Scattering amplitudes and 4-jet production in k_T-factorization

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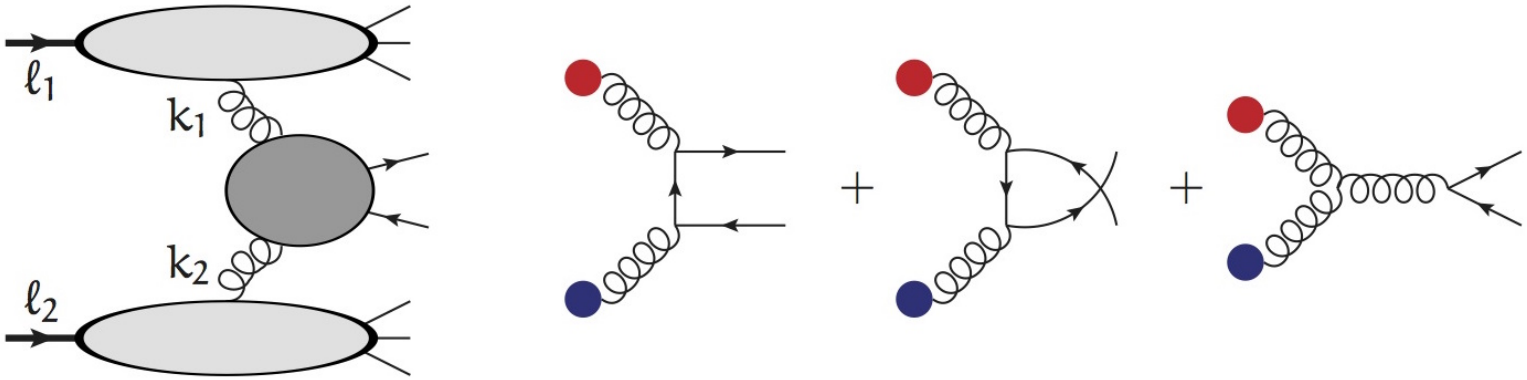
of Krzysztof Kutak

High-Energy-factorisation: original formulation

High-Energy-Factorisation (or k_T-factorization)
(Catani,Ciafaloni,Hautmann, 1991 / Collins,Ellis, 1991)



$$\sigma_{h_1,h_2\to q\bar{q}} = \int d^2k_{1\perp}\, d^2k_{2\perp}\, \frac{dx_1}{x_1}\, \frac{dx_2}{x_2}\, \mathcal{F}_g(x_1,k_{1\perp})\, \mathcal{F}_g(x_2,k_{2\perp})\, \hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s}, \frac{k_{1\perp}}{m}, \frac{k_{2\perp}}{m}\right)$$

where the $\mathcal{F}_g$'s are the gluon densities, obeying BFKL, BK, CCFM evolution equations.

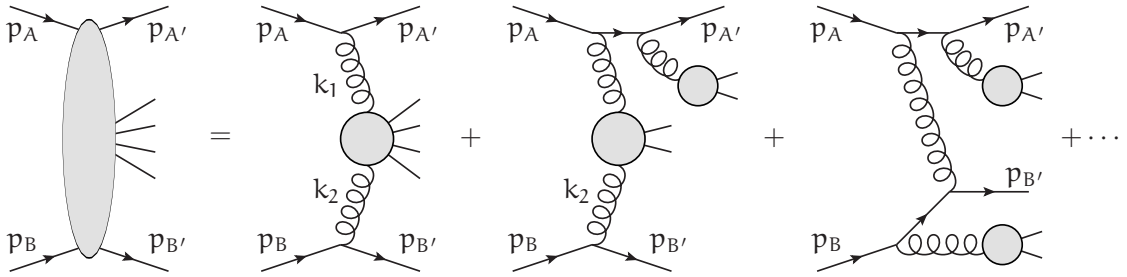
Non negligible transverse momentum is associated to small-x physics.

Momentum parameterisation:

$$k_1^\mu = x_1\, p_1^\mu + k_{1\perp}^\mu \quad , \quad k_2^\mu = x_2\, p_2^\mu + k_{2\perp}^\mu \quad \text{for} \quad p_i \cdot k_i = 0 \quad k_i^2 = -k_{i\perp}^2 \quad i = 1,2$$

Off-shell amplitudes: gluons

Problem: general partonic processes must be described by gauge invariant amplitudes
Off-shell gauge-invariant amplitudes obtained by embedding them into on-shell processes. For off-shell gluons: represent g^* as coming from a $\bar{q}qg$ vertex, with the quarks taken to be on-shell



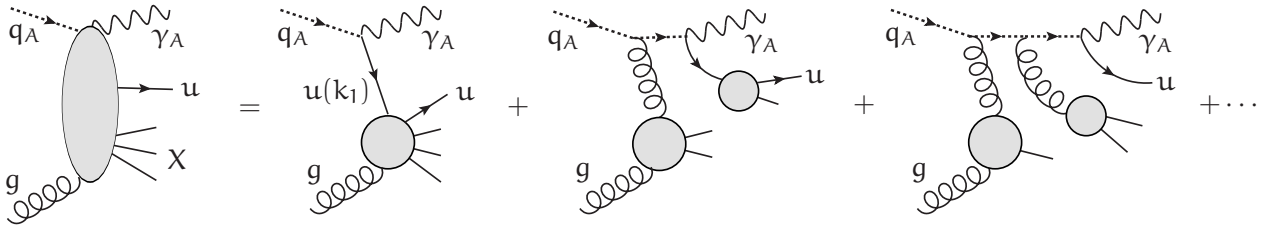
Prescriptions: *K. Kutak, P. Kotko, A. van Hameren, T. Salwa (2013)*

