IntraBeam Scattering Calculation

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SuperB Workshop

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Calculations procedure

1. Evaluate equilibrium emittances ε_i and radiation damping times τ_i at low bunch charge
2. Evaluate the IBS growth rates $1/T_i(\varepsilon_i)$ for the given emittances, averaged around the lattice, using K. Bane approximation (EPAC02)
3. Calculate the "new equilibrium" emittance from:

$$\varepsilon_i' = \frac{1}{1-\tau_i/T_i}\varepsilon_i$$

- For the vertical emittance use* :

$$\varepsilon_y' = (1-r_\varepsilon)\frac{1}{1-\tau_y/T_y}\varepsilon_y + r_\varepsilon\frac{1}{1-\tau_x/T_x}\varepsilon_y$$

- where r_ε varies from 0 (ε_y generated from dispersion) to 1 (ε_y generated from betatron coupling)

4. Iterate from step 2

* K. Kubo, S.K. Mtingwa, A. Wolski, "Intrabeam Scattering Formulas for High Energy Beams," Phys. Rev. ST Accel. Beams **8**, 081001 (2005)

Bane's approximation

- K. Bane, "A Simplified Model of Intrabeam Scattering," Proceedings of EPAC 2002, Paris, France (2002)

$$\frac{1}{T_p} \approx \frac{r_0^2 c N (\log)}{16\gamma^3 \epsilon_x^{3/4} \epsilon_y^{3/4} \sigma_s \sigma_p^3} \left\langle \sigma_H \, g(a/b) \, (\beta_x \beta_y)^{-1/4} \right\rangle$$

$$\frac{1}{T_{x,y}} \approx \frac{\sigma_p^2 \langle \mathcal{H}_{x,y} \rangle}{\epsilon_{x,y}} \frac{1}{T_p}$$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{\mathcal{H}_x}{\epsilon_x} + \frac{\mathcal{H}_y}{\epsilon_y}$$

$$(\log)_i = Coulomb \log$$

$$g(\alpha) \approx \alpha^{(0.021 - 0.044 \ln \alpha)} \qquad [\text{for } 0.01 < \alpha < 1]$$

$$a = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_x}{\epsilon_x}}, \qquad b = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_y}{\epsilon_y}}$$

Completely-Integrated Modified Piwinski formulae

K. Kubo, S.K. Mtingwa, A. Wolski, "Intrabeam Scattering Formulas for High Energy Beams," Phys. Rev. ST Accel. Beams **8**, 081001 (2005).

$$\frac{1}{T_p} \approx 2\pi^{3/2} A(\log)\left\langle \frac{\sigma_H^2}{\sigma_p^2}\left(\frac{g(b/a)}{a} + \frac{g(a/b)}{b}\right)\right\rangle,$$

$$\frac{1}{T_h} \approx 2\pi^{3/2} A(\log)\left\langle -ag\left(\frac{b}{a}\right) + \frac{\mathcal{H}_h \sigma_H^2}{\varepsilon_h}\left(\frac{g(b/a)}{a} + \frac{g(a/b)}{b}\right)\right\rangle$$

$$\frac{1}{T_v} \approx 2\pi^{3/2} A(\log)\left\langle -bg\left(\frac{a}{b}\right) + \frac{\mathcal{H}_v \sigma_H^2}{\varepsilon_v}\left(\frac{g(b/a)}{a} + \frac{g(a/b)}{b}\right)\right\rangle,$$

$$g(\omega) = \sqrt{\pi/\omega}\left[P^0_{-1/2}\left(\frac{\omega^2+1}{2\omega}\right) \pm \frac{3}{2} P^{-1}_{-1/2}\left(\frac{\omega^2+1}{2\omega}\right)\right],$$

$$(\log) \approx \ln\left[\frac{\gamma^2 \sigma_v \varepsilon_h}{r_0 \beta_h}\right]$$

