# Comments on hadronic form factors in exclusive *B* and *D* decays

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# $B$ & $D$ leptonic and semileptonic (exclusive) channels:

$B \to l\nu_l$,
$B \to \pi l\nu_l,\ \eta l\nu_l, \rho l\nu_l, \omega l\nu_l$,..
$(l = e, \mu, \tau)$

$B \to D$ $(b \to c)$

- not accessible at LHCb
- important for CKM determination,

hypothetical new physics effects (charged Higgs, CP violation,..)

- current precision:

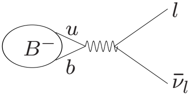
$BR(B^- \to \tau\bar{\nu}) = (1.65 \pm 0.38 \pm 0.38) \times 10^{-4}$ [Belle '08] $\Rightarrow \sim \pm 4\%$
$BR(\bar{B}^0 \to \pi^- l\bar{\nu}) = (1.36 \pm 0.09) \times 10^{-4}$ [PDG '08]$\Rightarrow \sim \pm 2 \div 4\%$

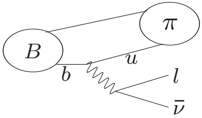
$\pm\delta BR$ projected at SuperB (75 ab$^{-1}$)

- need hadronic form factors with comparable accuracy
- form factors needed also for charmless and rare decays

as an input (e.g., $B \to PP$ , $B \to K^* ll$)

# Hadronic form factors




$\Rightarrow |V_{ub}|$

$$\Gamma(B^- \to l\bar{\nu}_l) = \frac{G_F^2 |V_{ub}|^2}{8\pi} m_l^2 m_B \left(1 - \frac{m_l}{m_B}\right)^3 f_B^2$$ decay constant

$$d\Gamma(\bar{B}^0 \to \pi^+ l\bar{\nu})/dq^2 = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} p_\pi^3 |f_{B\pi}^+(q^2)|^2$$ form factor

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- $B \to \tau\nu_\tau$ dominates
- $B \to \pi\mu\nu_\mu, \pi e\nu_e$, semileptonic region:
  $0 < q^2 < (m_B - m_\pi)^2 = q^2_{max} \simeq 26.4$ GeV$^2$
  $B$ rest frame, $p_\pi \to 0$ at $q^2 \to q^2_{max}$

# $B$ and $D_{(s)}$ decay constants [in MeV]

a sample of recent results:

| method | $f_B$ | $f_D$ | $f_{D_s}$ |
|---|---|---|---|
| exp. $\oplus$ CKM | $(242 \pm 28)\frac{3.99\times 10^{-4}}{V_{ub}}$ *[Belle '08]* | $205.8 \pm 8.5 \pm 2.5$ *[CLEO'08], $V_{cd} = V_{us}$* | $259.5 \pm 6.6 \pm 3.1$ *[CLEO'09], $V_{cs} = V_{ud}$* |
| lattice | $190\pm 13$ *[HPQCD,'09]* | $207 \pm 4$ *[HPQCD,UKQCD '08]* | $241 \pm 3$ *[HPQCD,UKQCD '08]* |
| QCD SR | $210 \pm 19$ *[Jamin-Lange '01]* $206\pm 20$ *[Penin-Steinhauser'01]* | - $195 \pm 20$ *[Penin-Steinhauser'01]* $203\pm 20$ *[Narison '02]* | $235\pm 24$ *[Narison '02]* |
| OPE bound | - | <230 | <270 |

- $f_{B_s}/f_B$, SR in agreement with lattice QCD
- still some tension of exp. vs lattice $f_{D_s}$ ( and vs the bound)
- already some tension in $f_B$ ?

