Laboratori Nazionali del Gran Sasso
Thursday 25, 2015

II. Double beta decay nuclear matrix elements

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OUTLINE

0νββ

0νεε

0νεβ

$m_{\beta\beta}$

0νββ
NMEs

ν mass scale

CP-phases

Nuclear structure

Study of the 0νββ-decay is one of the highest priority issues in particle and nuclear physics

ν

0νββ-decay:
⇐ No | Yes ⇒

ν

US

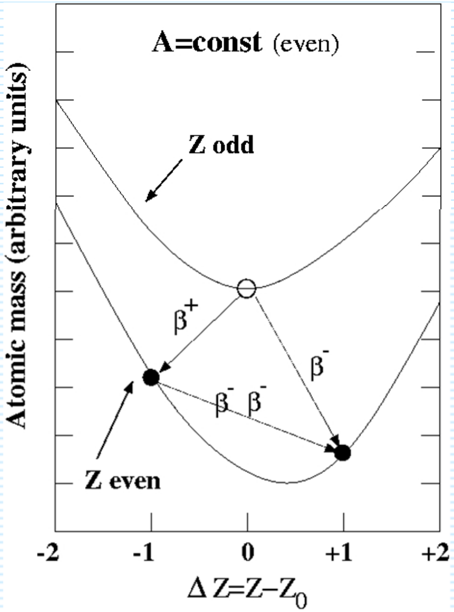
APS Joint Study on the Future of Neutrino Physics (physics/0411216)
We recommend, as a high priority, a phased program of sensitive searches for neutrinoless double beta decay (first in the list of recommendations)

Europe

ASPERA road map:

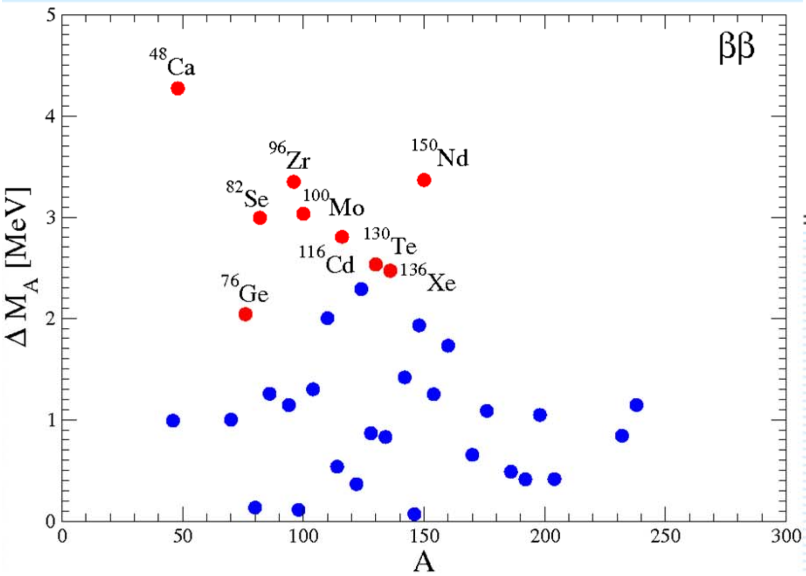
- *Requirement for construction and operation of two double-beta decay experiments with a European lead role or shared equally with non-European partners (GERDA, COBRA, CUORE, SuperNEMO)*
- *We finally reiterate the importance of assessing and reducing the uncertainty in our knowledge of the corresponding nuclear matrix elements, experimentally and theoretically.*

The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei



$$\frac{1}{T_{1/2}^{0\nu}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{01}(E_0, Z) \left|M^{0\nu}\right|^2$$

transition	$G^{01}(E_0,Z)$ $\times 10^{14} y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\lvert M^{0\nu}\rvert^2$
$^{150}Nd \rightarrow {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \rightarrow {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \rightarrow {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \rightarrow {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \rightarrow {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \rightarrow {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \rightarrow {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \rightarrow {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \rightarrow {}^{76}Se$	0.79	2.040	8	?



The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory

Neutrinoless double beta decay of ^{110}Pd

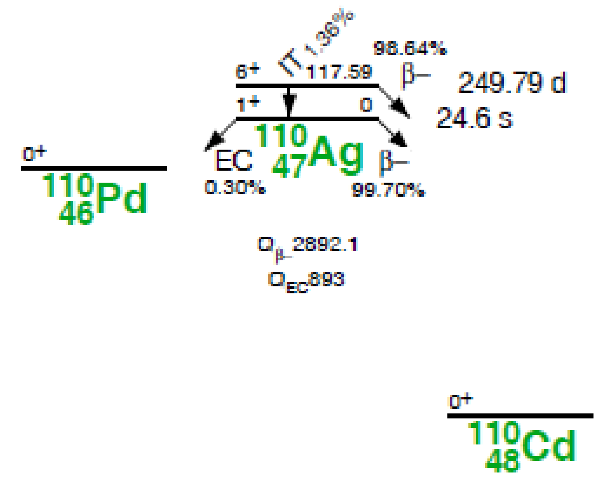
With its high natural abundance, the new results reveal ^{110}Pd to be an excellent candidate for double-*β* decay studies

Q-Value and Half-Lives for the Double-Beta-Decay Nuclide ^{110}Pd

D. Fink, et al.

Phys. Rev. Lett. 108 (2012) 062502.

	^{82}Se	^{110}Pd
Z	34	46
Abund. (%)	8.73	11.72
Q [keV]	2 995	2 017.8
$G^{0\nu}$ [10^{-15} yr^{-1}]	10.16	4.815
$0\nu\beta\beta$ NME	4.64	5.76
$T^{2\nu}_{1/2}$ [yr]	0.92 10^{20}	1.5(6) 10^{20}(SSD)

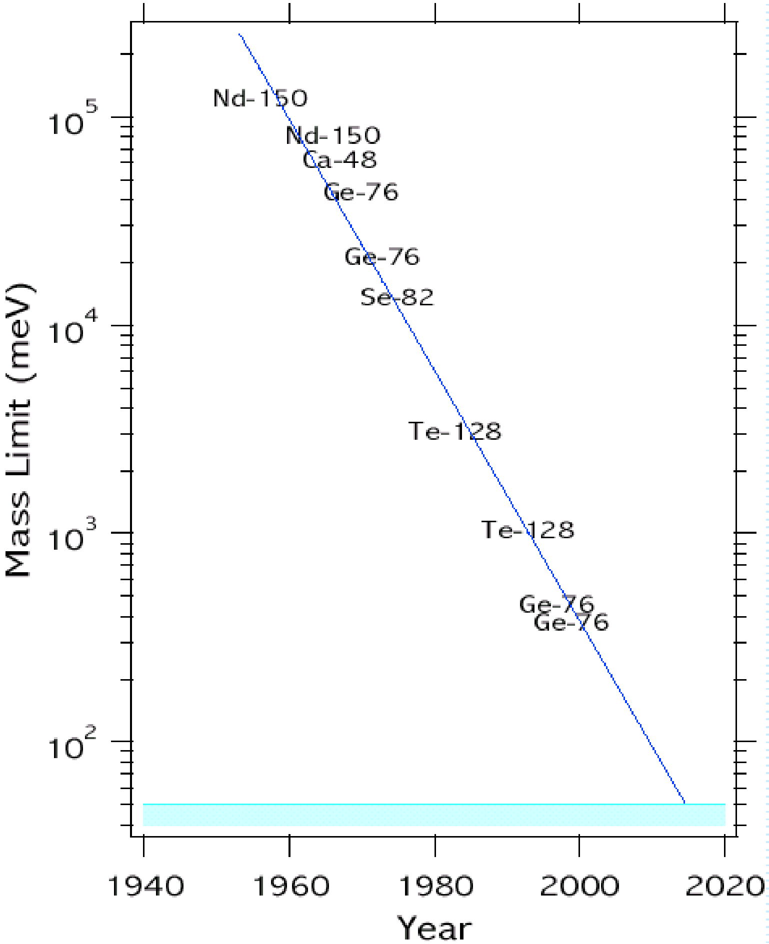


If (or when) the 0νββ decay is observed two theoretical problems must be resolved

S.R. Elliott, P. Vogel,
Ann.Rev.Nucl.Part.Sci. 52, 115 (2002)

1) *What is the **mechanism of the decay**, i.e., what kind of virtual particle is exchanged between the affected nucleons (quarks).*

2) *How to relate the observed decay rate to the fundamental parameters, i.e., what is the value of the corresponding **nuclear matrix elements**.*



The $0\nu\beta\beta$-decay: A nuclear physics problem

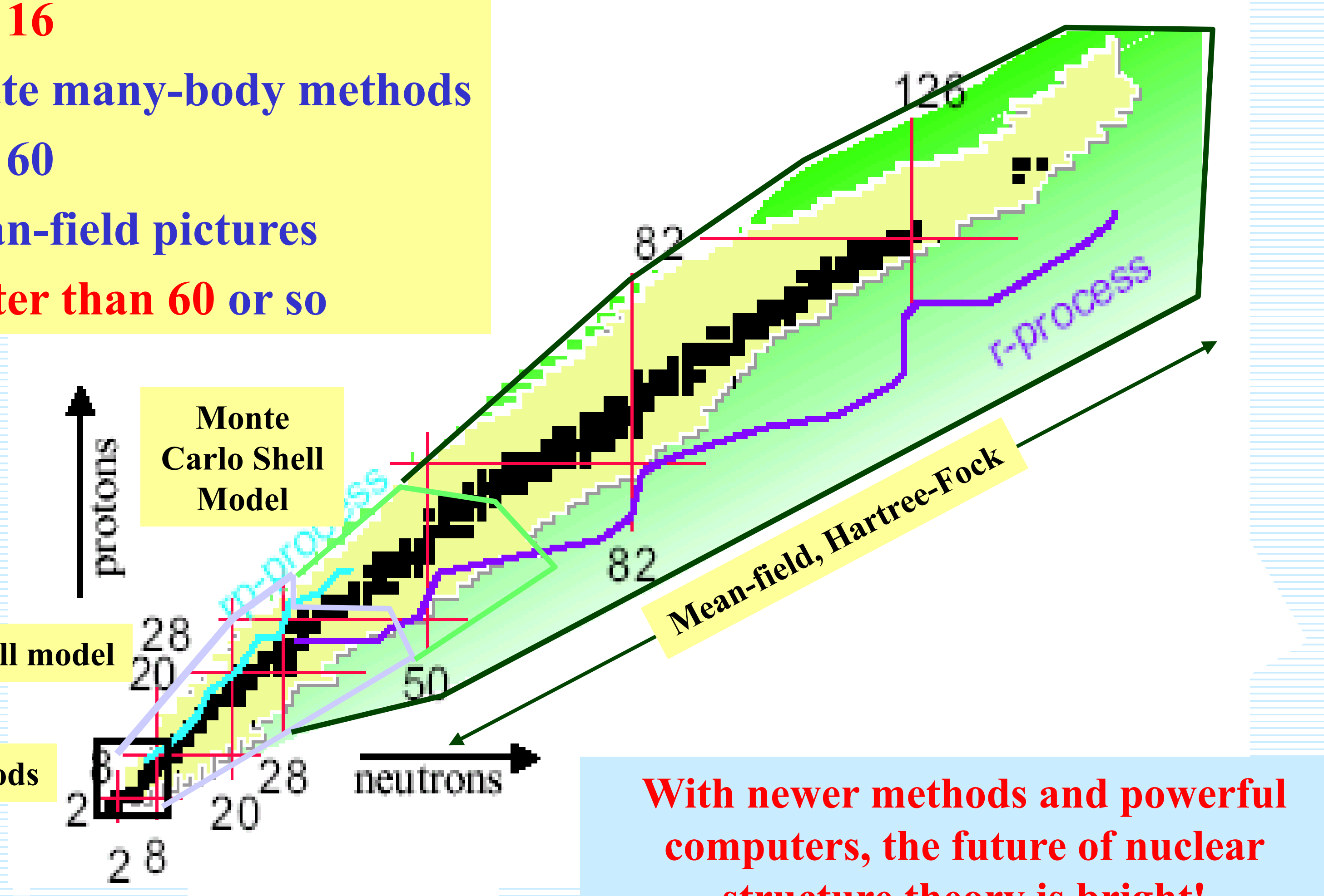
In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge directly the quality of the result.

Nuclear Structure

- Exact methods exist up to A=4
- Computationally exact methods for A up to 16
- Approximate many-body methods for A up to 60
- Mostly mean-field pictures for A greater than 60 or so



With newer methods and powerful computers, the future of nuclear structure theory is bright!

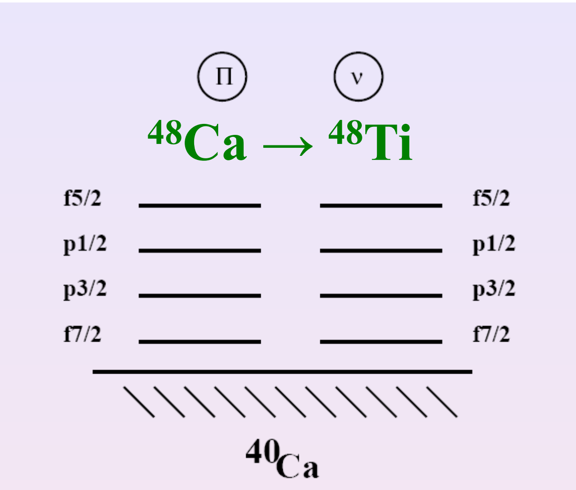
$0\nu\beta\beta$-decay NMEs: QRPA and other approaches

Nuclear Shell Model

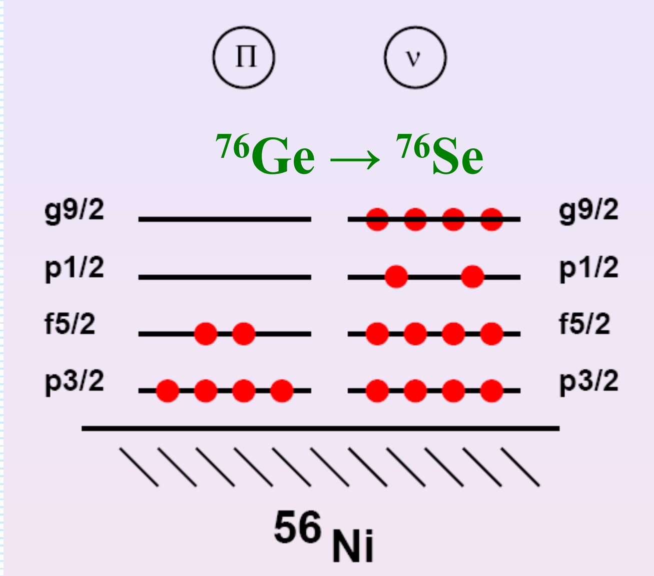
In NSM a limited valence space is used but all configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few 0νββ-decay calculations

- **Define a valence space**
- **Derive an effective interaction** $H\,\Psi = E\,\Psi \rightarrow H_{eff}\,\Psi_{eff} = E\,\Psi_{eff}$
- **Build and diagonalize Hamiltonian matrix (10^{10})**
- **Transition operator** $<\Psi_{eff}\,|\,O_{eff}\,|\,\Psi_{eff}>$
- **Phenomenological input: Energies of states, systematics of B(E2) and GT trans.**

$$H = \sum_a \varepsilon_a a_a^+ a_a - \sum_{abcd} \frac{\langle j_a j_b; JT | V | j_c j_d; JT \rangle_A}{\sqrt{(1+\delta_{ab})(1+\delta_{cd})}} \left[\left[a_a^+ \otimes a_b^+ \right]^{JT} \otimes \left[\tilde{a}_c \otimes \tilde{a}_d \right]^{JT} \right]_{00}^{00}$$



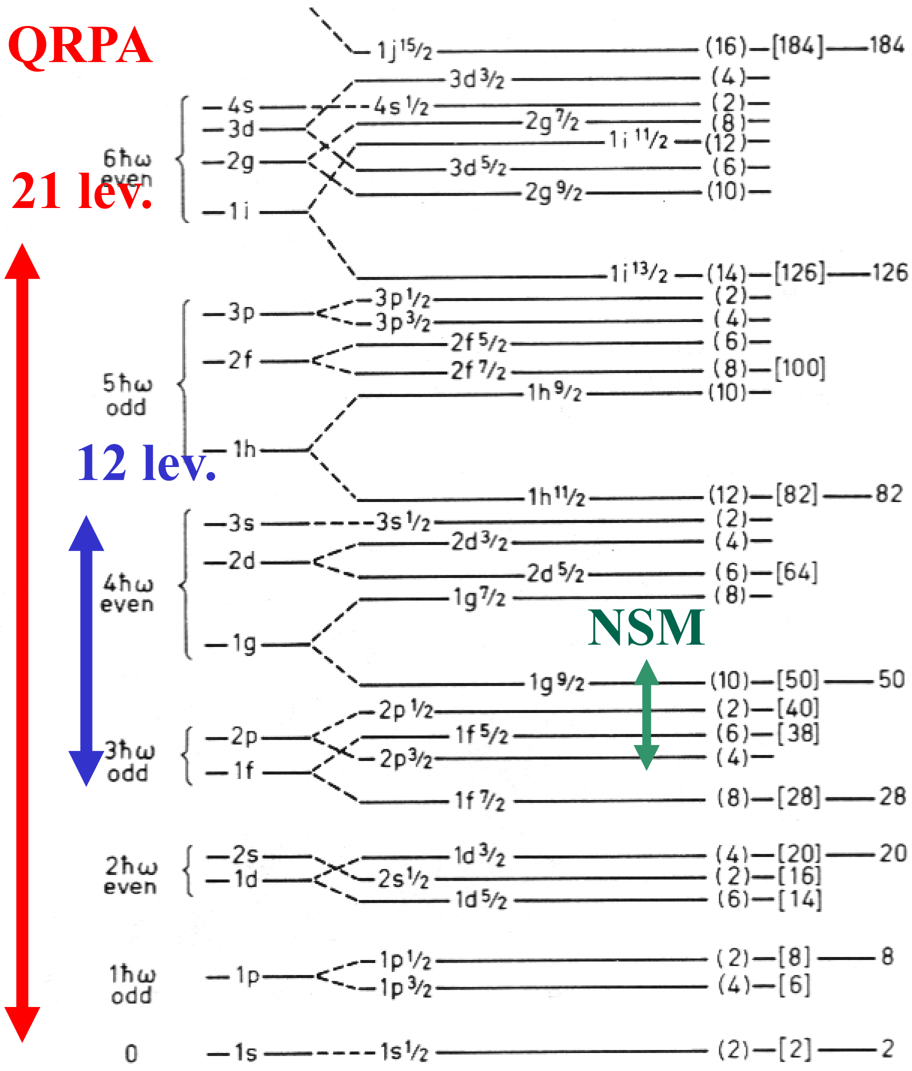
Small calculations



$^{76}Se_{42}$ in the valence
6 protons and 14 neutrons

Quasiparticle Random Phase Approximation (QRPA)

In QRPA a large valence space is used, but only a class of configurations is included. Describe collective states, but not details of dominantly few particle states. Relative simple, thus more 0nbb-decay calculations



- **Large model space** **(up 23 s.p.l, ^{150}Nd – 60 active prot. and 90 neut.)**
- **Spin-orbit partners included**
- **Possibility to describe all multipolarities of the intermed. nucl. J^{π} ($\pi=\pm1$, J=0…9)**

$$H = H_0 + g_{ph} V_{ph} + g_{pp} V_{pp}$$

quasiparticle mean field

Residual interaction

The Interacting Boson Model[¶]

- The low-lying states of the nucleus, composed by n and z valence nucleons, are modeled in terms of (n+z)/2 bosons.
- The bosons have either L = 0 (*s* boson) or L = 2 (*d* boson).
- The bosons can interact through one-body and two-body forces giving rise to bosonic wave functions.
- Any observable can be calculated using these wave functions provided that the relevant operator is employed.

[¶] F. Iachello and A. Arima, *The Interacting Boson Model*, Cambridge University Press, 1987

Projected Hartree-Fock-Bogoliubov Model

PHFB Model

States of good angular momentum J

$$\left|\Psi_M^J\right\rangle = \frac{2J+1}{8\pi^2 a_J}\int d\Omega\, D_{MK}^J(\Omega)\hat{R}(\Omega)\left|\Phi_K\right\rangle$$

Axially symmetric HFB intrinsic state

$$\left|\Phi_0\right\rangle = \prod_{im}\left(u_{im} + v_{im} b_{im}^+ b_{i\bar{m}}^+\right)$$

where

$$b_{im}^+ = \sum_m C_{i\alpha m} a_{im}^+ \qquad b_{i\bar{m}}^+ = \sum_m (-1)^{l+j-m} C_{i\alpha m} a_{i-m}^+$$

Hamiltonian:

$$H = H_{sp} + V(P) + \zeta_{qq} V(QQ)$$

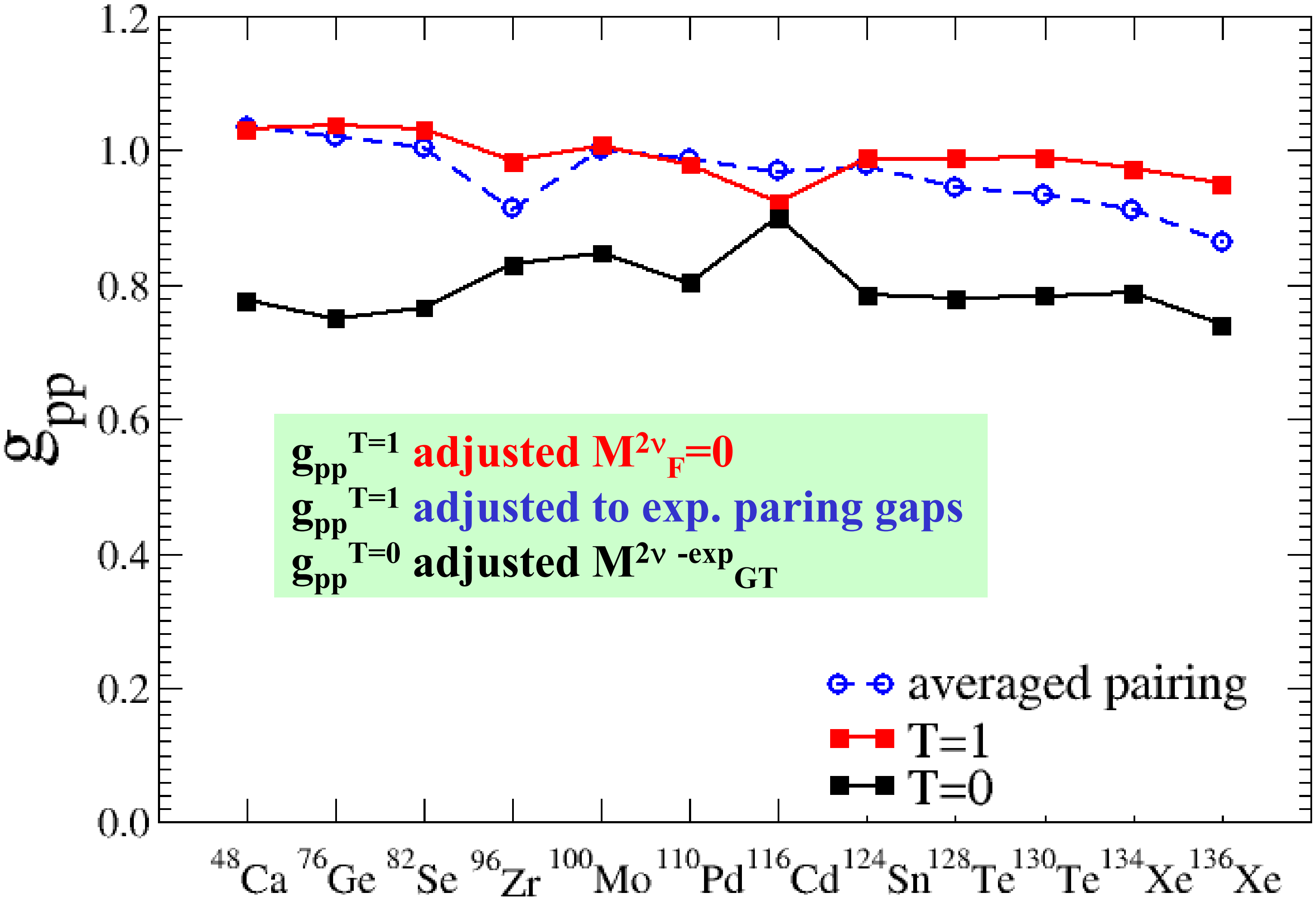
Only quadrupole interaction, GT interaction is missing

QRPA and isospin symmetry restoration

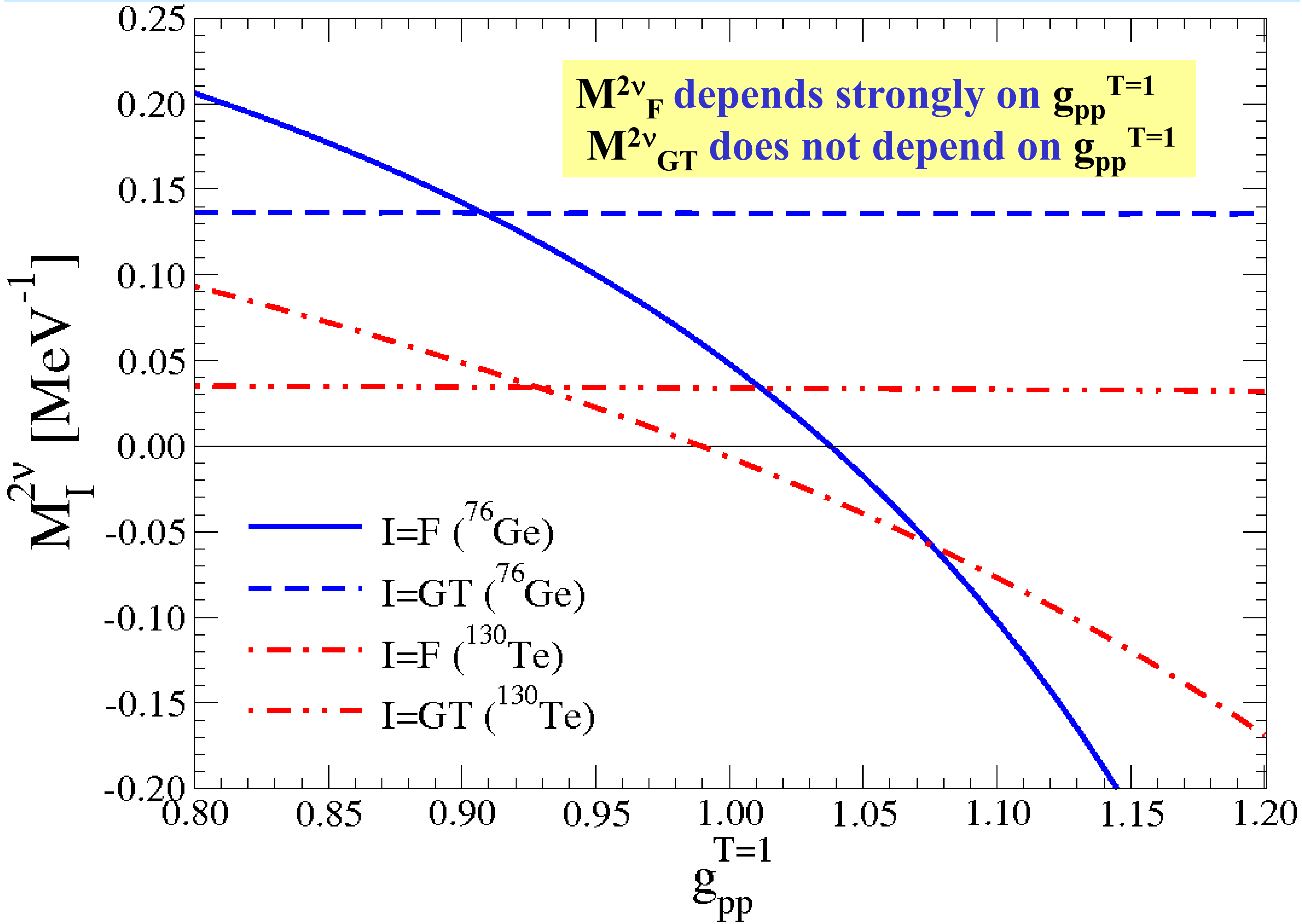
F.Š., V. Rodin, A. Faessler, and P. Vogel

PRC 87, 045501 (2013)

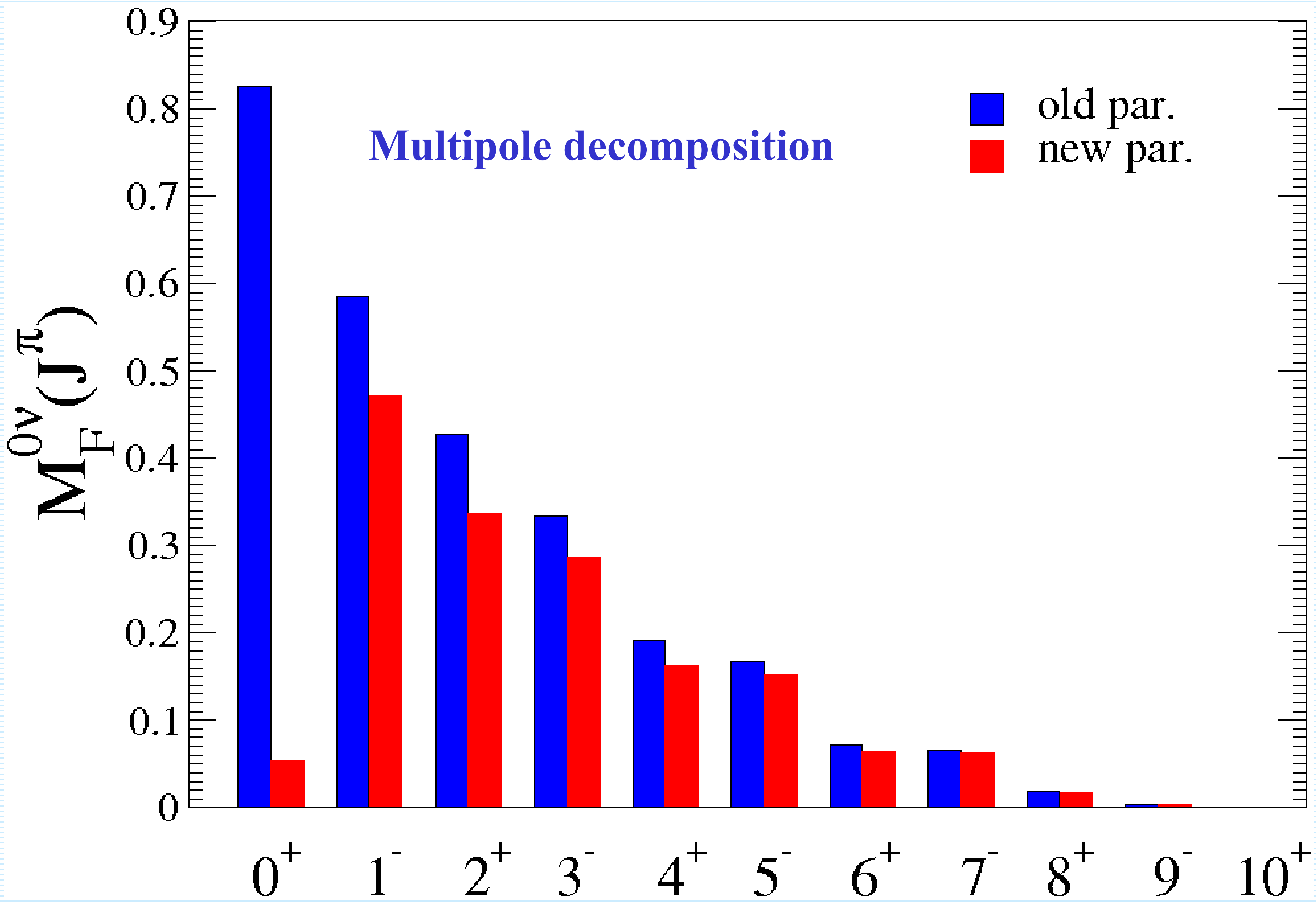
Close values ■ and ■ => no new parameter