Any legs via recursion relations: *P. Kotko (2014), A. van Hameren (2014)*

Applications:
- production of forward dijets initiated with gluons : $gg^* \to gg$
- production of forward dijets initiated with quarks : $q\bar{q}^* \to gg$
- **Test of TMDs in multi-jet production** : $p\,p \to n$ **(= 4 in this talk)** jets

Off-shell amplitudes: quarks

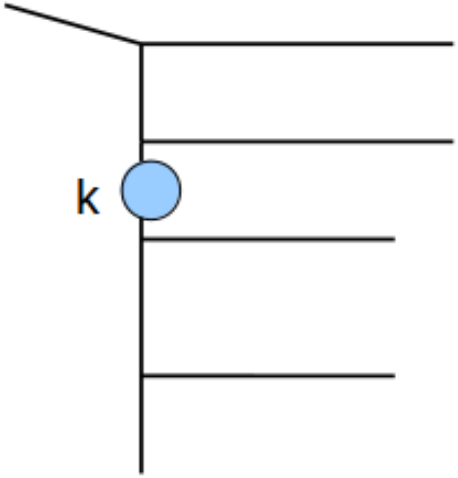
For off-shell quarks: represent q^* as coming from a $\gamma\bar{q}q$ vertex, with a 0 momentum and $\bar{q}$ on shell (and vice-versa)



- embed the scattering of the quark with whatever set of particles in the scattering of an auxiliary quark-photon pair, q_A and γ_A carrying momenta $p^\mu_{q_A} = k_1^\mu$, $p^\mu_{\gamma_A} = 0$
- Let q_A-propagators of momentum k be $\frac{i\,\not{p}_1}{p_1 \cdot k}$ and assign the spinors $|p_1\rangle, |p_1]$ to the A-quark.
- Assign the polarization vectors $\epsilon^\mu_+ = \frac{\langle q|\gamma^\mu|p_1]}{\sqrt{2}\langle p_1 q\rangle}$, $\epsilon^\mu_- = \frac{\langle p_1|\gamma^\mu|q]}{\sqrt{2}[p_1 q]}$ to the auxiliary photon, with q a light-like auxiliary momentum.
- Multiply the amplitude by $x_1\sqrt{-k_{1\perp}^2/2}$ and use ordinary Feynman rules everywhere else.

K. Kutak, T. Salwa, A. van Hameren, Phys.Lett. B727 (2013) 226-233

Our PDFs: the KMR prescription



Survival probability without emissions

Kimber, Martin, Ryskin prescription, '01 :

μ

$$T_s(\mu^2, k^2) =$$

$$exp\left(- \int_{\mu^2}^{k^2} \frac{dk'^2}{k'^2} \frac{\alpha_s(k'^2)}{2\pi} \times \sum_{a'} \int_0^{1-\Delta} dz' P_{aa'}(z') \right)$$

$$\Delta = \frac{\mu}{\mu + k}, \quad \mu = \text{hard scale}$$

$$\mathcal{F}(x, k^2, \mu^2) \sim \partial_{\lambda^2} \left(T_s(\lambda^2, \mu^2) \, x \, g(x, \lambda^2) \right) \Big|_{\lambda^2 = k^2}$$

DLC2016v2 (Double Log Coherence)

K. Kutak, R. Maciula, M. S., A. Szczurek, A. van Hameren, JHEP 1604 (2016) 175

K. Kutak, R. Maciula, M. S., A. Szczurek, A. van Hameren, arXiv:1605.08240

Conjectured formulas for 2 and 4 jets production:

$$\sigma_{2-jets} = \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d^2k_{T1} d^2k_{T2} \mathcal{F}_i(x_1, k_{T1}, \mu_F) \mathcal{F}_j(x_2, k_{T2}, \mu_F)$$
$$\times \frac{1}{2\hat{s}} \prod_{l=i}^{2} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{2-jet} (2\pi)^4 \delta \left(P - \sum_{l=1}^{2} k_i \right) \overline{|\mathcal{M}(i^*, j^* \to 2\,\text{part.})|^2}$$

$$\sigma_{4-jets} = \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d^2k_{T1} d^2k_{T2} \mathcal{F}_i(x_1, k_{T1}, \mu_F) \mathcal{F}_j(x_2, k_{T2}, \mu_F)$$
$$\times \frac{1}{2\hat{s}} \prod_{l=i}^{4} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{4-jet} (2\pi)^4 \delta \left(P - \sum_{l=1}^{4} k_i \right) \overline{|\mathcal{M}(i^*, j^* \to 4\,\text{part.})|^2}$$

- PDFs and matrix elements well defined.
- No factorization rigorous proof around (not even in the collinear case, actually)
- Reasonable description of data justifies this formula *a posteriori*

Our framework

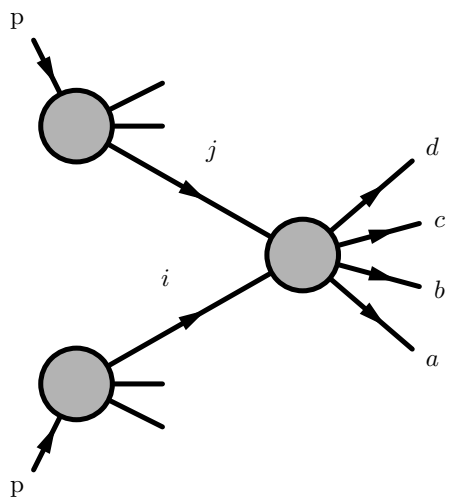
AVHLIB (A. van Hameren) : https://bitbucket.org/hameren/avhlib

- complete Monte Carlo program for tree-level calculations
- any process within the Standard Model
- any initial-state partons on-shell or off-shell
- employs numerical Dyson-Schwinger recursion to calculate helicity amplitudes
- automatic phase space optimization

- **Flavour scheme**: $N_f = 5$
- **Running** α_s from the MSTW68cl PDF sets
- **Massless quarks approximation** $E_{cm} = 7/8\,TeV \Rightarrow m_{q/\bar{q}} = 0$.
- **Scale** $\mu_R = \mu_F \equiv \mu = \frac{H_T}{2} \equiv \frac{1}{2}\sum_i p_T^i$, (sum over final state particles)

We don't take into account correlations in DPS: $D(x_1, x_2, \mu) = f(x_1, \mu)\, f(x_2, \mu)$.
There are attempts to go beyond this approximation:
Golec-Biernat, Lewandowska, Snyder, M.S., Stasto, Phys.Lett. B750 (2015) 559-564
Rinaldi, Scopetta, Traini, Vento, JHEP 1412 (2014) 028

4-jet production: Single Parton Scattering (SPS)



We take into account all the (according to our conventions) 20 channels.