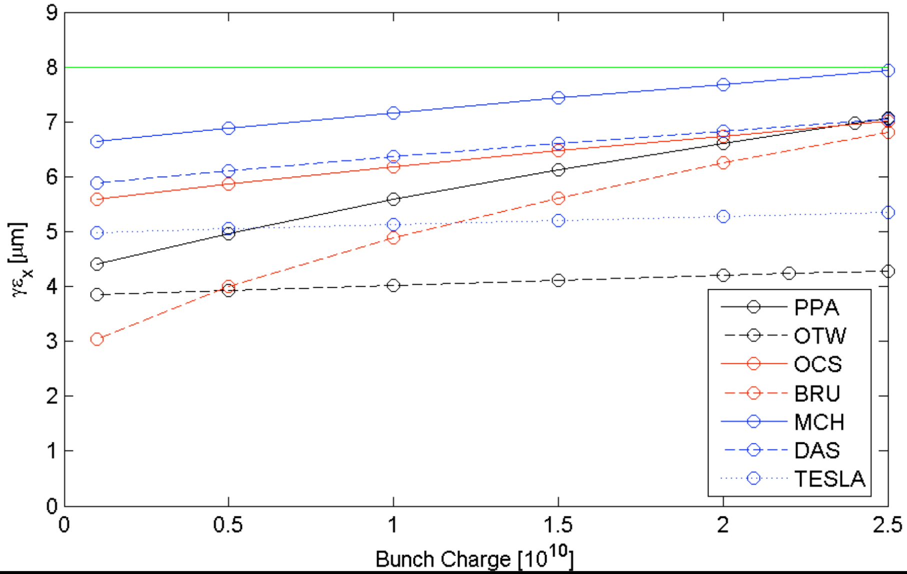
ILC Damping Ring OCS Parameters

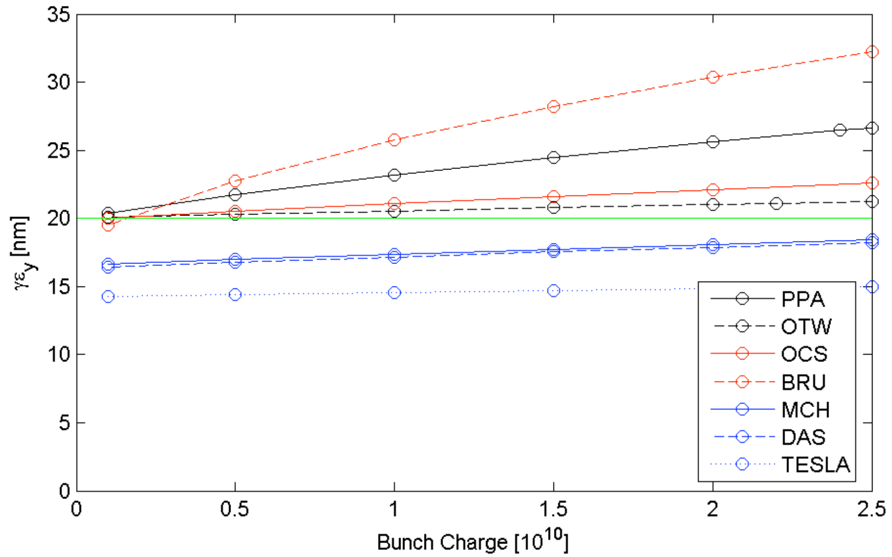
Energy (GeV)	5
Circumference (m)	6114
N particles/bunch	$2x10^{-10}$
Damping time τ_x (ms)	22
Emittance $\gamma\varepsilon_x$ (nm)	5500
Emittance $\gamma\varepsilon_y$ (nm)	20
Momentum compaction	$1.6\ x10^{-4}$
Energy spread	$1.3x10^{-3}$
Bunch length (mm)	6.0

To check the code the IBS effect calculated for the ILC damping ring OCS lattice has been compared with the results for the configuration options

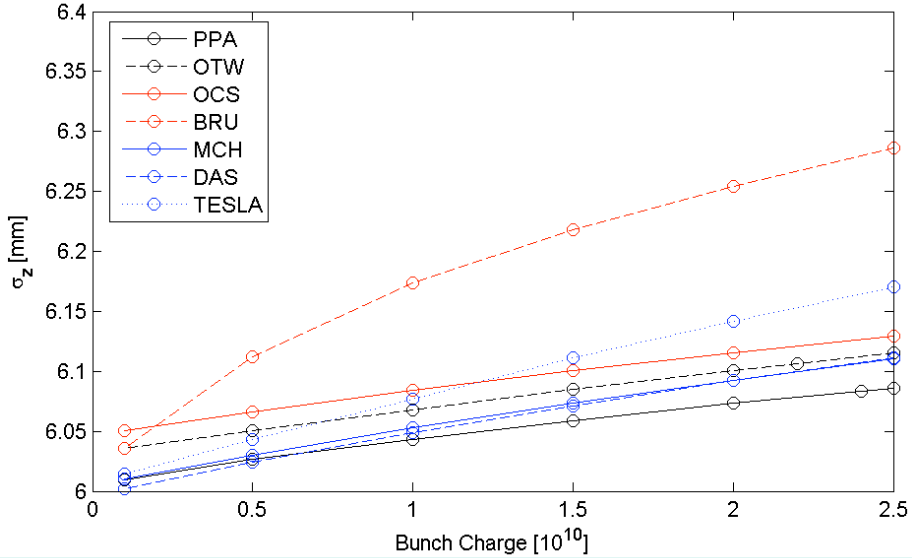
A. Wolski, J. Gao, S. Guiducci eds., "Configuration Studies and Recommendations for the ILC Damping Rings", LBNL–59449, February 2006 https://wiki.lepp.cornell.edu/ilc/pub/Public/DampingRings/ConfigStudy/DRConfigRecommend.pdf

