Coherent Synchrotron Radiation Calculations for the SuperB

Gabriele Bassi
University of Liverpool and the Cockcroft Institute, UK

1. SuperB LER Parameters for Longitudinal Dynamics
2. Longitudinal Dynamics with Steady State CSR Wake: Vlasov-Fokker-Planck Equation
3. A Method of Solution: Monte Carlo Particle Method
4. Numerical Studies
5. Discussion
6. Back-up Slides

SuperB LER parameters for longitudinal dynamics

Parameter	Symbol	Value	Unit
Energy reference particle	E_0	4	GeV
Circumference	C_0	1649.780	m
RF frequency	f_{rf}	513.89	MHz
RF Voltage	V_{rf}	8	MeV
Bunch length	σ_z	4.09	mm
Particles per bunch	N	5.52×10^{10}	
Momentum compaction	η	$3.26\ \times10^{-4}$	
1^{st} radiation integral	I_1	0.5379	m^{-1}
2^{nd} radiation integral	I_2	0.3167	m^{-1}
3^{rd} radiation integral	I_3	0.01793	m^{-2}
4^{th} radiation integral	I_4	1.29×10^{-3}	m^{-1}
Radius of curvature 1^{st} dipole	R_1	15.431	m
Radius of curvature 2^{nd} dipole	R_2	21.108	m
Radius of curvature 3^{rd} dipole	R_3	113.684	m
Radius of curvature 4^{th} dipole	R_4	139.640	m
Radius of curvature 5^{th} dipole	R_5	172.736	m
Length of dipoles 1^{st} dipole	L_1	0.45	m
Length of dipoles 2^{nd} dipole	L_2	0.45	m
Length of dipoles 3^{rd} dipole	L_3	5.4	m
Length of dipoles 4^{th} dipole	L_4	5.4	m
Length of dipoles 5^{th} dipole	L_5	5.4	m

Longitudinal Dynamics with Steady State CSR Wake

Vlasov-Fokker-Planck Equation

$$\frac{\partial\rho}{\partial s} - \eta p\frac{\partial\rho}{\partial z} + \frac{eV_{rf}}{E_0C_0}\sin\frac{\omega_{rf}}{c}z\frac{\partial\rho}{\partial p} - \frac{\partial}{\partial p}D_1(p)\rho + \frac{1}{2}\frac{\partial}{\partial p}D_2(p)\frac{\partial}{\partial p}D_2(p)\rho - \frac{2\pi Rr_0}{\gamma C_0}W(z,s)\frac{\partial\rho}{\partial p} = 0,$$

where $D_1(p) = -u_0(1+p)^2/C_0$, $D_2(p) = \sigma_{u_0}(1+p)/C_0$ and the steady state CSR wake for $\gamma \gg 1$ and a magnet of radius R is

$$W(z,s) = \frac{2}{(3R^2)^{1/3}}\int_{-\infty}^{z}\frac{1}{(z-z')^{1/3}}\frac{\partial\lambda(z',s)}{\partial z'}dz'.$$

Here s is path length, $z = c(t-t_0)$ with t the arrival time at s, and $p = (E-E_0)/E_0$ and $\lambda(z,s) = \int dp\rho(z,p,s)$.

A Method of Solution: Monte Carlo Particle Method I

Monte Carlo method: particle tracking with density estimation for wake calculation via a Fourier method.

Density estimation: from scattered (z, p) points at s $\longrightarrow$ smooth/global representation of λ (and $\frac{\partial \lambda}{\partial z}$) via a Fourier method.

Transformation for Fourier expansion: $\bar{z} = (z/\Delta z + 1)/2$ where Δz is the extend of the grid in z (Assume λ has compact support in z), thus $\bar{z} \in [0, 1]$.