Here u and d stand for different quark flavours in the initial (final) state.

We do not introduce K factors, amplitudes@LO.

$\sim$ 95 % of the total cross section

There are 19 different channels contributing to the cross section at the parton-level:

$$gg \to 4g\,, gg \to q\bar{q}\,2g\,, qg \to q\,3g\,, q\bar{q} \to q\bar{q}\,2g\,, qq \to qq\,2g\,, qq' \to qq'\,2g\,,$$
$$gg \to q\bar{q}q\bar{q}\,, gg \to q\bar{q}q'\bar{q}'\,, qg \to qgq\bar{q}\,, qg \to qgq'\bar{q}'\,,$$
$$q\bar{q} \to 4g\,, q\bar{q} \to q'\bar{q}'\,2g\,, q\bar{q} \to q\bar{q}q\bar{q}\,, q\bar{q} \to q\bar{q}q'\bar{q}'\,, q\bar{q} \to q'\bar{q}'q'\bar{q}'\,,$$
$$q\bar{q} \to q'\bar{q}'q''\bar{q}''\,, qq \to qqq\bar{q}\,, qq \to qqq'\bar{q}'\,, qq' \to qq'q\bar{q}\,,$$

Introducing Double Parton Scattering

DPS $\equiv$ the simultaneous occurrence of two partonic hard scatterings in the same proton-proton collision

Double parton scattering cross section:

$$\sigma^D = \mathcal{S} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2, b; t_1, t_2)\, \Gamma_{kl}(x_1', x_2', b; t_1, t_2)\, \hat{\sigma}(x_1, x_1')\, \hat{\sigma}(x_2, x_2')\, dx_1 dx_2 dx_1' dx_2' d^2 b$$

Usual assumption: separation of longitudinal and transverse DOFs:

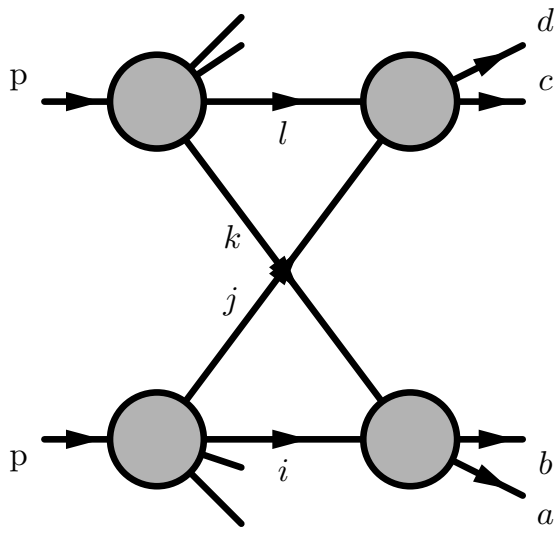
$$\Gamma_{ij}(x_1, x_2, b; t_1, t_2) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F^{ij}(b) = D_h^{ij}(x_1, x_2; t_1, t_2)\, F(b)$$

- Longitudinal correlations, most often ignored or assumed to be negligible, especially at small x : $D_h^{ij}(x_1, x_2; t_1, t_2) = D^i(x_1; t_1)\, D^j(x_2; t_2)$
- Transverse correlation, assumed to be independent of the parton species, taken into account via $\sigma_{eff}^{-1} = \int d^2 b\, F(b)^2 \approx 15 mb$ (CDF and D0)

Usual final kind-of-crafty formula:

$$\sigma^D = \frac{\mathcal{S}}{\sigma_{eff}} \sum_{i_1, j_1, k_1, l_1; i_2, j_2, k_2, l_2} \sigma(i_1 j_1 \to k_1 l_1) \times \sigma(i_2 j_2 \to k_2 l_2)$$

4-jet production: Double parton scattering (DPS)



$$\sigma = \sum_{i,j,a,b;k,l,c,d} \frac{\mathcal{S}}{\sigma_{eff}} \sigma(i,j \to a,b)\, \sigma(k,l \to c,d)$$

$$\mathcal{S} = \begin{cases} 1/2 & \text{if} \quad i\,j = k\,l \quad \text{and} \quad a\,b = c\,d \\ 1 & \text{if} \quad i\,j \neq k\,l \quad \text{or} \quad a\,b \neq c\,d \end{cases}$$

$$\sigma_{eff} = 15\, mb\,,$$

Experimental data may hint at different values of σ_{eff}; main conclusions not affected

In our conventions, 9 channels from $2 \to 2$ SPS events,

$$\begin{aligned} \#1 &= gg \to gg\,, \quad \#6 = u\bar{u} \to d\bar{d} \\ \#2 &= gg \to u\bar{u}\,, \quad \#7 = u\bar{u} \to gg \\ \#3 &= ug \to ug\,, \quad \#8 = uu \to uu \\ \#4 &= gu \to ug\,, \quad \#9 = ud \to ud \\ \#5 &= u\bar{u} \to u\bar{u} \end{aligned}$$