Results for ILC damping ring configuration options



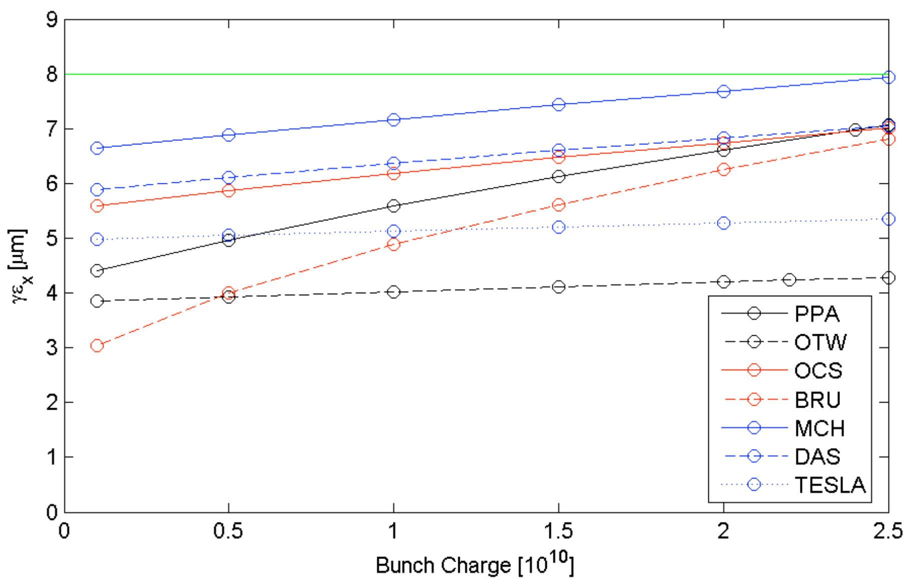


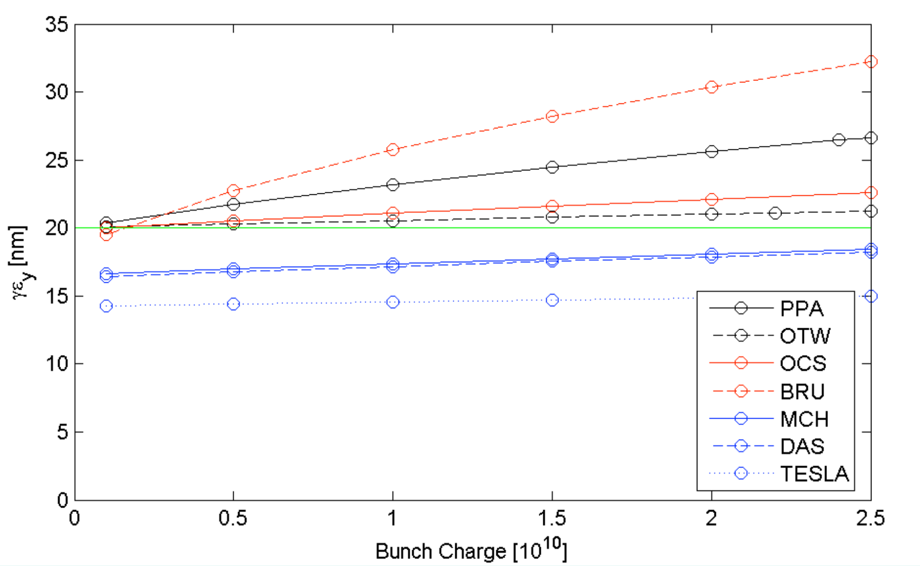
	OCS conf. options	OCS this calculation
N/bunch	$2\ 10^{10}$	$2\ 10^{10}$
$\varepsilon_x^{ibs}/\varepsilon_x^0$	1.2	1.23
$\varepsilon_y^{ibs}/\varepsilon_y^0$ @r_ε=0.5	1.1	1.12
$\sigma_p^{ibs}/\sigma_p^0$	1.01	1.01
$\sigma_l^{ibs}/\sigma_l^0$	1.01	1.01