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- lattice QCD: "Yes, we can !"
  [appendix A of SuperB report '07 ]

# SuperB precision: a challenge for QCD ?

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- Can QCD provide $\sim \pm 1 \div 2\%$ accuracy
  for the hadronic decay constants?
- lattice QCD: "Yes, we can !"
  [appendix A of SuperB report '07 ]
- lattice (HPQCD-UKQCD) :
  $f_{D_{(s)}}$ has almost achieved that level (see table)
  (discussions within lattice community)

# "non-lattice" QCD (approximate analytic methods):

- QCD sum rules, based on two-point correlation functions
  provide reliable estimates of $f_{B,D}$ but with a limited accuracy:
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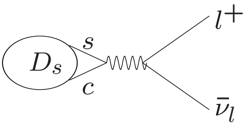
    minimized by Borel transform., fixing the hadron mass from SR
  - currently: $\sim 10\%$ error
    no way to get $< 5\%$ accuracy level in future

# OPE bounds for $f_{D_s}$ and $f_D$

*[A.K., hep/ph-0812.3747]*




the hadronic matrix element:

$$(m_c + m_s)\langle 0|\bar{s}i\gamma_5 c|D_s\rangle = f_{D_s} m_{D_s}^2$$

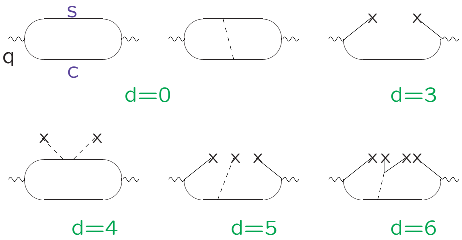
- Correlation function of two charmed-strange currents:

$$j(x) = (m_c + m_s)\bar{s}(x)i\gamma_5 c(x)$$

$$\Pi(q^2) = i\int d^4x e^{iqx}\langle 0 \mid T\{j(x)j^\dagger(0)\} \mid 0\rangle$$
$$= \frac{f_{D_s}^2 m_{D_s}^4}{m_{D_s}^2 - q^2} + \sum_{h=D^*K,\dots} \frac{\langle 0 \mid j|h\rangle\langle h|j^\dagger \mid 0\rangle}{m_h^2 - q^2}$$

$s \to d$, $D_s \to D$,

# OPE diagrams




- inputs: $\bar{m}_c$ , $\bar{m}_s$ $\langle \bar{q}q \rangle$ (GMOR), $\langle \bar{s}s \rangle$, $d = 5, 6$ condensates
- including $O(\alpha_s^2)$ correction to heavy-light correlator

  *[K.Chetyrkin, M. Steinhauser (2000)]*

# Deriving the bound

- calculate $\Pi(q^2)$ and apply Borel transformation:

$$\Pi(M^2) = \sum_{n=0,1,2} \int\limits_{(m_c+m_d)^2}^{\infty} ds \left(\frac{\alpha_s}{\pi}\right)^n \rho^{(n)}(s) e^{-s/M^2}$$
$$+ \sum_{n=0,1} \left(\frac{\alpha_s}{\pi}\right)^n \Pi^{(n)}_{\langle \bar{q}q\rangle}(M^2) + \sum_{d=4,5,6} \Pi_d(M^2)\,. \qquad (1)$$

- equate to the hadronic sum and use the positivity of it:

$f_D^2 m_D^4 e^{-m_D^2/M^2} + .... = \Pi(M^2; m_c, m_s, \alpha_s, \text{cond.},\ \mu, )$

- the same OPE as QCD SR , with no duality assumption involved

$\Rightarrow\ f_D < \sqrt{\Pi(M^2)/(m_D^4 e^{-m_D^2/M^2})}$

$M > 1.0$ GeV$^2$ and $\mu > 1.5$ GeV, OPE convergence

# Results

$f_D < 230$ MeV , $f_{D_s} < 270$ MeV

- the estimated error of 10 (20) MeV for $(f_D)_{up}$ $((f_{D_s})_{up})$ added
- lattice results obey the bounds
- experimental $f_D <$ bound
- experimental $f_{D_s}$ almost saturate the bound
  an unnatural $SU(3)_{fl}$ violation !
- no constraining bound for $f_B$

# What else can be done to improve $f_{B,D_{(s)}}$ determination

- investigate the processes of the experimental background

( e.g. $D_s \to l\nu_l\gamma$ , $B^- \to l\nu_l\gamma$ at low $E_\gamma$ )