$\Rightarrow$ 45 channels for the DPS; only 14 contribute to $\geq 95\%$ of the cross section :

$$(1,1),(1,2),(1,3),(1,4),(1,8),(1,9),(3,3)$$
$$(3,4),(3,8),(3,9),(4,4),(4,8),(4,9),(9,9)$$

Hard jets

We reproduce all the LO results (only SPS) for $p\,p \to n\,jets\,, \quad n = 2, 3, 4$ published in
BlackHat collaboration, Phys.Rev.Lett. 109 (2012) 042001
S. Badger et al., Phys.Lett. B718 (2013) 965-978

Asymmetric cuts for hard central jets

$$p_T \geq 80\,\text{GeV}\,, \quad \text{for leading jet}$$
$$p_T \geq 60\,\text{GeV}\,, \quad \text{for non leading jets}$$
$$|\eta| \leq 2.8\,, \quad R = 0.4$$

PDFs set: MSTW2008LO@68cl

$$\sigma(\geq 2\,\text{jets}) = 958^{+316}_{-221} \quad \sigma(\geq 3\,\text{jets}) = 93.4^{+50.4}_{-30.3} \quad \sigma(\geq 4\,\text{jets}) = 9.98^{+7.40}_{-3.95}$$

Cuts are too hard to pin down DPS and/or benefit from HEF: 4-jet case

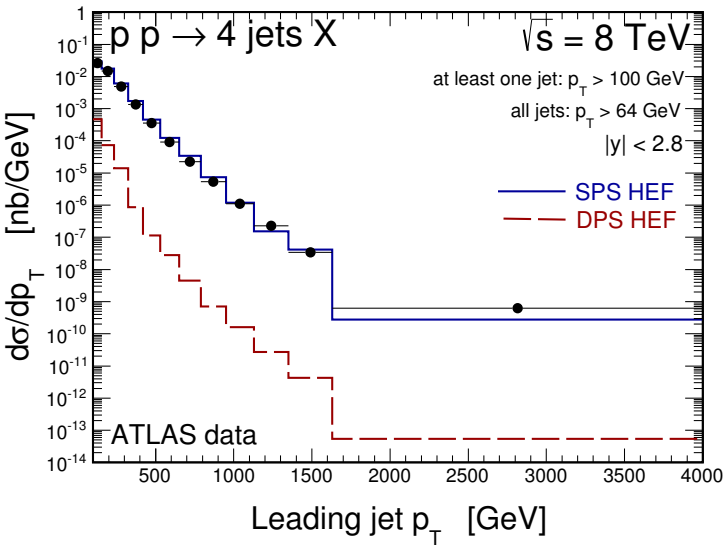
Collinear case $\begin{cases} 9.98^{+7.40}_{-3.95} & SPS \\ 0.094^{+0.06}_{-0.036} & DPS \end{cases}$ HEF case $\begin{cases} 10.0^{+6.9}_{-5.3} & SPS \\ 0.05^{+0.054}_{-0.029} & DPS \end{cases}$

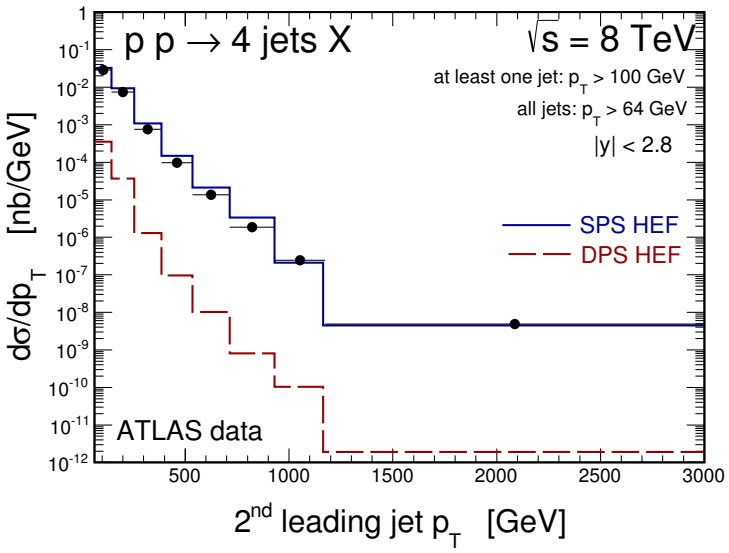
Differential cross section

Most recent ATLAS paper on 4-jet production in proton-proton collision:

$$p_T(1) \geq 100 GeV \, , p_T(2,3,4) \geq 64 GeV \, , |\eta| \geq 2.8, \Delta R \geq 0.65$$

ATLAS, JHEP 1512 (2015) 105





- All channels included and running α_s @ NLO
- Good agreement with data
- DPS effects are manifestly too small for such hard cuts: this could be expected.

DPS effects in collinear and HEF

Inspired by Maciula, Szczurek, Phys.Lett. B749 (2015) 57-62
DPS effects are expected to become significant for lower p_T cuts, like the ones of the CMS collaboration, Phys.Rev. D89 (2014) no.9, 092010

$$p_T(1,2) \geq 50\,\text{GeV}\,, \quad p_T(3,4) \geq 20\,\text{GeV}\,, \quad |\eta| \leq 4.7\,, \quad R = 0.5$$

CMS collaboration :	$\sigma_{tot} = 330 \pm 5\,(\text{stat.}) \pm 45\,(\text{syst.})\,nb$
LO collinear factorization :	$\sigma_{SPS} = 697\,nb\,, \quad \sigma_{DPS} = \mathbf{125\,nb}\,, \quad \sigma_{tot} = 822\,nb$
LO k_T-factorization :	$\sigma_{SPS} = 548\,nb\,, \quad \sigma_{DPS} = \mathbf{33\,nb}\,, \quad \sigma_{tot} = 581\,nb$

In HE factorization DPS gets suppressed and does not dominate at low p_T

Counterintuitive result from well-tested perturbative framework $\Rightarrow$ phase space effect ?