1D orthogonal series estimator of the probability density $\bar{\lambda}(\bar{z})$

$$\bar{\lambda}_J(\bar{z}) := \sum_{j=0}^{J} \theta_j \phi_j(\bar{z}), \quad \theta_j = \int_0^1 \phi_j(\bar{z}) f(\bar{z}) d\bar{z},$$

$$\phi_0(\bar{z}) = 1, \quad \phi_j(\bar{z}) = \sqrt{2}\cos(\pi j \bar{z}), \quad j = 1, 2, ...$$

A Method of Solution: Monte Carlo Particle Method II

According to the fact that $\bar{\lambda}(\bar{z})$ is a probability density we have

$$\theta_j = E\{\phi_j(\bar{Z})\},$$

it follows that a natural estimate of E is the sample mean

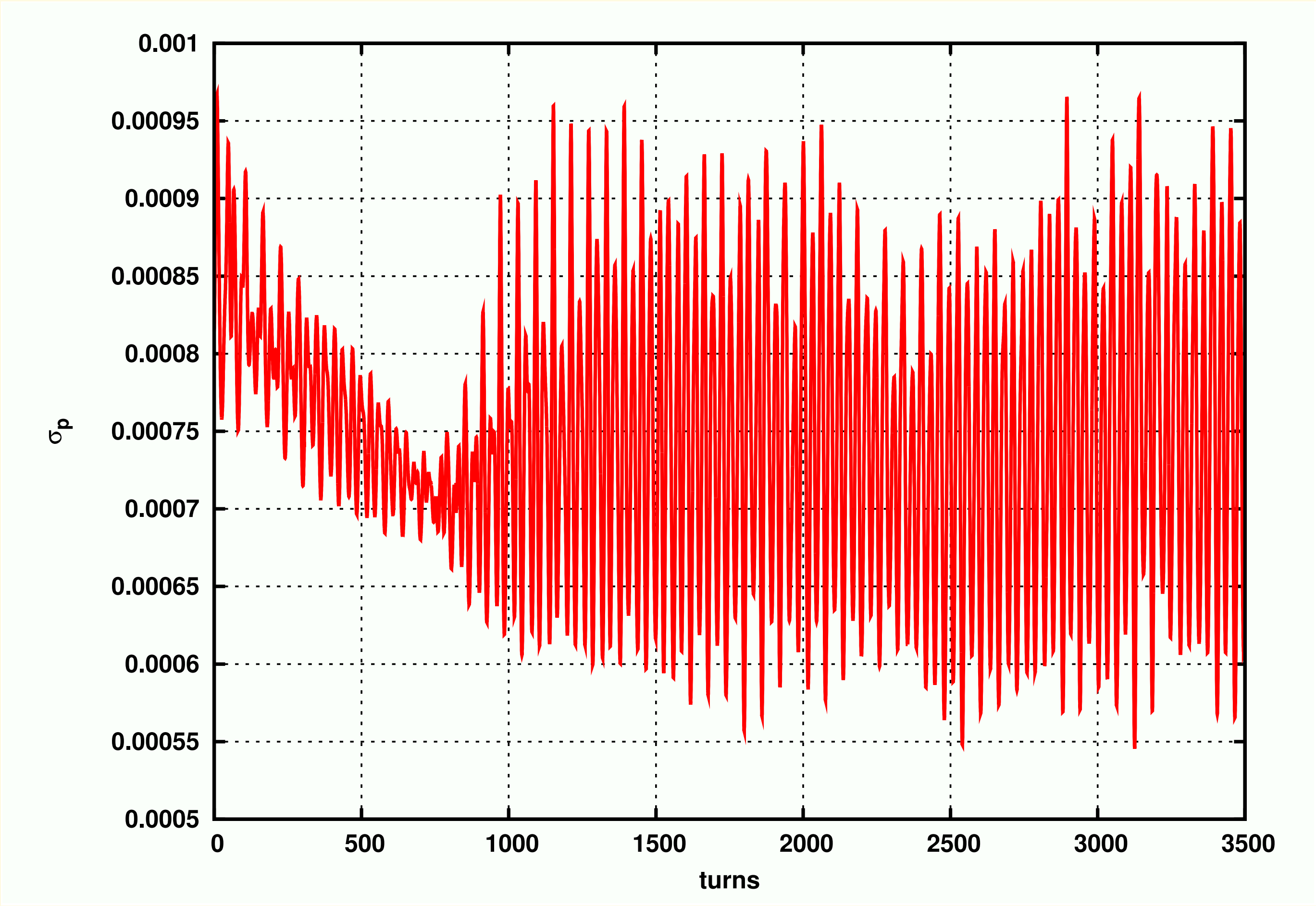
$$\hat{\theta}_j := \frac{1}{N}\sum_{n=1}^{N}\phi_j(\bar{Z}_n),$$