A. Wolski

Results for ILC damping ring configuration options





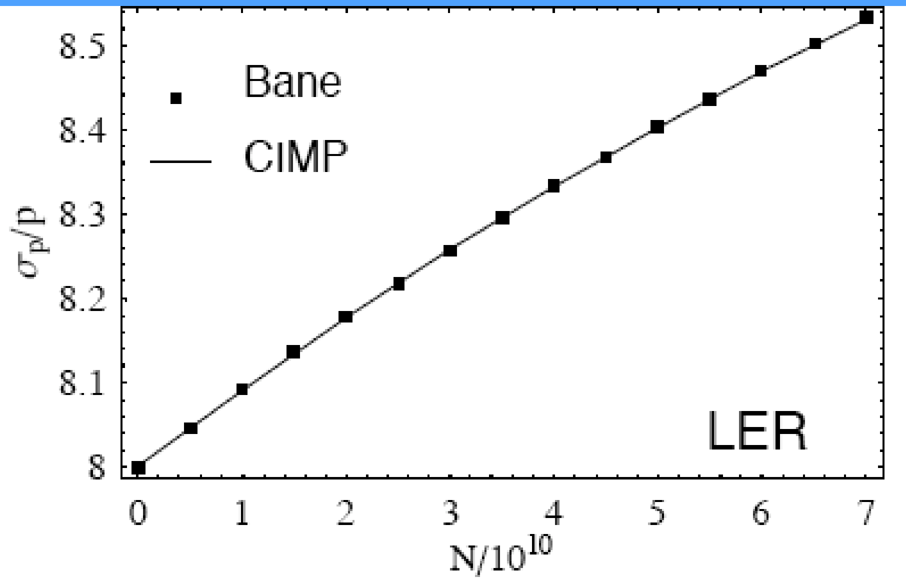
N	0.1e10	1.0e10	2.0e10
$\varepsilon_x/\varepsilon_{x0}$	1.01	1.12	1.23
$\varepsilon_y/\varepsilon_{y0}$	1.01	1.06	1.12
σ_p/σ_{p0}	1.00	1.003	1.006
ε_x(nm)	0.56	0.62	0.68
ε_y (pm)	2.01	2.12	2.23
σ_p	0.0013	0.0013	0.0013
σ_l (mm)	6.0	6.0	6.1

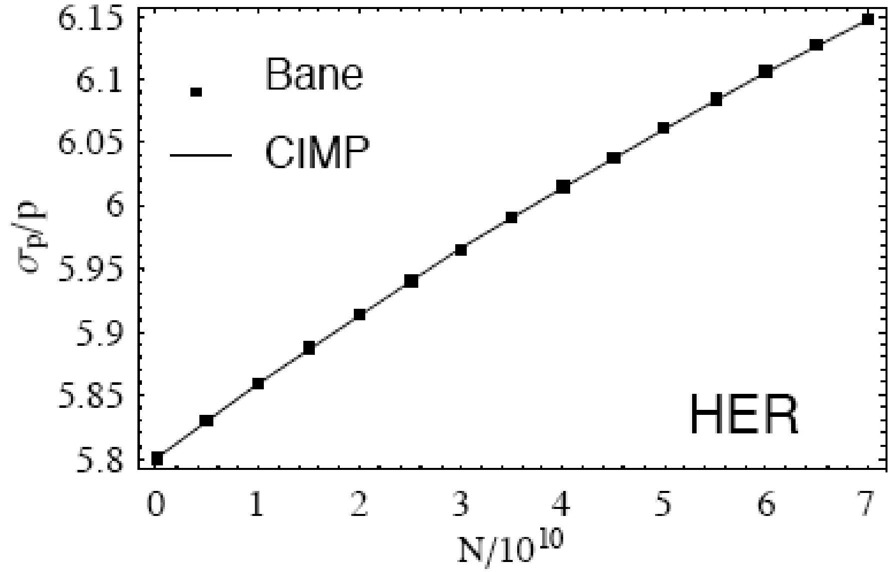
A. Wolski

SuperB parameters

	E[Gev]	ε_h[nm]	ε_v [pm]	σ_s [mm]	σ_p/p	τ_h/ τ_s [ms]
HER	7	1.6	4	5	5.8e-4	40/20
LER	4	2.8	7	5	8e-4	40/20