An old problem: higher order corrections to 2-jet production

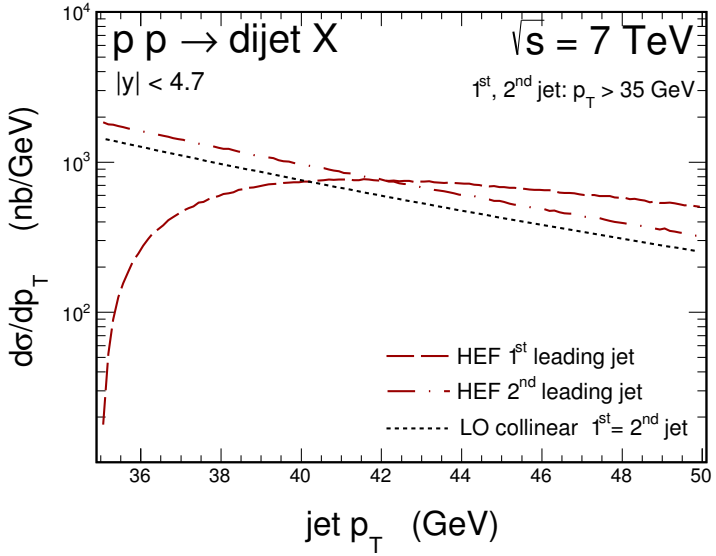


Figure: The transverse momentum distribution of the leading (long dashed line) and subleading (long dashed-dotted line) jet for the dijet production in HEF.

NLO corrections to 2-jet production suffer from instability problem when using symmetric cuts: Frixione, Ridolfi, Nucl.Phys. B507 (1997) 315-333

Symmetric cuts rule out from integration final states in which the momentum imbalance due to the initial state non vanishing transverse momenta gives to one of the jets a lower transverse momentum than the threshold.

ATLAS data vs. theory (nb) @ LHC7 for 2,3,4 jets. Cuts are defined in Eur.Phys.J. C71 (2011) 1763; theoretical predictions from Phys.Rev.Lett. 109 (2012) 042001

#jets	*ATLAS*	*LO*	*NLO*
2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	$1193(3)^{+130}_{-135}$
3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	$54.5(0.5)^{+2.2}_{-19.9}$
4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	$5.54(0.12)^{+0.08}_{-2.44}$

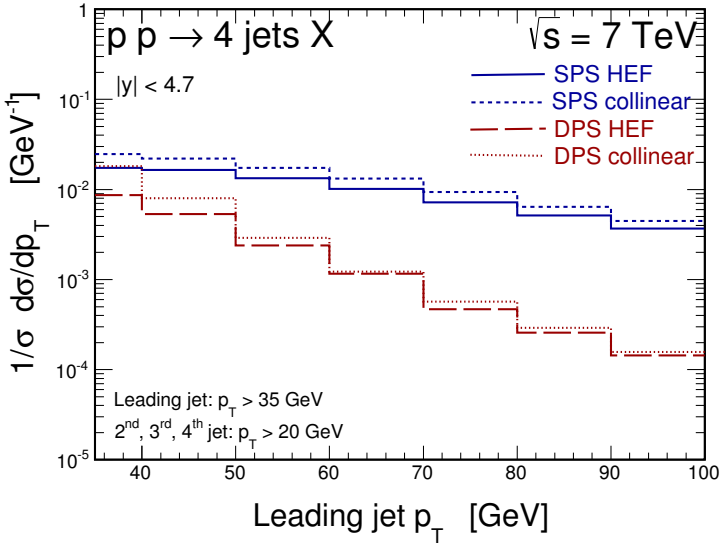
Reconciling HE and collinear factorisation: asymmetric p_T cuts

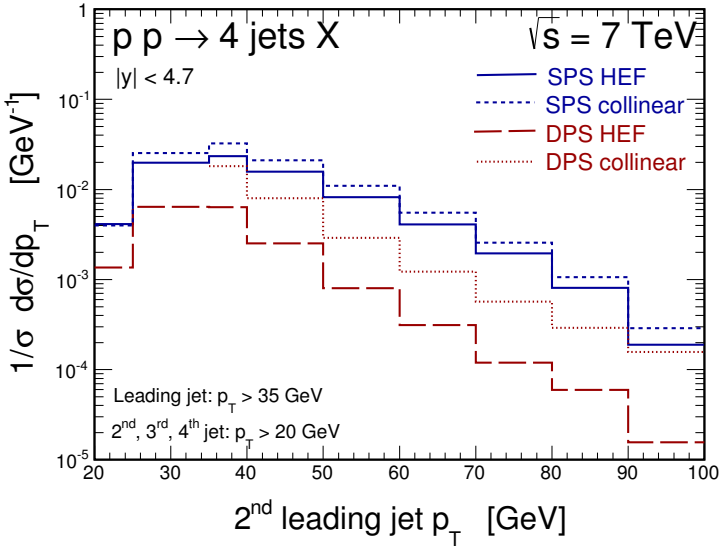
In order to open up wider region of soft final states and thereof expected that the DPS contribution increases

$p_T(1) \geq 35\text{GeV}\,, \quad p_T(2,3,4) \geq 20\,\text{GeV}\,, |\eta| < 4.7\,, \quad \Delta R > 0.5$

LO collinear factorization : $\sigma_{SPS} = 1969\,nb\,, \quad \sigma_{DPS} = 514\,nb\,, \quad \sigma_{tot} = 2309\,nb$

LO HEF k_T-factorization : $\sigma_{SPS} = 1506\,nb\,, \quad \sigma_{DPS} = 297\,nb\,, \quad \sigma_{tot} = 1803\,nb$