therefore a representation for λ is

$$\lambda(\bar{z}) \approx \sum_{j=0}^{J}\hat{\theta}_j\phi_j(\bar{z}).$$

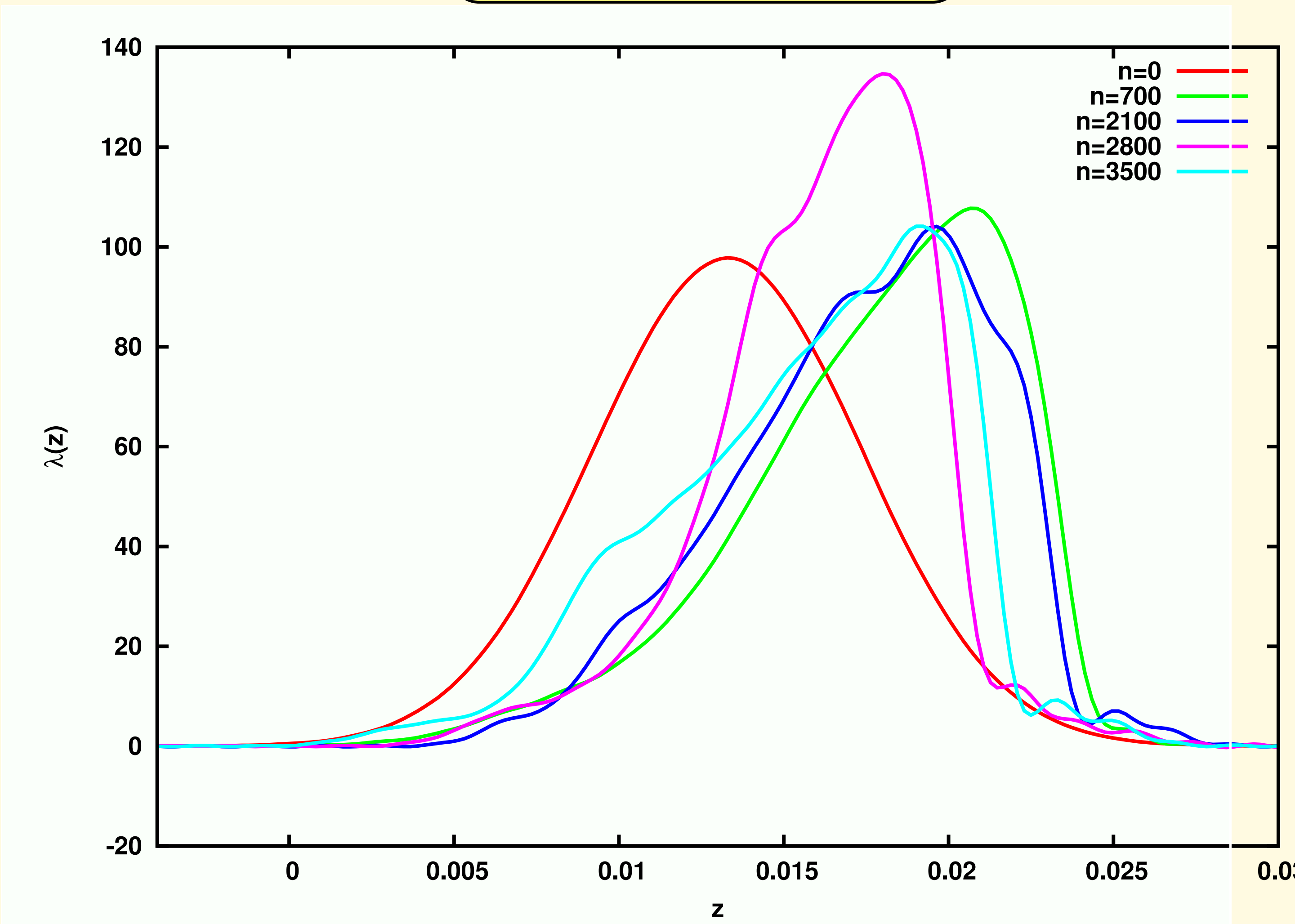
Remark: $\partial\lambda/\partial z$ follows by differentiating the representation for λ. As a diagnostic tool one can estimate ρ with a 2D Fourier expansion.

