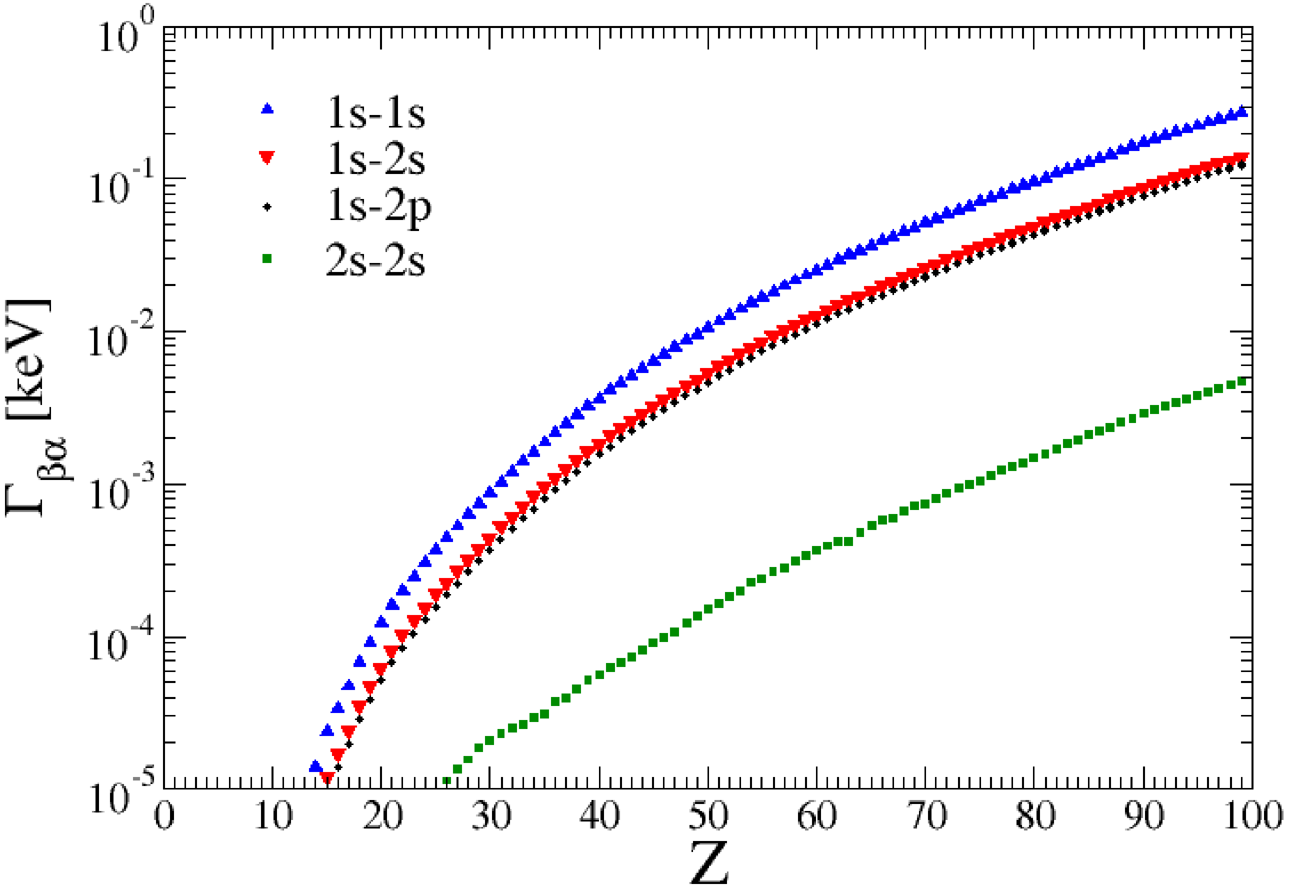
Widths of atomic excited states (2 holes)

$$\Gamma^*_{\beta\gamma} = \sum_\alpha \frac{\nu_\alpha - \delta_{\alpha\gamma}}{2(2l_\beta+1)} \eta_{\beta\alpha} \Gamma_{\beta\alpha} + \sum_\alpha \frac{\nu_\alpha - \delta_{\alpha\beta}}{2(2l_\gamma+1)} \eta_{\gamma\alpha} \Gamma_{\gamma\alpha}$$

$$\eta_{\beta\alpha} = \frac{\omega^{*3}_{\beta\alpha}}{\omega^3_{\beta\alpha}} \frac{d^{*2}_{\beta\alpha}}{d^2_{\beta\alpha}}$$

$$d^*_{\beta\alpha} = \int \frac{1}{a_\beta^{3/2}} R_{nl}(\frac{r}{a_\beta}) \frac{1}{a_\alpha^{3/2}} R_{n'l'}(\frac{r}{a_\alpha}) r^3 dr$$