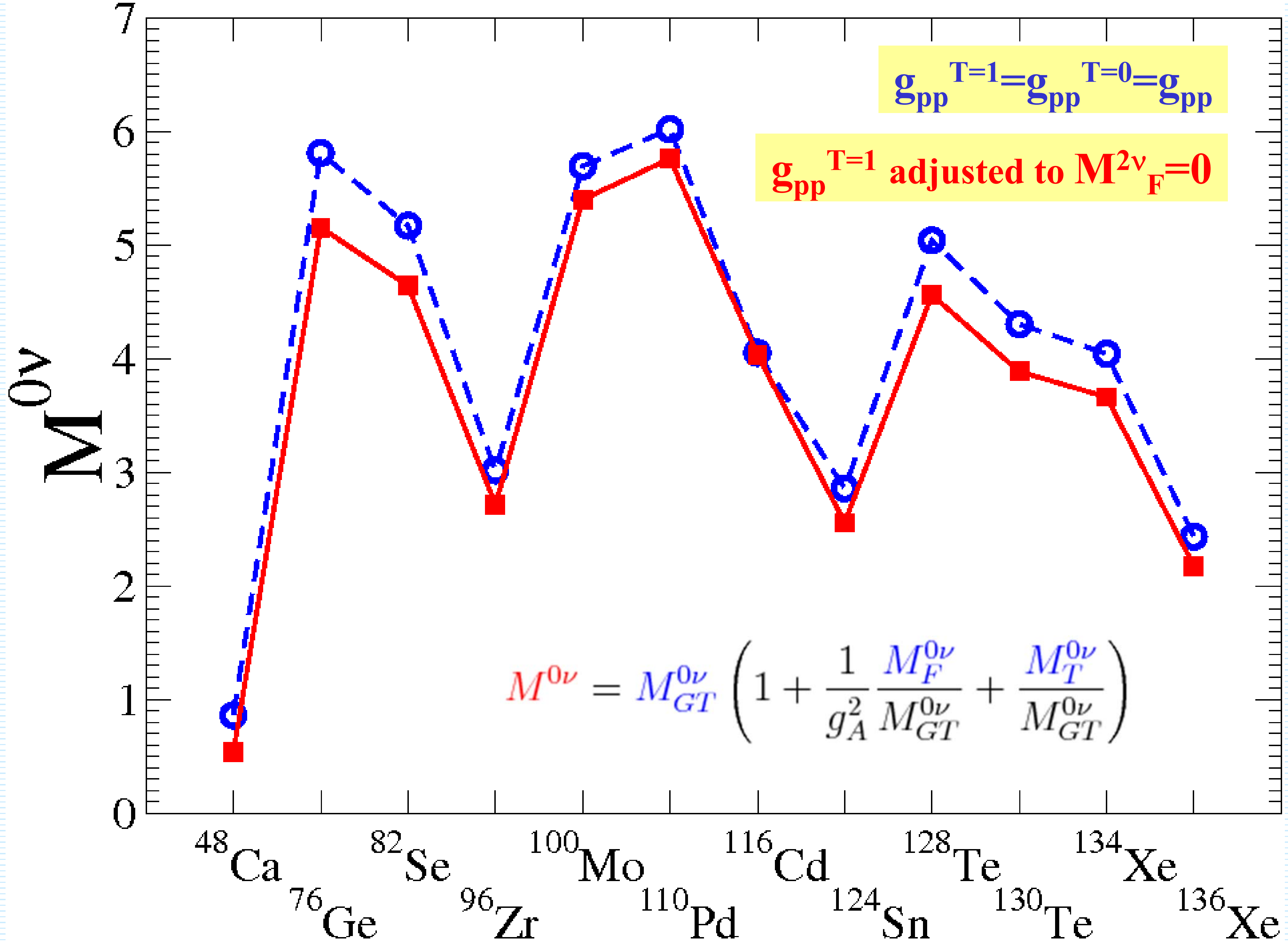
Separation of g_{pp} into $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$



$M^{2\nu}_F$ depends strongly on $g_{pp}^{T=1}$
$M^{2\nu}_{GT}$ does not depend on $g_{pp}^{T=1}$
$M^{2\nu}_I$ [MeV^{-1}]
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
$g_{pp}^{T=1}$
I=F (^{76}Ge)
I=GT (^{76}Ge)
I=F (^{130}Te)
I=GT (^{130}Te)

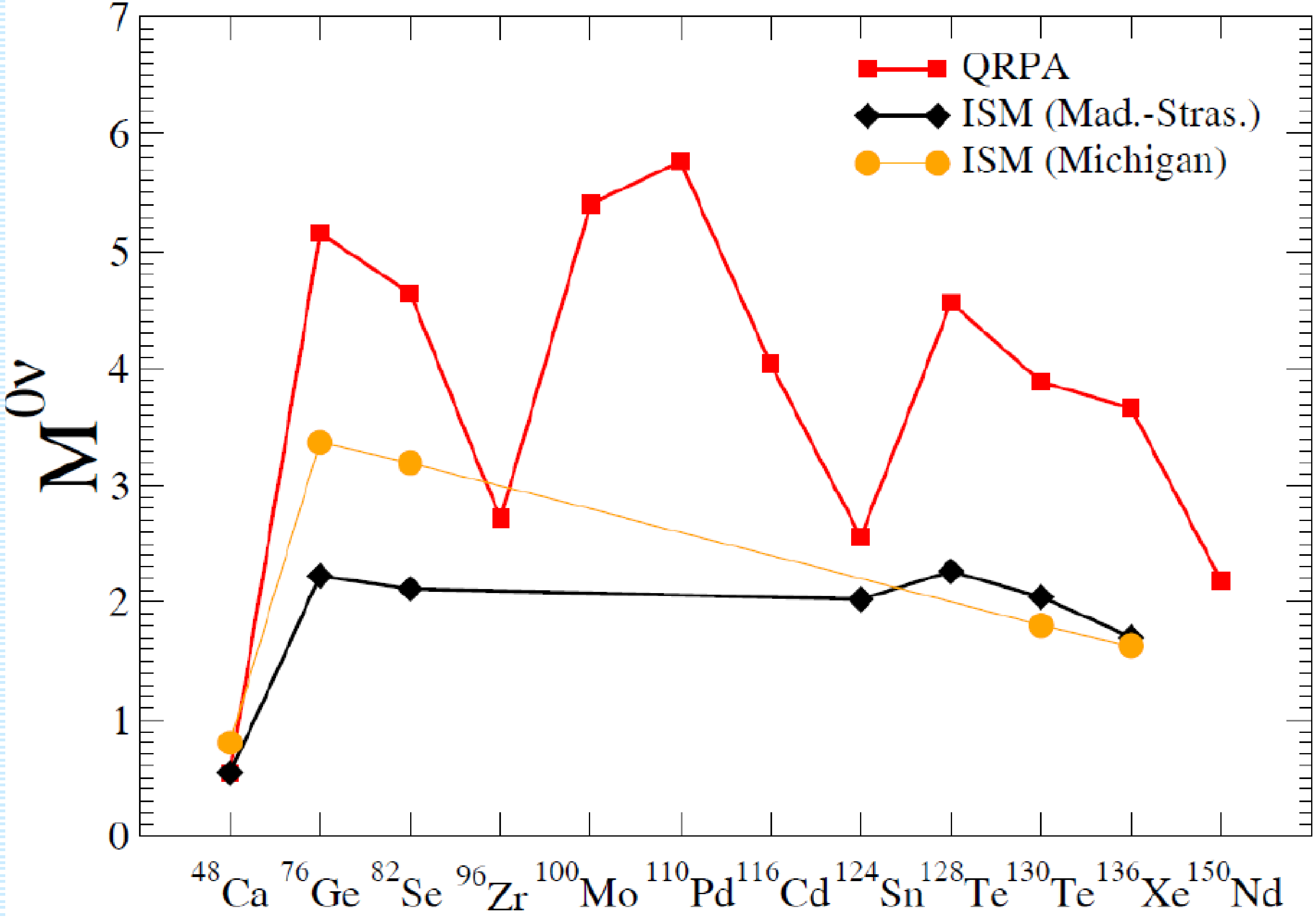


Multipole decomposition
old par.
new par.
$M_F^{0\nu}(J^\pi)$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0^+ 1^- 2^+ 3^- 4^+ 5^- 6^+ 7^- 8^+ 9^- 10^+



$g_{pp}^{T=1}=g_{pp}^{T=0}=g_{pp}$
$g_{pp}^{T=1}$ adjusted to $M^{2\nu}_F=0$
$M^{0\nu}$
$M^{0\nu} = M^{0\nu}_{GT}\left(1+\frac{1}{g_A^2}\frac{M^{0\nu}_F}{M^{0\nu}_{GT}}+\frac{M^{0\nu}_T}{M^{0\nu}_{GT}}\right)$
0
1
2
3
4
5
6
7
^{48}Ca
^{76}Ge
^{82}Se
^{96}Zr
^{100}Mo
^{110}Pd
^{116}Cd
^{124}Sn
^{128}Te
^{130}Te
^{134}Xe
^{136}Xe

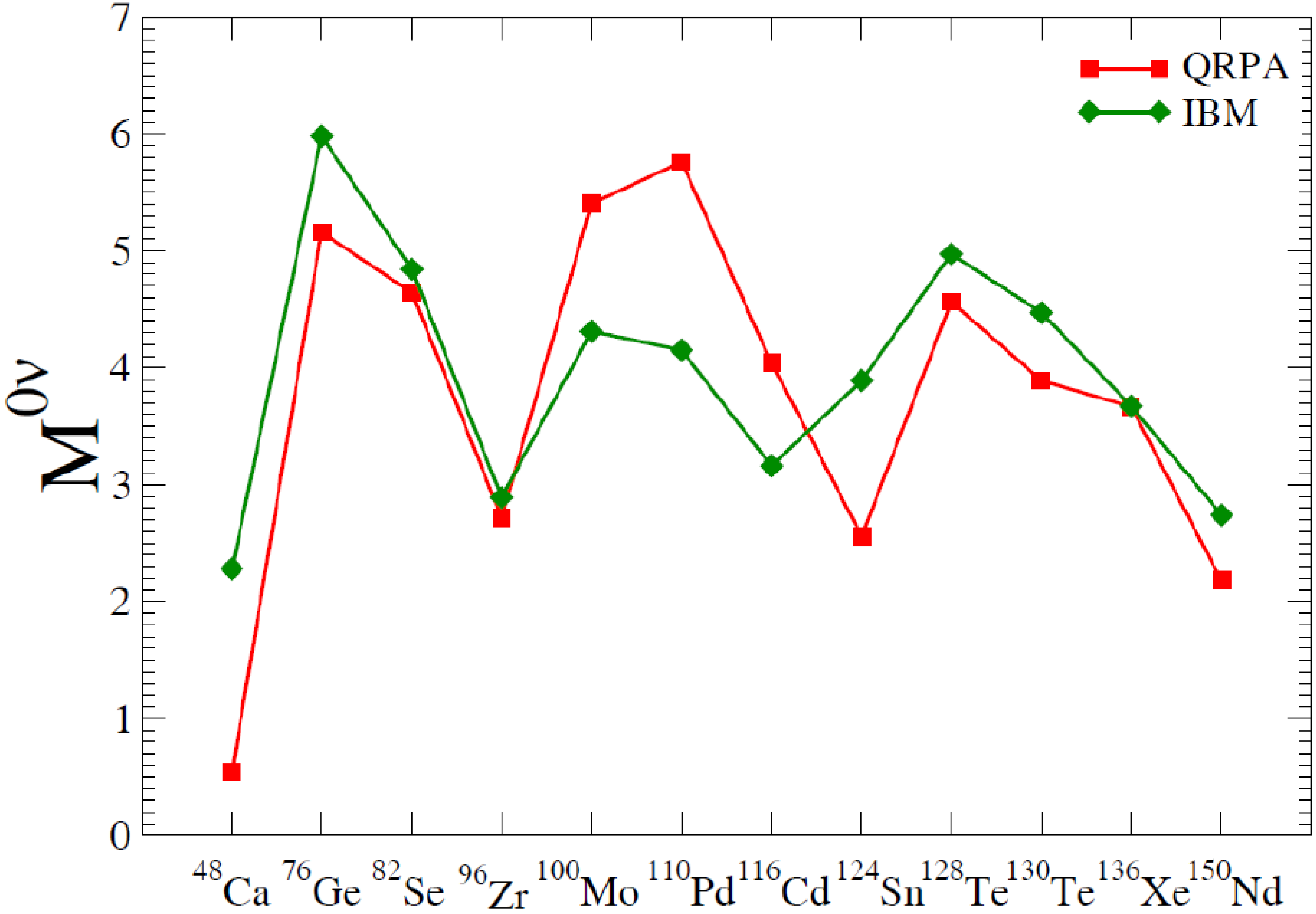
QRPA versus Interacting Shell Model



ISM: Menendez et al. NPA 818 (2009) 139

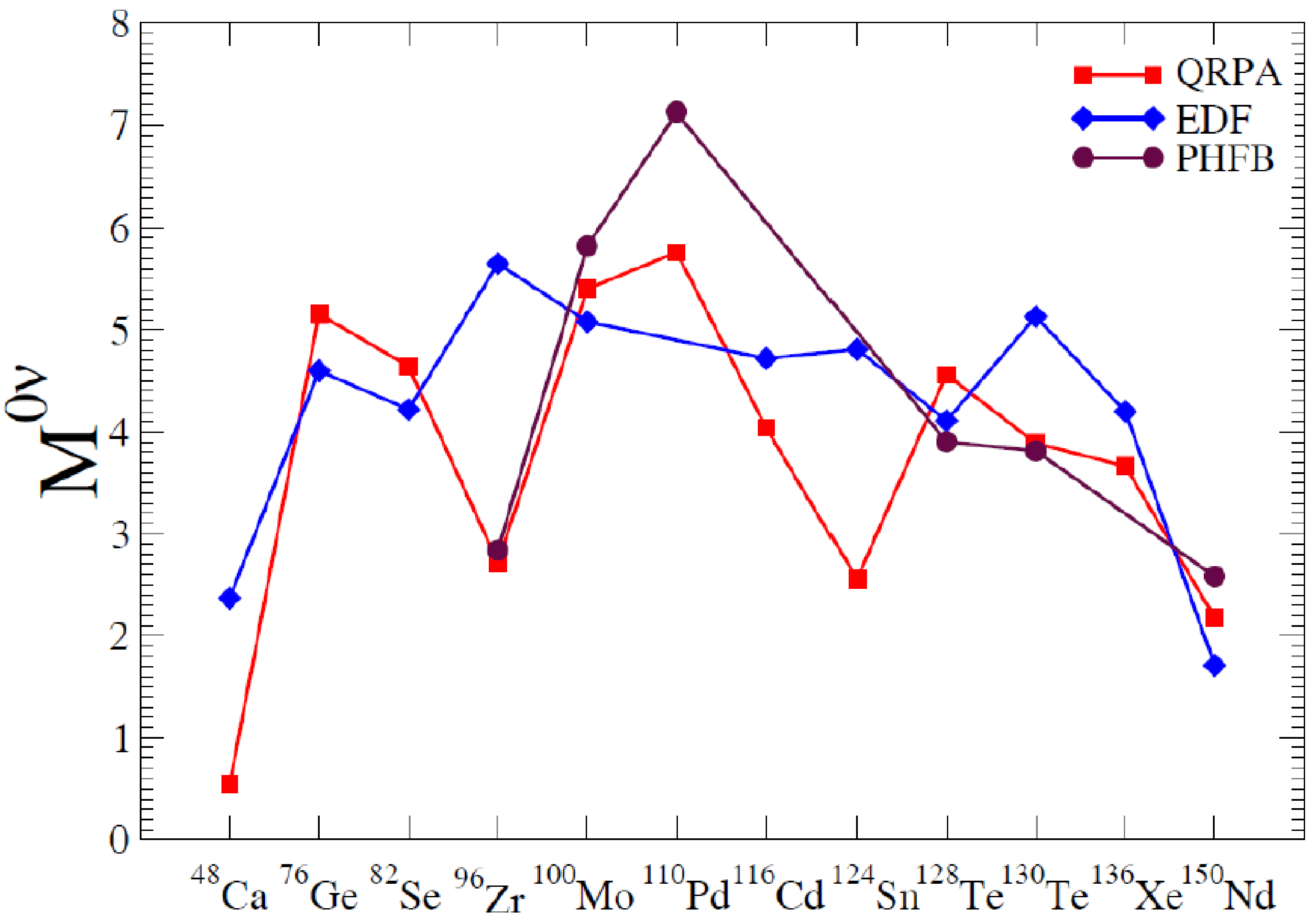
ISM: Brown, Horoi etc., Phys.Rev.Lett. 113 (2014) 262501 …

QRPA versus IBM



IBM: Barea, Kotila, Iachello, PRC (2013) 014315

QRPA versus EDF/PHFB



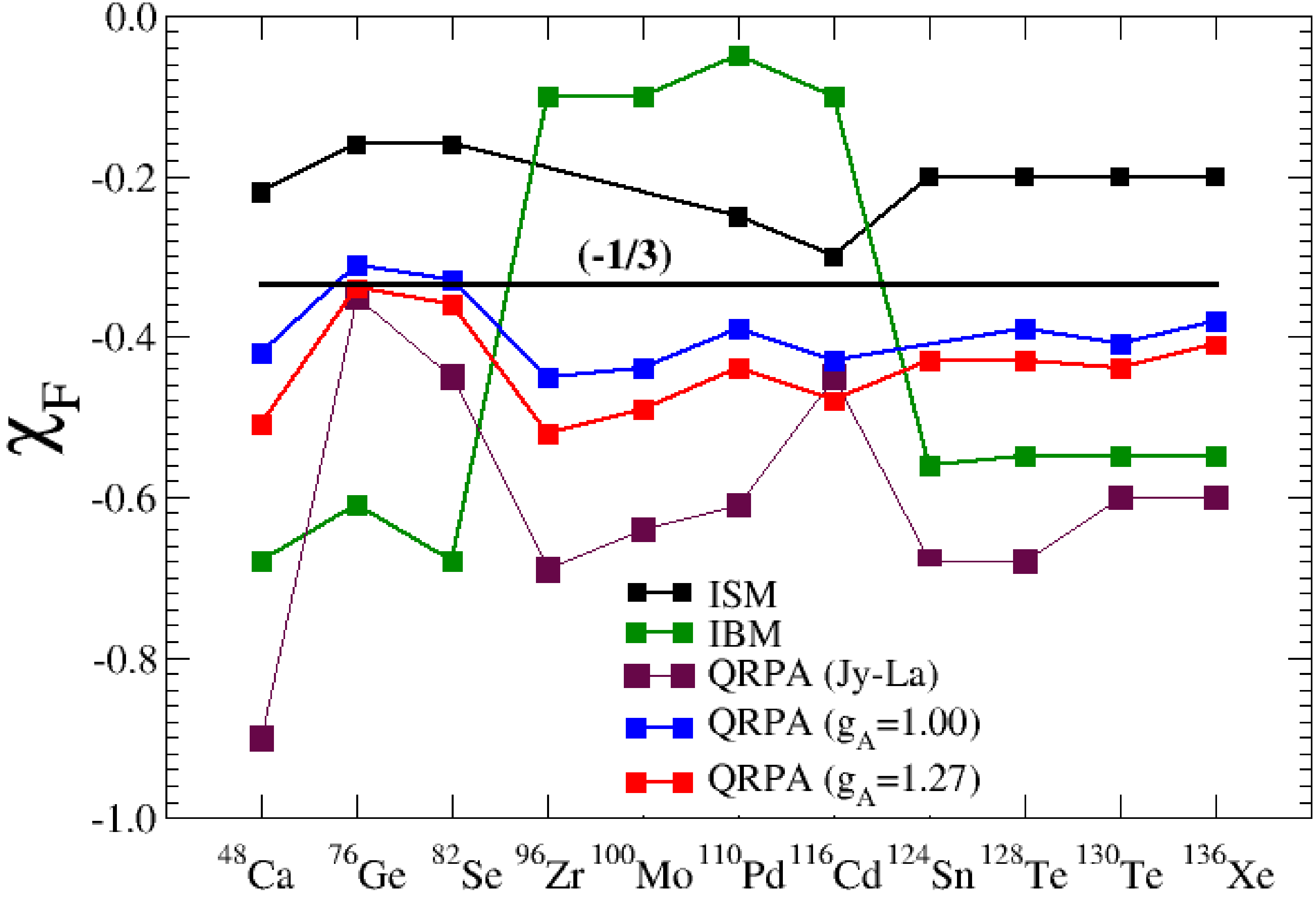
EDF: Rodrigez, Martinez-Pinedo, PRL (2010) 105

PHFB: K. Rath et al., PRC 85 (2012) 014308

$\chi_F = M^{0\nu}_F / M^{0\nu}_{GT} \approx -1/3$

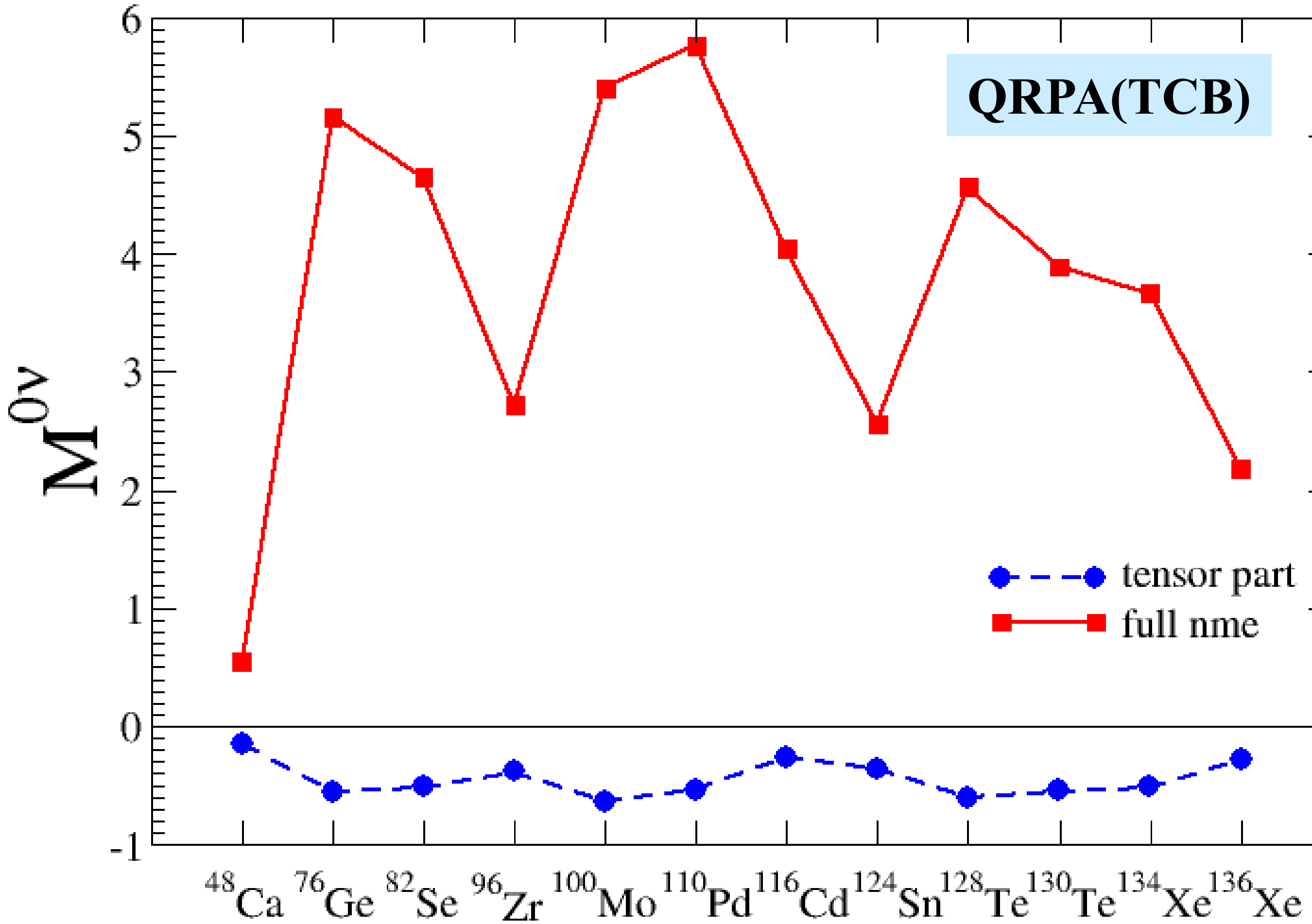
Fermi : $1 = \Omega(S=0) + \Omega(S=1)$

Gamow-Teller: $\sigma.\sigma = -3\,\Omega(S=0) + \Omega(S=1)$



Tensor part of the $0\nu\beta\beta$ NME

(some disagreement)

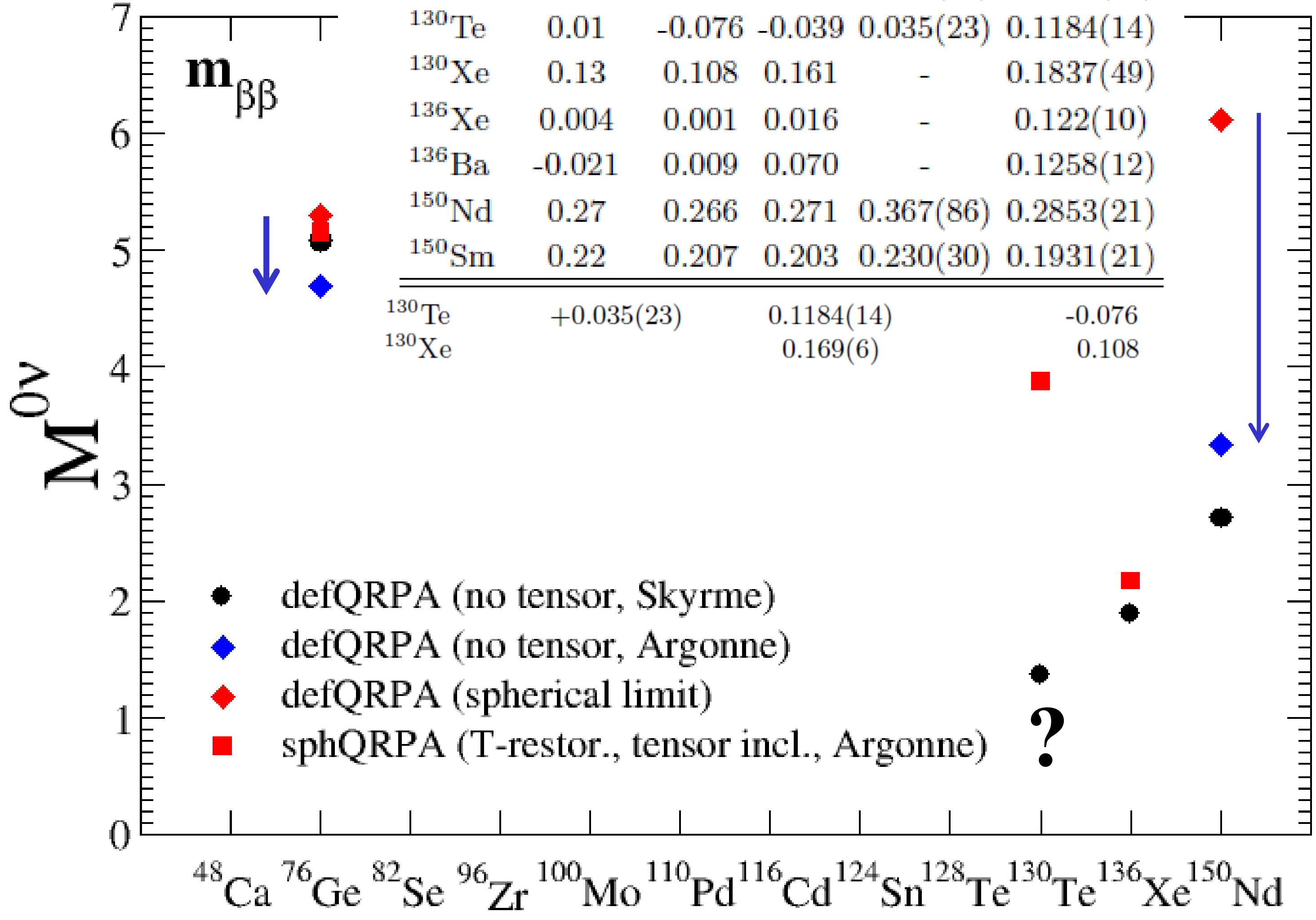


ISM: effect is small, QRPA(J): negligable; PHFB, EDF: not calculated; QRPA(TBC), IBM: up to 10%

Deformed QRPA

	this work	Ref. [24]		Exp.	
		Sk3	SG2	Ref. [25]	Ref. [26]
^{76}Ge	0.184[a]	0.161	0.157	0.095(30)	0.2623(9)
^{76}Se	-0.018	-0.181	-0.191	0.163(33)	0.3090(37)
^{130}Te	0.01	-0.076	-0.039	0.035(23)	0.1184(14)
^{130}Xe	0.13	0.108	0.161	-	0.1837(49)
^{136}Xe	0.004	0.001	0.016	-	0.122(10)
^{136}Ba	-0.021	0.009	0.070	-	0.1258(12)
^{150}Nd	0.27	0.266	0.271	0.367(86)	0.2853(21)
^{150}Sm	0.22	0.207	0.203	0.230(30)	0.1931(21)

^{130}Te	+0.035(23)	0.1184(14)	-0.076
^{130}Xe		0.169(6)	0.108



Skyrme int: Mustonen, Engel, PRC 87 (2013) 064302
Argonn int: Fang, Faessler, Rodin, F.Š. , PRC 83 (2011) 034320

Anatomy of the 0$\nu\beta\beta$-decay NMEs

The 0νββ-decay NME (light ν exchange mech.)

The 0νββ-decay half-life

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 \ ,$$

NME= sum of Fermi, Gamow-Teller and tensor contributions

$$M'^{0\nu} = \left(\frac{g_A}{1.25}\right)^2 \ \langle f| - \frac{M_F^{0\nu}}{g_A^2} + M_{GT}^{0\nu} + M_T^{0\nu}|i\rangle$$

Neutrino potential (about $1/r_{12}$)

$$H_K(r_{12}) = \frac{2}{\pi g_A^2} R \int_0^\infty \ f_K(qr_{12}) \ \frac{h_K(q^2) q dq}{q + E^m - (E_i + E_f)/2}$$

$$f_{F,GT}(qr_{12}) = j_0(qr_{12}), \qquad f_T(qr_{12}) = -j_2(qr_{12})$$

Form-factors: finite nucleon size

$$h_F = g_V^2(q^2)$$

$$h_{GT} = g_A^2 \left[1 - \frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} + \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right]$$

$$h_T = g_A^2 \left[\frac{2}{3}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2} - \frac{1}{3}\left(\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\right)^2\right]$$

Induced pseudoscalar coupling (pion exchange)

$$M_{K=F,GT,T} = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \ \sum_{pnp'n'} (-1)^{j_n+j_{p'}+J+\mathcal{J}} \sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\}$$

$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J}\rangle$$

$$\times \langle 0_f^+ || [c_{p'}^+ \tilde{c}_{n'}]_J || J^\pi k_f\rangle \langle J^\pi k_f | J^\pi k_i\rangle \langle J^\pi k_f || [c_p^+ \tilde{c}_n]_J || 0_i^+\rangle$$

Jastrow f. s.r.c.