Numerical Sudies I



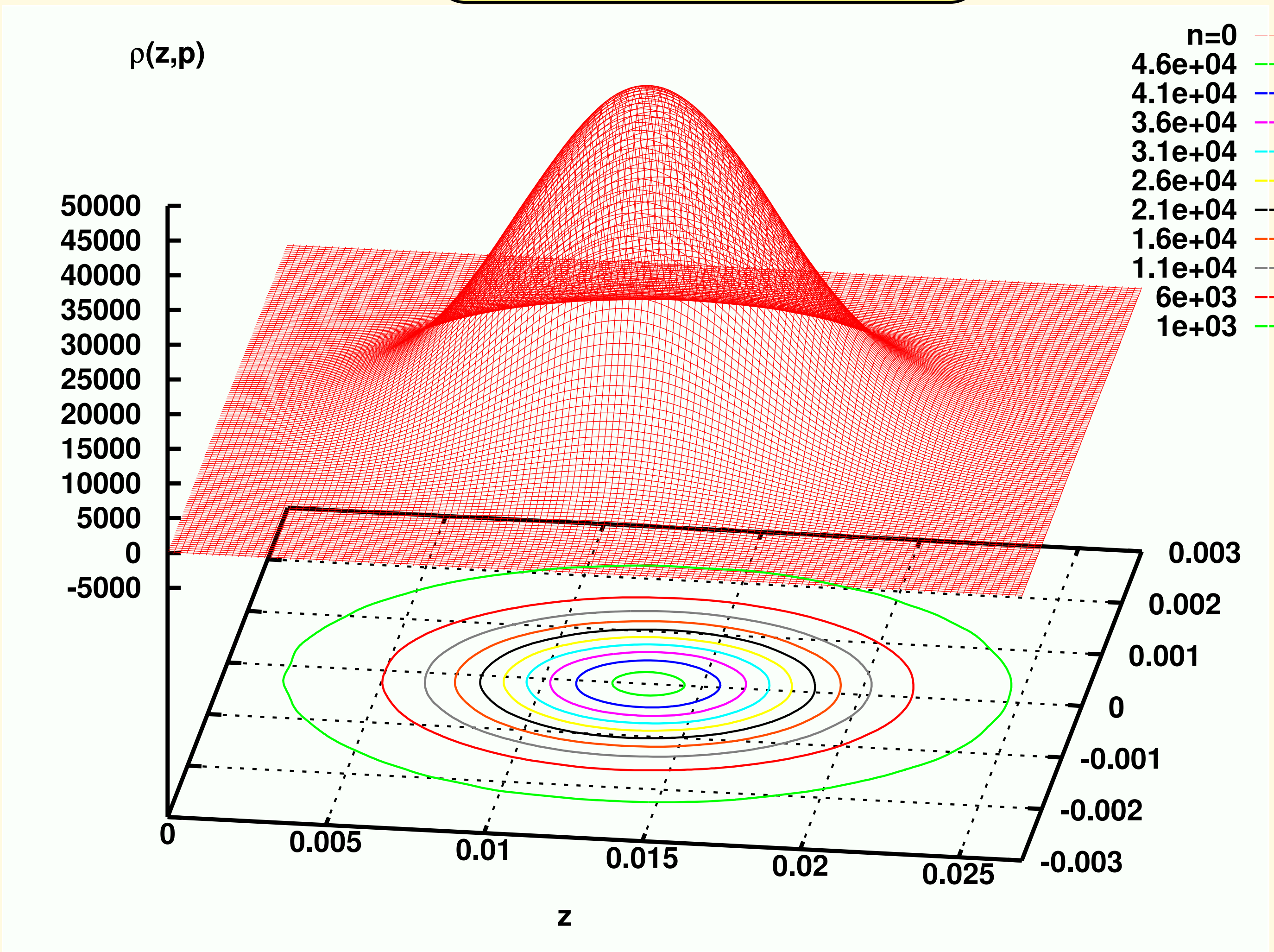
N=2.76×10^{10}. Standard deviation σ_p vs number of turns.