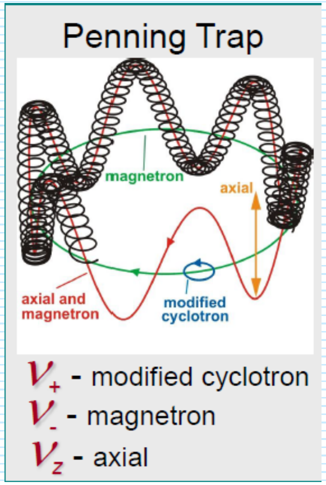
$$d_{\beta\alpha} = \int \frac{1}{a^{3/2}} R_{nl}(\frac{r}{a}) \frac{1}{a^{3/2}} R_{n'l'}(\frac{r}{a}) r^3 dr$$



Improved Q-value measurements
Klaus Blaum (MPI Heidelberg)

nucl. tr.	Q_{old}	$E = B + E_\gamma$	Orbit.	$\Delta = Q(old) - E$	Q_{new}	$\Delta = Q(new) - E$
$^{112}Sn \rightarrow ^{112}Cd$	1919.5(4.8)	1901.7	KL_1	17.8(4.8)	1919.82(16)	18.12(16)
		1924.4	KK	-4.9(4.8)		-4.56(16)
$^{152}Gd \rightarrow ^{152}Sm$	54.6(3.5)	54.79+0	KL_1	-0.19(3.50)	55.70(18)	0.91(18)
$^{164}Er \rightarrow ^{164}Dy$	23.3(3.9)	18.09	l_1L_1	5.21(3.90)		

^{152}Gd→^{152}Sm **(Eliseev, et al., F.Š, M. Krivoruchenko, PRL 106, 052504 (2011))**
(F.Š., Krivoruchenko, Faessler, PPNP 66, 446 (2011))



$$\Gamma_{\varepsilon\varepsilon} = |V_{\varepsilon\varepsilon}|^2 \frac{\Gamma}{\Delta^2 + \Gamma^2/4} = |V_{\varepsilon\varepsilon}|^2 R$$

$$V_{\varepsilon\varepsilon} = m_{\beta\beta} \frac{\sqrt{2} g_A^2 G_\beta^2}{(4\pi)^2 R_{nucl}} \overline{f}_a \overline{f}_b M^{0\nu}$$

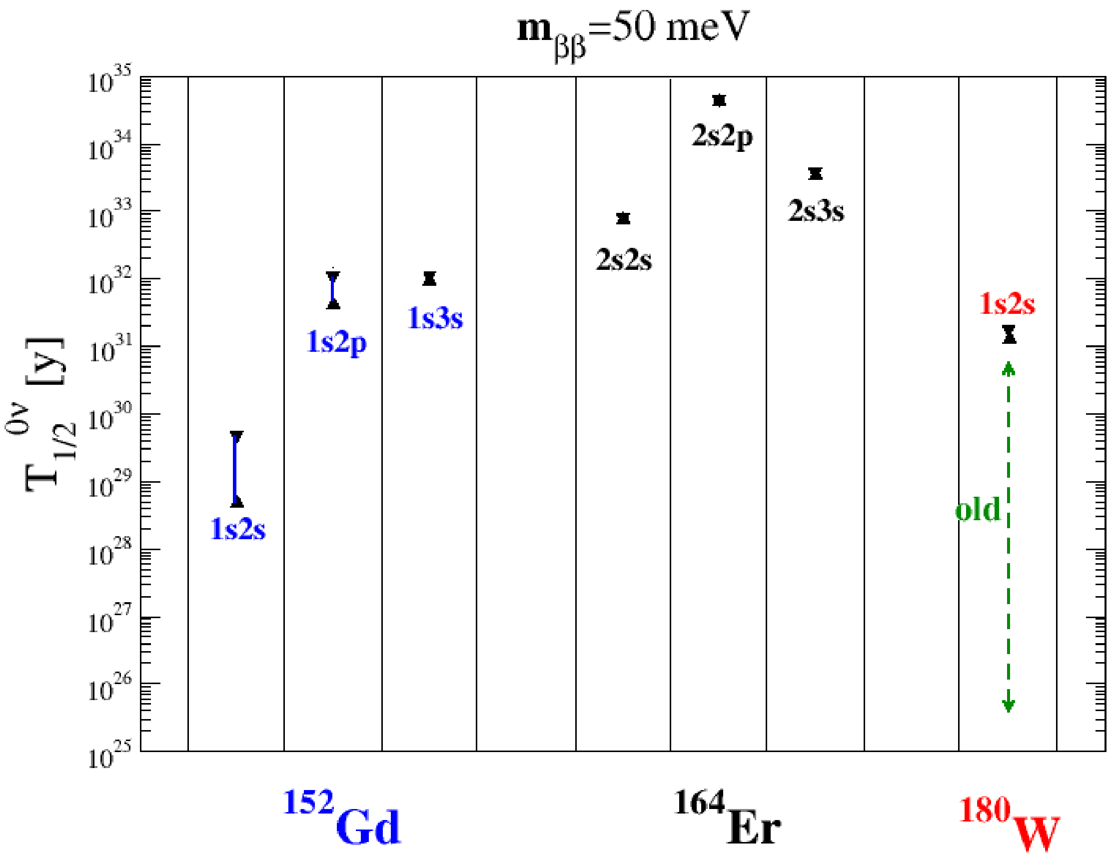
$$T_{1/2}^{0\nu} = 4 \times 10^{26} \left(\frac{1 \text{ eV}}{m_{\beta\beta}} \right)^2 \text{ years.}$$