IBS momentum spread vs. number of particles/bunch





σ_p/σ_{p0}= 1.05, @ N=5.5•10^{10}

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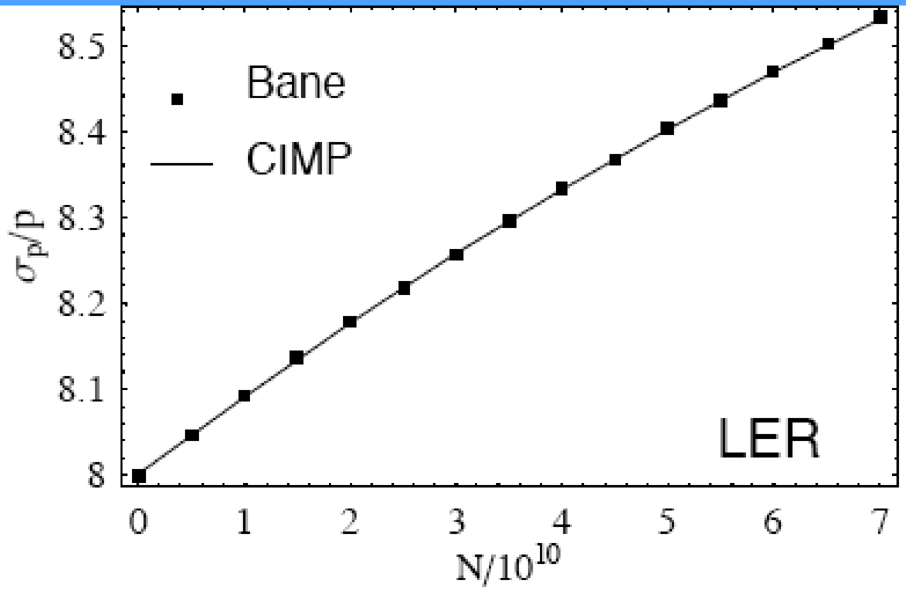
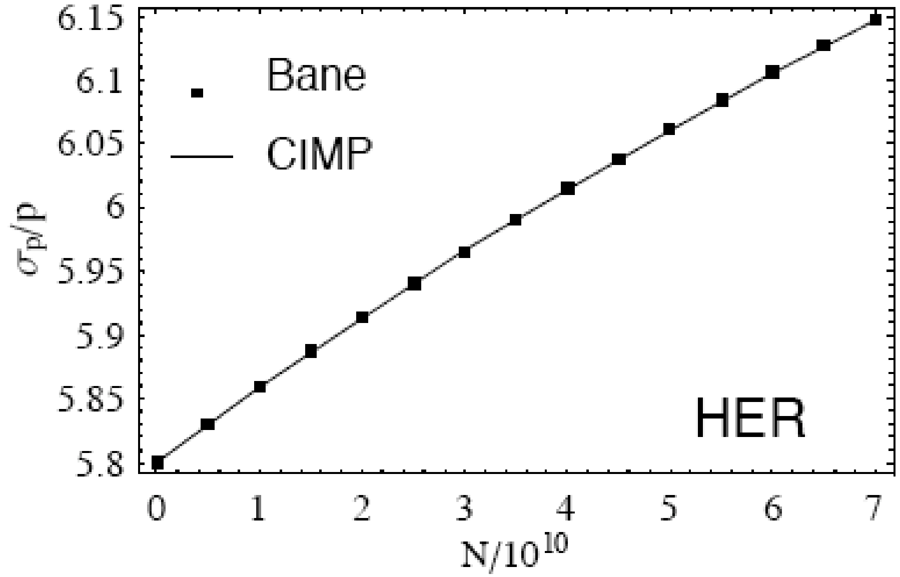
IBS horizontal emittance vs.number of particles/bunch