Numerical Sudies II



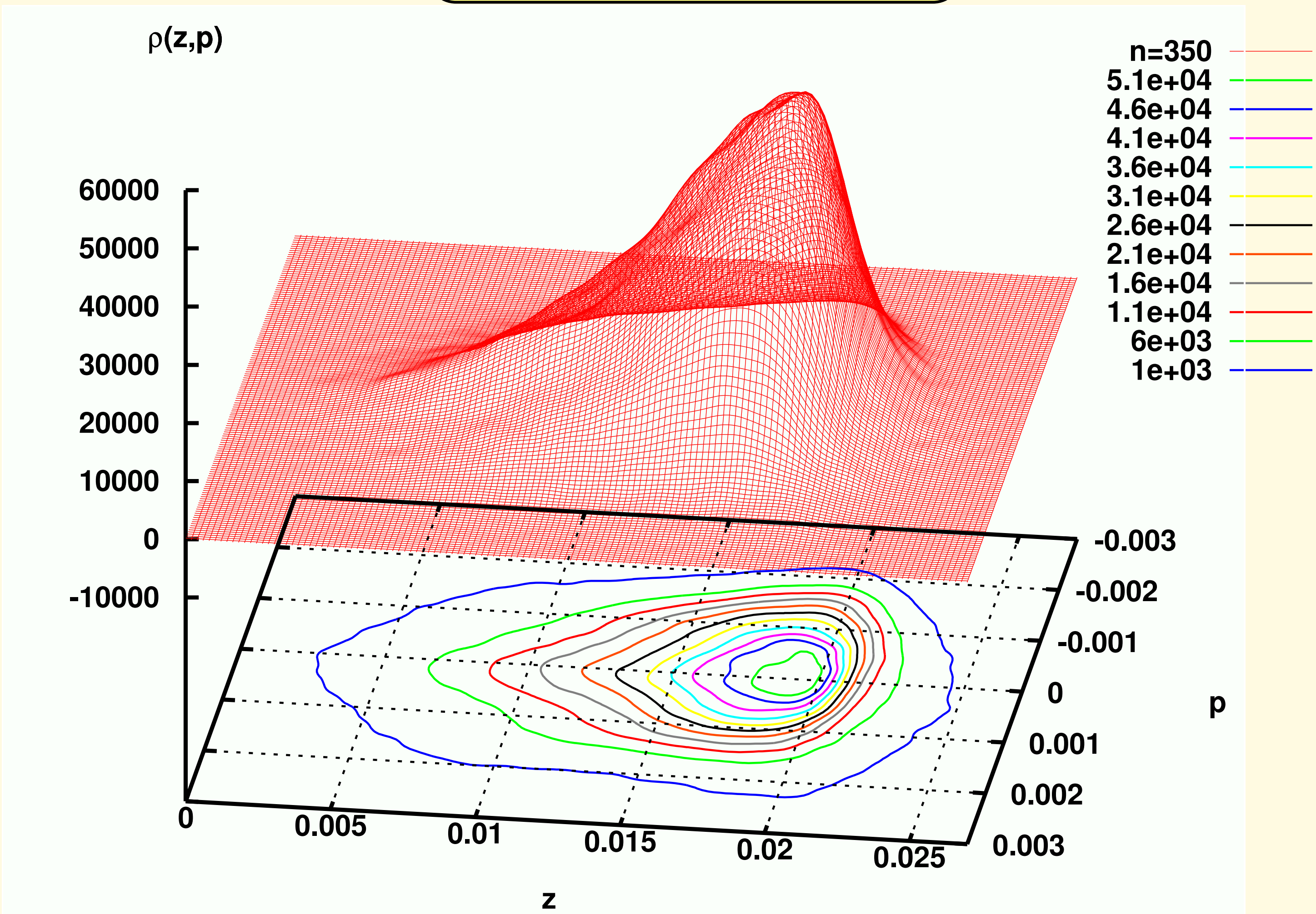
$N=2.76\times10^{10}$. Longitudinal density after n turns.