J^π = $0^+, 1^+, 2^+$... $0^-, 1^-, 2^-$...

List of reasons, why QRPA-like $0\nu\beta\beta$–decay NME are different

Quasiparticle mean field
fixing of pp,nn (pn) pairing

Many-body approximations
QRPA, RQRPA, SRQRPA

Choice of NN interaction
Schem., realistic (Bonn, Paris …

the closure approximation

p-h interaction ($g_{ph} \cong 1$)
fixed to GT resonance

The size of model space

p-p interaction (g_{pp})
fixed to $2\nu\beta\beta$–decay

two-nucleon s.r.c. (~ 10-20%)

finite size of nucleon (~10%)
form factors

h.o.t. of nucleon curr. (~30%)
Induced PS, weak magnetism

the overlap factor
the BCS overlap

the axial-vector coupling
g_A=1.0 or 1.25

Nuclear shape
Spherical - deformed

2νββ-decay in the QRPA

$$M_{GT}^{2\nu} = \sum_m \frac{<0_f^+||\tau^+\sigma||1_m^+><1_m^+||\tau^+\sigma||0_i^+>}{E_m - E_i + \Delta}$$

Shell model

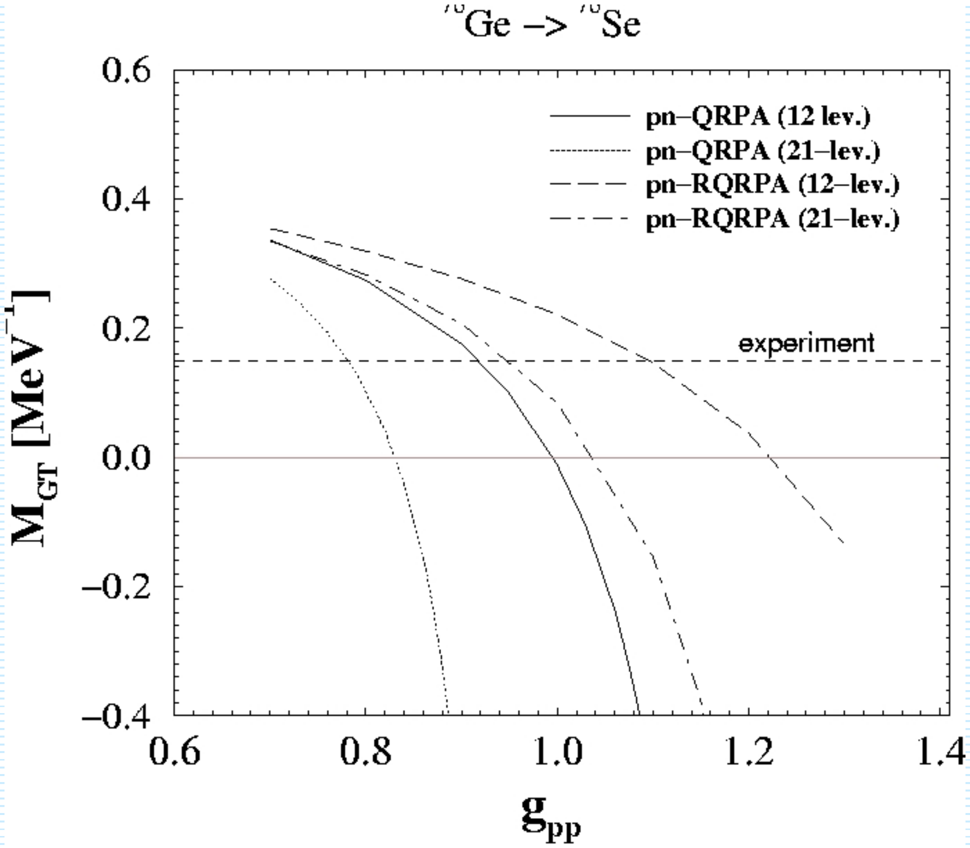
Small model space, effective w.f. and operators

Isotope	$T_{1/2}$(th.)[y]	$T_{1/2}$(exp.)[y]
^{48}Ca	$3.7\ 10^{19}$	$4.2\ 10^{19}$
^{76}Ge	$1.2\ 10^{21}$	$1.4\ 10^{21}$
^{82}Se	$3.4\ 10^{19}$	$9.0\ 10^{19}$
^{130}Te	$1.3\ 10^{20}$	$6.1\ 10^{20}$
^{136}Xe	$1.9\ 10^{20}$	$8.1\ 10^{20}$

Strasbourg group, SuperNEMO meeting, Dec. 2003

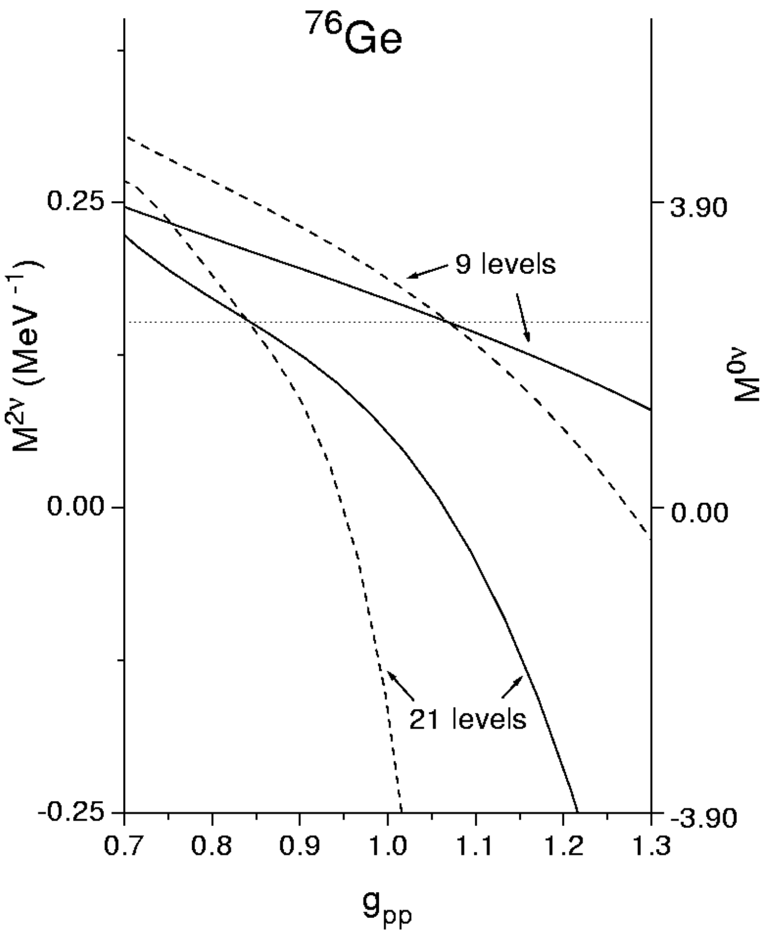
QRPA

$$H = H_0 + g_{ph} H_{ph} + g_{pp} H_{pp}$$

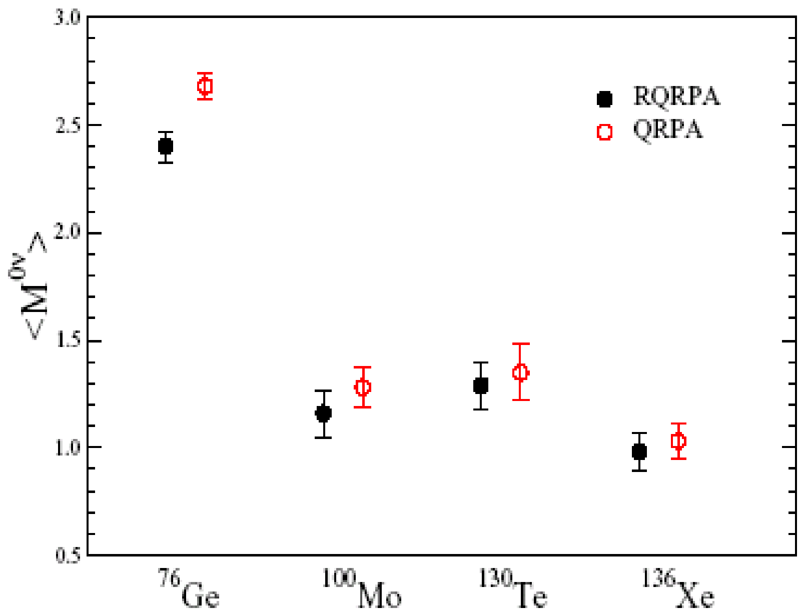


The 0νββ-decay NME: g_{pp} fixed to 2νββ-decay

Each point: (3 basis sets) x (3 forces) = 9 values

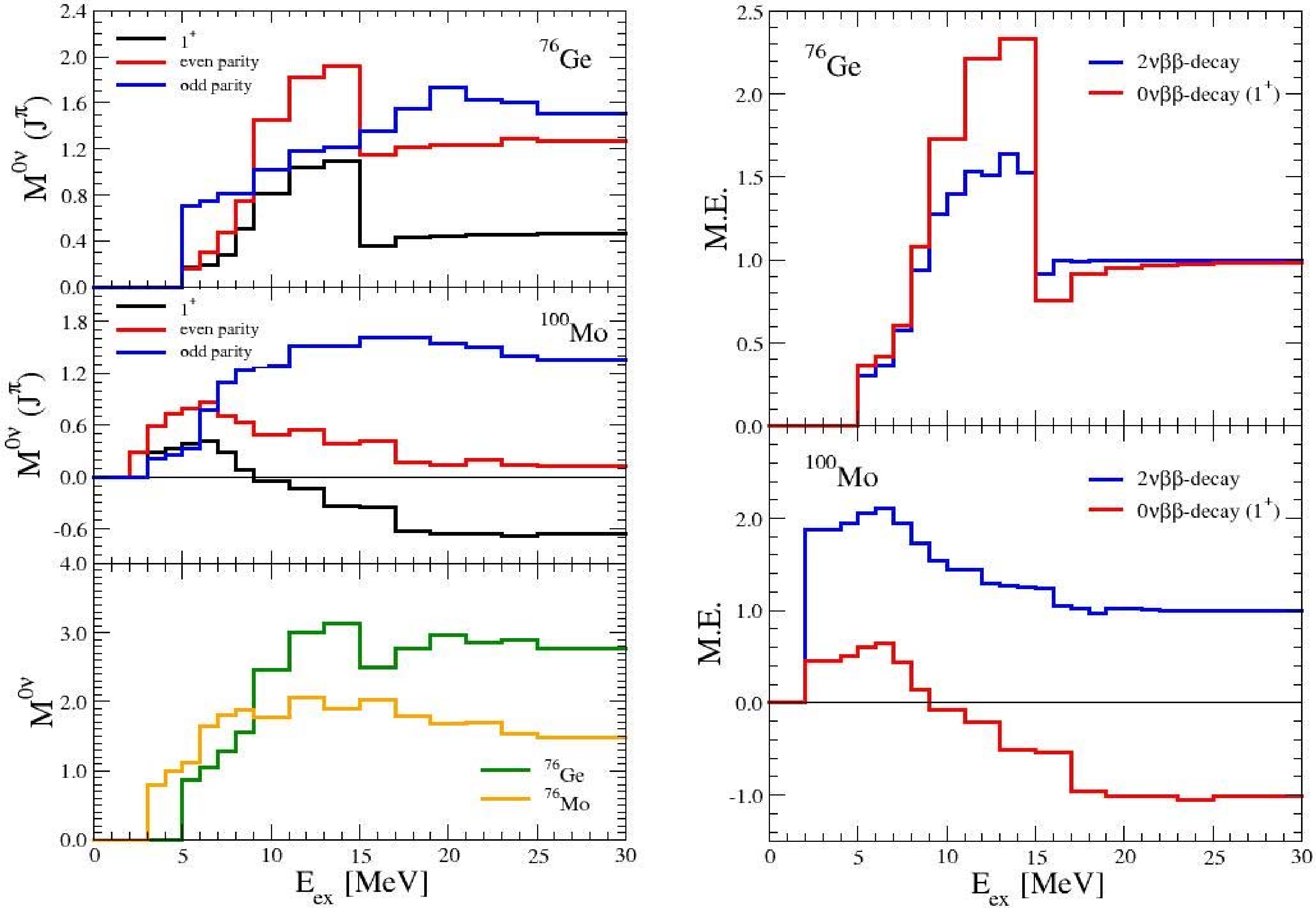


By adjusting of g_{pp} to 2νββ-decay half-life the dependence of the 0νββ-decay NME on other things that are not a priori fixed is essentially removed



Rodin, Faessler, Šimkovic, Vogel, Phys. Rev. C 68, 044302 (2003)

The importance of transition through higher-lying states of (A,Z+1) nucleus



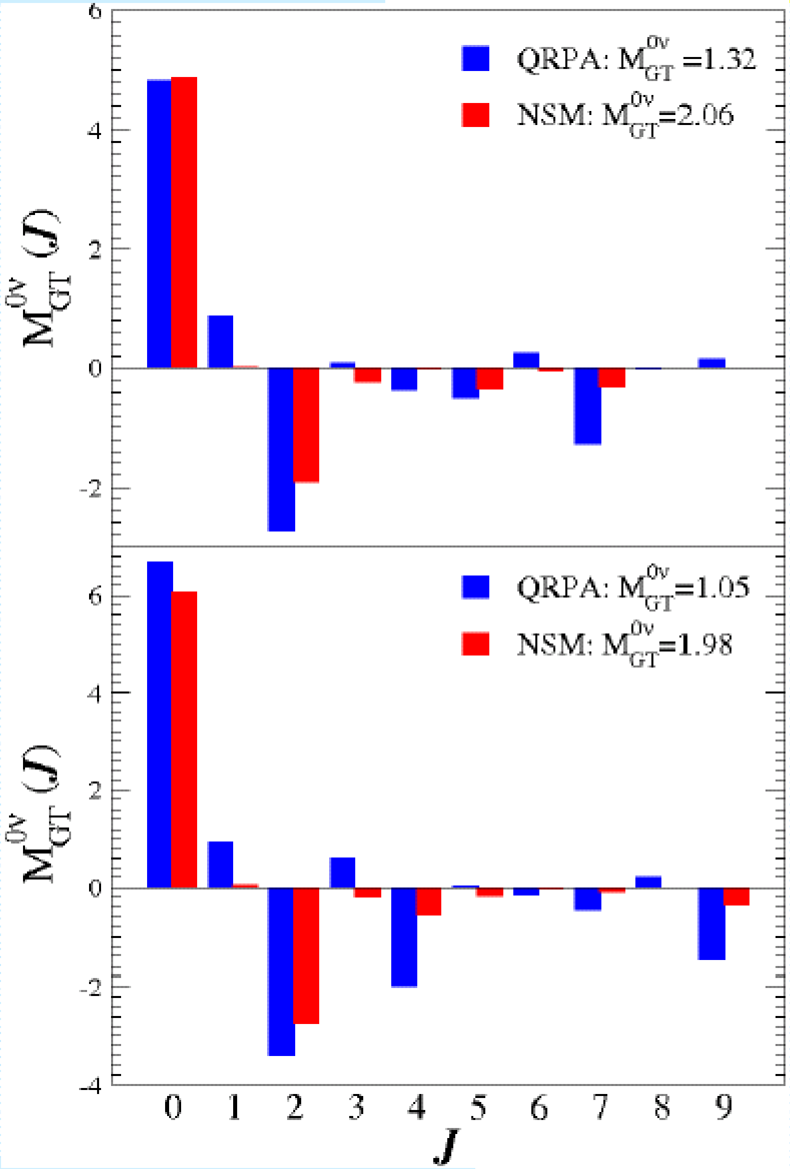
Rodin, Faessler, Šimkovic, Vogel, nucl-th/0503063

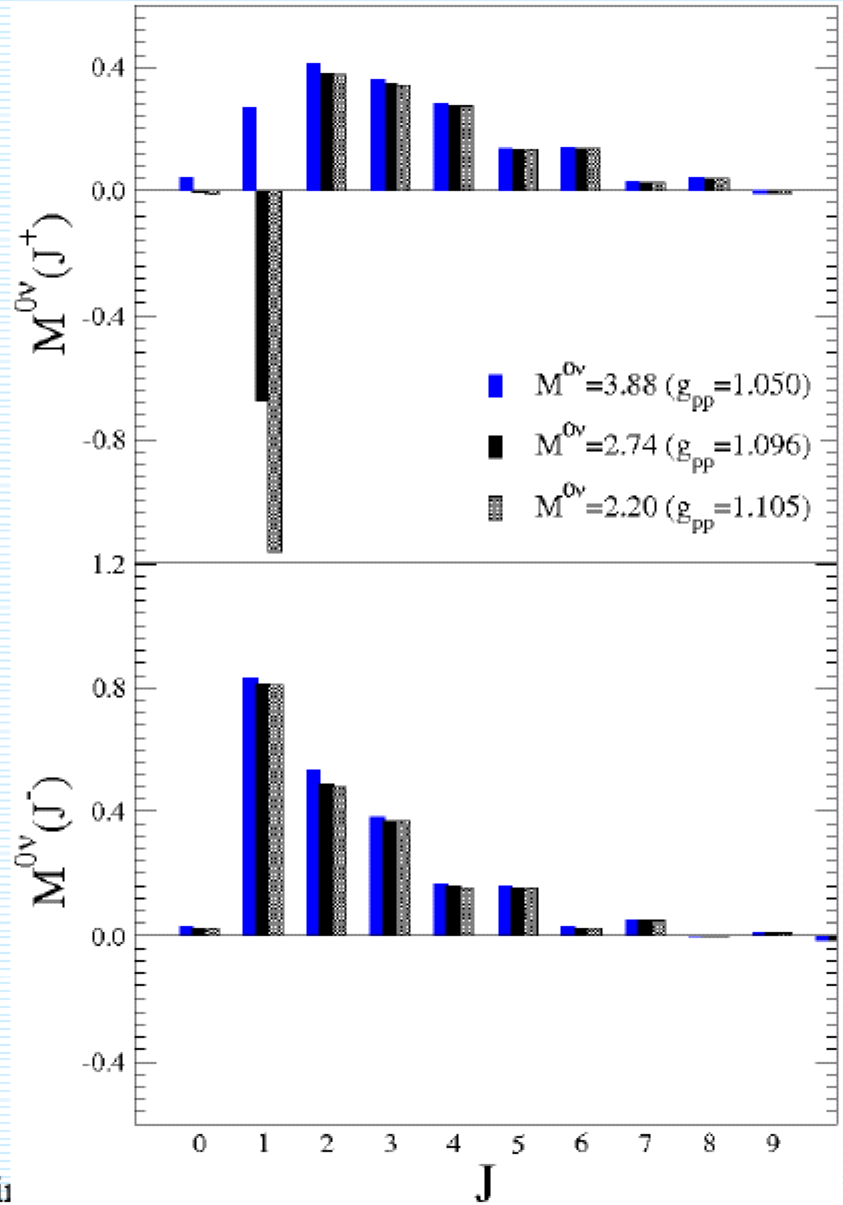
Dominance of Pairing mode (J=0)

Two types of decompositions (Particle-particle) and (particle-hole)

Sensitivity to g_{pp} of 1^+ state

$\langle p(1), p'(2); \mathcal{J} \,||\, f(r_{12}) O_K f(r_{12}) \,||\, n(1), n'(2); \mathcal{J} \rangle$



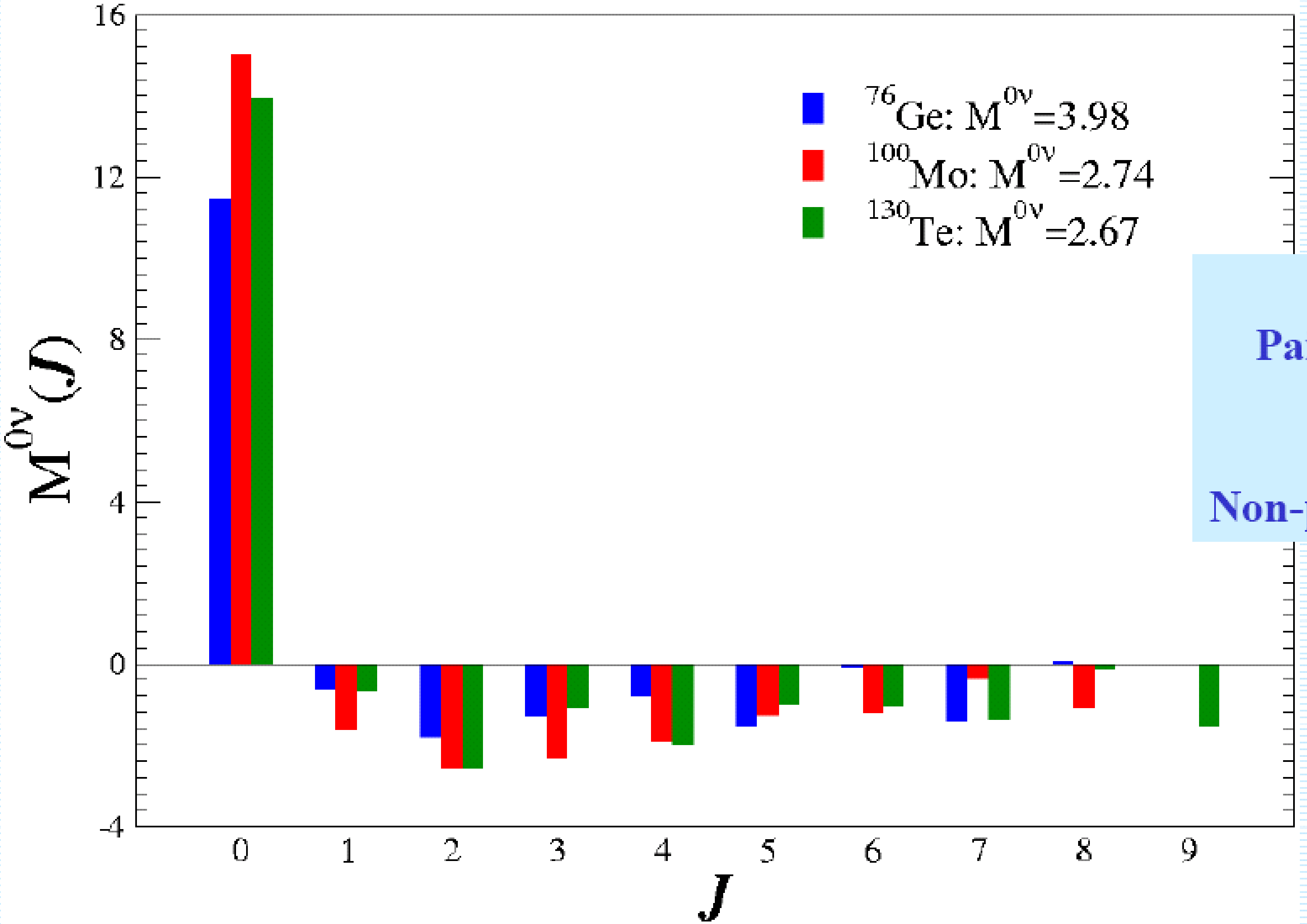


$\langle 0_f^+ || [\widetilde{c_{p'}^+ \tilde{c}_{n'}}]_{\mathcal{J}} || J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_f || [c_p^+ \tilde{c}_n]_{\mathcal{J}} || 0_i^+ \rangle$

A comparison with NSM for the same model space

Decomposition in in pp and nn channels

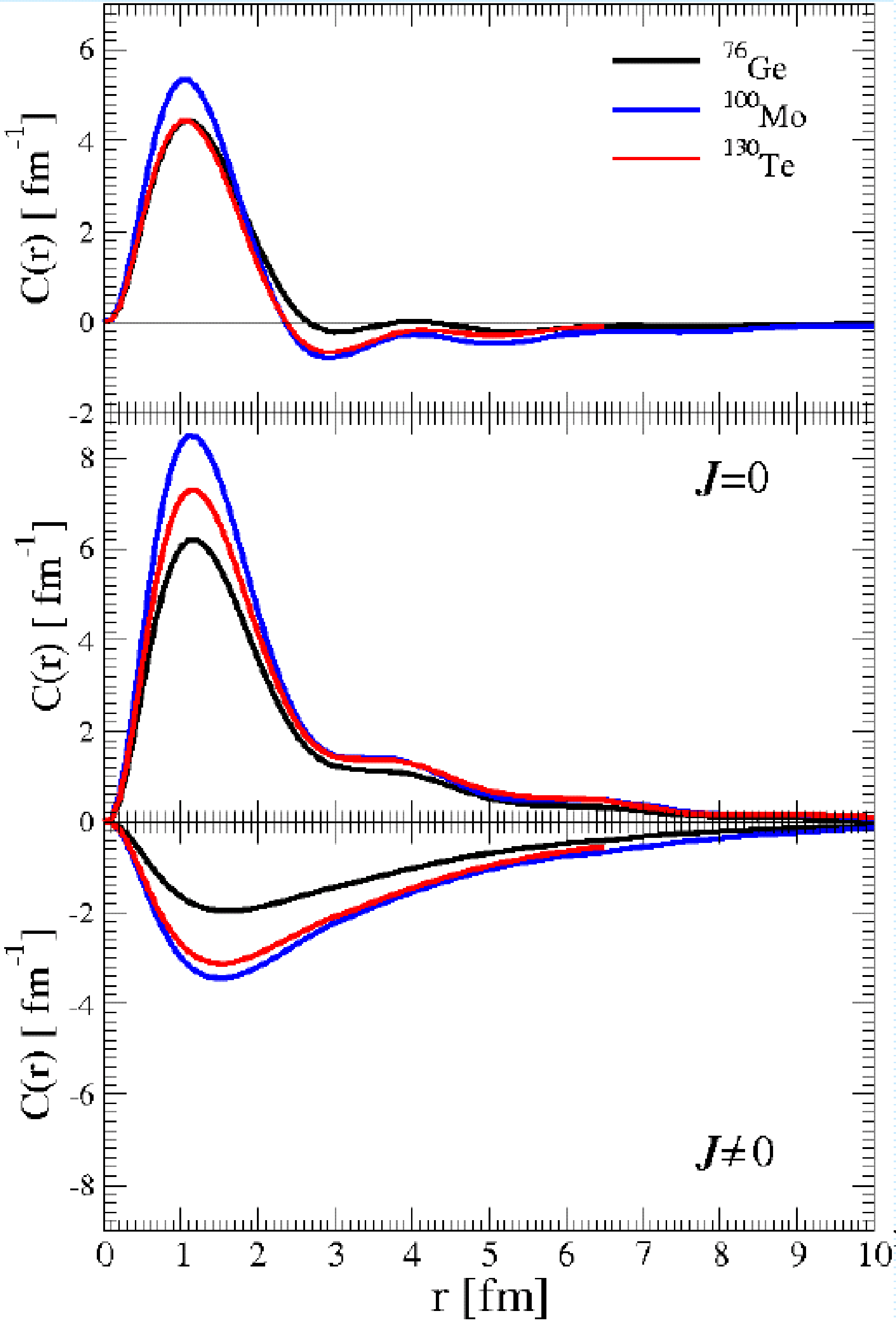
$$\langle p(1), p'(2); \mathcal{J} \parallel f(r_{12}) O_K f(r_{12}) \parallel n(1), n'(2); \mathcal{J} \rangle$$



J=0
Pairing mode

J≠0
Non-pairing mode

F. Š., A. Faessler V.Rodin,, P. Vogel, J. Engel, arXiv 0710.2055 (2007)



nkovic

r-dependence of the 0νββ-decay NME

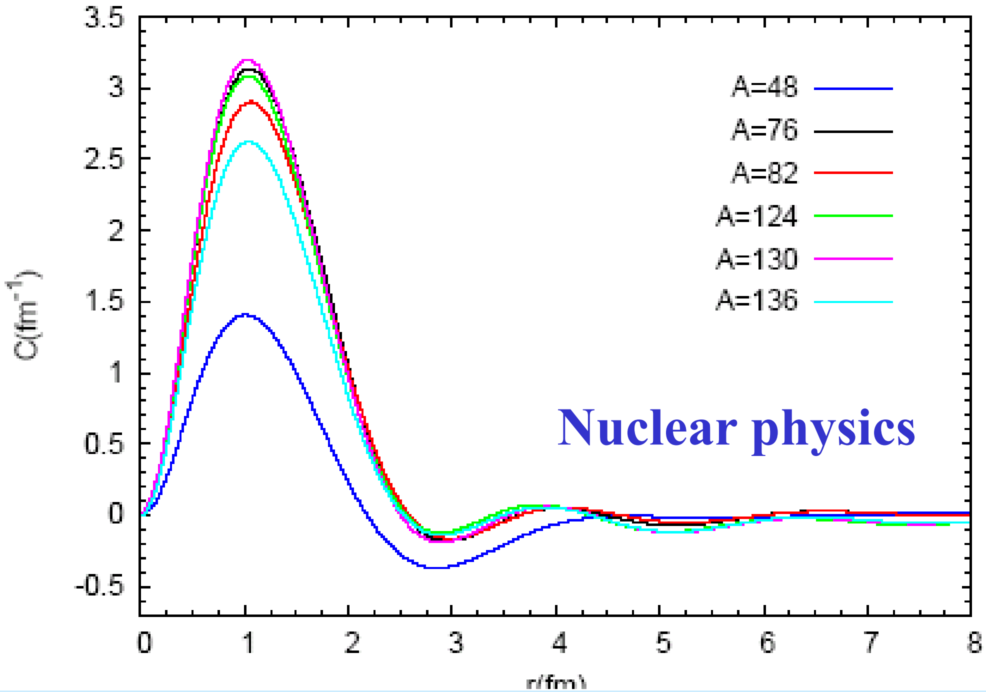
The radial dependence of M^{0n} for the three indicated nuclei. The contributions summed over all components shown in the upper panel. The `pairing' $J = 0$ and `broken pairs' $J \neq 0$ parts are shown separately below. Note that these two parts essentially cancel each other for r > 2-3 fm. This is a generic behavior. Hence the treatment of small values of *r* and large values of *q* are quite important.