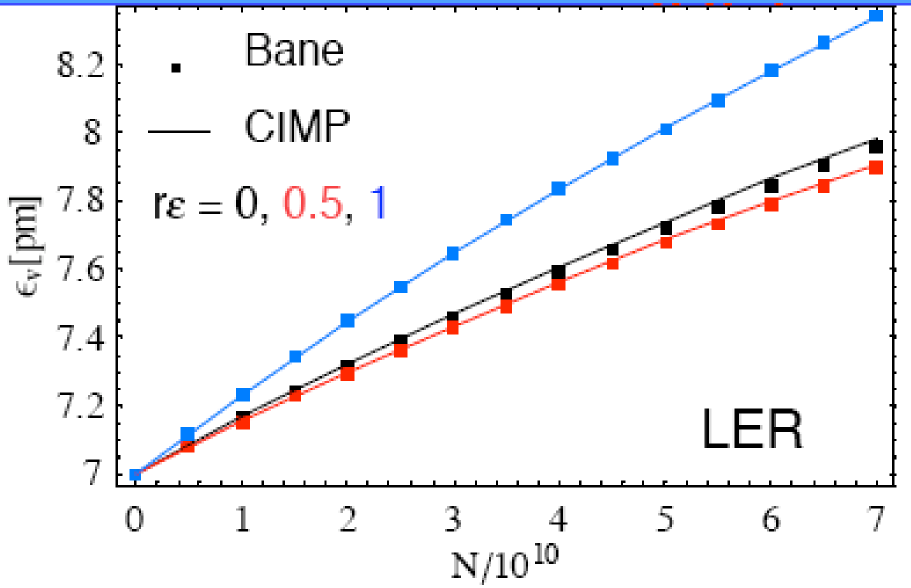


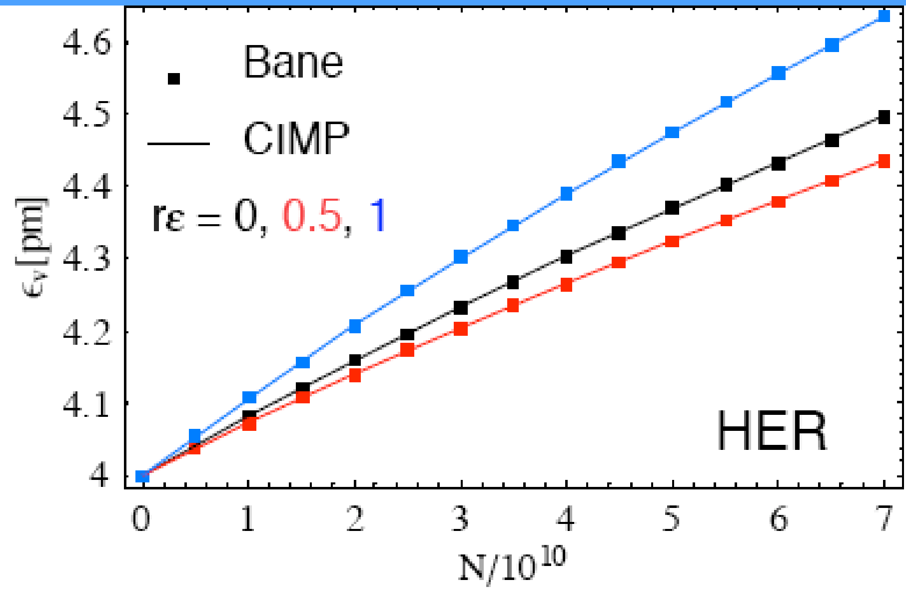


$\varepsilon_h/\varepsilon_{h0}$=1.14, @ N=5.52•$10^{10}$

$\varepsilon_h/\varepsilon_{h0}$=1.11, @ N=5.52•$10^{10}$

IBS vertical emittance vs. number of particles/bunch for r_ε = 0, 0.5, 1





N=5.52•10^{10}	
$\varepsilon_v/\varepsilon_{v0}$	r_ε
1.10	0
1.11	0.5
1.15	1

N=5.52•10^{10}	
$\varepsilon_v/\varepsilon_{v0}$	r_ε
1.08	0
1.10	0.5
1.13	1

LER lattice - IBS emittance growth @ N=5.5e10 with and without wigglers

	No IBS		With IBS	
B_{wig} (T)	0.	0.85	0.	0.85
V_{RF} (MV)	5.4	6.3	5.4	6.3
$\tau_{x,y}$ (ms)	39.0	32.4	39.0	32.4
τ_p (ms)	19.5	16.2	19.5	16.2
$\varepsilon_x/\varepsilon_{x0}$			1.15	1.18
$\varepsilon_y/\varepsilon_{y0}$			1.10	1.12
σ_p/σ_{p0}			1.06	1.06
ε_x	2.7e-9	2.3e-9	3.1e-9	2.7e-9
ε_y	7.0e-12	6.3e-12	7.7e-12	7.0e-12
σ_l	5.0	4.7	5.3	5.0
σ_p	8.1e-4	8.2e-4	8.6e-4	8.7e-4