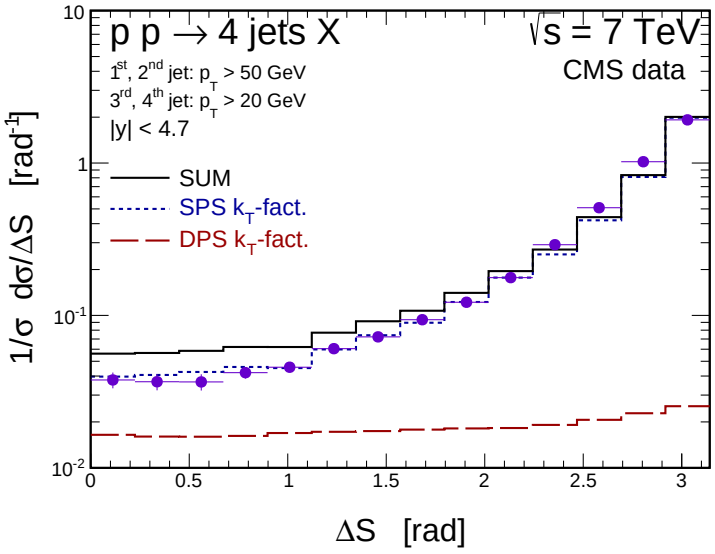


DPS dominance pushed to even lower p_T but restored in HE factorization as well

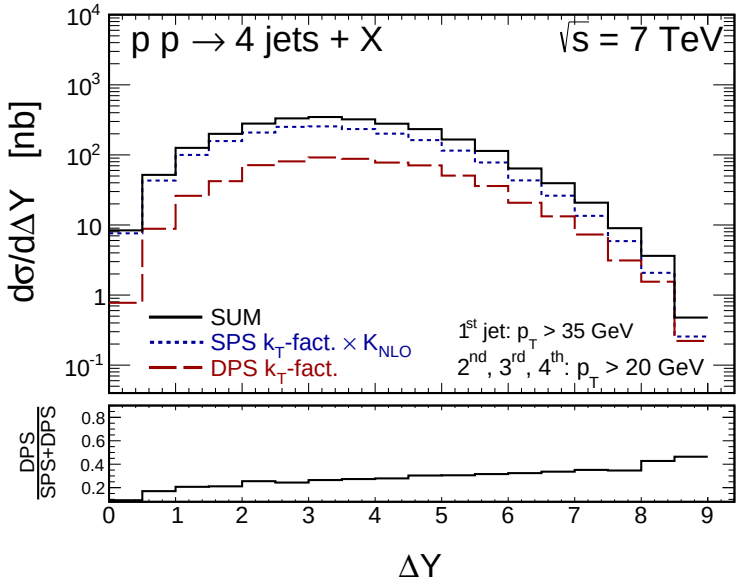
Pinning down double parton scattering: ΔS

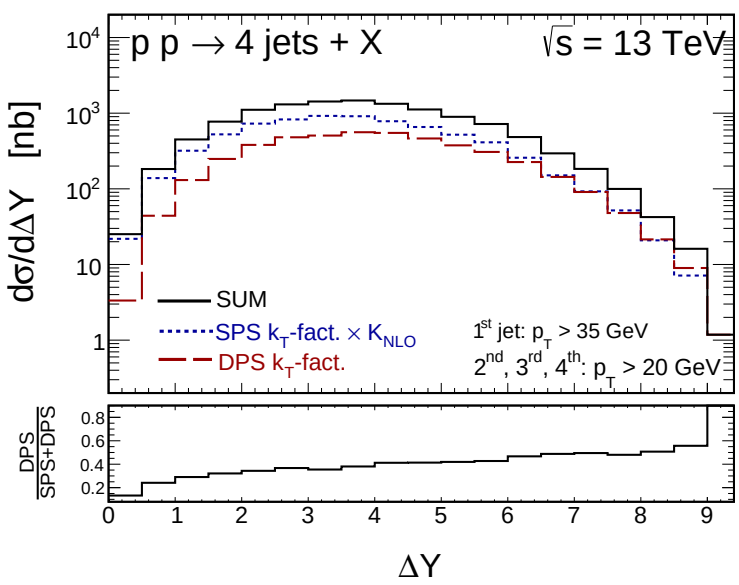
$$\Delta S = \arccos\left(\frac{\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}}) \cdot \vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})}{|\vec{p}_T(j_1^{\text{hard}}, j_2^{\text{hard}})| \cdot |\vec{p}_T(j_1^{\text{soft}}, j_2^{\text{soft}})|}\right), \quad \vec{p}_T(j_i, j_k) = p_{T,i} + p_{T,j}$$

We roughly describe the data via pQCD effects within HEF, which are (partially as well) described by parton-showers and soft MPIs by CMS.
CMS collaboration Phys.Rev. D89 (2014) no.9, 092010



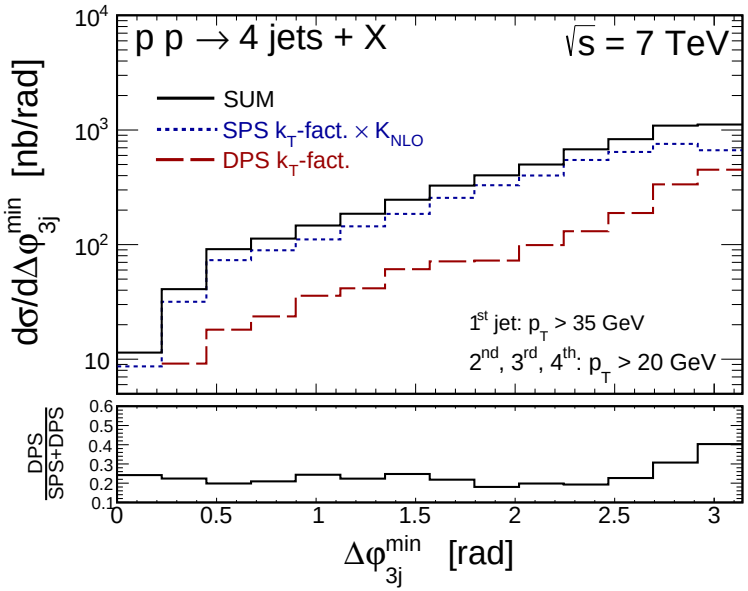
Pinning down double parton scattering: large rapidity separation