QRPA

F.Š, Faessler, Rodin, Vogel, Engel
PRC 77, 045503 (2008)

Large Scale Shell Model

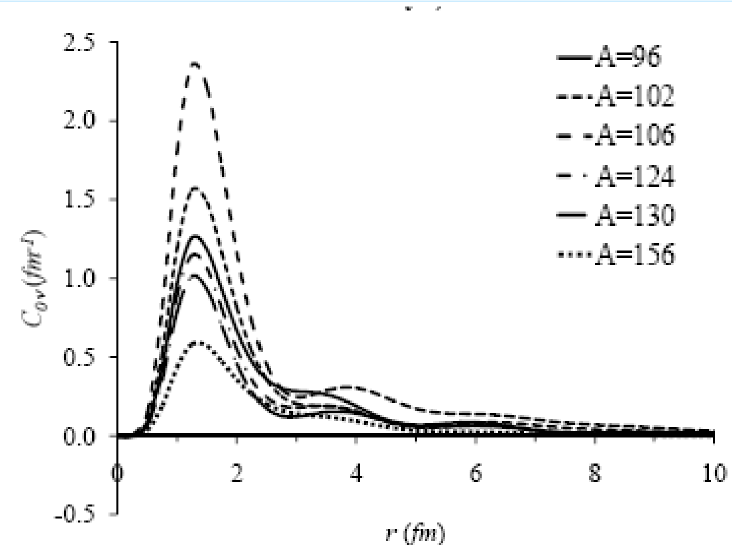
Menendez, Poves, Caurier, Nowacki,
Arxive:0901.3760 [nucl-th]



Nucleon physics

PHFB

P.Rath, R. Chandra, K. Chaturverdi,
P.Raina, J.G. Hirsch,
to be published in PRC



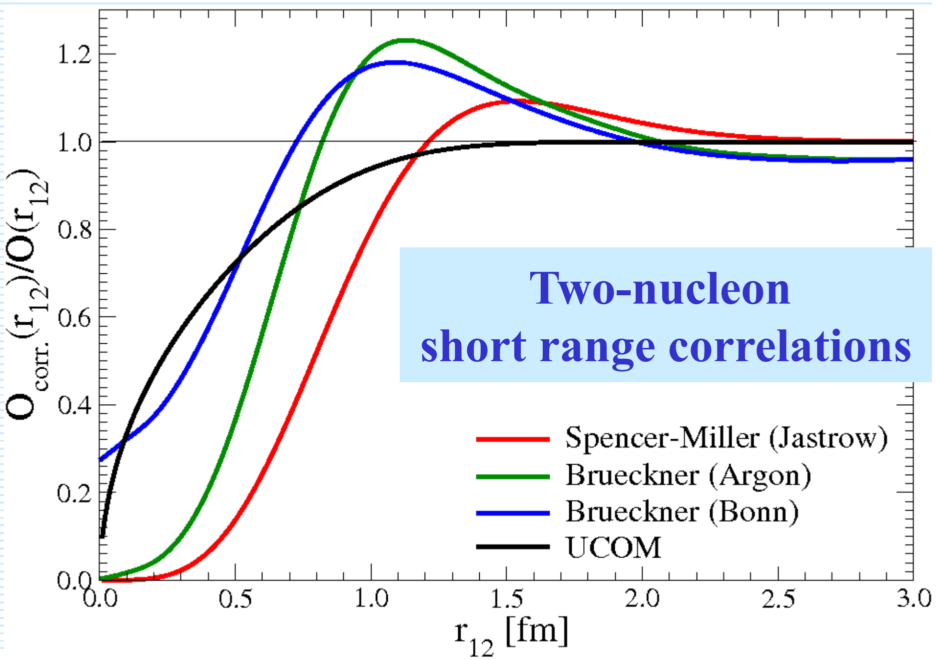
A consistent approach for the 0νββ-decay (pairing, s.r.c, g.s.c. calculated with the same NN potential- BonnCD, Argon)

Neutrino potential: I(r)/r

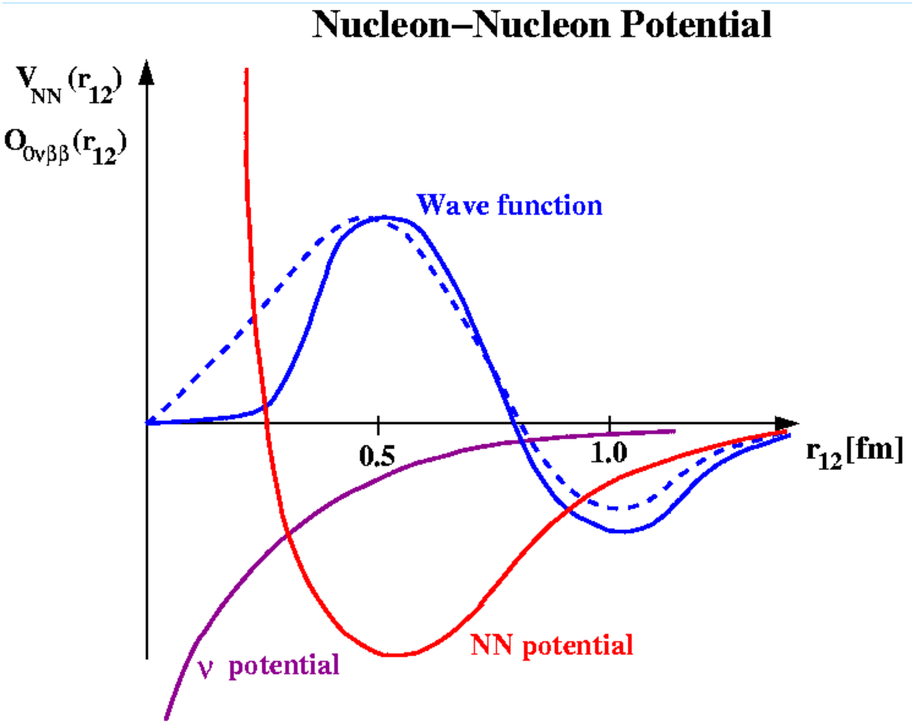
$$I(r) = \frac{2}{\pi}\int_0^\infty \frac{\sin(qr)}{(q+E_{aver})}\frac{dq}{(1+q^2/E_{cut}^2)^4}$$

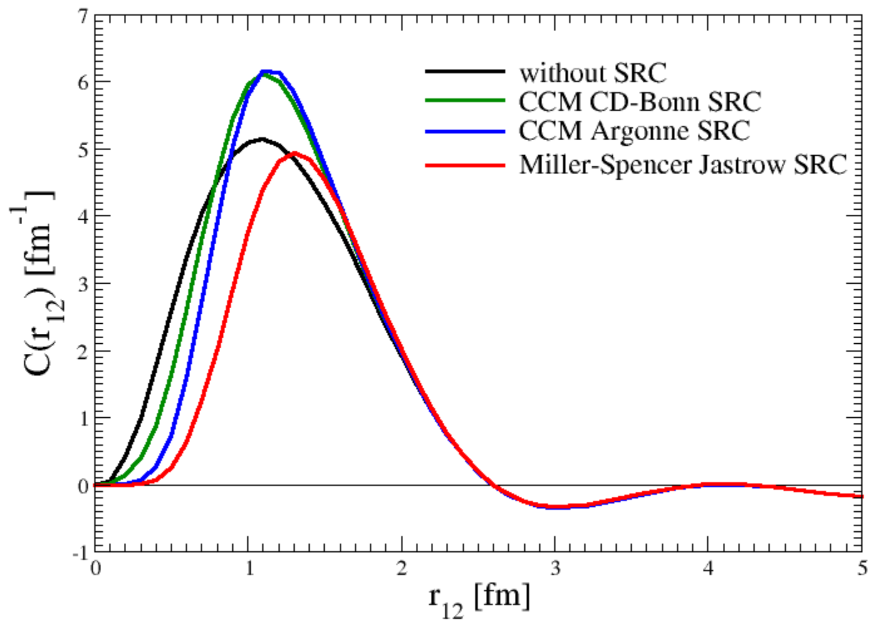
$$|\Psi>_{corr.} = f(r_{12})\,|\Psi>$$

$$O_{corr.}(r_{12}) = f(r_{12})O(r_{12})f(r_{12})$$



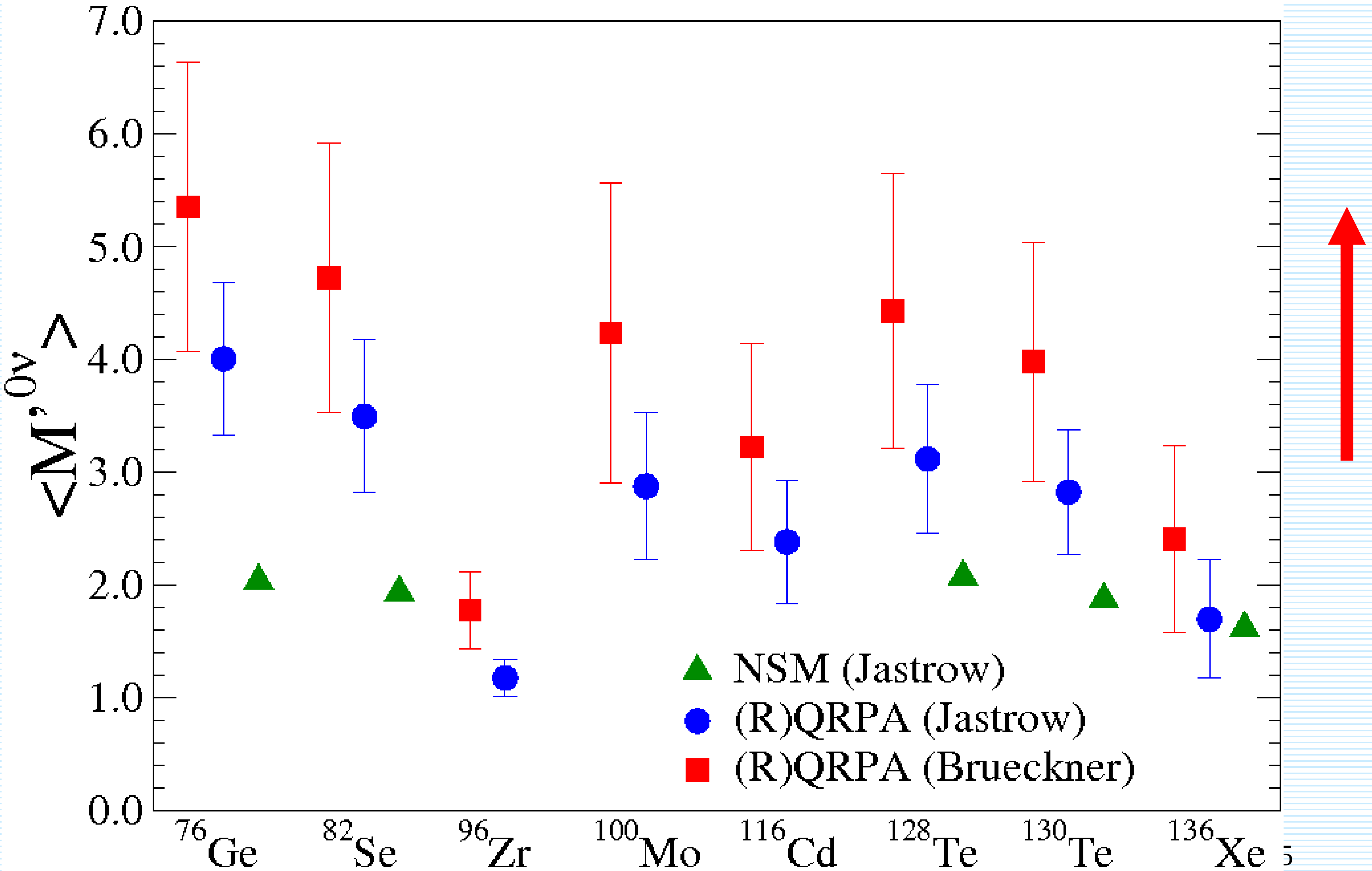
Two-nucleon short range correlations





Neutrinoless double beta decay matrix elements

F.Š., Faessler, Muether, Rodin, Stauf, PRC 79, 055501 (2009)



It is of interest to see the contribution of individual orbits to the 0νββ matrix element. Within QRPA and its generalization this can be done by using the basic formula:

sum over virtual intermediate states

sum over proton and neutron orbits

$$M_K = \sum_{J^\pi, k_i, k_f, \mathcal{J}} \sum_{pnp'n'} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \times$$

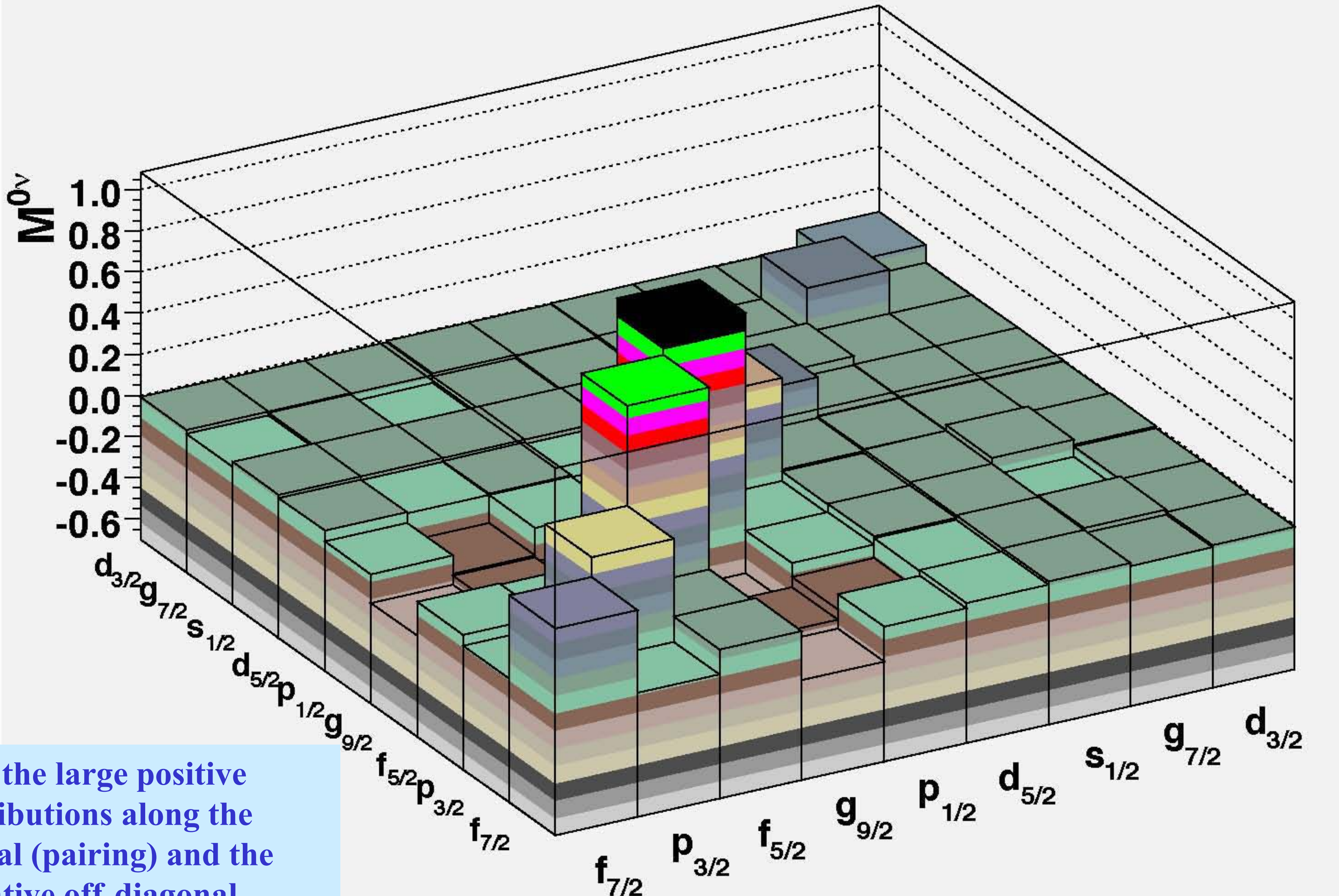
$$\sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\} \times$$

$$\langle p(1), p'(2); \mathcal{J} \parallel \bar{f}(r_{12}) O_K \bar{f}(r_{12}) \parallel n(1), n'(2); \mathcal{J} \rangle \times$$

$$\langle 0_f^+ || [c_{p'}^+ \tilde{c}_{n'}]_J || J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_f i || [c_p^+ \tilde{c}_n]_J || 0_i^+ \rangle$$

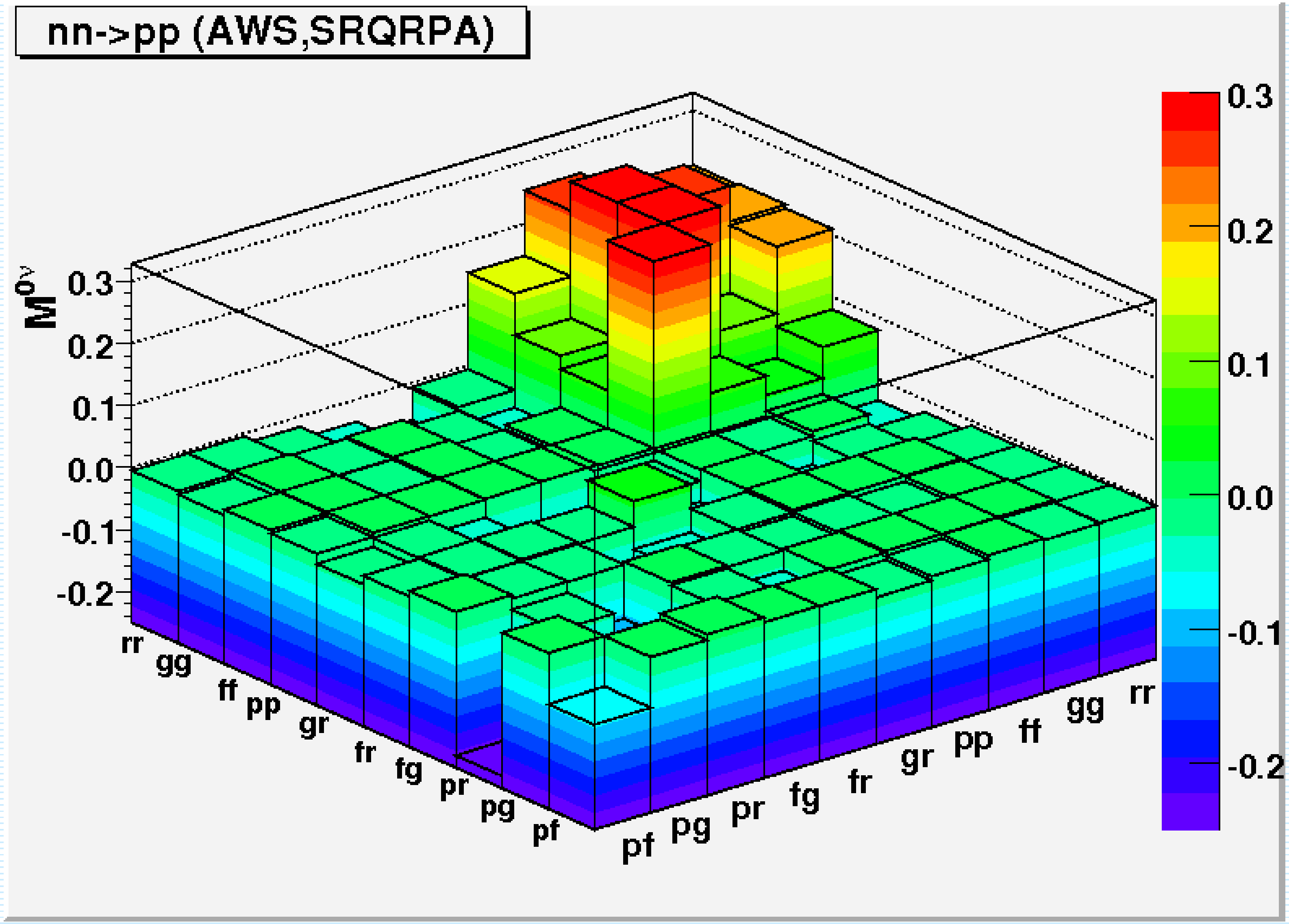
Summing over all indeces except *n,n'* (or *p,p'*) will tell give us the required contribution. Note that it can be positive or negative.

Contribution of individual neutron orbits to $M^{0\nu}$ for ^{76}Ge $0\nu\beta\beta$ decay

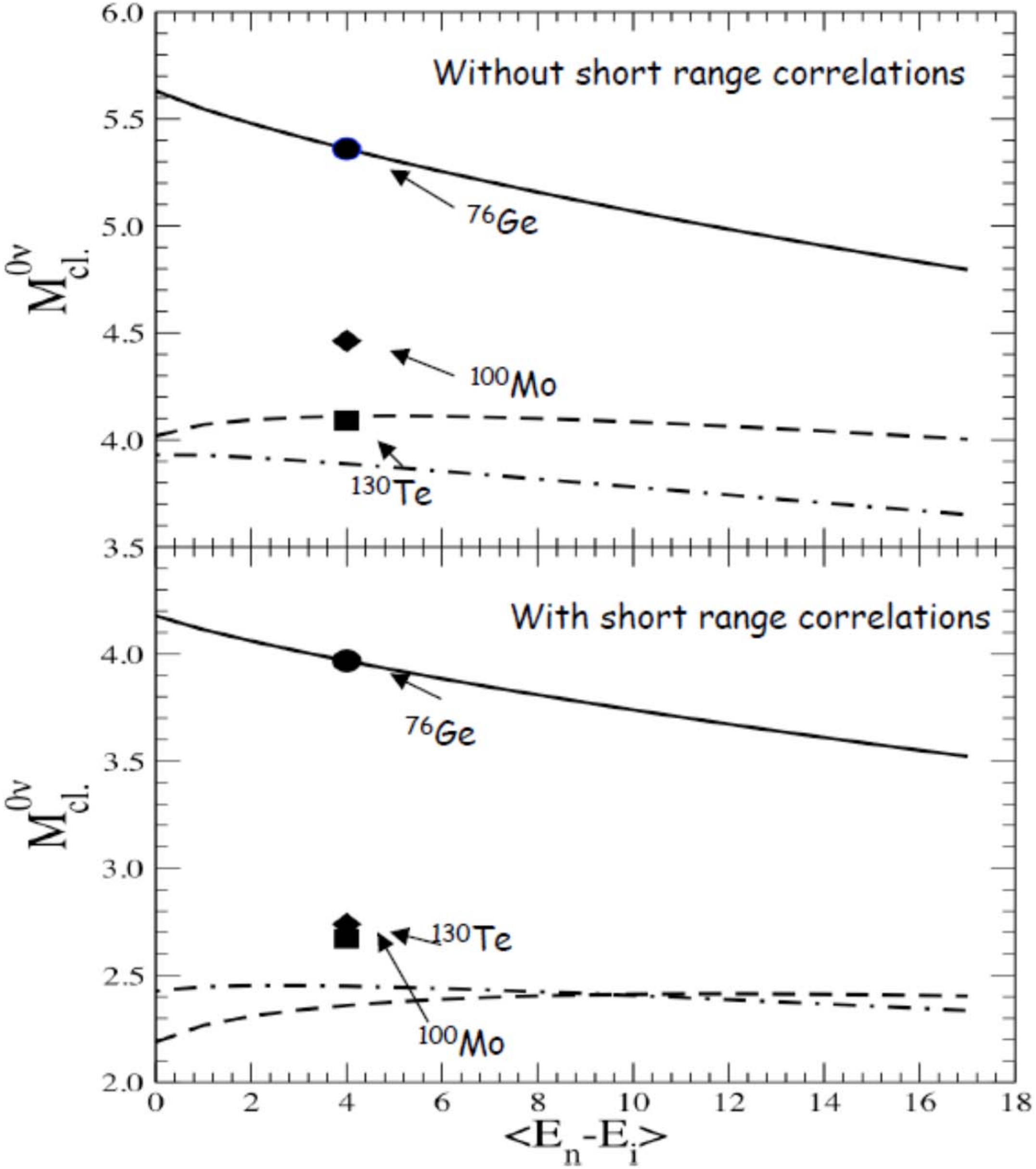


Note the large positive contributions along the diagonal (pairing) and the negative off-diagonal contributions (higher seniority). The valence orbits dominate, but

Contribution of different configurations to $M^{0\nu}$ for ^{76}Ge $0\nu\beta\beta$ decay



Closure approximation



How good is the closure approximation?

Comparison between the QRPA $M^{0\nu}$ with the proper energies of the virtual intermediate states (symbols with arrows) and the closure approximation (lines) with different $\langle E_n - E_i \rangle$.

Note the mild dependence on $\langle E_n - E_i \rangle$ and the fact that the exact results are reasonably close to the closure approximation results for $\langle E_n - E_i \rangle < 20$ MeV.

Graph by F. Simkovic

Constraining the $0\nu\beta\beta$-decay NMEs

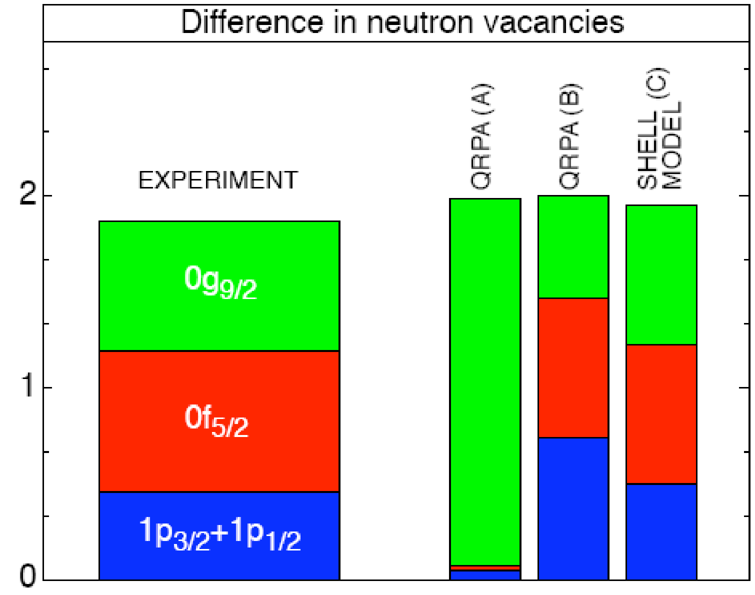
Nucleons that change from neutrons to protons are valence neutrons

Proton, neutron removing transfer reaction

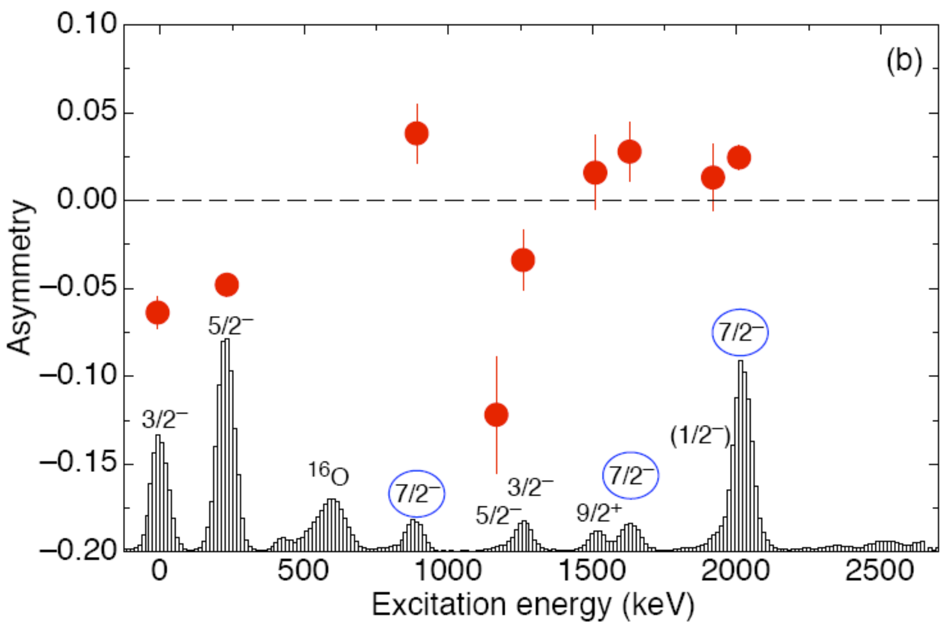
$^{76}Ge \rightarrow {}^{76}Se$

J. Schiffer, B. Kay, P.Grabmayr et al

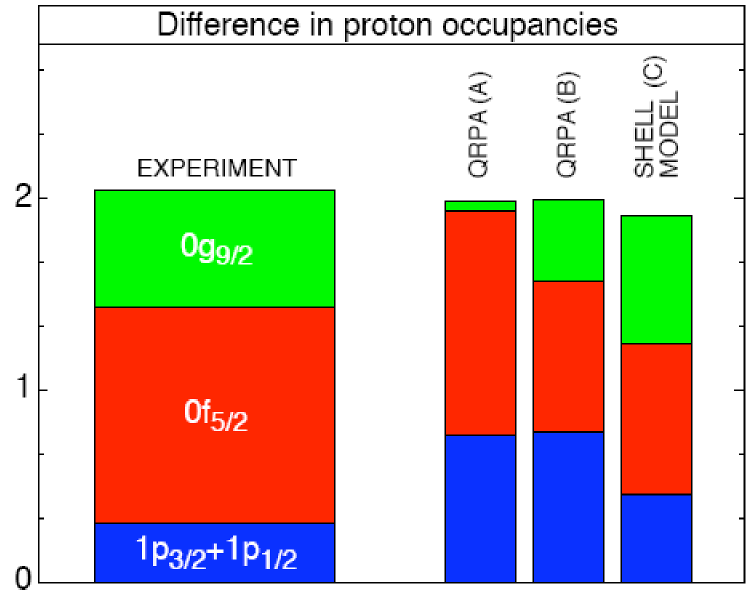
J.P. Schiffer *et al.*, Phys. Rev. Lett. **100**, 112501 (2008)



$$n_j^{exp} = \langle 0^+_{init} | \Sigma_m c^+_{j,m} c_{j,m} | 0^+_{init} \rangle$$



Kay et. Al, PRC 79, 021301 (2009)



QRPA(A) ≡ BCS (WS)

QRPA(B) ≡BCS (AWS) Suhonen, Civitarese, PLB 668, 277 (2008)

ovic

How can we take into account theoretically the constraint represented by the experimentally determined occupancies?