(hadronic form factors from QCD at large $E_\gamma$ )

- to improve quark-hadron duality approximation:

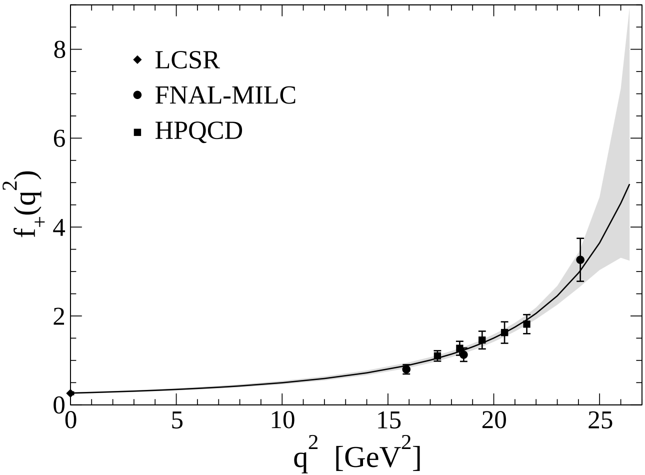
better data on the hadron spectroscopy in $D_{(s)}$ and $B$ channels :

  - identify and study radial excitations:
    (wide) $D'$ -resonances , $D' \to D\pi\pi$ , $D^*\pi$ etc.,
    e.g. in semileptonic $B \to D^{(*)}Xl\nu_l$ channels ,
  - radial excitations of $B$: a task for LHCb

# $B \to \pi$ form factor and $|V_{ub}|$

- the current status accumulated in a single figure:

*from [ Bourrely, Caprini, Lellouch, 0807.222 hep-ph ]*



- $q^2 = 0$ : Light-cone sum rules (LCSR) , recent update

*[ G.Duplancic, A.K., B.Melic, Th.Mannel, N.Offen (2007)*

$$f^+_{B\pi}(0) = 0.26^{+0.04}_{-0.03}$$

- large $q^2$ lattice *[FNAL-MILC, HPQCD]*:

$f^+(q^2)$ with errors at the level of $\pm 12\%$

- the curve: analyticity $\oplus$ "conformal mapping" $q^2 \to z \oplus$ $z$-expansion $\Rightarrow$ model independent shape parametrizations

- fitting theory $\oplus$ exp. $dBR/dq^2$ [BaBar, Belle ] to $z$-parameterization

# Recent $|V_{ub}|$ determinations from $B \to \pi l \nu_l$

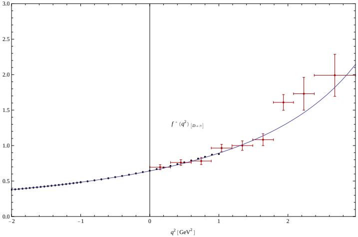
| [ref.] | $f^+_{B\pi}(q^2)$ calculation | $f^+_{B\pi}(q^2)$ input | $\lvert V_{ub}\rvert \times 10^3$ |
|---|---|---|---|
| Okamoto et al. '05 | lattice ($n_f = 3$) | - | 3.78±0.25±0.52 |
| HPQCD '06 | lattice ($n_f = 3$) | - | 3.55±0.25±0.50 |
| Flynn et al '07 | - | lattice $\oplus$ LCSR | $3.47 \pm 0.29 \pm 0.03$ |
| Ball, Zwicky '04 | LCSR | - | $3.5 \pm 0.4 \pm 0.1$ |
| DKMMO '07 | LCSR | - | $3.5 \pm 0.4 \pm 0.2 \pm 0.1$ |
| Bourrely, Caprini, Lellouch '08 | - | lattice$\oplus$ LCSR | $3.54 \pm 0.24$ |

- $f^{+}$'s from "lattice " and "non-lattice" (LCSR ) have comparable uncertainties,
- this will change , if lattice calculations achieve their goal of $\sim 1 \div 2\%$ accuracy [ App.A SuperB report '07]
- LCSR uncertainties : the same counting as for $f_B$ SR
higher twist effects to be investigated, duality error larger
- $< 10\%$ accuracy is hardly achievable with LCSR ,
future aim: to better assess OPE/input/duality uncertainties
- other "non-lattice" tools:
  - effective theories (HQET,SCET)