Remeasured Q-value: **^{112}Sn, ^{74}Se, ^{136}Ce, ^{96}Ru, ^{152}Gd, ^{162}Er, ^{168}Yb, ^{106}Cd, ^{156}Dy, ^{180}W**

need to be remeasured: ^{124}Xe, ^{130}Ba, ^{184}Os, ^{190}Pt

$0\nu\varepsilon\varepsilon$ half-lives

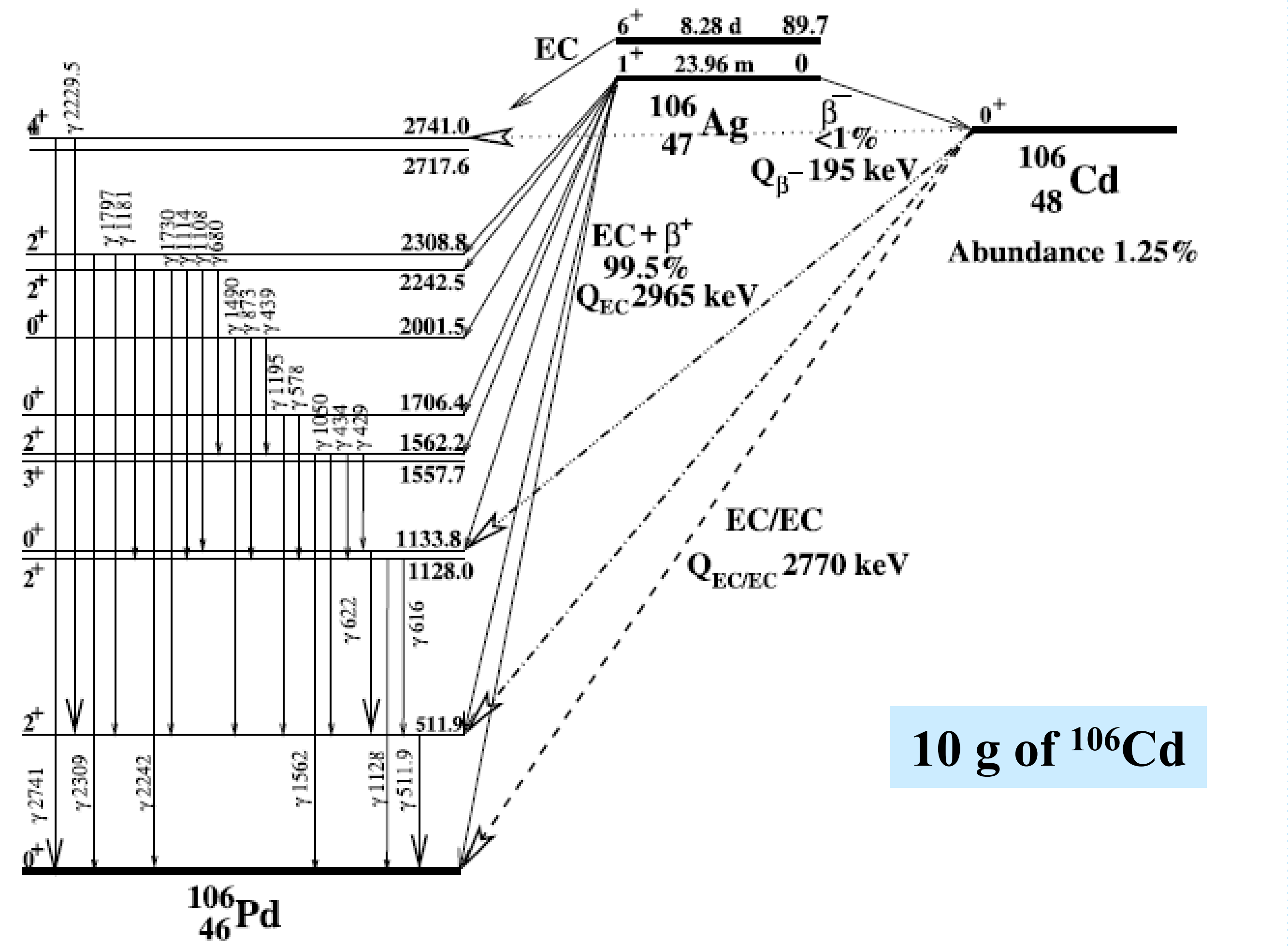
$m_{\beta\beta}$=50 meV



Nucleus	$(n2jl)_a$	$(n2jl)_b$	E_a	E_b	E_C	Γ_{ab} (keV)	Δ (keV)	$T^{\min}_{1/2}$ (y)	$T^{\max}_{1/2}$ (y)
^{152}Gd	110	210	46.83	7.74	0.34	2.3×10^{-2}	-0.83 ± 0.18	4.7×10^{28}	4.8×10^{29}
	110	211	46.83	7.31	0.32	2.3×10^{-2}	-1.27 ± 0.18	4.2×10^{31}	1.1×10^{32}
	110	310	46.83	1.72	0.11	3.2×10^{-2}	-7.07 ± 0.18	9.4×10^{31}	1.1×10^{32}