12 wigglers

λ_{wig} = 0.4m

L_{wig} = 2.45m

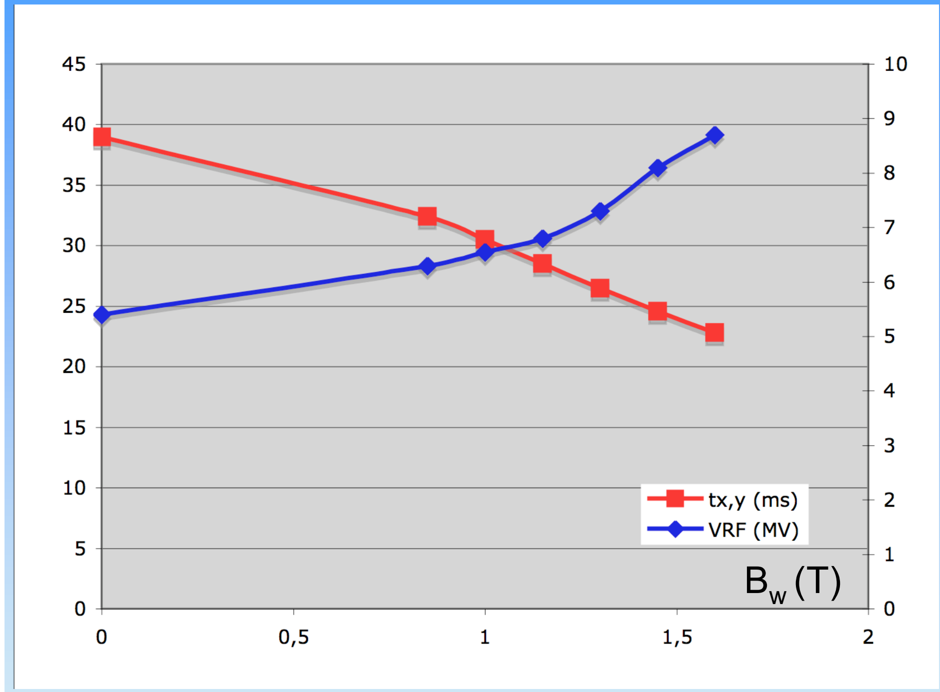
Nominal values

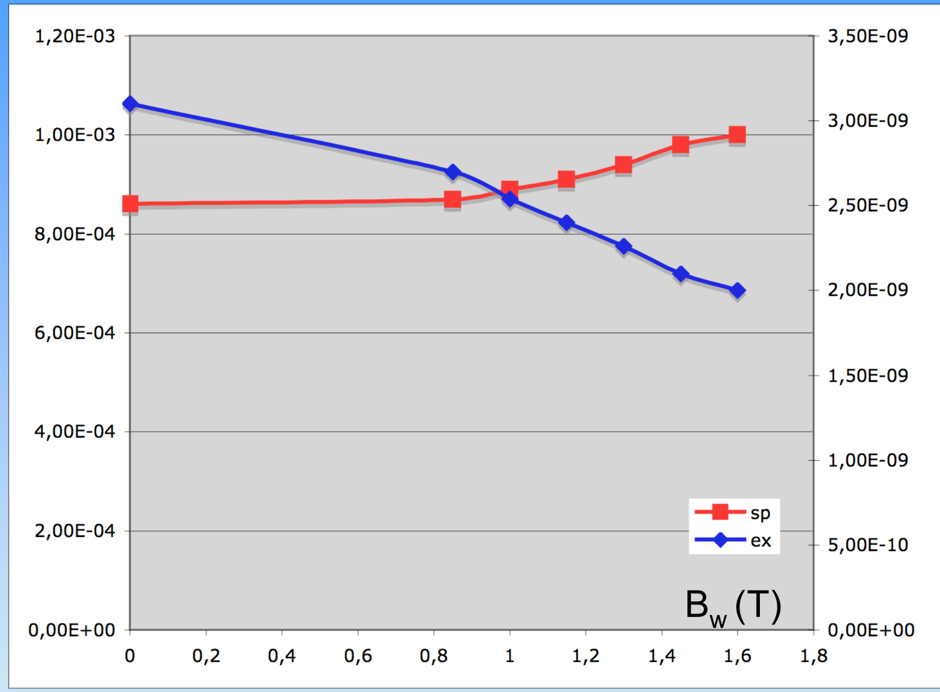
LER lattice - IBS emittance growth for different wiggler fields

B_{wig} (T)	0.	1.0	1.3	1.6
V_{RF} (MV)	5.4	6.6	7.3	8.7
$\tau_{x,y}$ (ms)	39.0	30.5	26.5	22.8
τ_p (ms)	19.5	15.3	13.3	11.4
$\varepsilon_x/\varepsilon_{x0}$	1.15	1.18	1.18	1.18
$\varepsilon_y/\varepsilon_{y0}$	1.10	1.12	1.11	1.11
σ_p/σ_{p0}	1.06	1.06	1.04	1.03
ε_{x0}	2.7e-9	2.1e-9	1.9e-9	1.7e-9
ε_{y0}	7.0e-12	6.3e-12	6.3e-12	6.3e-12
σ_{p0}	8.1e-4	8.4e-4	9.0e-4	9.9e-4
σ_{l0}	5.0	4.7	4.8	4.8
ε_x	3.1e-9	2.5e-9	2.3e-9	2.0e-9
ε_y	7.7e-12	7.0e-12	7.0e-12	7.0e-12
σ_p	8.6e-4	8.9e-4	9.4e-4	1.0e-3
σ_l	5.3	5.0	5.0	5.0