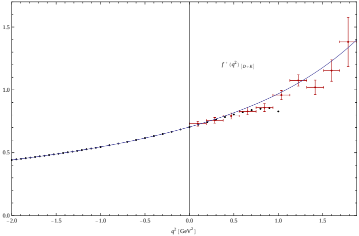
    *[ LCSR in SCET F. De Fazio, Th. Feldmann T.Hurth '06*
  - LCSR with $B$ meson distribution amplitudes
    [ A.K., N. Offen, Th. Mannel '06]

# $D \to \pi, K$ form factors from LCSR

preliminary: [Ch.Klein, A.K., Th. Mannel, N.Offen, paper in prepar.]




$f^+_{D\pi}(0) = 0.62\ ^{+0.09}_{-0.08}$, $f^+_{D\pi}(q^2)$




$f^+_{DK}(0) = 0.70\ ^{+0.10}_{-0.09}$, $f^+_{DK}(q^2)$

points: CLEO'08 + CKM , curve: LCSR z-param. fit

# How to improve "non-lattice" determinations of $f^+$'s

- more accurate data on $q^2$ distribution
  $\Rightarrow$ refine/adjust the input
- radial excitations of $B$ and $D$
  $\Rightarrow$ a better control over quark-hadron duality ansatz

# Other semileptonic channels

| Channel ($l = e, \mu$) | BR[$10^{-4}$] [HFAG] [PDG] | hadronic : form factor(s) |
|---|---|---|
| $B^0 \to \pi^- l^+ \nu_l$<br>$B^+ \to \pi^0 l^+ \nu_l$ | $1.39 \pm 0.06 \pm 0.06$<br>$0.74 \pm 0.11$ | $f^+_{B\pi}$ |
| $B^0 \to \pi^- \tau^+ \nu_l$ | - | $f^+_{B\pi}$, $f^0_{B\pi}$ |
| $B^+ \to \eta l^+ \nu_l$ | $0.84 \pm 0.27 \pm 0.21$ | $f^+_{B\eta}$ |
| $B^0 \to \rho^- l^+ \nu_l$<br>$B^+ \to \rho^0 l^+ \nu_l$ | $2.38 \pm 0.20 \pm 0.32$<br>$1.24 \pm 0.23$ | $A_1^{B\rho}$, $A_2^{B\rho}$, $V^{B\rho}$ |
| $B^+ \to \omega l^+ \nu_l$ | $1.3 \pm 0.4 \pm 0.2 \pm 0.3$ | $A_1^{B\omega}$ , $A_2^{B\omega}$, $V^{B\omega}$ |

# Hadronic uncertainties with $\rho$, $K^*$

- $\rho(770)$ ($K^*(890)$) are $P$-wave resonant states of $2\pi$ ($K\pi$)
  not precisely defined in QCD, only in quark model
- their experimental identification involves:
- Breit-Wigner resonance with energy dependent width
- model of nonresonant hadronic background
  (radial excitations $\rho'$,....; $K^*$, .... involved)

- an example :

$\tau \to K^* \nu_\tau$ identification, [Belle 0706.023[hep/ex]]
involving a model with 2 radial excitations of $K^*$

- these models originate in hadron phenomenology

and have no direct connection to QCD (recent AdS/QCD ?)
( a topical task: refresh/improve/update these models)
e.g. vector mesons in the pion/kaon form factor [ C.Bruch, A.K., J.Kühn '04 ]

- $\omega$ and $\phi$ are narrow and hence somewhat better

- a measurement of $BR(B \to \rho ...)$, $BR(B \to K^* ... )$ always

contains a "systematic" uncertainty, due to resonance identification , optimistically few % ,

- puts a certain limit on our abilities to calculate/measure

obsrevables involving $\rho, K^*$ with $\sim 1\%$ percent accuracy