^{164}Er	210	210	9.05	9.05	0.22	8.6×10^{-3}	-6.82 ± 0.12	7.5×10^{32}	8.4×10^{32}
	210	211	9.05	8.58	0.23	8.3×10^{-3}	-7.28 ± 0.12	4.2×10^{34}	4.6×10^{34}
	210	310	9.05	2.05	0.11	1.8×10^{-2}	-13.92 ± 0.12	3.5×10^{33}	3.9×10^{33}
^{180}W	110	110	63.35	63.35	1.26	7.2×10^{-2}	-11.24 ± 0.27	1.3×10^{31}	1.8×10^{31}

$0\nu\varepsilon\varepsilon$ experiments in Modane and Grand Sasso

New level
2737 keV
(J^π=?)



10 g of ^{106}Cd

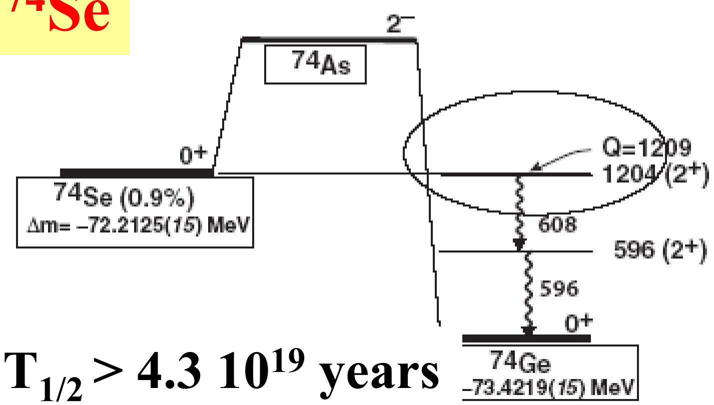
$T_{1/2}^{2\nu\varepsilon\varepsilon}$ (^{106}Cd) > 3.6 10^{20} y

TGV Coll., Rukhadze et al., NPA 852, 197 (2011)

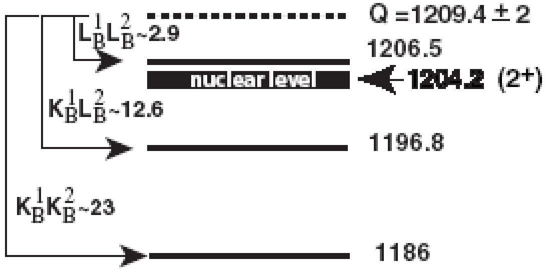
$T_{1/2}^{0\nu\varepsilon\varepsilon}$ (^{106}Cd) > 1.1 10^{20} y