N=5.5e10

Insertion of Wigglers

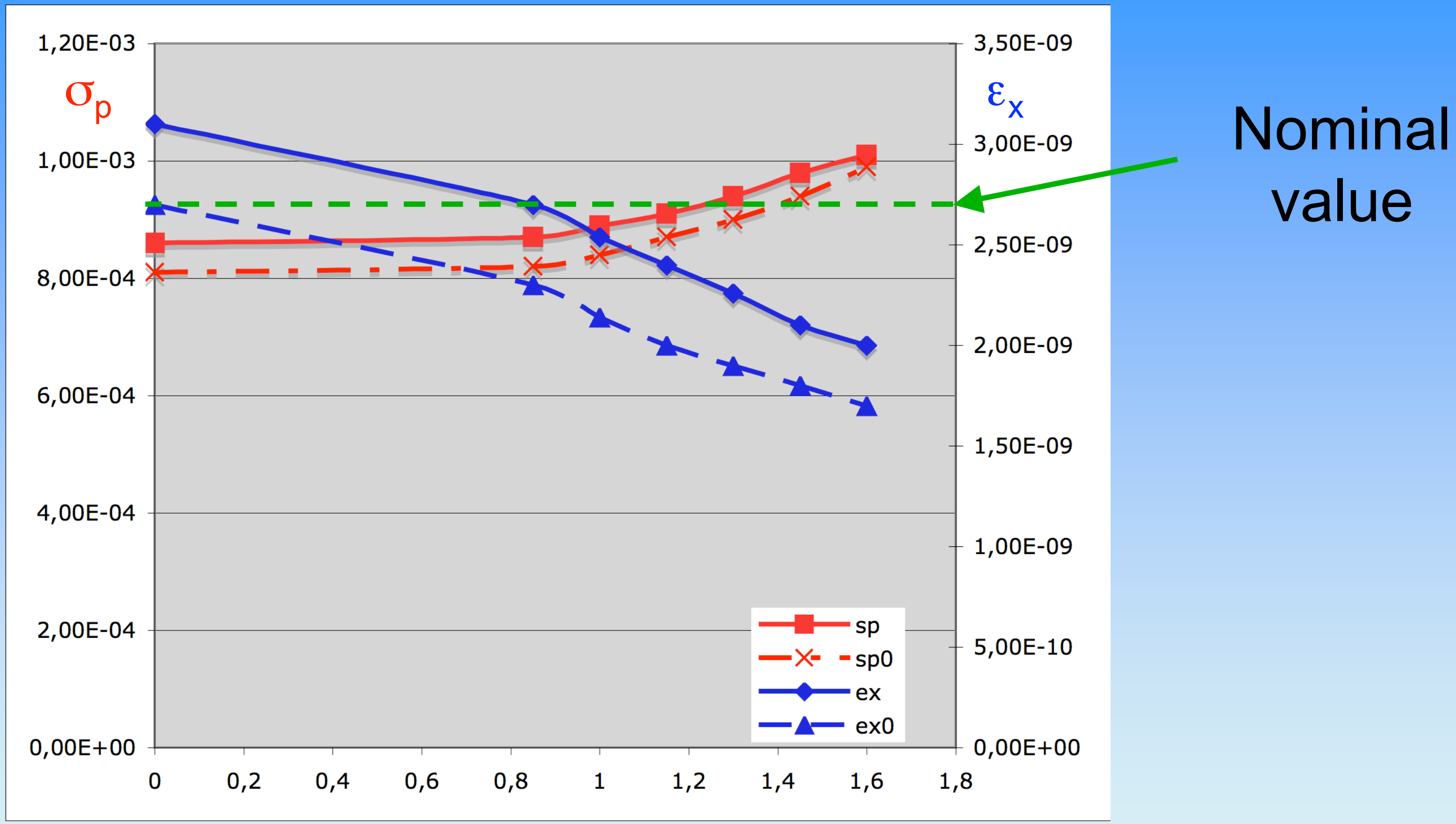




Damping time $\tau_{x,y}$ and RF voltage V_{RF} vs. wiggler field B_w

Relative energy spread σ_p and emittance ε_x vs. wiggler field B_w

$N = 5.5e\text{-}10$, $\sigma_l = 5\text{mm}$, $\varepsilon_y = 7\text{pm}$



Emittance and sigmap vs. wiggler field with (—) and without (---) IBS

Conclusions

- The effect of IBS on the transverse emittance is reasonably small for both rings
- It can be compensated by adding 12 low field wigglers to get the nominal transverse emittance at the expenses of an increased energy spread and RF voltage
- Another possibility to get the nominal emittance is to increase the phase advance in the arc cells
- An interesting study: develop a MonteCarlo code to study the beam size distribution in the presence of IBS