Numerical Sudies III



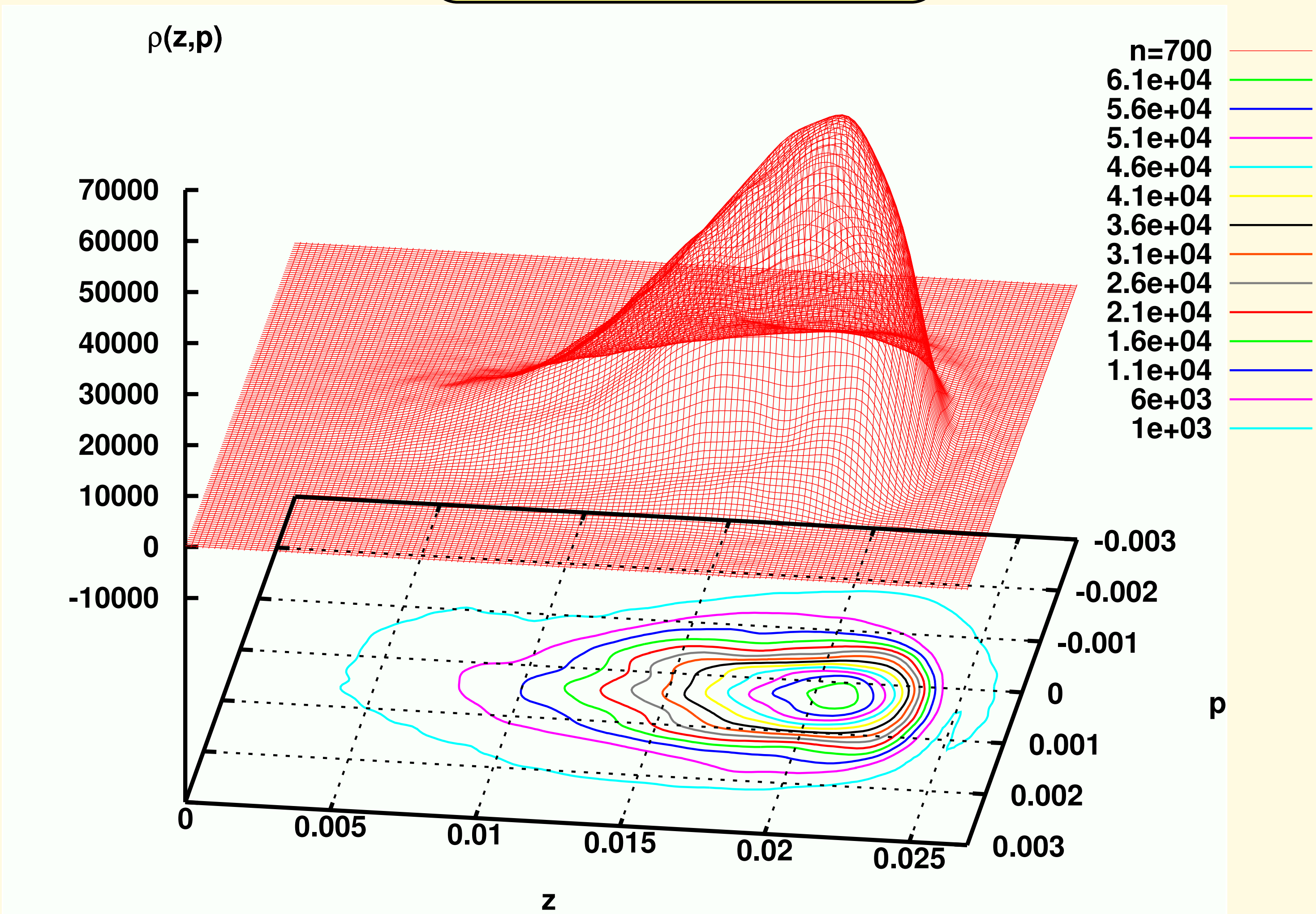
N=2.76×10^{10}. Initial Phase space density.

Numerical Sudies IV



N=2.76×10^{10}. Phase space density after 350 turns.

Numerical Sudies V



N=2.76×10^{10}. Phase space density after 700 turns.