^{74}Se

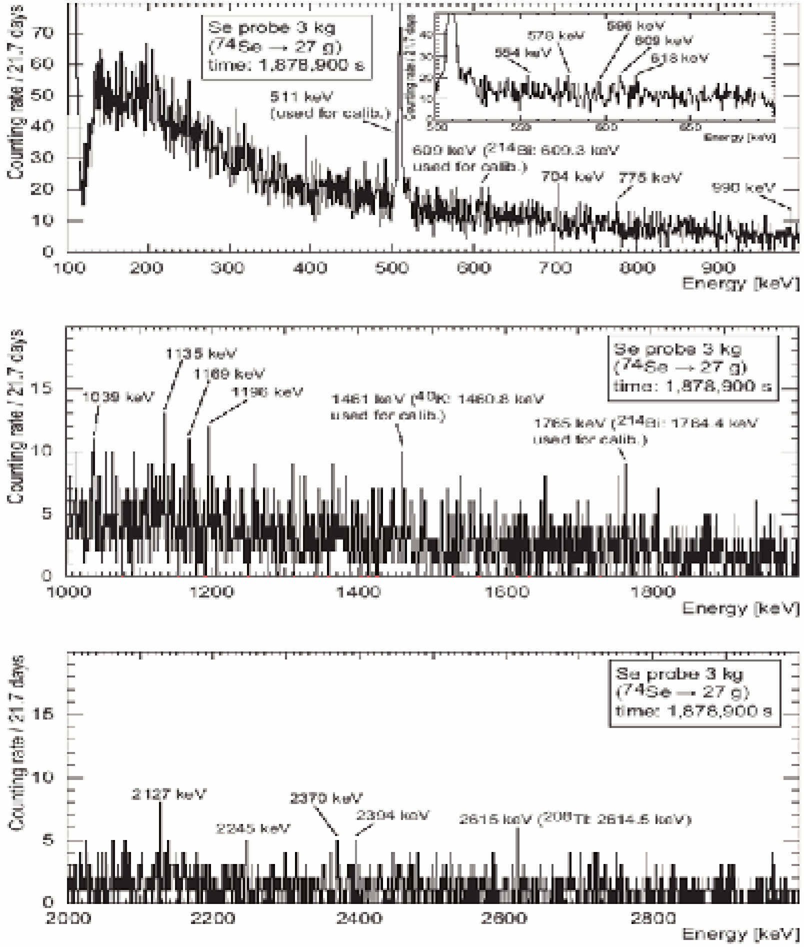
Experiment in Bratislava!



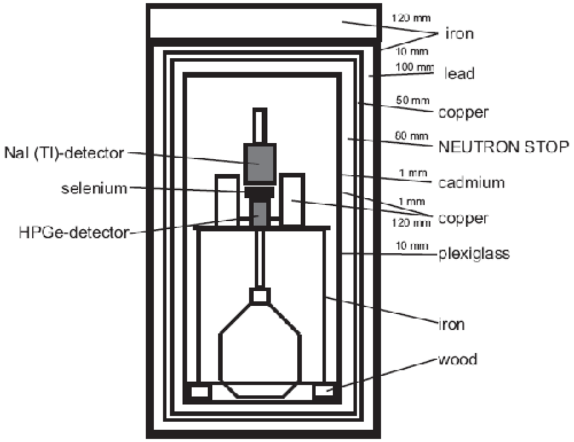
$T_{1/2} > 4.3\ 10^{19}$ years



Muenster and Bratislava groups



Frekers, Puppe, Thies, Povinec, F.Š., Staníček, Sýkora, NPA 860, 1 (2011) .



A comparison

Resonance enhancement of neutrinoless double electron capture
M.I. Krivoruchenko, F. Š., D. Frekers, and A. Faessler,
Nucl. Phys. A 859, 140-171 (2011)

$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

$e^- + e^- + (A,Z) \rightarrow (A,Z-2)^{**}$

Perturbation theory

$$\frac{1}{T^{0\nu}_{1/2}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{01}(E_0,Z)\left|M^{0\nu}\right|^2$$

Breit-Wigner form

$$\Gamma^{0\nu ECEC}(J^\pi) = \frac{|V_{\alpha\beta}(J^\pi)|^2}{(M_i - M_f)^2 + \Gamma^2_{\alpha\beta}/4}\Gamma_{\alpha\beta}$$

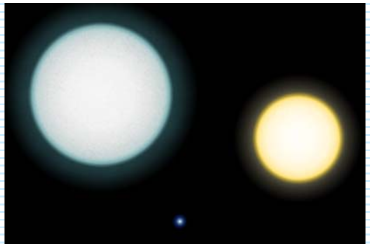
- 2νββ-decay background can be a problem
- Uncertainty in NMEs factor ~2, 3
- $0^+ \rightarrow 0^+, 2^+$ transitions
- Large Q-value
- ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, ^{136}Xe …
- Many exp. in construction, potential for observation in the case of inverted hierarchy (2020)