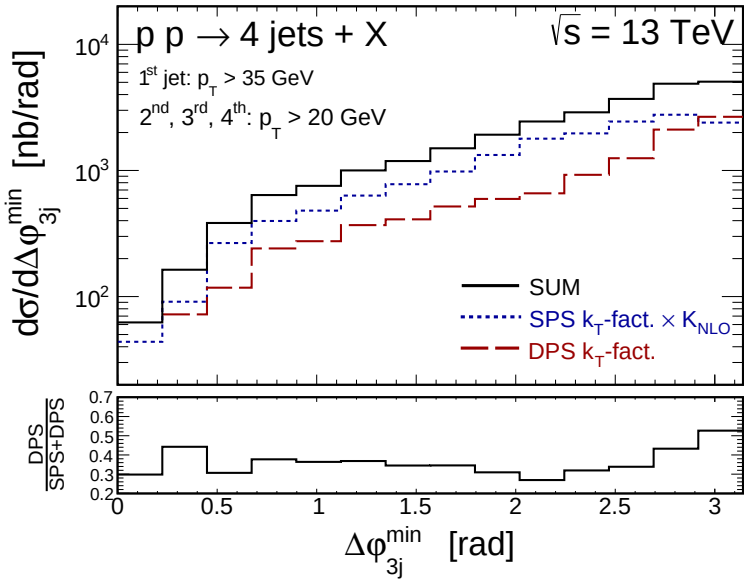




- It is interesting to look for kinematic variables which could make DPS apparent.
- The maximum rapidity separation in the four jet sample is one such variable, especially at 13 GeV.
- for $\Delta Y > 6$ the total cross section is dominated by DPS.

Pinning down double parton scattering: $\Delta\phi_3^{min}$ - azimuthal separation





- Definition: $\Delta\phi_3^{min} = min_{i,j,k[1,4]}\left(\left|\phi_i - \phi_j\right| + \left|\phi_j - \phi_k\right|\right)\ , \quad i \neq j \neq k$
- Proposed by ATLAS in *JHEP 12 105 (2015)* for high p_T analysis
- High values favour configurations closer to back-to-back, i.e. DPS
- In the high bins the total cross section is twice as much as the SPS one

Summary and conclusions

- We have a complete framework for the evaluation of cross sections from amplitudes with off-shell quarks and TMDs via KMR procedure obtained from NLO collinear PDFs
- HE factorisation reproduces well ATLAS data @ 7 and 8 TeV for hard central inclusive 4-jet production. Essential agreement with collinear predictions.
- HE factorisation smears out the DPS contribution to the cross section for less central jet, pushing the DPS-dominance region to lower p_T, but asymmetric cuts are in order: initial state transverse momentum generates asymmetries in the p_T of final state jet pairs.
- It would be interesting to have an experimental analysis with cuts which are *asymmetric* **and** *soft*.
- Further insight into HE factorisation prediction will come with progress in NLO results and with the addition of final state paton showers. Work in progress...