Discussion

- Strong distortion in the distribution density due to CSR effects for N $= 2.76\times10^{10}$ (half nominal value). CSR modeled with steady state CSR wake.
- Next steps
  - Improve 1D model including transient effects and shielding.
  - Use 2D Vlasov-Maxwell-Fokker-Planck model, if necessary.

Possible contributions to the TDR from the Cockcroft Institute

- Coherent Synchrotron Radiation effects
- Space Charge effects
- Beam-Beam Interaction
- Intrabeam Scattering
- etc..

Self Consistent Vlasov-Maxwell Treatment

Collaborators

James A. Ellison, Klaus Heinemann, Math and Stat, UNM, Albuquerque, New Mexico, USA

Balsa Terzic, Beam Physics and Astrophysics Group, NIU, DeKalb, Illinois, USA

Robert Warnock, SLAC, Stanford University, Stanford, California, USA

Wave equation in lab frame with "2D" planar source:

$$(\partial_Z^2 + \partial_X^2 + \partial_Y^2 - \partial_u^2)\mathcal{E} = H(Y)\mathcal{S}(\mathbf{R}, u), \quad \mathcal{E}(\mathbf{R}, Y = \pm g, u) = 0.$$

where $u = ct$, $\mathcal{E}(\mathbf{R}, Y, u) = (E_Z, E_X, B)$, $\mathbf{R} = (Z, X)$.

Vlasov equation in beam frame:

$$f_s - \kappa(s)xf_z + F_z f_{p_z} + p_x f_x + [\kappa(s)p_z + F_x]f_{p_x} = 0$$

where $\mathbf{V} = \bar{v}(\mathbf{t}(s) + p_x\mathbf{n}(s))$,

$$\begin{aligned} F_z &= \frac{e}{\bar{v}\bar{E}}\mathbf{V}\cdot\mathbf{E}, \\ F_x &= \frac{e}{\bar{E}\bar{\beta}^2}\Big[-\bar{X}'(s)E_Z + \bar{Z}'(s)E_X + \bar{v}B)\Big], \end{aligned}$$

and $\mathbf{E} = (E_Z, E_X)$ and B are evaluated at $\mathbf{R} = \bar{\mathbf{R}}(s) + x\mathbf{n}(s)$ and $u = (s - z)/\bar{\beta}$.

Field Calculation and Density Estimation

Field formula:

$$\mathcal{E}(\mathbf{R},u) := \int_{-g}^{g} H(Y)\mathcal{E}(\mathbf{R},Y,u)dY \stackrel{H(Y)=\delta(Y)}{=} -\frac{1}{2\pi}\sum_{k=0}^{\infty} a_k \int_{-\infty}^{u-kh} dv \int_{-\pi}^{\pi} d\theta \; \mathcal{S}(\hat{\mathbf{R}},v,k)$$

where $\hat{\mathbf{R}} = \mathbf{R} + \sqrt{(u-v)^2-(kh)^2}(\cos\theta,\sin\theta)$ and $a_k = (-1)^k(1-\delta_{k0}/2)$.

- localization in θ for $v \ll u - kh \Longrightarrow \int \theta$ with superconvergent trapezoidal rule
- non uniform behavior in $v \Longrightarrow \int v$ with adaptive Gauss-Kronrod rule

Density estimation: from scattered beam frame points at s $\rightarrow$ smooth/global lab frame charge/current density via a 2D Fourier method.

1D Example: 1D orthogonal series estimator of $f(x)$, $x \in [0,1]$

$$f_J(x) := \sum_{j=0}^{J} \theta_j \phi_j(x), \quad \theta_j = \int_0^1 \phi_j(x) f(x)dx, \quad \phi_0(x) = 1, \phi_j(x) = \sqrt{2}\cos(\pi j x), j = 1,2,...$$

According to the fact that $f(x)$ is a probability density

$$\theta_j = E\{I_{\{X\in[0,1]\}}\phi_j(X)\}, \quad \text{therefore a natural estimate is} \quad \hat{\theta}_j := \frac{1}{N}\sum_{n=1}^{N} I_{\{X_n\in[0,1]\}}\phi_j(X_n)$$