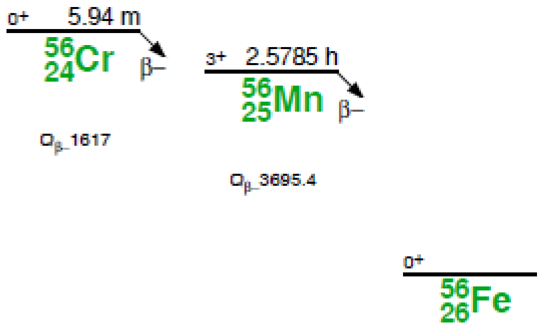
- 2νεε-decay strongly suppressed
- NMEs need to be calculated
- $0^+ \rightarrow 0^+, 0^-, 1^+, 1^-$ transitions
- Small Q-value
- Q-value needs to be measured at least with 100 eV accuracy
- ^{152}Gd, looking for additional
- small experiments yet

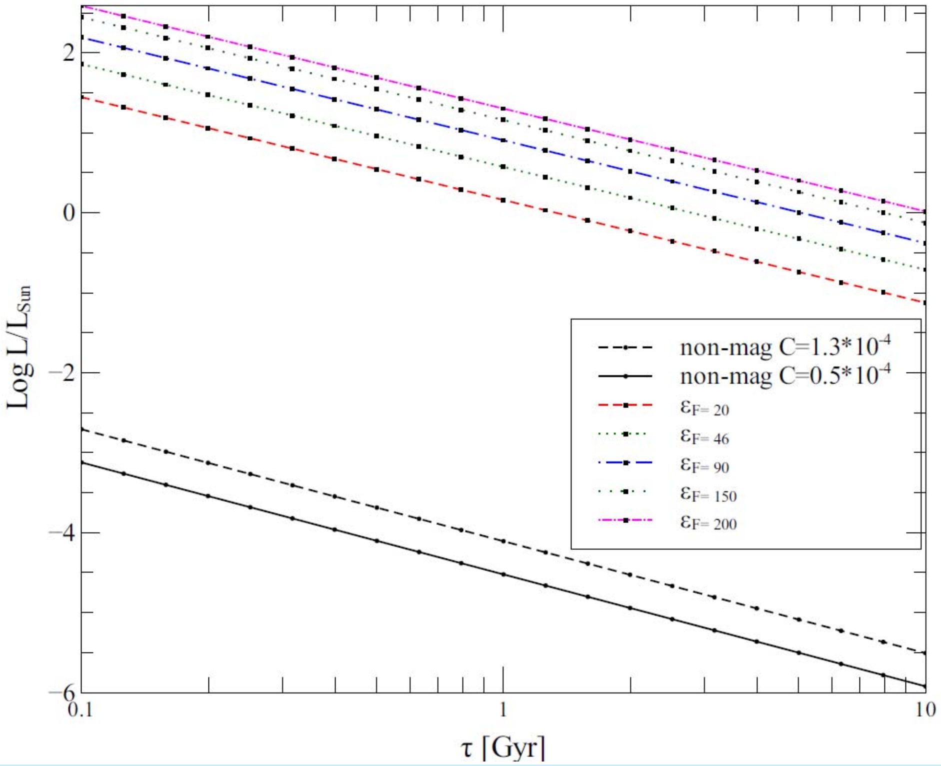
Universe as a laboratory to study LN violation

Belyaev, Ricci, Simkovic, Truhlik, arXiv: 1212.3155, Truhlik, MEDEX13 presentation