The experiment fixes $n_j^{exp} = \langle 0^+_{init} | \Sigma\, c_{j,m}^+\, c_{j,m}\, | 0^+_{init} \rangle$ and the same for the final nucleus

particle creation and annihilation operators

In BCS $n_j^{BCS} = v_j^2 \times (2j+1)$ depends only on v_j which in turn depends on the mean field eigenenergies

In QRPA the ground state includes correlations and thus

$$n_j^{QRPA} = (2j+1)\times[v_j^2 + (u_j^2 - v_j^2)\xi_j]$$

$\xi_j = (2j+1)^{-1/2} \langle 0^+_{qrpa} | [a^+_j a_j]^0 | 0^+_{qrpa} \rangle$ depends on the quasiparticle content of the correlated ground state

quasiparticle creation and annihilation operators

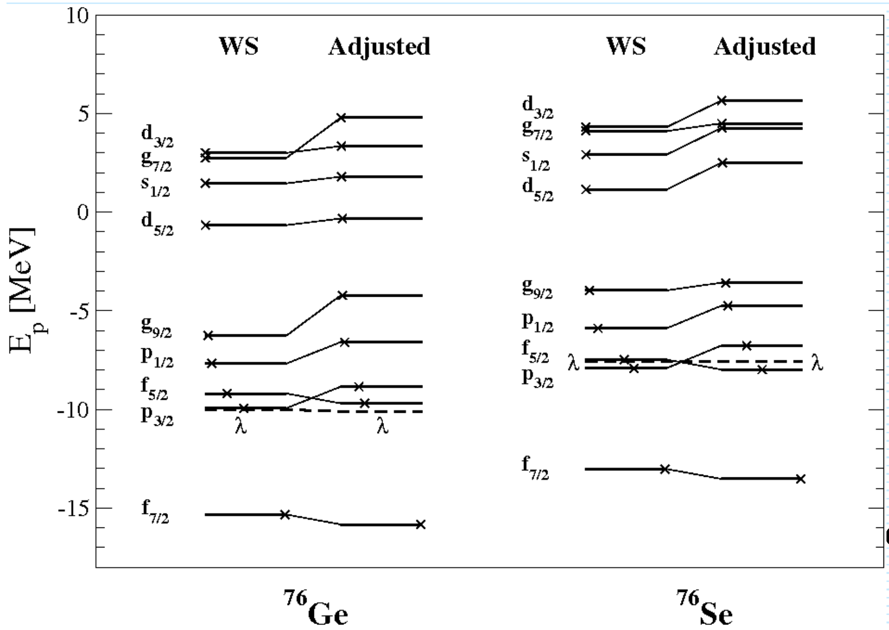
Initial and adjusted mean field levels

While n_j^{exp} and n_j^{BCS} are constrained by $\Sigma n_j = N$ (or Z) the n_j^{QRPA} are not constrained by that requirement. The particle number is not conserved, even on average. Thus the QRPA must be modified to remedy this ⇒ Selfconsistent Renormalized QRPA

$^{76}Ge \rightarrow ^{76}Se$	prev.	new
Jastrow s.r.c.	4.24(0.44)	3.49(0.23)
UCOM s.r.c.	5.19(0.54)	4.60(0.39)

	^{76}Ge				^{76}Se			
neut.	BCS	Q	S	exp	BCS	Q	S	exp
p	5.65	5.27	4.64	4.9±0.2	5.57	5.05	4.12	4.4±0.2
$f_{5/2}$	5.54	5.12	4.34	4.6±0.4	5.53	5.00	3.63	3.8±0.4
$f_{7/2}$	7.91	7.67	7.62	-	7.90	7.54	7.37	-
$s_{1/2}$	0.01	0.05	0.07	-	0.01	0.04	0.08	-
$d_{3/2}$	0.03	0.14	0.15	-	0.02	0.14	0.16	-
$d_{5/2}$	0.09	0.30	0.36	-	0.07	0.27	0.39	-
$g_{7/2}$	0.14	0.53	0.48	-	0.12	0.56	0.58	-
$g_{9/2}$	4.63	4.78	6.35	6.5±0.3	2.78	3.55	5.66	5.8±0.3
prot.								
p	2.23	2.34	1.75	1.77±0.15	2.77	2.76	2.28	2.08±0.15
$f_{5/2}$	1.61	2.27	2.08	2.04±0.25	2.95	2.97	3.03	3.16±0.25
$f_{7/2}$	7.83	7.19	7.13	-	7.76	7.12	7.06	-
$s_{1/2}$	0.00	0.02	0.03	-	0.00	0.03	0.04	-
$d_{3/2}$	0.01	0.07	0.07	-	0.01	0.09	0.09	-
$d_{5/2}$	0.01	0.12	0.15	-	0.02	0.17	0.18	-
$g_{7/2}$	0.02	0.19	0.16	-	0.03	0.31	0.27	-
$g_{9/2}$	0.29	0.85	0.62	0.23±0.25	0.46	1.15	1.04	0.84±0.25

F.Š., A. Faessler, P. Vogel, PRC 79, 015502 (2009)



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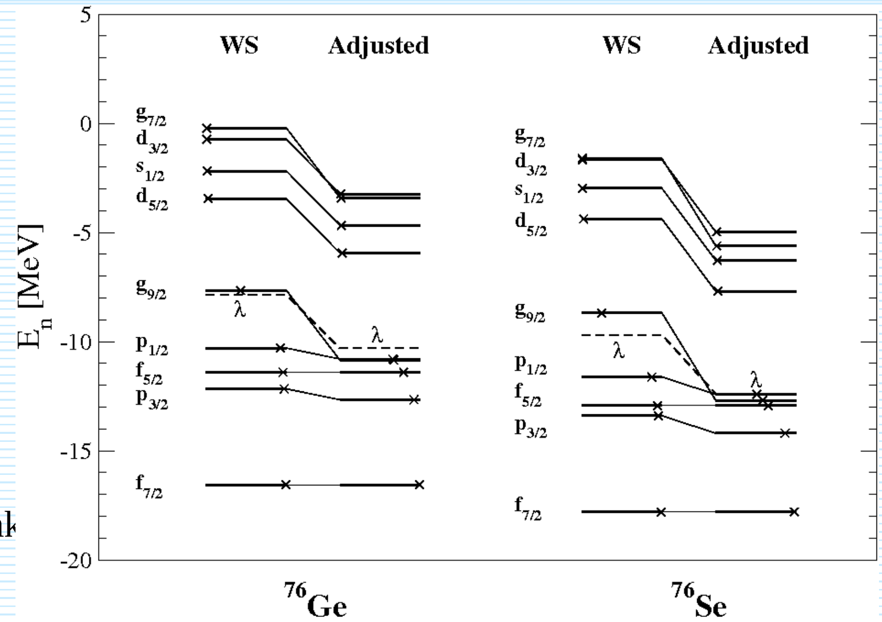
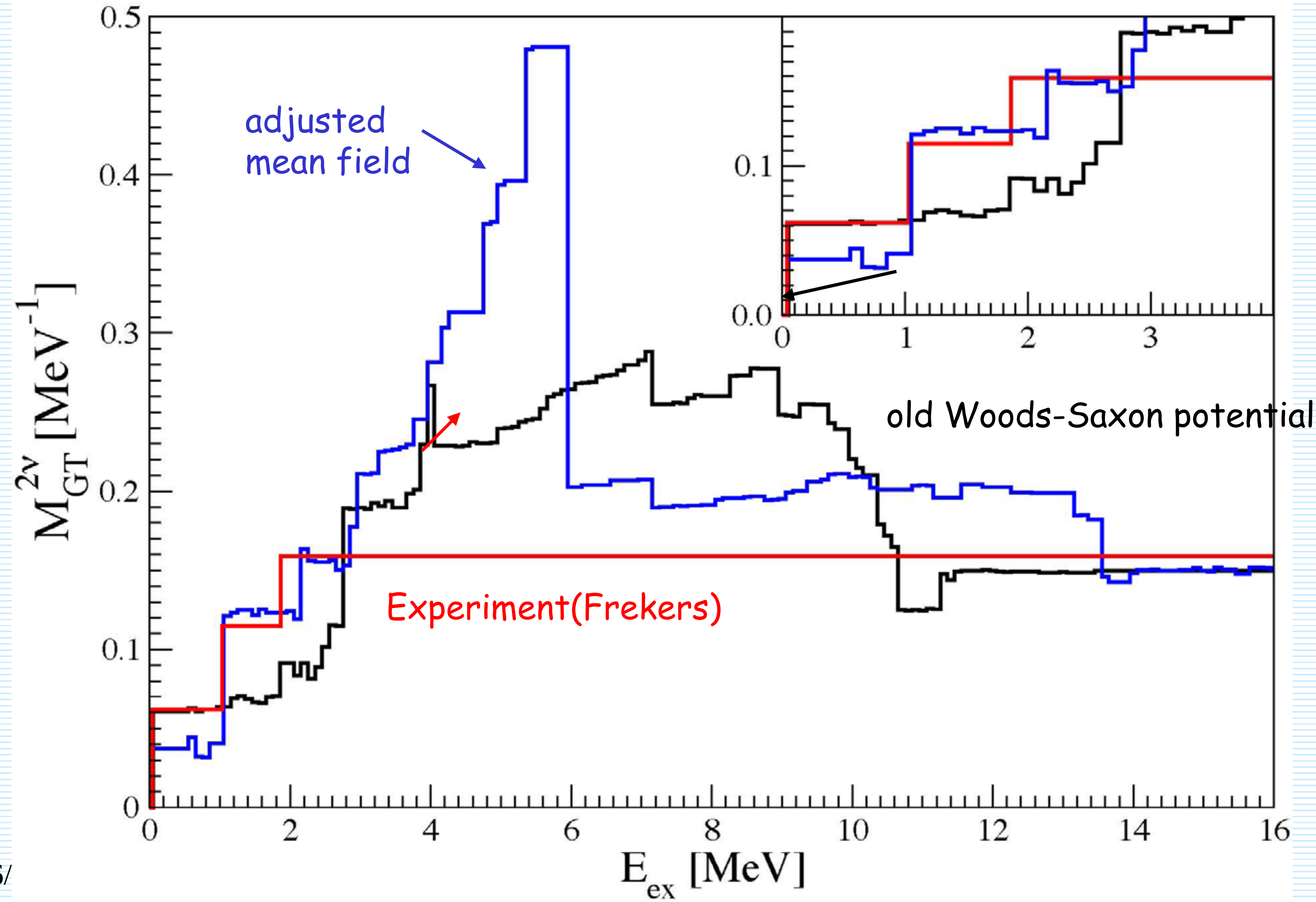
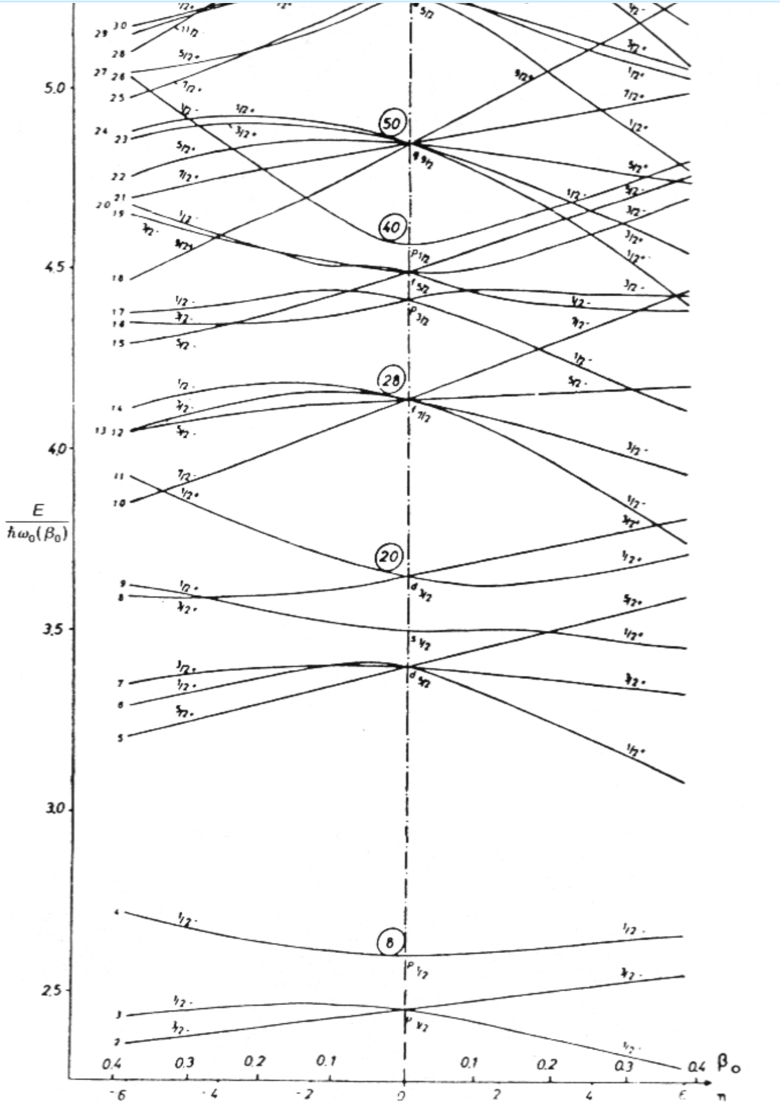


Staircase plot (running sum) of the contributions to the $2\nu\beta\beta$ decay ($^{76}Ge \rightarrow ^{76}Se$)



Shell structure of the mean field changed



Deformation

Anisotropic harmonic oscillator

Nuclear deformation

$$\beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Z r_c^2}$$

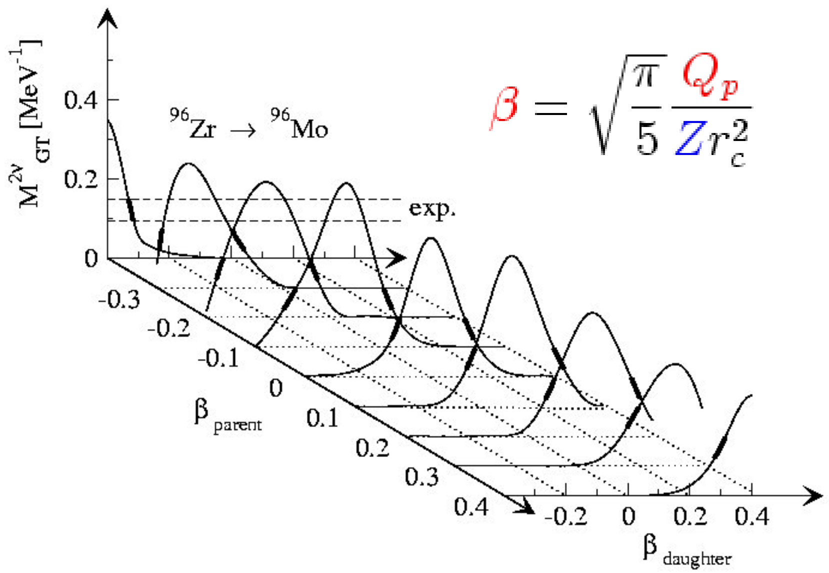
Exp. I **(nuclear reorientation method)**
Exp.II **(based on measured E2 trans.)**
Theor. I **(Rel. mean field theory)**
Theor. II **(Microsc.-Macrosc. Model of Moeller and Nix)**

Till now, in the QRPA-like calculations of the 0νββ-decay NME spherical symetry was assumed

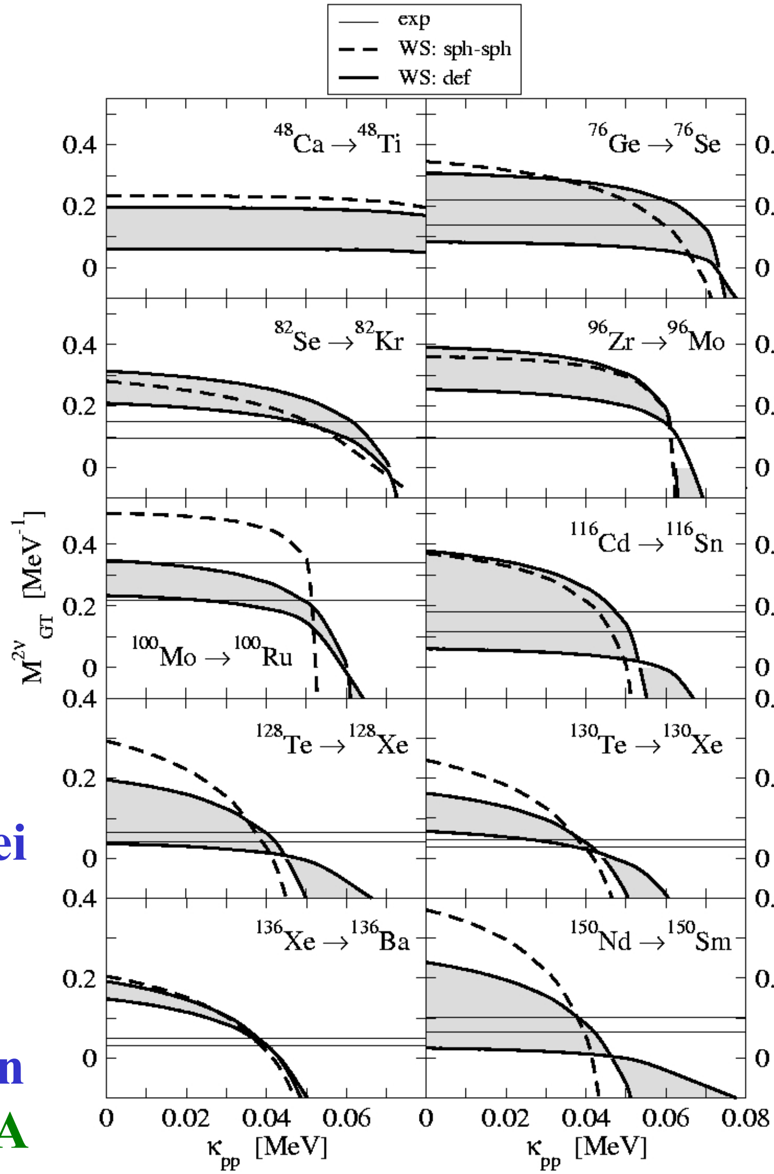
The effect of deformation on NME has to be considered

Nucl.	Exp. I	Exp. II	Theor. I	Theor. II
^{48}Ca	0.00	0.101	0.00	0.00
^{48}Ti	+0.17	0.269	-0.01	0.00
^{76}Ge	+0.09	0.26	0.16	0.14
^{76}Se	+0.16	0.31	-0.24	-0.24
^{82}Se	+0.10	0.19	0.13	0.15
^{82}Kr		0.20	0.12	0.07
^{96}Zr		0.081	0.22	0.22
^{96}Mo	+0.07	0.17	0.17	0.08
^{100}Mo	+0.14	0.23	0.25	0.24
^{100}Ru	+0.14	0.22	0.19	0.16
^{116}Cd	+0.11	0.19	-0.26	-0.24
^{116}Sn	+0.04	0.11	0.00	0.00
^{128}Te	+0.01	0.14	-0.00	0.00
^{128}Xe		0.18	0.16	0.14
^{130}Te	+0.03	0.12	0.03	0.00
^{130}Xe		0.17	0.13	-0.11
^{136}Xe		0.09	0.00	0.00
^{136}Ba		0.12	0.00	0.00
^{150}Nd	+0.37	0.28	0.22	0.24
^{150}Sm	+0.23	0.19	0.18	0.21

New Suppression Mechanism of the DBD NME



$$\beta = \sqrt{\frac{\pi}{5}} \frac{Q_p}{Z r_c^2}$$



The suppression of the NME depends on relative deformation of initial and final nuclei

F.Š., Pacearescu, Faessler.
NPA 733 (2004) 321

Systematic study of the deformation effect on the 2νββ-decay NME within deformed QRPA

Alvarez,Sarriguren, Moya,Pacearescu, Faessler, F.Š., Phys. Rev. C 70 (2004) 321

QRPA with realistic forces in deformed nuclei

M. Saleh Yousef, V. Rodin, A. Faessler, F.Š, PRC 79 (2009) 014314

$$\langle p\rho_p \bar{n}\rho_n | G | p'\rho_{p'} \overline{n'}\rho_{n'} \rangle = \sum_J \sum_{(N_0 lj)_p} \sum_{(N_0 lj)_n} \sum_{(N_0 lj)_{p'}} \sum_{(N_0 lj)_{n'}} B^{(p)}_{(N_0 lj)_p} B^{(n)}_{(N_0 lj)_n} B^{(p')}_{(N_0 lj)_{p'}} B^{(n')}_{(N_0 lj)_{n'}}$$

$$\times (-1)^{j_n - \Omega_n} (-1)^{j_n' - \Omega_n'} C^{JK}_{j_p \Omega_p j_n \Omega_n} C^{JK}_{j_{p'} \Omega_{p'} j_{n'} \Omega_{n'}}$$

$$\times \langle (N_0 lj)_p (N_0 lj)_n, J | G | (N_0 lj)_{p'} (N_0 lj)_{n'}, J \rangle$$

G-matrix elements in
spherical single particle basis
Bonn CD potential

Effect of nuclear deformation on 0νββ-decay

SuperNEMO (SNO+): about 56 kg of ^{150}Nd => 0.1 eV

0νββ-decay of ^{150}Nd in different models with half-lives for $m_{\beta\beta}$*=50 meV*

	QRPA [6] [a]	this work ($\beta_2 = 0$) [b]	this work	pseudo-SU(3) [8]	PHFB [9]	IBM-2 [10]
$M^{0\nu}$	5.17	5.78	**3.16**	1.57	1.61	2.32
$T^{0\nu}_{1/2}$, 10^{25} y ($\langle m_{\beta\beta}\rangle = 50$ meV)	1.72	1.38	**4.60**	18.7	17.7	8.54

[1] Rodin, Faesler, F.Š., Vogel, NPA 766 (2006), spherical QRPA
[2] Fang, Faessler, Rodin, F.Š., to be published in PRC, deformed QRPA
[3] Hirsch et al., NPA 582 (1995), pseudo SU(3) approach
[4] K. Chaturvedi et al, PRC 78 (2008), PHFB approach
[5] Barea and Iachello, PRC 79 (2009), IBM approach

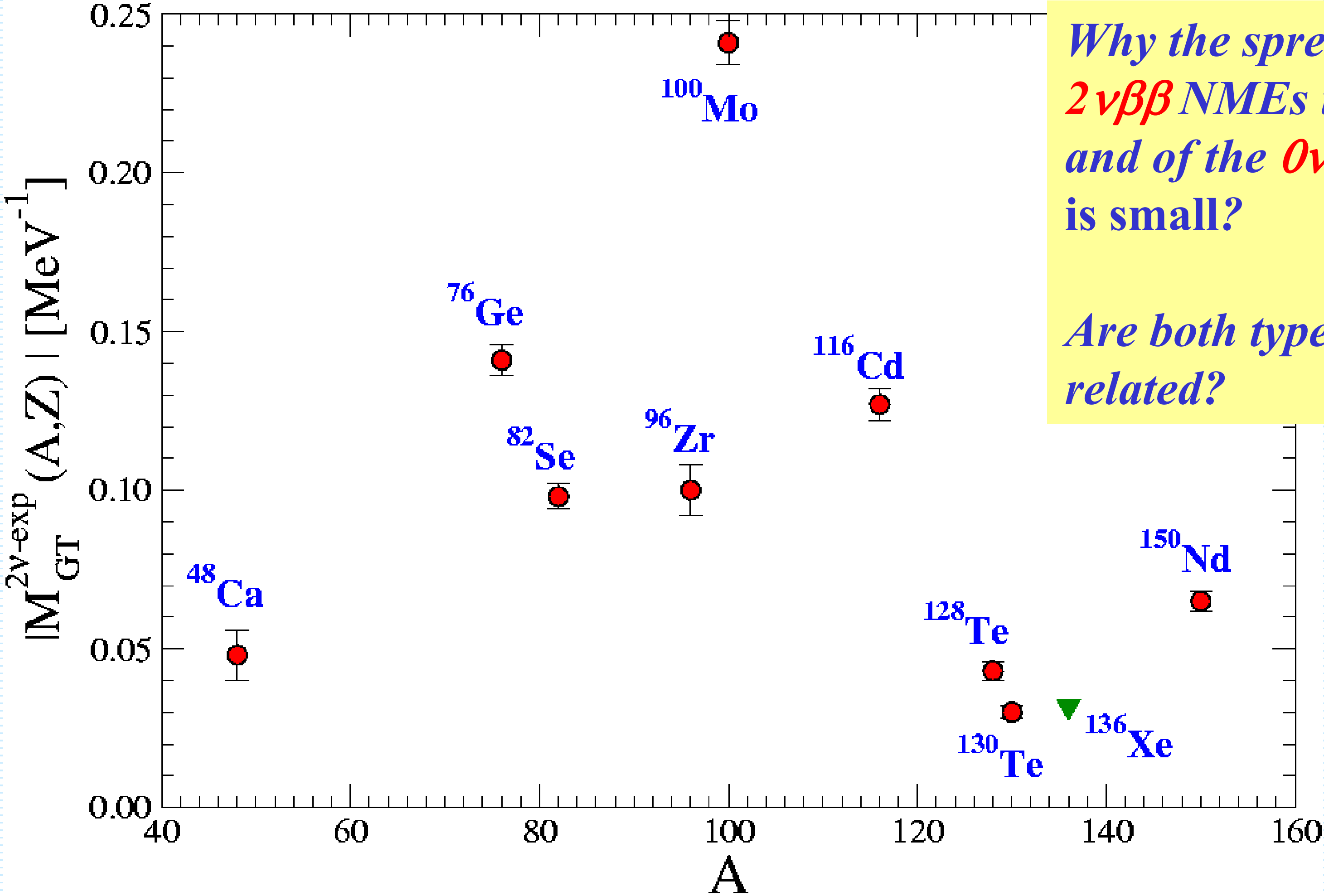
On the relation between 0νββ-decay and 2νββ-decay (GT) NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{0\nu} = M^{0\nu}_{GT}\left(1 + \frac{1}{g_A^2}\frac{M^{0\nu}_F}{M^{0\nu}_{GT}} + \frac{M^{0\nu}_T}{M^{0\nu}_{GT}}\right)$$

2νββ-decay NMEs

$$\frac{1}{T_{1/2}^{2\nu-exp}} = G^{2\nu}(E_0, Z)\ g_A^4\ |M_{GT}^{2\nu}|^2$$



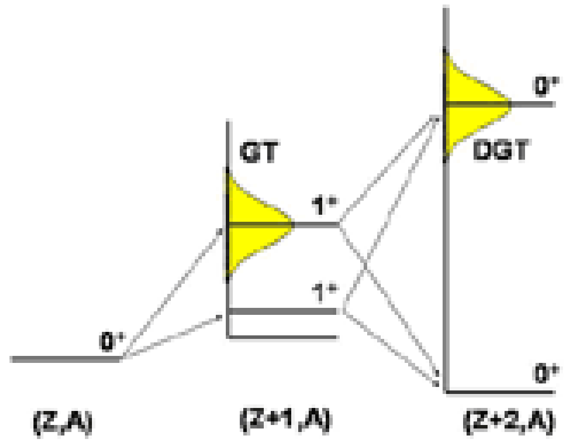
***Why the spread of the 2νββ NMEs is large and of the 0νββ* NMEs is small?**

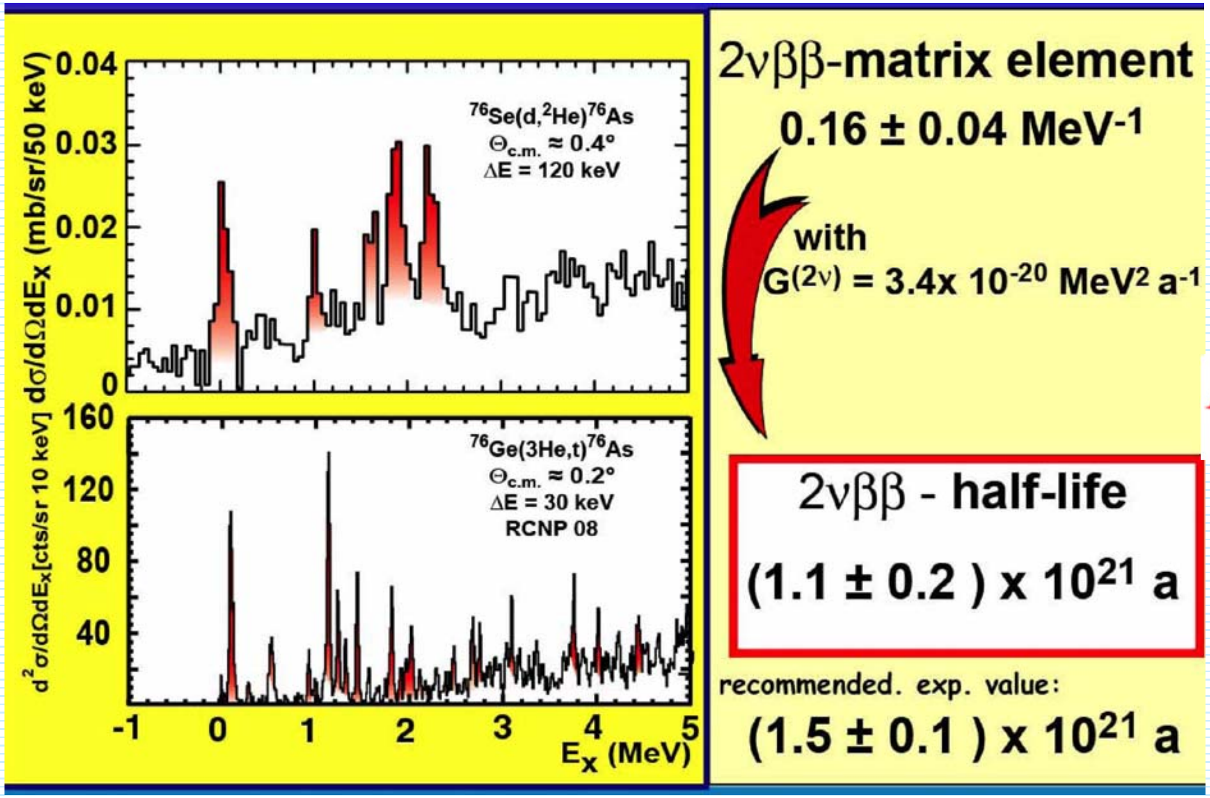
Are both type of NMEs related?