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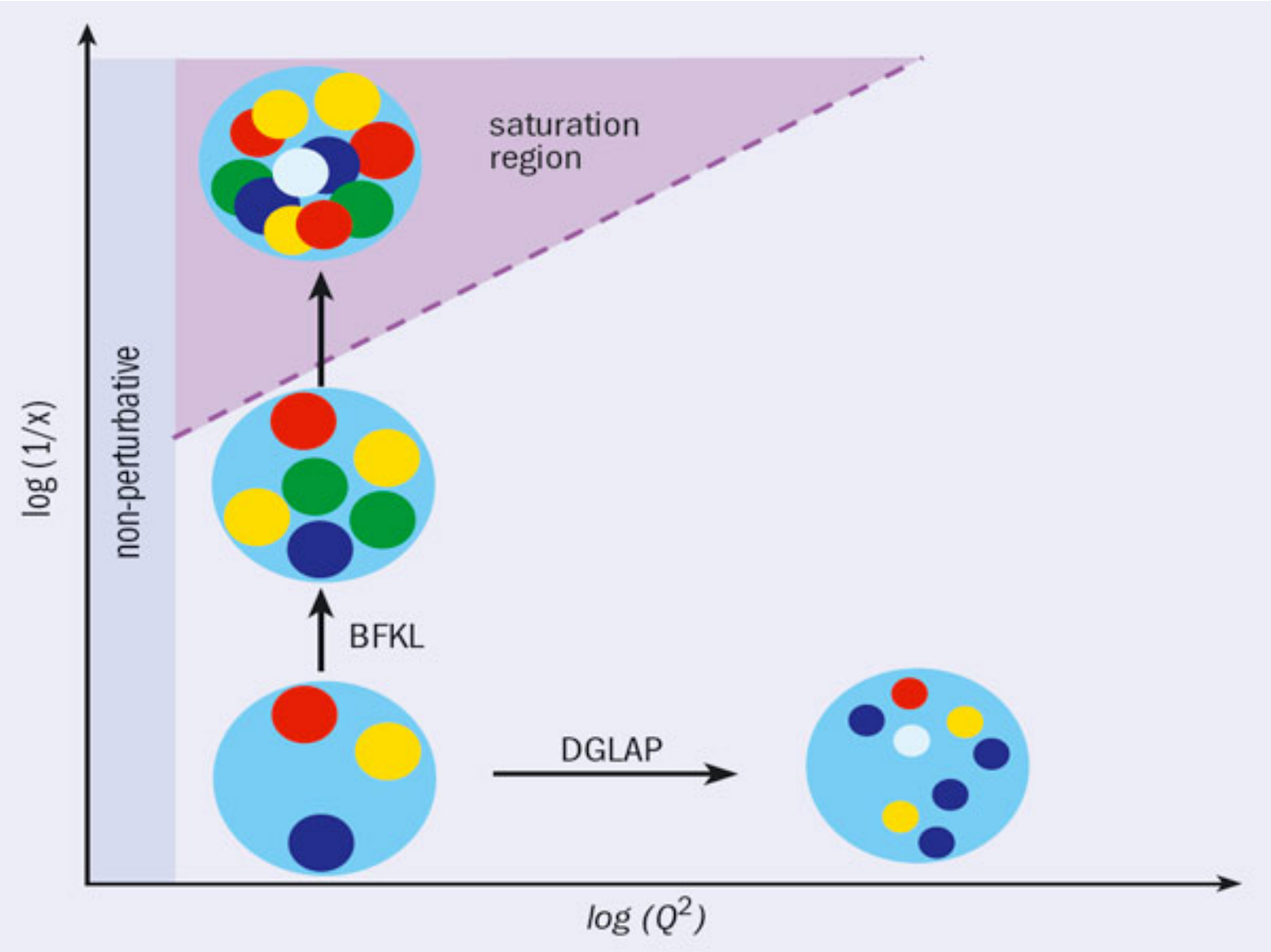
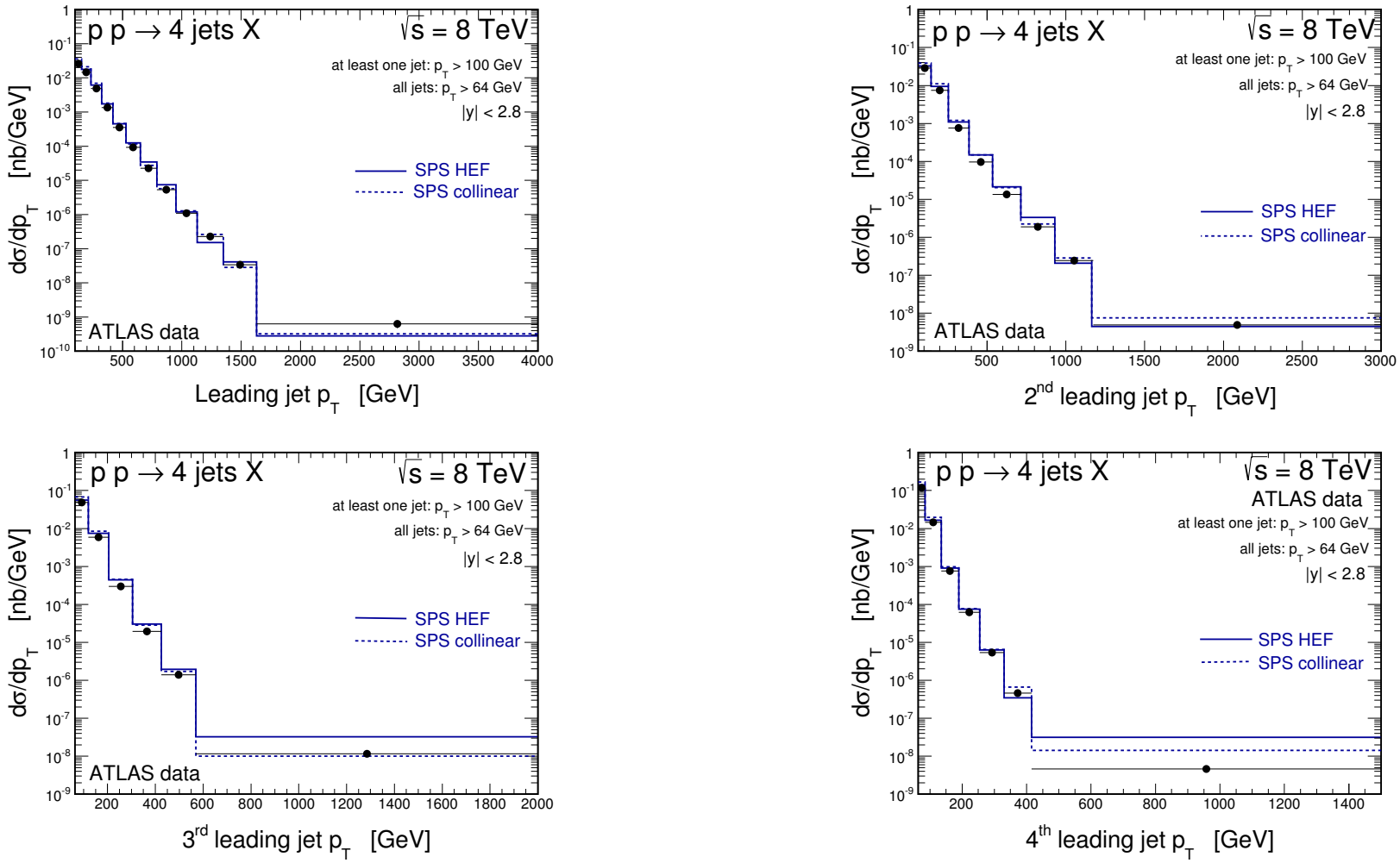
Thank you for your attention !

Smal-x physics vs. DGLAP framework



Comparing collinear factorization and HEF



Collinear factorization performs slightly better for intermediate values and HEF does a better job for the last bins, except for the 4th jet.