Cooling of strongly magnetized iron White dwarfs







$$e^- + {}^{56}Fe \rightarrow {}^{56}Cr + e^+$$

$$e^- + e^- + {}^{56}Fe \rightarrow {}^{56}Cr$$

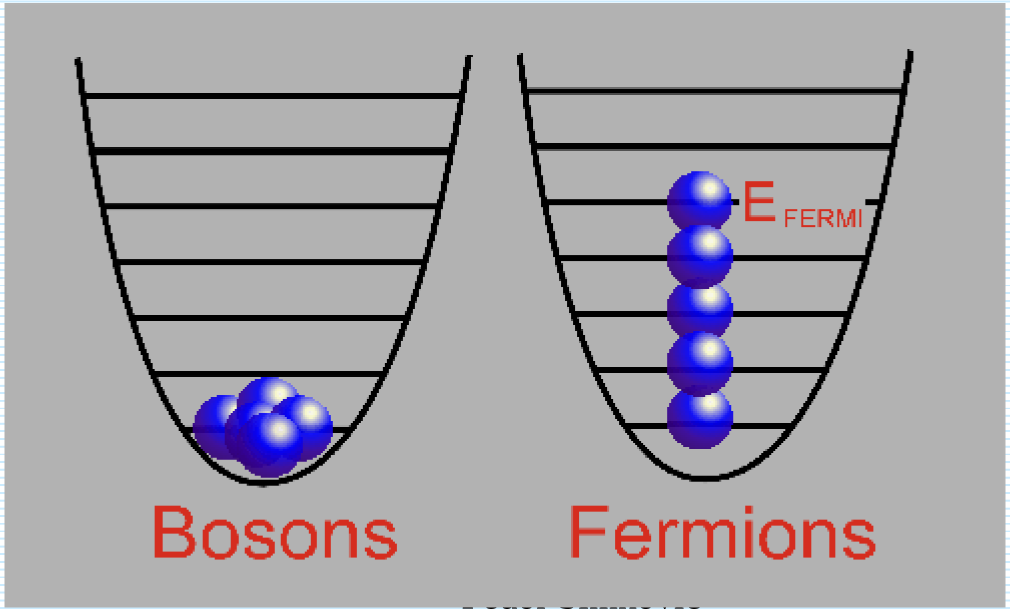
(Partly)bosonic or fermionic neutrinos?

Fermions:

No two fermions can occupy the same state, so in the ground state (T=0), fermions stack from The lowest energy level to higher Energy levels, leaving no holes.

Bosons:

In the ground state (T=0) all bosons occupy lowest energy state.



Mixed statistics for neutrinos

Definnition of mixed state

$$\begin{aligned} |\nu > &= \hat{a}^\dagger |0> \\ &\equiv \cos\delta \;\; \hat{f}^\dagger|0> \; + \; \sin\delta \;\; \hat{b}^\dagger|0> \\ &= \cos\delta \;\; |f> \; + \; \sin\delta \;\; |b> \end{aligned}$$

with commutation Relations

$$\begin{aligned} \hat{f}\hat{b} &= e^{i\phi}\hat{b}\hat{f} \qquad \hat{f}^\dagger\hat{b}^\dagger = e^{i\phi}\hat{b}^\dagger\hat{f}^\dagger \\ \hat{f}\hat{b}^\dagger &= e^{-i\phi}\hat{b}^\dagger\hat{f} \qquad \hat{f}^\dagger\hat{b} = e^{-i\phi}\hat{b}\hat{f}^\dagger \end{aligned}$$

Amplitude for 2νββ

$$\begin{aligned} A^{2\nu} &= [\cos\delta^4 + \cos\delta^2 \sin\delta^2 (1-\cos\phi)] A^f \; + \; [\cos\delta^4 + \cos\delta^2 \sin\delta^2 (1+\cos\phi)] A^b \\ &= \cos\chi^2 A^f \; + \; \sin\chi^2 A^b \end{aligned}$$

Decay rate

$$\begin{aligned} W^{2\nu} &= \cos\chi^4 \; W^f \; + \; \sin\chi^4 \; W^b \\ &= (1-b^2) \; W^f \; + \; b^2 \; W^b \end{aligned}$$

Partly bosonic neutrino requires knowing NME or log ft values for HSD or SSD

(calculations coming up soon)

Looking for a signature of bosonic ν

2νββ–decay half-lives $(0^+ \rightarrow 0^+_{g.s.},\ 0^+ \rightarrow 0^+_1,\ 0^+ \rightarrow 2^+_1)$

- **HSD – NME needed**
- **SSD – log ft_{EC}, log ft_β needed**

$$\frac{T^{2\nu-SSD}_{1/2}(2^+_f)}{T^{2\nu-SSD}_{1/2}(0^+_f)} = 2.41 \times 10^4 \quad \text{fermionic } \nu \qquad T^{2\nu}_{1/2}(2^+) = 1.73 \times 10^{23} years$$