Differencies among 2νββ-decay NMEs: up to factor 10

The cross sections of (t,^{3}He) and (d,^{2}He) reactions give B(GT$^\pm$) for β^+ and β^-, product of the amplitudes (B(GT)$^{1/2}$) entering the numerator of $M^{2\nu}_{GT}$

$$M^{2\nu}_{GT} = \sum_m \frac{M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)}{Q_{\beta\beta}/2 + m_e + E_x(1^+_m) - E_0}$$





Grewe, …Frekers at al, PRC 78, 044301 (2008)

Closure 2νββ-decay NME

$$M^{2\nu}_{GT-cl} = \sum_m M^{(+)}_{GT}(m)\ M^{(-)}_{GT}(m)$$

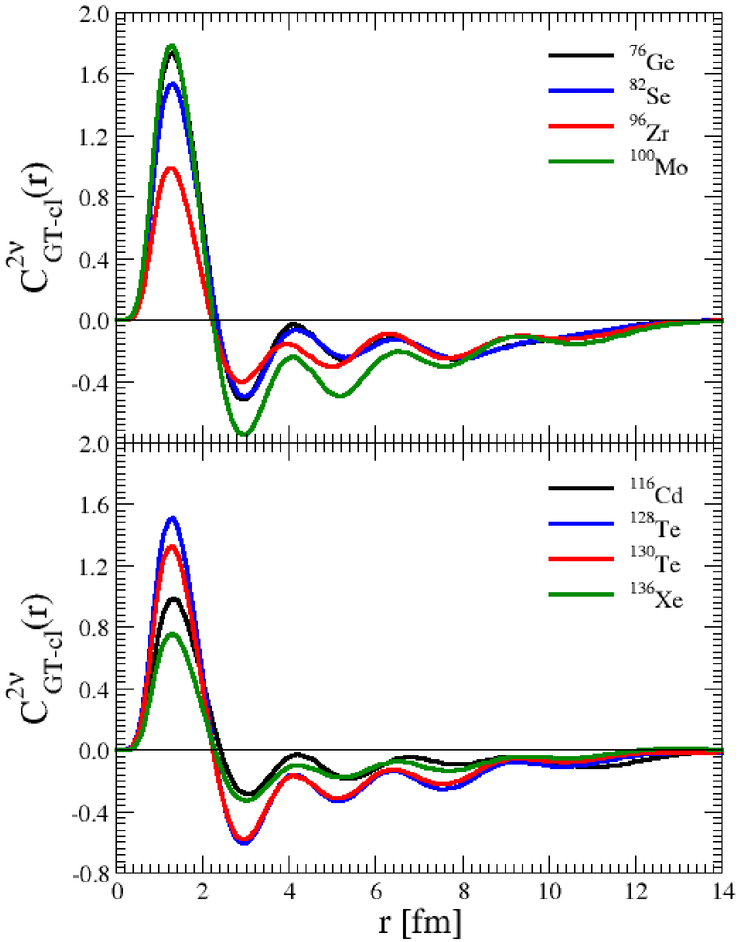
SSD hypothesis

$$g_A^2\ M^{2\nu}_{GT-cl} = \frac{3\ D}{\sqrt{ft_{EC}\, ft_{\beta^-}}}$$

Going to relative coordinates:

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r)dr$$

***r*- relative distance of two nucleons**



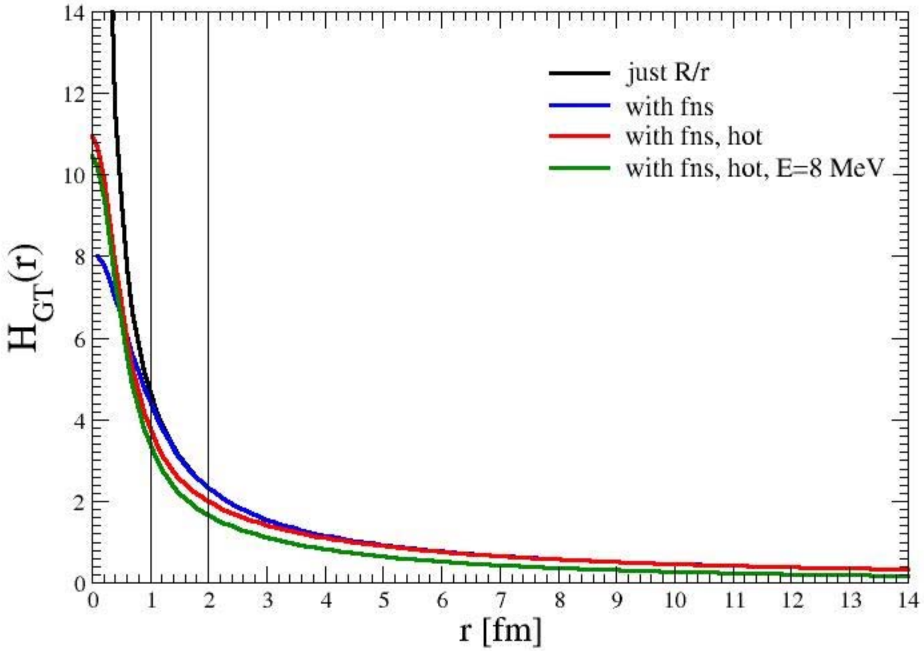
A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{0\nu}_{GT} = \int_0^\infty H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r)dr$$

Neutrino potential

$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}} f^2_{FNS}(q^2) g_{HOT}(q^2)dr$$



Neutrino potential prefer short distances

Closure 2νββ GT NME

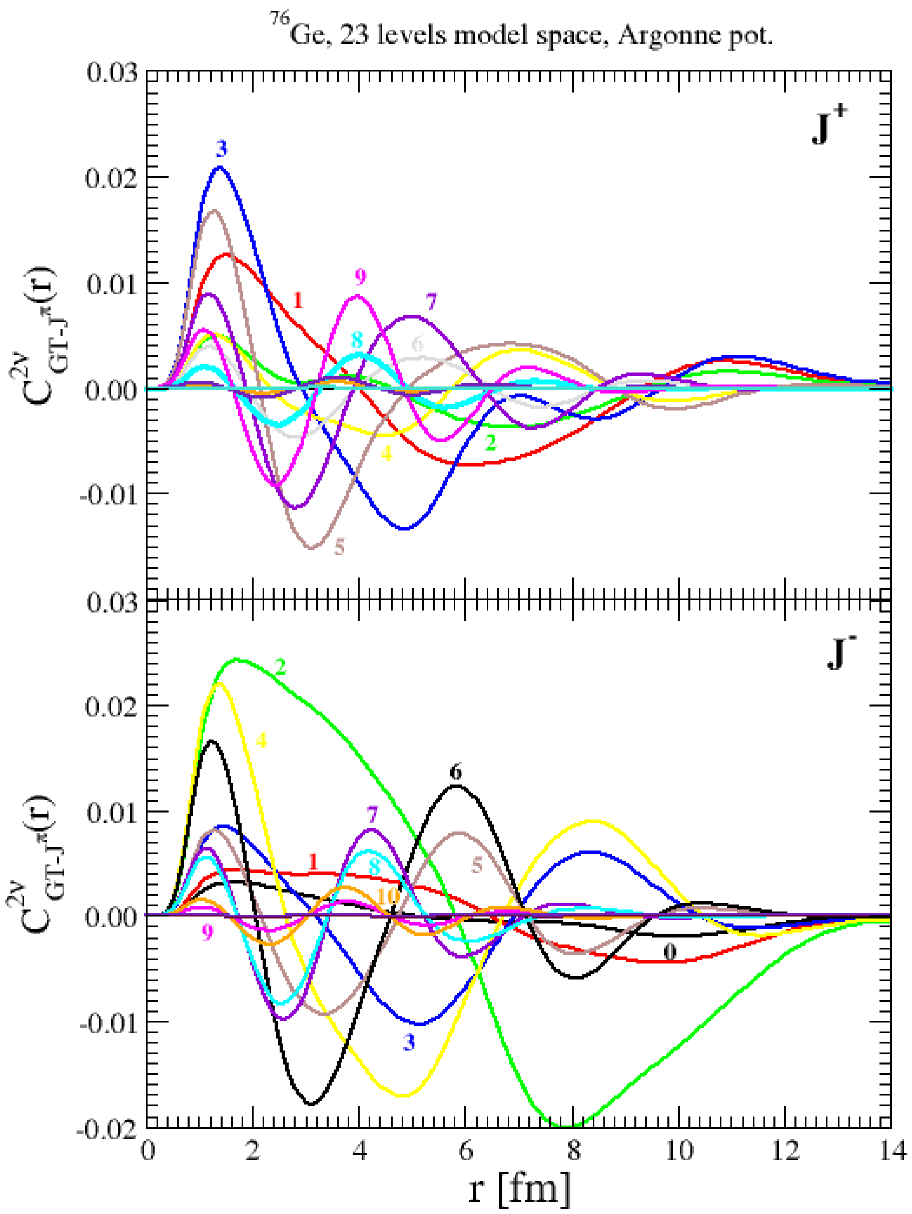
The only non-zero contribution from $J^{\pi}=1^{+}$

$$M^{2\nu}_{GT-cl} =$$

$$\sum_{J^{\pi},m} < 0^{+}_{f}|\tau^{+}\vec{\sigma}|J^{\pi},m > \cdot < J^{\pi},m|\tau^{+}\vec{\sigma}|0^{+}_{i} >$$

$$= \sum_{m} < 0^{+}_{f}|\tau^{+}\vec{\sigma}|1^{+},m > \cdot < 1^{+},m|\tau^{+}\vec{\sigma}|0^{+}_{i} >$$

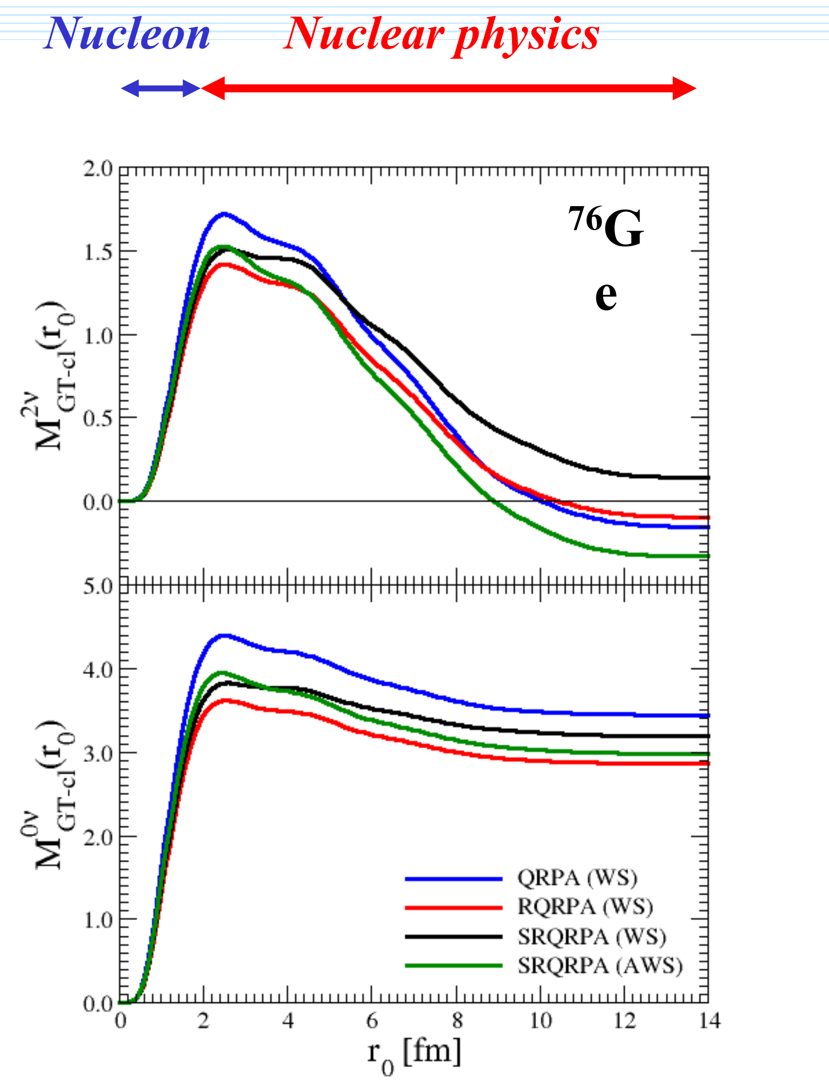
$$M^{2\nu}_{GT-cl} = \sum_{J^{\pi}} \int_{0}^{\infty} C^{2\nu}_{GT-J^{\pi}}(r)dr$$



F

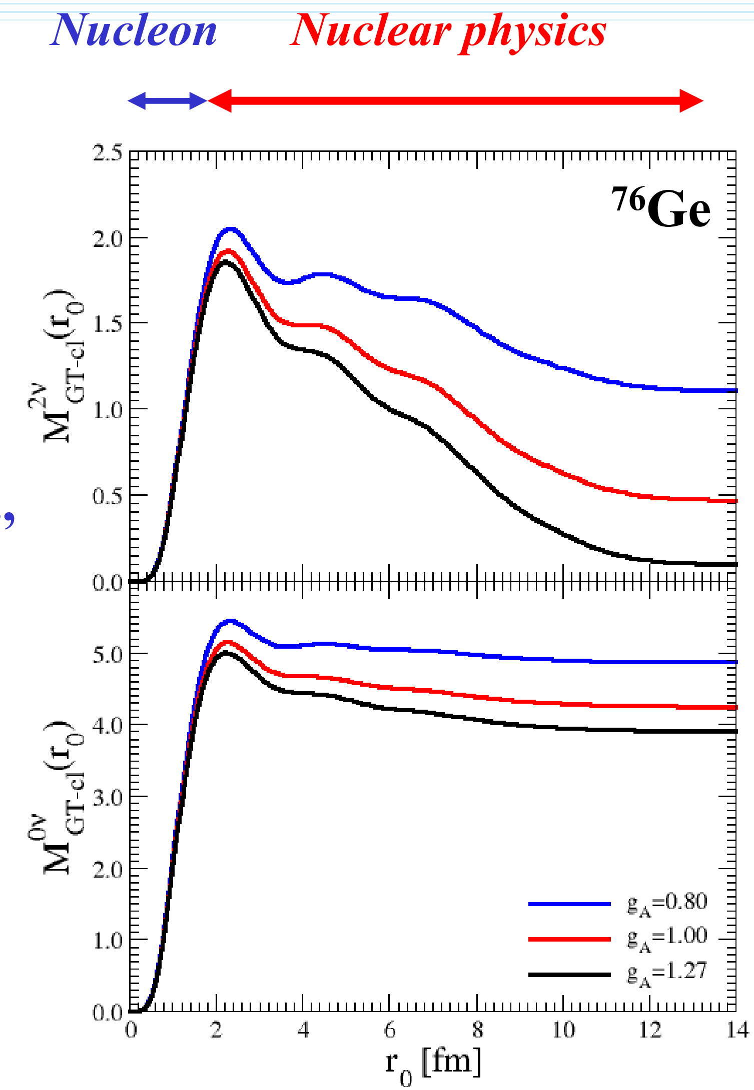
$M^{0\nu}_{GT}$ depends weakly on g_A/g_{pp} and QRPA approach unlike $M^{2\nu}_{GT}$

$$M^{0\nu}_{GT}(r_0) = \int_0^{r_0} H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r) dr$$



Different QRPA-like approches

F.Š.,



Dependence on axial-vector coupling

Phenomenological estimation of $M^{0\nu}{}_{GT}$

Nucleus	$T_{1/2}^{2\nu-exp}$ [y] [years]	$\lvert M_{GT}^{2\nu-exp}\rvert$ $[MeV^{-}1]$	SSD $\lvert M_{GT-cl}^{2\nu}\rvert$	SSD $\lvert M^{0\nu-ph}\rvert$	ChER $\lvert M_{GT}^{2\nu}\rvert$ $[MeV^{-}1]$	ChER $\lvert M_{GT-cl}^{2\nu}\rvert$	ChER $\lvert M^{0\nu-ph}\rvert$
^{48}Ca	4.4×10^{19}	0.046	-	-	0.083	0.220	1.98
^{76}Ge	1.5×10^{21}	0.0.136	-	-	0.159	0.522	5.46
^{96}Zr	2.3×10^{19}	0.090	-	-	-	0.222	3.45
^{100}Mo	7.1×10^{18}	0.231	0.350	4.02	-	-	-
^{116}Cd	2.8×10^{19}	0.126	0.349	4.21	0.064	0.305	3.67
^{128}Te	1.9×10^{24}	0.126	0.033	0.41	-	-	-

Neutrino potential

$$H(r) = R\frac{2}{\pi}\int_0^\infty j_0(qr)\frac{q}{q+\overline{E}}f_{FNS}^2(q^2)g_{HOT}(q^2)dr$$

with Taylor expansion

$$\begin{aligned} j_0(qr) &= 1 - \frac{1}{6}(qr)^2 + \frac{1}{120}(qr)^4 - \cdots \\ &= 1 - \mathcal{F}(r) \end{aligned}$$

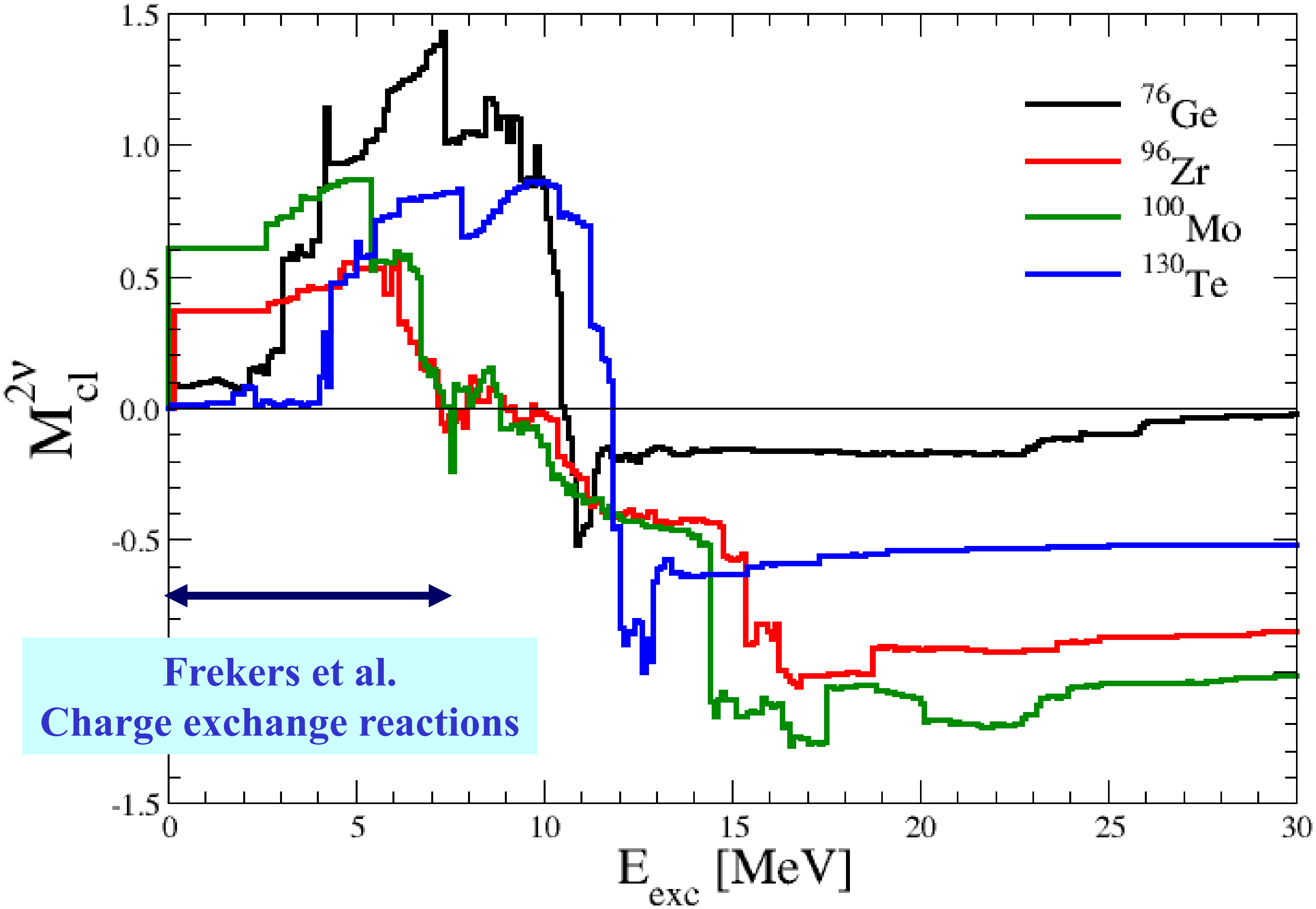
$$\begin{aligned} M_{GT}^{0\nu} &= H_{GT}(r=0)\ M_{GT-cl}^{2\nu} \\ &\quad - \int_0^\infty \mathcal{F}(r)C_{GT-cl}^{2\nu}(r)dr \\ &= M_{GT}^{0\nu-ph} - M_{GT}^{0\nu-rest} \end{aligned}$$

A: Phenomen. prediction:
Too large
(~ factor 2)

B: Need to be calculated
Not negligable

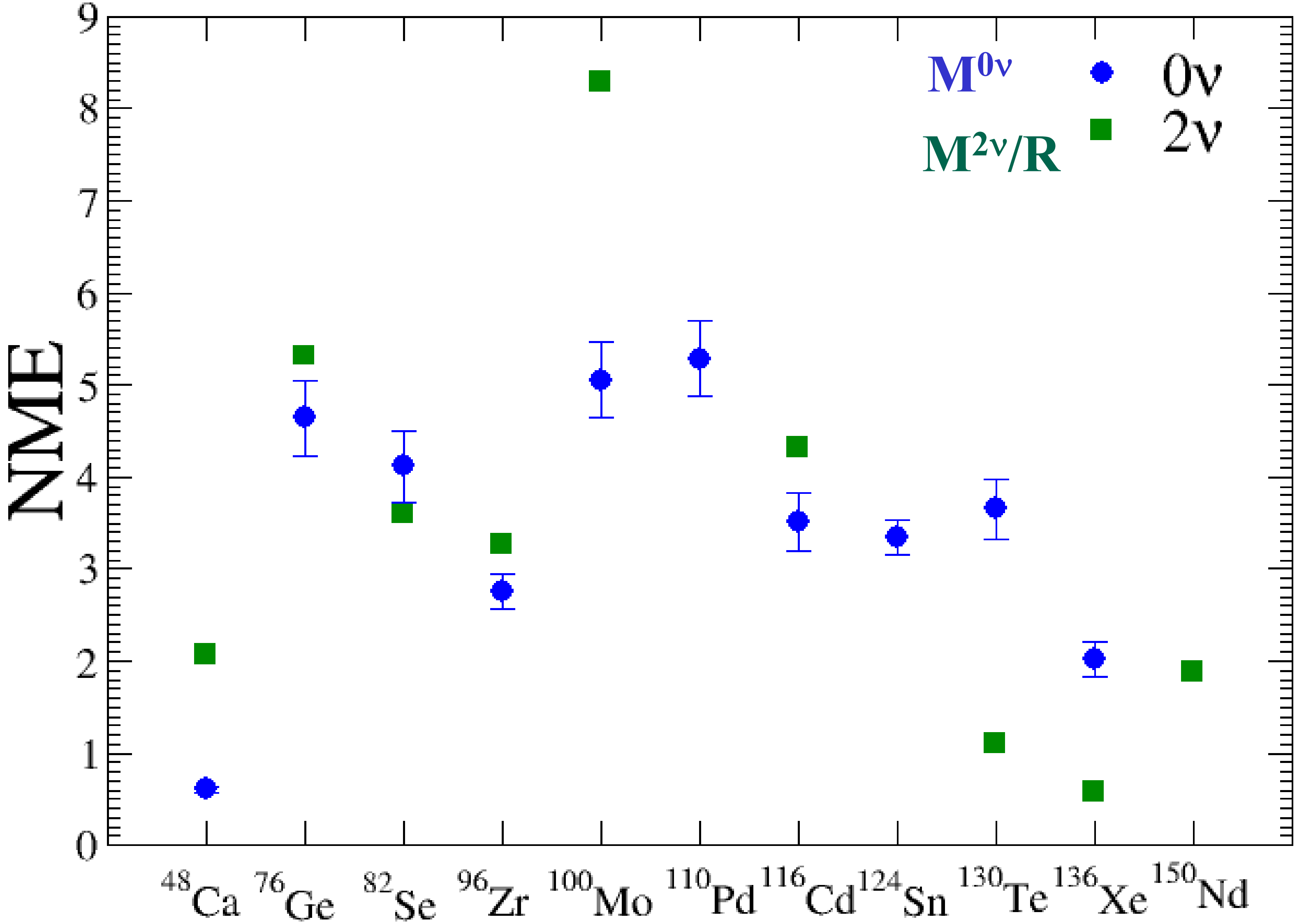
There is no proportionality between $M^{0\nu}{}_{GT}$ and $M^{2\nu}{}_{GT}$

There is no proportionality between 0νββ-decay and 2νββ-decay NM!!!



Frekers et al.
Charge exchange reactions

Is there a connection between 0νββ- and 2νββ-decay NME?

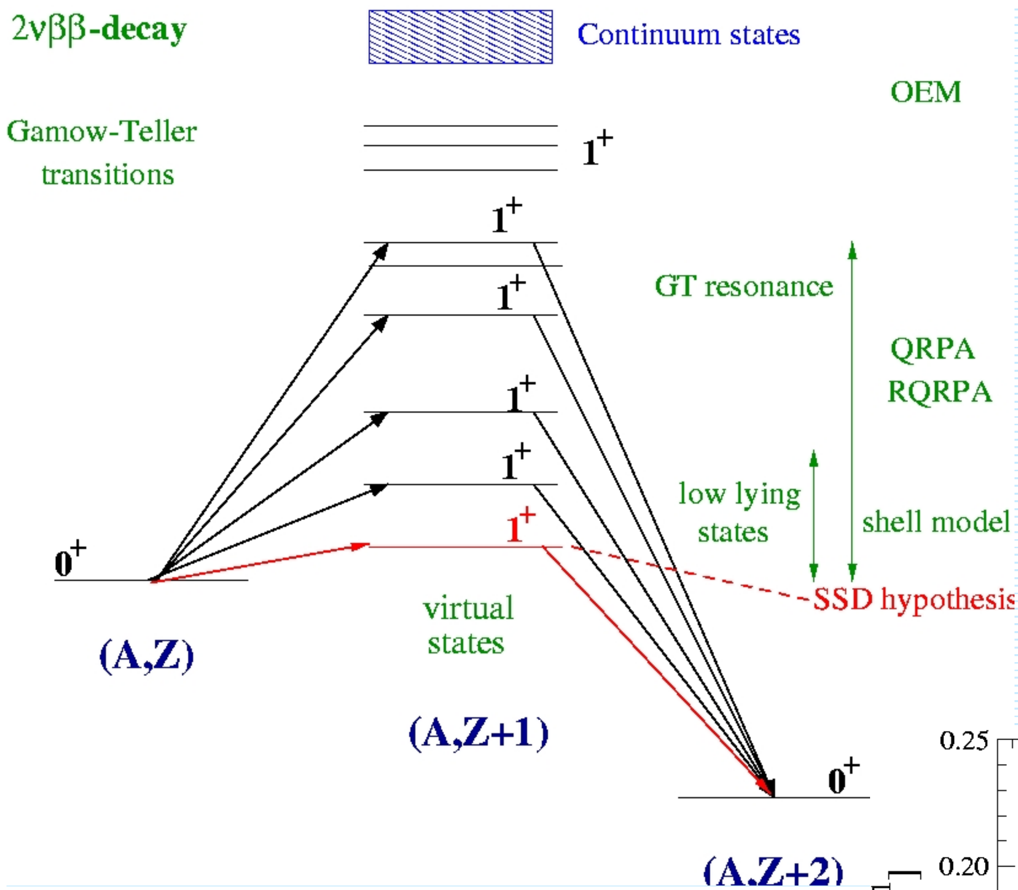


F. Š., Nucl. Phys. B, Proc. Suppl. (2014) – Proceeding of NOW14 conference

$2\nu\beta\beta$-decay

Both $2\nu\beta\beta$ and $0\nu\beta\beta$ operators connect the same states.
Both change two neutrons into two protons.

Explaining $2\nu\beta\beta$-decay is necessary but not sufficient

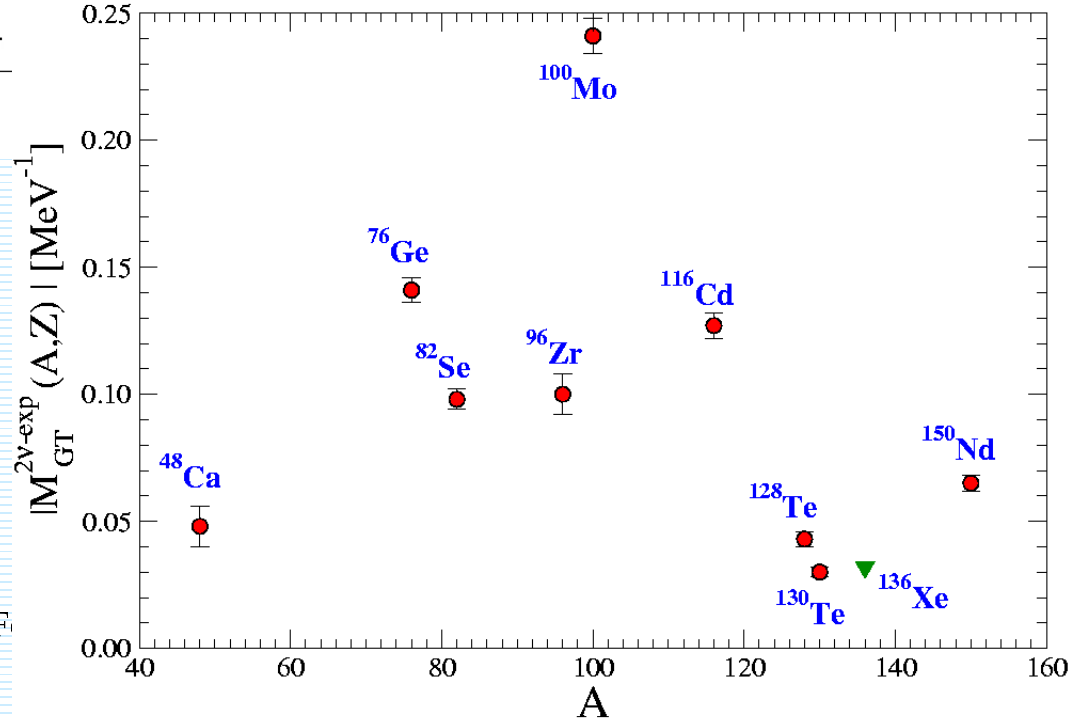


2νββ-decay nuclear matrix elements

$$\left(T_{1/2}^{2\nu}\right)^{-1} = G^{2\nu}\left|M_{GT}^{2\nu}\right|^2$$

Deduced from measured $T_{1/2}^{2\nu}$

$$M_{GT}^{2\nu} = \sum_m \frac{< 0_f^+||\tau^+\sigma||1_m^+ >< 1_m^+||\tau^+\sigma||0_i^+ >}{E_m - E_i + \Delta}$$



Differencies in NME: by factor ~ 10

2νββ-decay within the field theory

F.Š., G. Pantis, Phys. Atom. Nucl. 62 (1999) 585

Weak interaction Hamiltonian

$$\mathcal{H}^{\beta}(x) = \frac{G_F}{\sqrt{2}} 2\left[\bar{e}_L(x)\gamma_\alpha \nu_{eL}(x)\right] j_\alpha(x) + h.c.$$

2nbb-decay amplitude

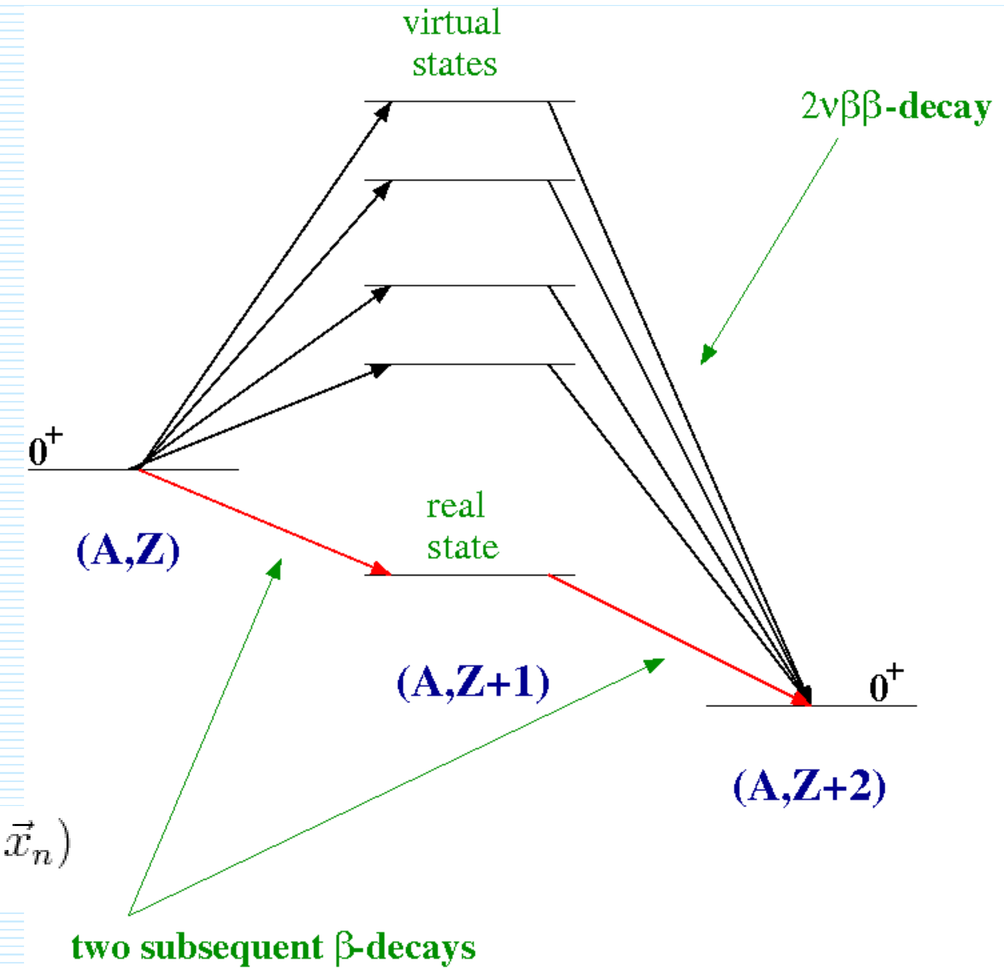
$$< f|S^{(2)}|i > =$$
$$\frac{(-i)^2}{2}\left(\frac{G_F}{\sqrt{2}}\right)^2 L_{\mu\nu}(p_1,p_2,k_1,k_2) J_{\mu\nu}(p_1,p_2,k_1,k_2)$$
$$-(p_1 \leftrightarrow p_2) - (k_1 \leftrightarrow k_2) + (p_1 \leftrightarrow p_2)(k_1 \leftrightarrow k_2)$$

Hadron part of amplitude

$$J_{\mu\nu}(p_1,p_2,k_1,k_2) = \int e^{-i(p_1+k_1)x_1} e^{-i(p_2+k_2)x_2}$$
$$_{out}<p_f|T(J_\mu(x_1)J_\nu(x_2))|p_i>_{in}\, dx_1 dx_2$$

$$T(J_\mu(x_1)J_\nu(x_2)) = J_\mu(x_1)J_\nu(x_2) \quad (two\ \beta - decays)$$
$$+ \ \Theta(x_{20}-x_{10})[J_\nu(x_2), J_\mu(x_1)] \quad (2\nu\beta\beta - decay)$$

A sum over intermediate nuclear states represents a sum over all meson and gamma exchange correlations of two beta decaying nucleons



$$J_\alpha(0,\vec{x}) = \sum_n \tau_n^+(\delta_{\alpha 4} + i g_A(\vec{\sigma})_k \delta_{\alpha k})\delta(\vec{x}-\vec{x}_n)$$

$$J^{2\beta 2\nu}_{\mu\nu}(p_1,p_2,k_1,k_2) = -i2M_{GT}\delta_{\mu k}\delta_{\nu k}$$
$$\times 2\pi\delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}), \ \ k = 1,2,3,$$

Integral representation of M_{GT}

$$M_{GT} = \frac{i}{2}\int_0^\infty (e^{i(p_{10}+k_{10}-\Delta)t} + e^{i(p_{20}+k_{20}-\Delta)t})M_{AA}(t)dt$$

with

$$M_{AA}(t) =< 0_f^+|\frac{1}{2}[A_k(t/2), A_k(-t/2)]|0_i^+ >$$

$$A_k(t) = e^{iHt}A_k(0)e^{-iHt}, \qquad A_k = \sum_i \tau_i^+(\vec{\sigma}_i)_k, \; k = 1, 2, 3.$$

$$A_k(t) = e^{itH}A_k(0)e^{-itH} = \sum_{n=0}^\infty \frac{(it)^n}{n!} \overbrace{[H[H...[H}^{n\ times}, A_k(0)]...]]$$

Completeness:
$\Sigma_n |n><n|=1$

$$< A'|J_\alpha(x_1)J_\beta(x_2)|A > \;=\; \sum_n < A'|J_\alpha(0,\vec{x}_1)|n >< n|J_\beta(0,\vec{x}_2)|A > \times \; e^{-i(E'-E_n)x_{10}}e^{-i(E_n-E)x_{20}}$$

$$\int_0^\infty e^{-iat}dt \Rightarrow \lim_{\epsilon\to 0}\int_0^\infty e^{-i(a-i\epsilon)t}dt = \lim_{\epsilon\to 0}\frac{-i}{a-i\epsilon}$$

$$M_{GT} = \sum_n \frac{< 0_f^+|A(0)_k|1_n^+ >< 1_n^+|A(0)_k|0_i^+ >}{E_n - E_i + \Delta}$$

Double beta decay is a two-body process

$$\mathbf{H = one - body + two - body}, \quad \mathrm{A}_k(0) = \text{one–body}$$

$$A_k(t) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \overbrace{[H[H...[H}^{n\ times}, A_k(0)]...]]$$

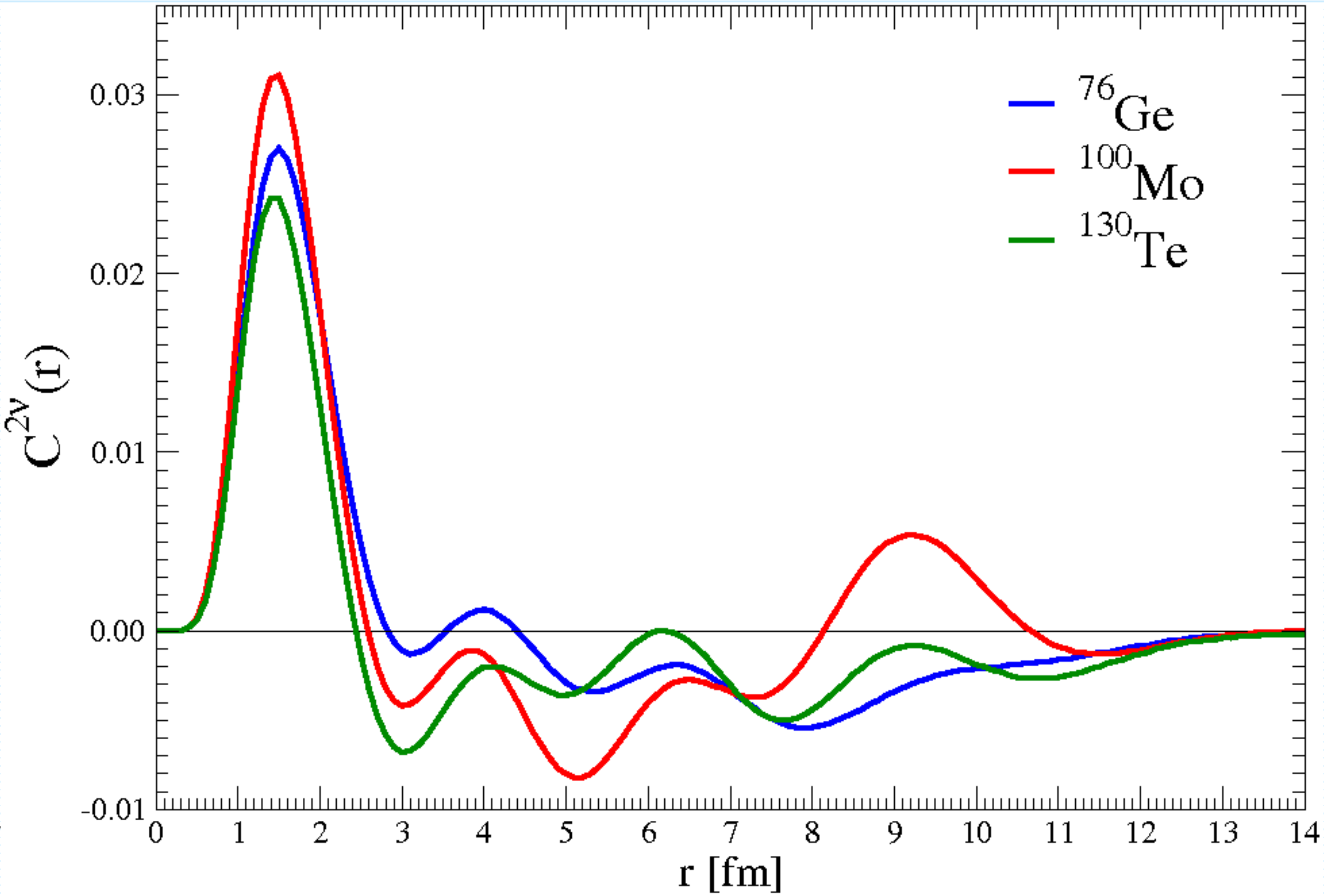
$$\begin{array}{lllllllll}
\text{n=0} & [H^{(0)}A] & [1] & & & & & & \\
 & & \downarrow & \searrow & & & & & \\
\text{n=1} & [H^{(1)}, A] & [1] & & [2] & & & & \\
 & & \downarrow & \searrow & \downarrow & \searrow & & & \\
\text{n=2} & [H^{(2)}, A] & [1] & & [2] & & [3] & & \\
 & & \downarrow & \searrow & \downarrow & \searrow & \downarrow & \searrow & \\
\text{n=3} & [H^{(3)}, A] & [1] & & [2] & & [3] & & [4]
\end{array}$$

If H $\approx$ one–body op. $\Longrightarrow$ $\mathbf{A_k(t)}$ is one–body op.

r_{12} -dependence of the $2\nu\beta\beta$-decay NME

$$M^{2\nu}_{GT} = \int C^{2\nu}(r)dr$$

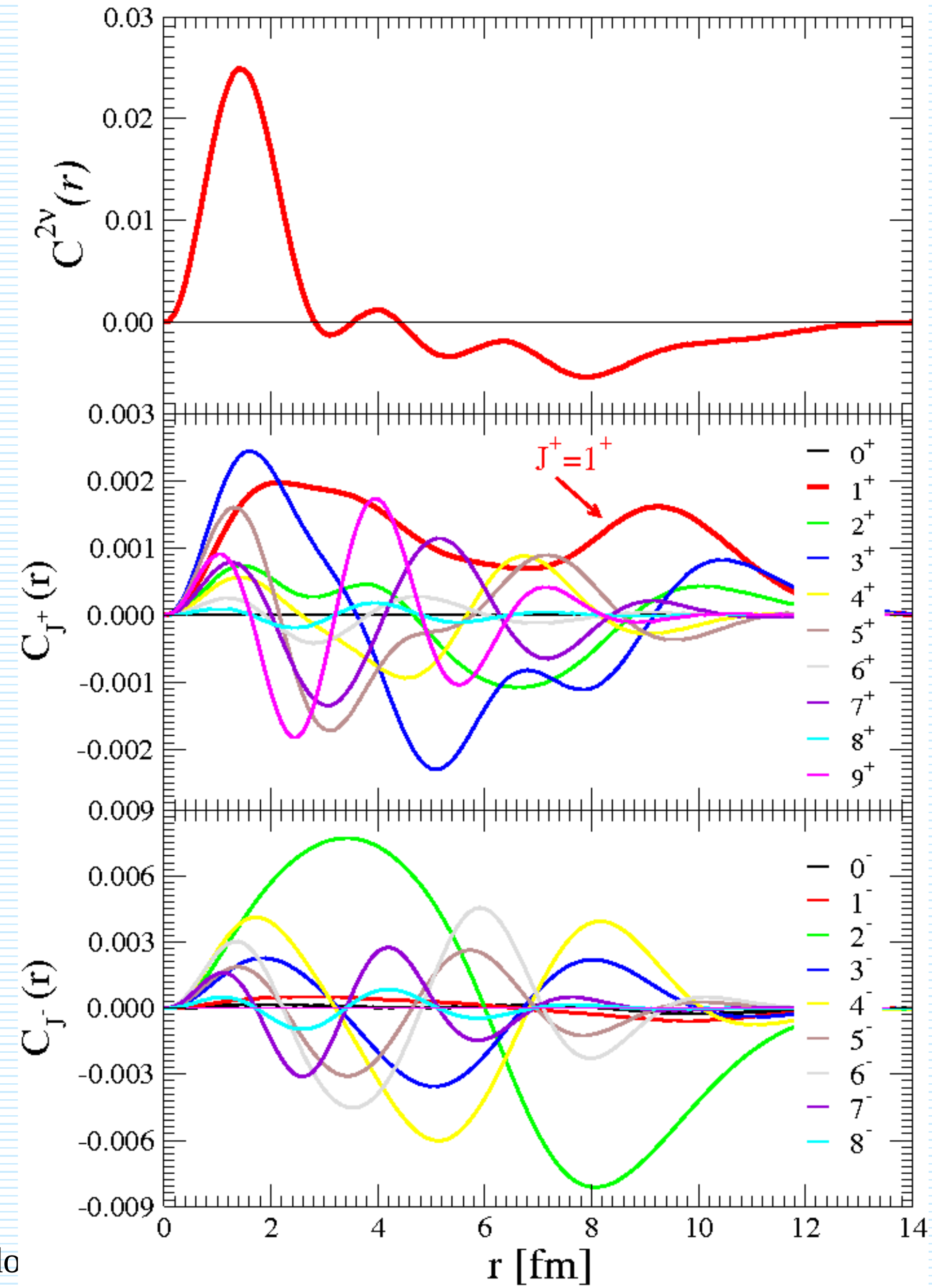
$$M^{2\nu}_{GT} = \sum_{J^\pi,k_i,k_f,\mathcal{J}} \sum_{pnp'n'} (-1)^{j_n+j_{p'}+J+\mathcal{J}} \times \sqrt{2\mathcal{J}+1} \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{array} \right\} \times \langle p(1), p'(2); \mathcal{J} \parallel \sigma(1)\cdot\sigma(2) \parallel n(1), n'(2); \mathcal{J} \rangle \times \langle 0^+_f || [\widetilde{c^+_{p'}\tilde{c}_{n'}}]_J || J^\pi k_f \rangle \langle J^\pi k_f | J^\pi k_i \rangle \langle J^\pi k_f i || [c^+_p \tilde{c}_n]_J || 0^+_i \rangle$$



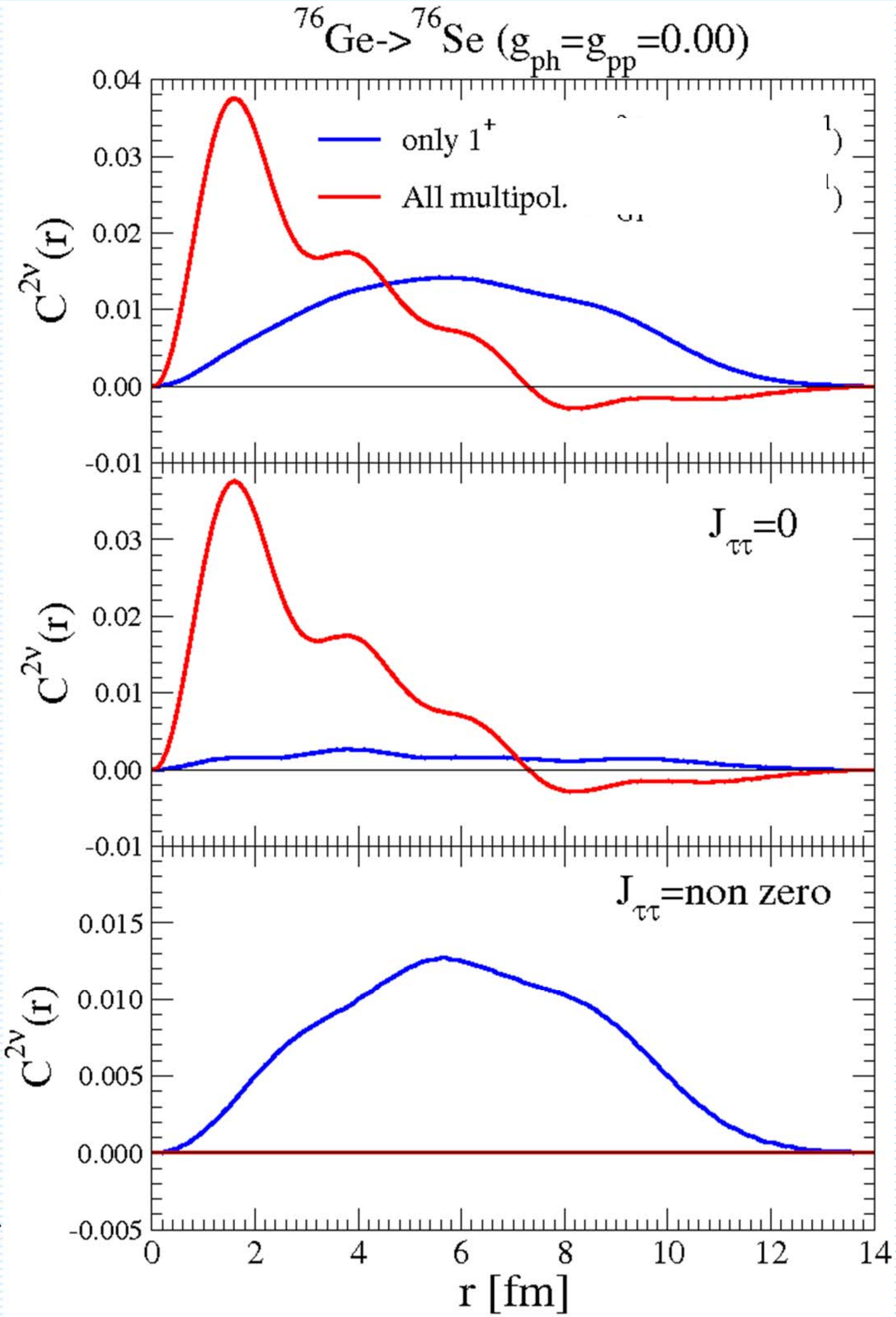
Decomposition of $C^{2\nu}$ on multipole contributions

$$M_{GT}^{2\nu} = \langle 0_f^+ | \tau^+ \boldsymbol{\sigma} \sum_{J^\pi, m} \frac{|J_m^\pi \rangle \langle J_m^\pi|}{E_m - (E_i + E_f)/2} \tau^+ \boldsymbol{\sigma} | 0_i^+ \rangle$$
$$= \sum_m \frac{\langle 0_f^+ || \tau^+ \boldsymbol{\sigma} || 1_{m'}^+ \rangle \langle 1_m^+ || \tau^+ \boldsymbol{\sigma} || 0_i^+ \rangle}{E_m - (E_i + E_f)/2}$$

$$\int C_J(r)\, dr = M_{GT}^{2\nu} \text{ for } J^\pi = 1^+$$
$$= 0 \quad \text{ for } J^\pi \neq 1^+$$

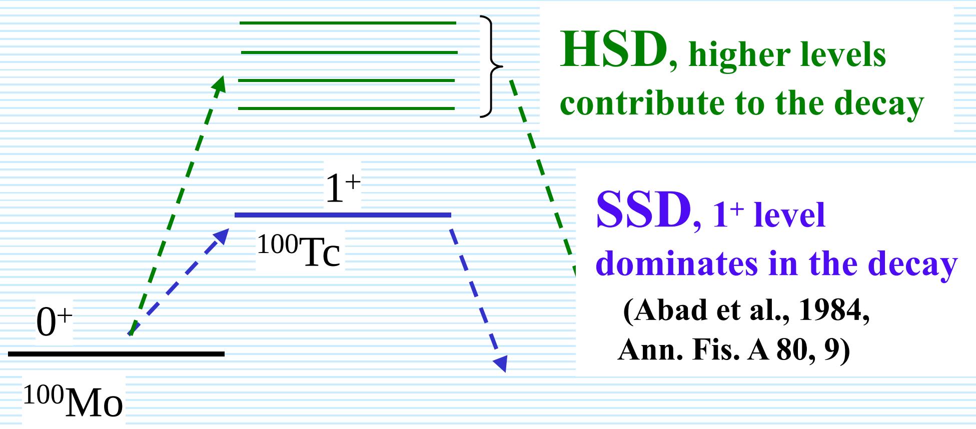


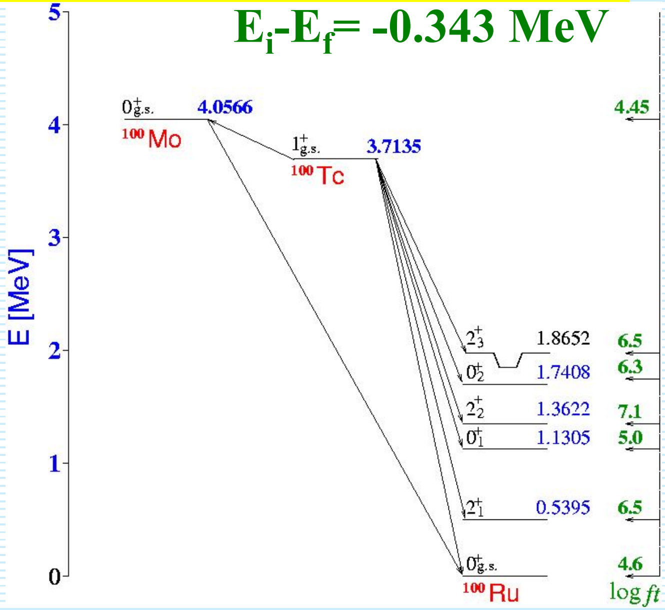
BCS limit (gph=gpp=0)
and
decomposition of M_{GT} on pairing and broken pairs contributions

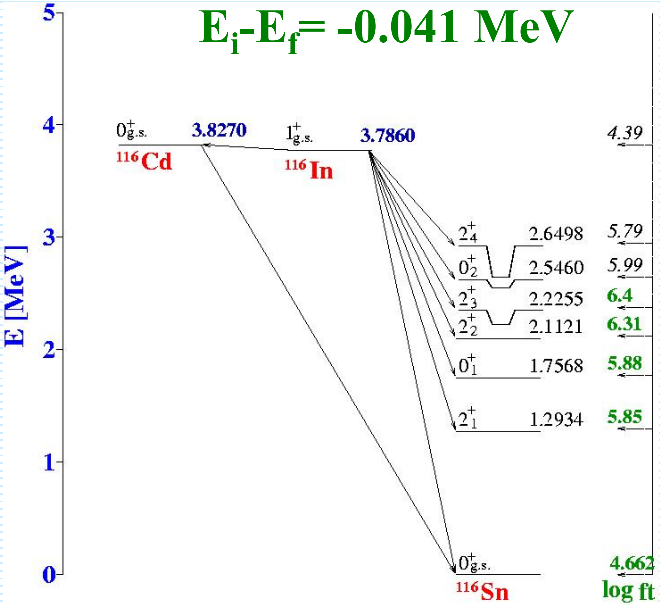


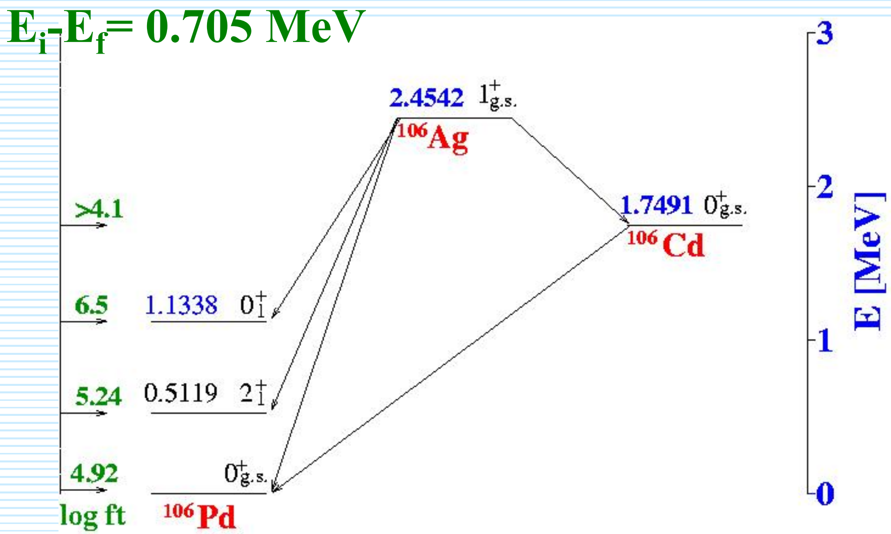
$$\langle p(1), p'(2); \mathcal{J} \parallel \sigma(1)\cdot\sigma(2) \parallel n(1), n'(2); \mathcal{J}\rangle$$

Single State Dominance (^{100}Mo, ^{106}Cd, ^{116}Cd, ^{128}Te …)









SSD – theoretical studies

$$M^K_{GT} = \sum_m \left(\frac{M^i_m(1^+) M^f_m(1^+)}{E_m - E_i + e_{10} + \nu_{10}} + \frac{M^i_m(1^+) M^f_m(1^+)}{E_m - E_i + e_{20} + \nu_{20}} \right) \qquad M^K_{GT} = M^L_{GT}(\nu_{10} \leftrightarrow \nu_{20})$$

$$\overset{\text{SSD}}{\Rightarrow} \quad \frac{M^i_1(1^+) M^f_1(1^+)}{E_1 - E_i + e_{10} + \nu_{10}} + \frac{M^i_1(1^+) M^f_1(1^+)}{E_1 - E_i + e_{20} + \nu_{20}} \Rightarrow 2\frac{M^i_1(1^+) M^f_1(1^+)}{E_1 - E_i + \Delta}$$

common approx.

$$e_{10} + \nu_{10} \approx e_{20} + \nu_{20} \approx (E_i - E_f)/2 \equiv \Delta$$

Isotope	f.s.	$T_{1/2}$(SSD)[y]	$T_{1/2}$(exp.)[y]
		$2\nu\beta^-\beta^-$	
^{100}Mo	$0_{g.s.}$	$6.8\ 10^{18}$	$6.8\ 10^{18}$
	0_1	$4.2\ 10^{20}$	$6.1\ 10^{18}$
^{116}Cd	$0_{g.s.}$	$1.1\ 10^{19}$	$2.6\ 10^{19}$
^{128}Te	$0_{g.s.}$	$1.1\ 10^{25}$	$2.2\ 10^{24}$
		EC/EC	
^{106}Cd	$0_{g.s.}$	$>4.4\ 10^{21}$	$>5.8\ 10^{17}$
^{130}Ba	$0_{g.s.}$	$5.0\ 10^{22}$	$4.0\ 10^{21}$

E_1-E_i ≈ 0 or neg. ⇒ sensitivity to lepton energies in energy denominators

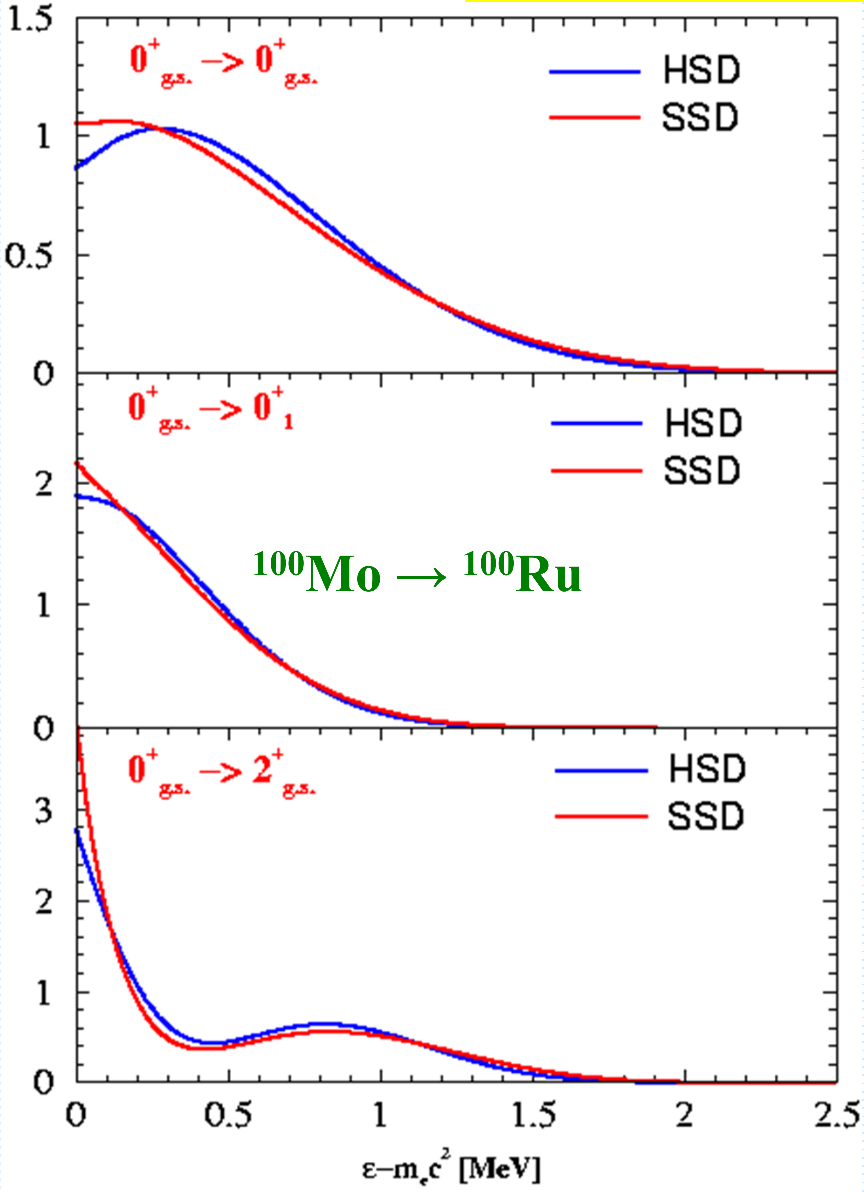
⇒ SSD and HSD offer different differential characteristics

Šimkovic, Šmotlák, Semenov
J. Phys. G, 27, 2233, 2001

Domin, Kovalenko, Šimkovic, Semenov, NPA 753, 337 (2005)

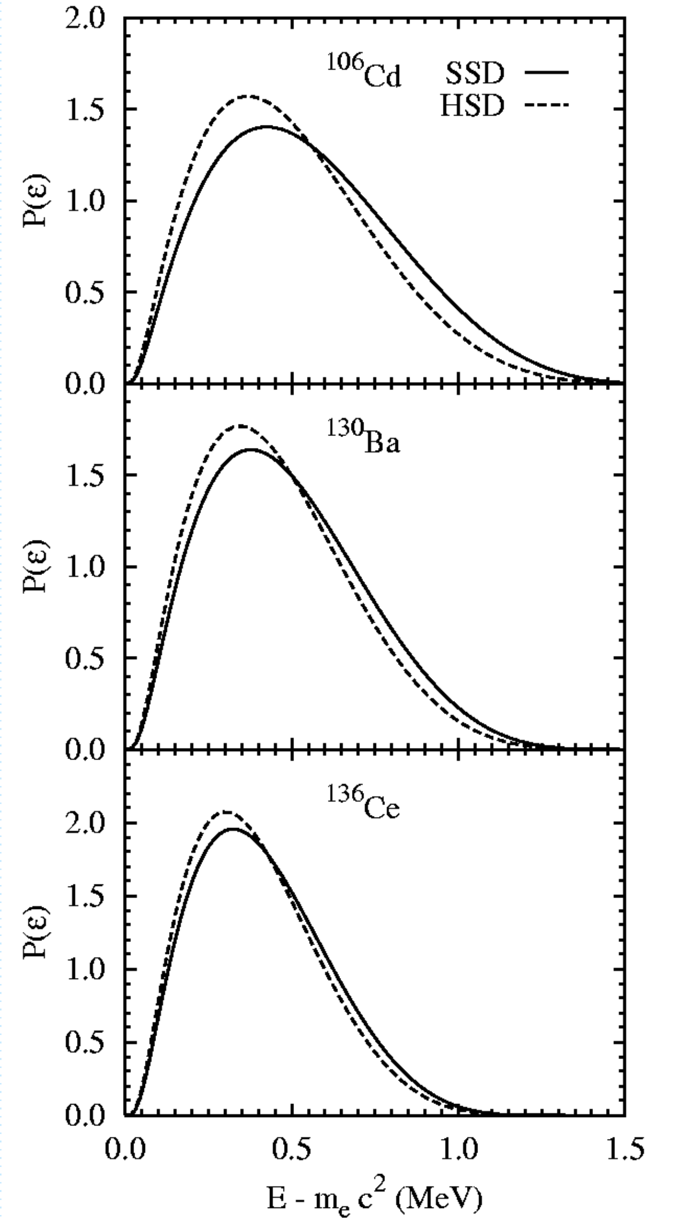
SSD differential characteristics

2νβ⁻β⁻–decay



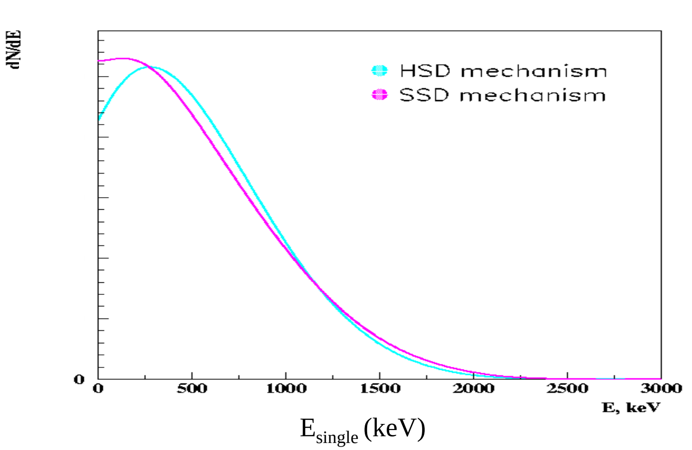
Do not depend on $M^i M^f$

2νEC/β⁺–decay



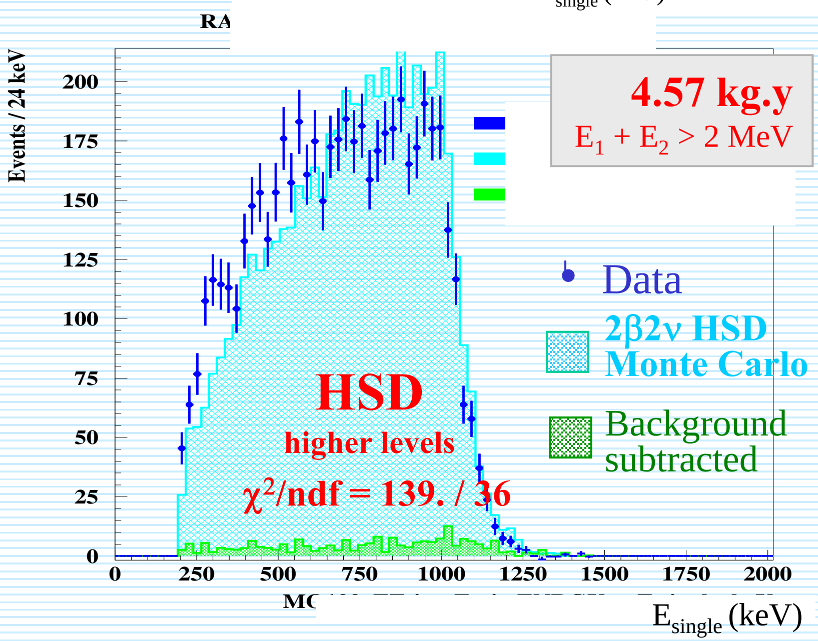
^{100}Mo 2β2ν: Experimental Study of SSD Hypothesis

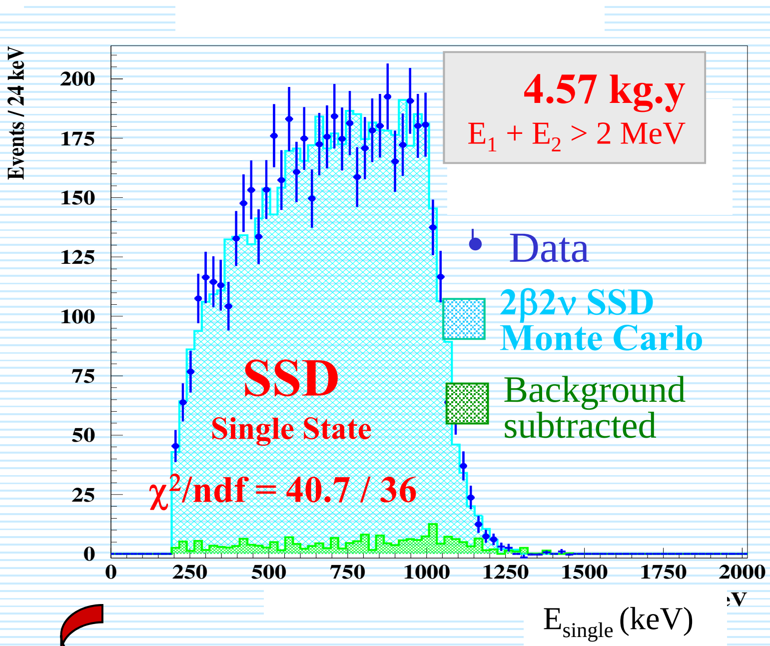
NEMO 3 exp.