$$= 403 \quad \text{bosonic } \nu \qquad = 2.74 \times 10^{21} years$$

$$T^{2\nu-exp}_{1/2}(2^+) > 1.6 \times 10^{21} years$$

Normalized differential characteristics

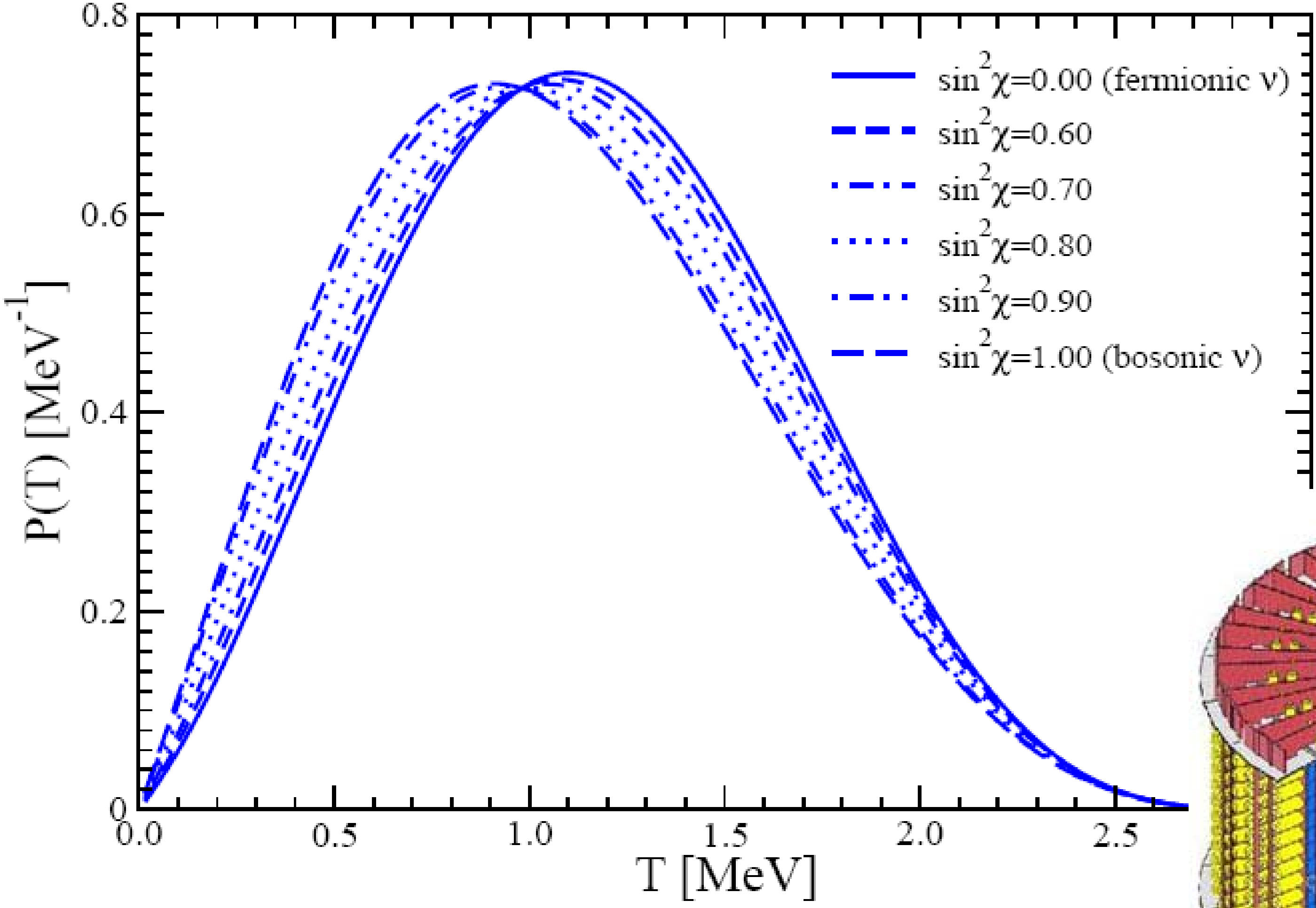
- **The single electron energy distribution**
- **The distribution of the total energy of two electrons**
- **Angular correlations of two electrons**

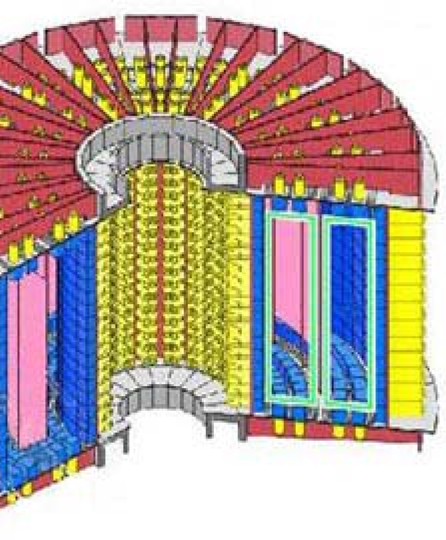
(free of NME and log ft)

Mixed ν excluded for $\sin^2\chi < 0.6$ (NEMO3 data)

$$W^{2\nu} = \cos\chi^4\, W^f + \sin\chi^4\, W^b$$
$$= (1-b^2)\, W^f + b^2\, W^b$$

^{100}Mo → ^{100}Ru (SSD)





Barabash, Dolgov, Dvornický, F. Š., Smirnov, NPB 783, 90 (2007)