Single electron spectrum different between SSD and HSD

Šimkovic, Šmotlák, Semenov J. Phys. G, 27, 2233, 2001





HSD: $T_{1/2} = 8.61 \pm 0.02\,(\text{stat}) \pm 0.60\,(\text{syst}) \times 10^{18}$ y

SSD: $T_{1/2} = 7.72 \pm 0.02\,(\text{stat}) \pm 0.54\,(\text{syst}) \times 10^{18}$ y

^{100}Mo 2β2ν single energy distribution in favour of Single State Dominant (SSD) decay

The DBD Nuclear Matrix Elements and the SU(4) symmetry

D. Štefánik, F.Š., A. Faessler, arXiv:1506.00835 [nucl-th], accepted in PRC

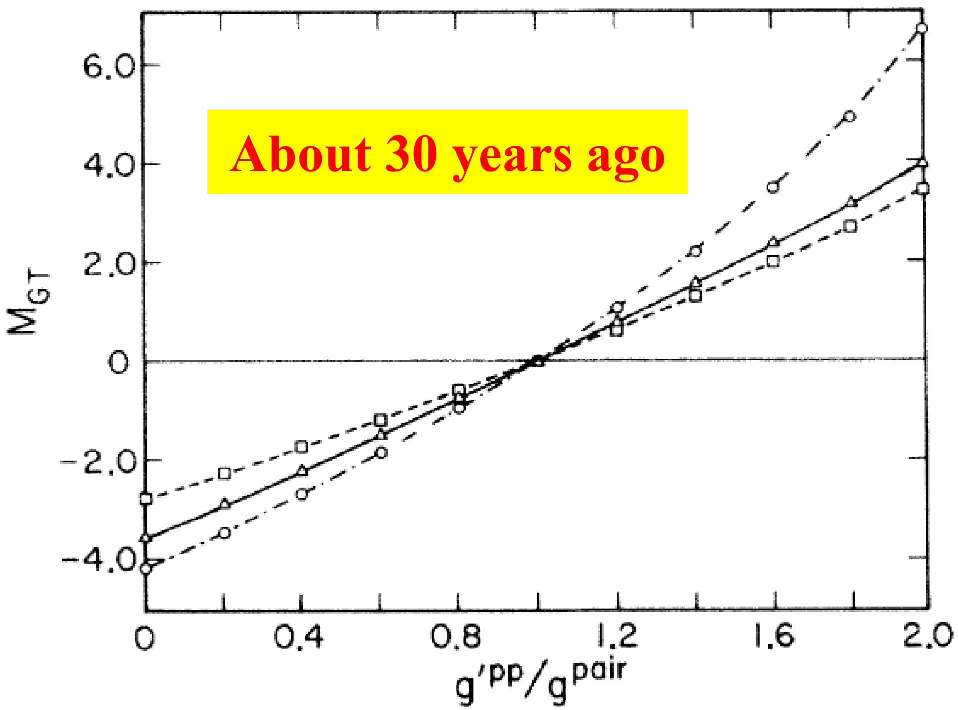
Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects

P. Vogel, M.R. Zirnbauer, PRL (1986) 3148

O. Civitarese, A. Faessler, T. Tomoda, PLB 194 (1987) 11

E. Bender, K. Muto, H.V. Klapdor, PLB 208 (1988) 53

…



The isospin is known to be a good approximation in nuclei

In heavy nuclei the SU(4) symmetry is strongly broken by the spin-orbit splitting.

What is beyond this behavior? Is it an artifact of the QRPA?

2νββ-decay rate

$$\left[T_{1/2}^{2\nu\beta\beta}(0^+)\right]^{-1} = \frac{m_e}{8\pi^7 \ln 2}(G_\beta m_e^2)^4 I^{2\nu}(0^+),$$

$$I^{2\nu}(0^+) = \frac{1}{m_e^9}\int_{m_e}^{E_i-E_f-m_e} F_0(Z_f,E_{e_1})p_{e_1}E_{e_1}dE_{e_1} \times \int_{m_e}^{E_i-E_f-E_{e_1}} F_0(Z_f,E_{e_2})p_{e_2}E_{e_2}dE_{e_2} \times \int_0^{E_i-E_f-E_{e_1}-E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}^{2\nu} dE_{\nu_1}.$$

$$\mathcal{A}^{2\nu} = g_V^4\left[\frac{1}{4}|M_F^K+M_F^L|^2+\frac{3}{4}|M_F^K-M_F^L|^2\right] - g_V^2 g_A^2 \mathrm{Re}\left\{M_F^{K*}M_{GT}^L + M_{GT}^{K*}M_F^L\right\} + \frac{g_A^4}{3}\left[\frac{3}{4}|M_{GT}^K+M_{GT}^L|^2+\frac{1}{4}|M_{GT}^K-M_{GT}^L|^2\right]$$

$$M_F^K = \sum_n \frac{K(0_n^+)}{2}F_n, \quad M_F^L = \sum_n \frac{L(0_n^+)}{2}F_n,$$

$$M_{GT}^K = \sum_n \frac{K(1_n^+)}{2}G_n, \quad M_{GT}^L = \sum_n \frac{L(1_n^+)}{2}G_n,$$

$$F_n = \langle 0_f^+ \| \sum_m \tau_m^- \| 0_n^+\rangle\langle 0_n^+ \| \sum_m \tau_m^- \| 0_i^+\rangle,$$

$$G_n = \langle 0_f^+ \| \sum_m \tau_m^-\sigma_m \| 1_n^+\rangle\langle 1_n^+ \| \sum_m \tau_m^-\sigma_m \| 0_i^+\rangle$$

$$K_n(J^+) = \frac{2}{(2E_n(J^+)-E_i-E_f)+\epsilon_K} + \frac{2}{(2E_n(J^+)-E_i-E_f)-\epsilon_K}$$

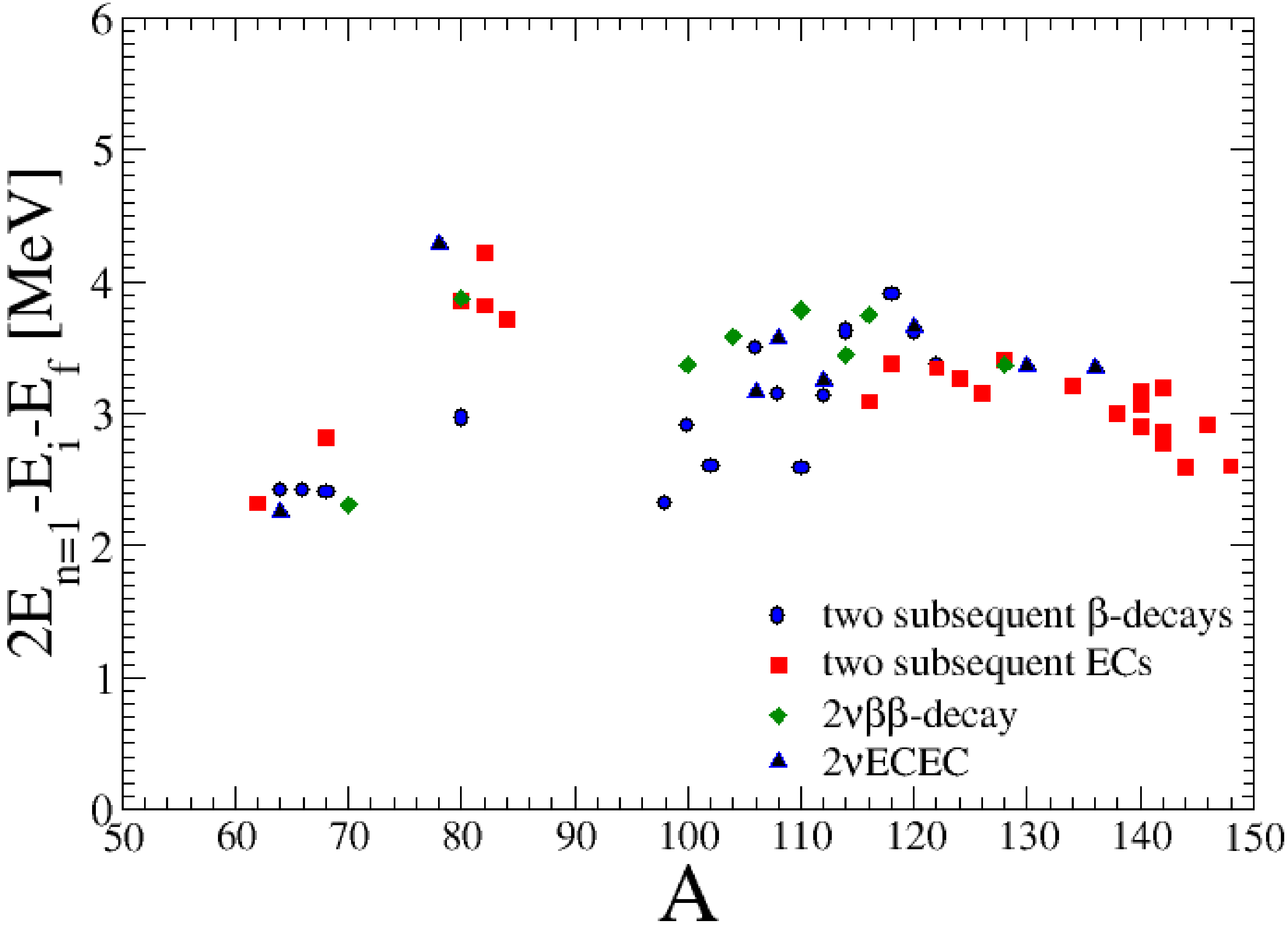
$$L_n(J^+) = \frac{2}{(2E_n(J^+)-E_i-E_f)+\epsilon_L} + \frac{2}{(2E_n(J^+)-E_i-E_f)-\epsilon_L}$$

$$\epsilon_K = E_{e_2}+E_{\nu_2}-E_{e_1}-E_{\nu_1}$$

$$\epsilon_L = E_{e_1}+E_{\nu_2}-E_{e_2}-E_{\nu_1}$$

In the limit $2E_n - E_i - E_f = 0$ ⇒ $\mathcal{A}^{2\nu} = 0$

What is the meaning of quantity $(2E_{n=1}-E_i-E_f)$?



s.p. mean-field

Conserves SU(4) symmetry

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair}\left(\sum_{M_T=-1,0,1} A^\dagger_{0,1}(0,M_T)A_{0,1}(0,M_T) + \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0)\right) + g_{ph}\sum_{a,b} E^\dagger_{a,b}E_{a,b}}_{H_0}$$

$$\underbrace{+ (g_{pair} - g^{T=0}_{pp}) \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0) + (g_{pair} - g^{T=1}_{pp}) A^\dagger_{0,1}(0,0)A_{0,1}(0,0)\,.}_{H_I}$$

H_I violates SU(4) symmetry

g_{pair}- strength of isovector like nucleon pairing (L=0, S=0, T=1, M_T=±1)
$g_{pp}^{T=1}$- strength of isovector spin-0 pairing (L=0, S=0, T=1, M_T=0
$g_{pp}^{T=0}$- strength of isoscalar spin-1 pairing (L=0, S=1, T=0)
g_{ph}- strength of particle-hole force

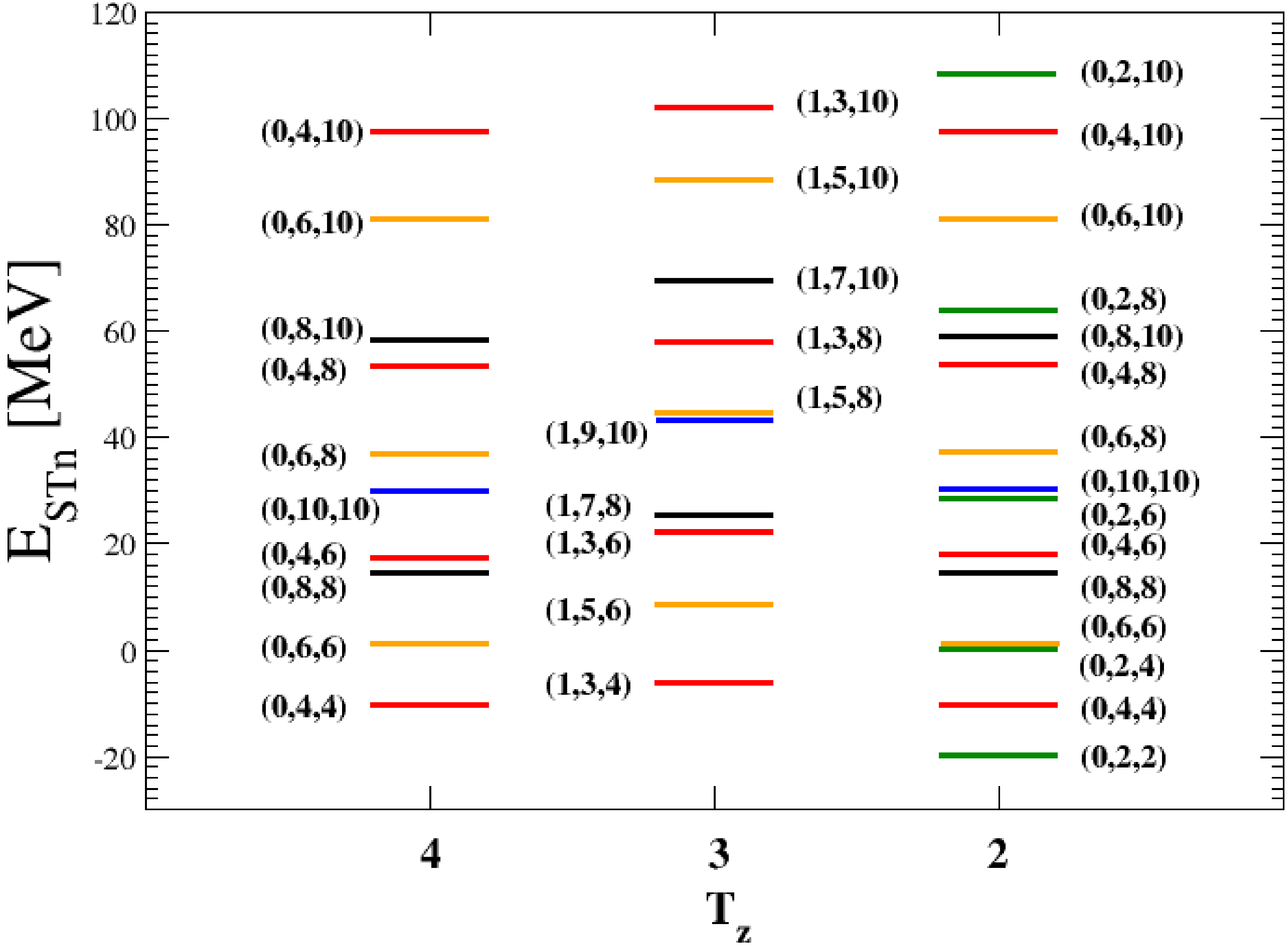
M_F and M_{GT} do not depend on the mean-field part of H and are governed by a weak violation of the **SU(4)** symmetry by the Particle-particle interaction of H

$$M^{2\nu}_F = -\frac{48\sqrt{\frac{33}{5}}\,(g_{pair} - g^{T=1}_{pp})}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}$$

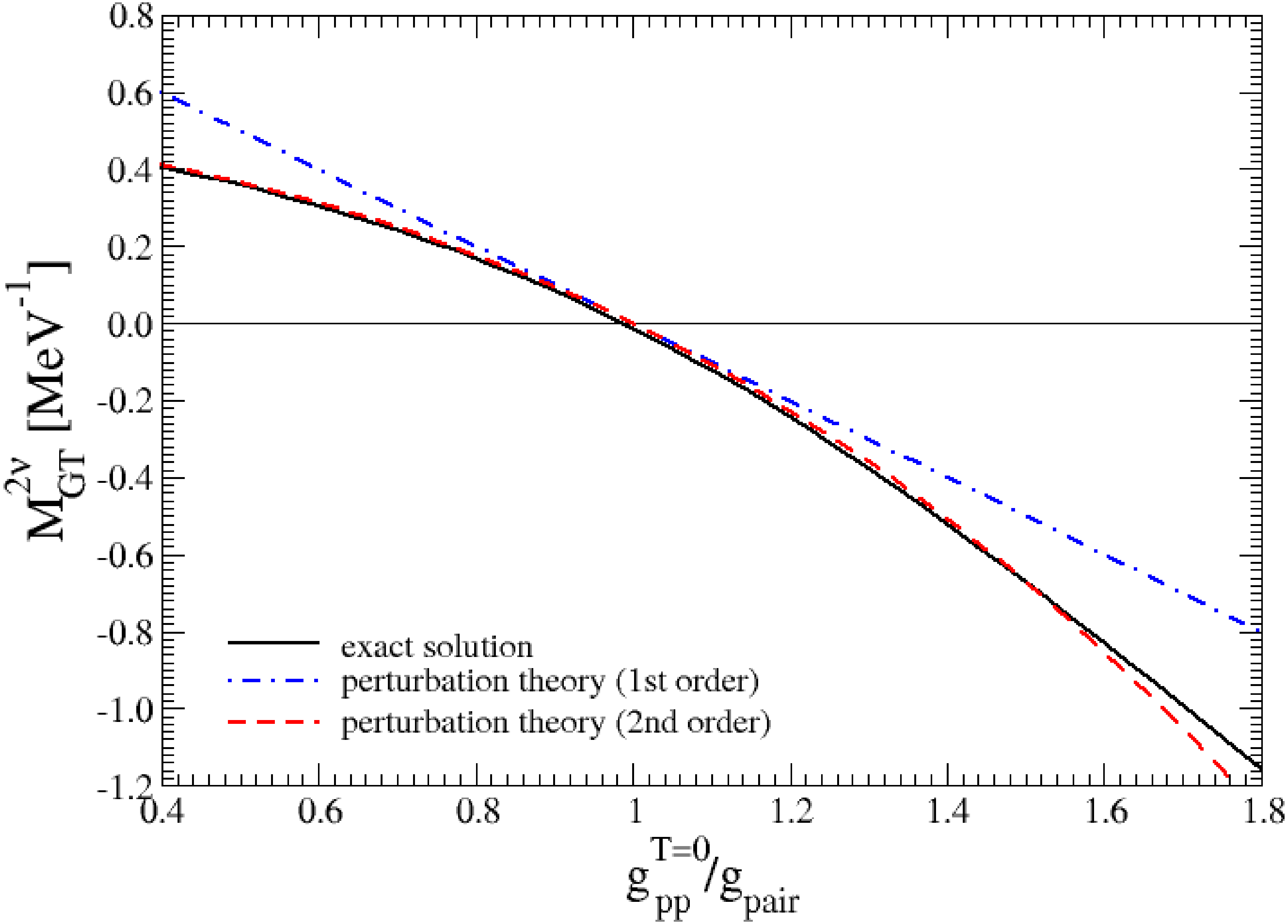
$$M^{2\nu}_{GT} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair} + 9g_{ph}}\left\{\frac{(g_{pair} - g^{T=0}_{pp})}{(10g_{pair} + 20g_{ph})} + \frac{2g_{ph}(g_{pair} - g^{T=1}_{pp})}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})}\right\}$$

Energies of excited states for the case of conserved SU(4) symmetry

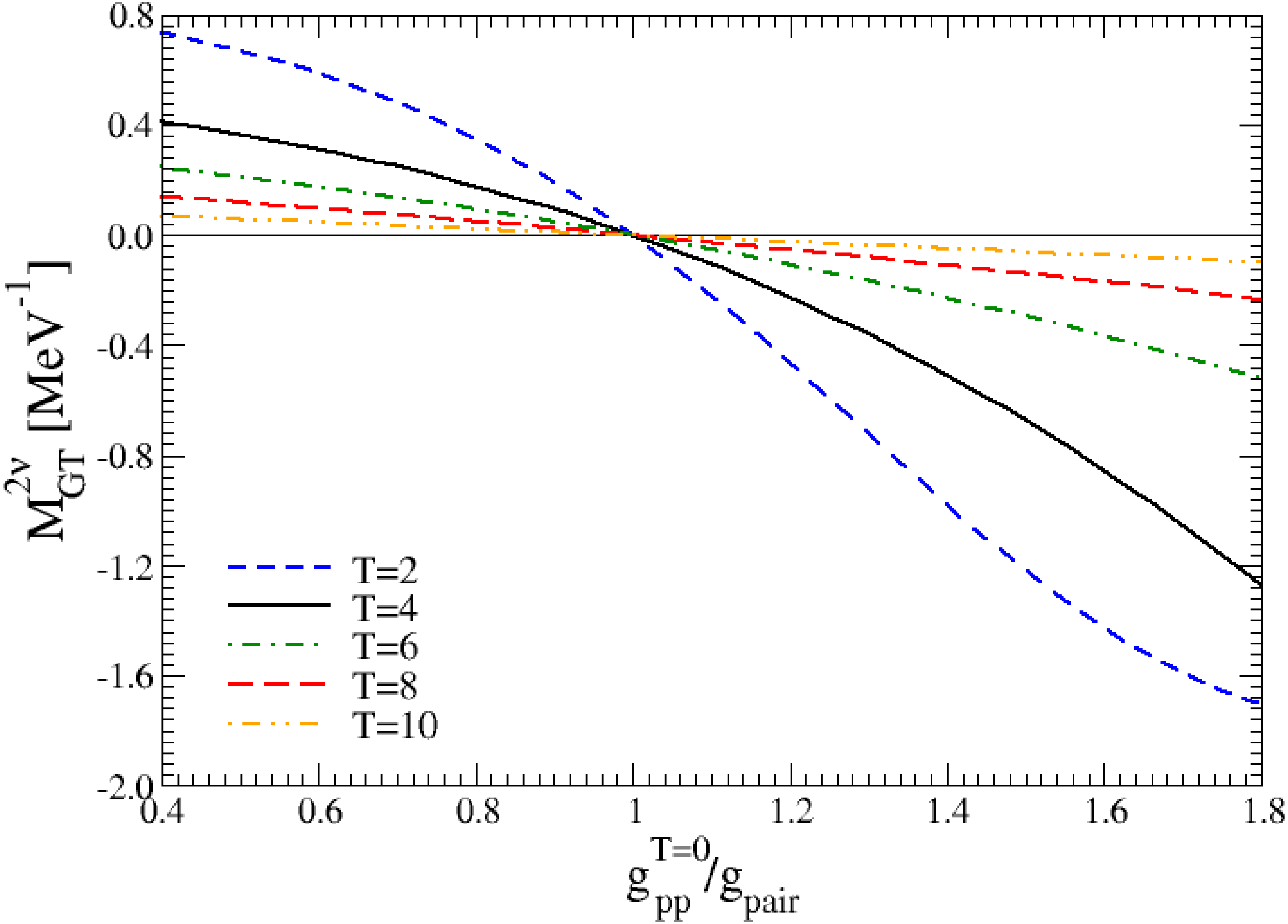
M_F=0, M_{GT}=0   (see SU(4) multiplets)



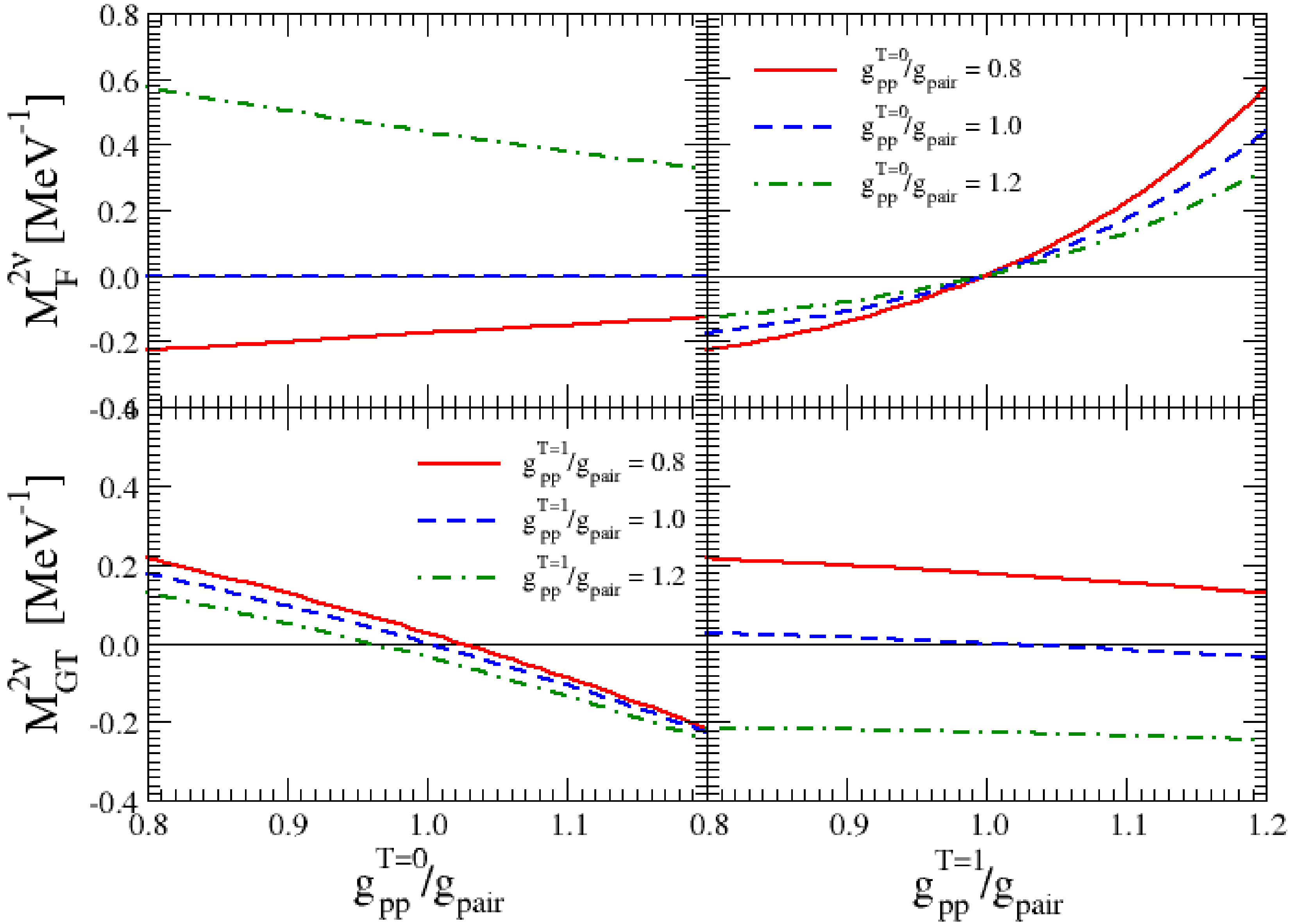
M_{GT} up to the second order of perturbation theory due to violation of the SU(4) symmetry by the particle-particle interaction of H



By assuming a fixed violation of the SU(4) symmetry by particle-particle int.
M_{GT} decreases by increase of isospin of the ground state



Results confirm dependence of M_F and M_{GT} on $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$ by the QRPA



3.Energy - weighted sum rule involving ΔZ=2 nuclei

We propose an energy-weighted sum rule with connection of nuclei participating in the double beta decay. We have

$$\sum_n (2E_n - E_i - E_f) \left\langle f \left| O_{F,GT} \right| n \right\rangle \left\langle n \left| O_{F,GT} \right| i \right\rangle = \left\langle f \left| \left[O_{F,GT}, \left[H, O_{F,GT} \right] \right] \right| i \right\rangle$$

For SO(8) model

$$\sum_n (2E_n - E_i - E_f) \left\langle f \left| \vec{\sigma}\tau^- \right| n \right\rangle \left\langle n \left| \vec{\sigma}\tau^- \right| i \right\rangle =$$
$$12(g_{pp}^{T=0} - g_{pair}) \left\langle f \left| A^\dagger_{0,1}(0,-1) A_{0,1}(0,1) \right| i \right\rangle - 2g_{ph} \left\langle f \left| \vec{\sigma}\tau^- \cdot \vec{\sigma}\tau^- \right| i \right\rangle - 6g_{ph} \left\langle f \left| T^- T^- \right| i \right\rangle$$

Instead of Conclusion:
There is a need for supporting experiments

Nuclear matrix elements:

• *Mean field*	*p and n removing transfer reactions*
• β^- *and* β^+ *strengths*	*Charge-changing reactions and muon capture*
• *deformation*	*Exp. to remeasure deformetion needed*
• *2νββ-decay*	*Double beta decay experiments*
• *??*	*Double charge exchange reactions (with pions and nuclei)*

Frank Avignone:

Nuclear Matrix Elements
are
as important as DATA