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Effective theory approach to LFV

[arXiv:0707.4058](https://arxiv.org/abs/0707.4058), JHEP 0712:061,2007; [arXiv:0803.0481](https://arxiv.org/abs/0803.0481), Phys.Rev.D78:033007,2008

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Motivation

👉 (1) Need a high scale ($> \text{SM}$) to generate Neutrino masses and mixings:

Oscillations	$m_\nu > \sqrt{\Delta m_{\text{atm}}^2} \sim 5 \times 10^{-2} \text{ eV}$
Cosmology	$\sum_i m_{\nu_i} < \sim 1 \text{ eV}$
Tritium	$m_{\nu_e} \lesssim 2.2 \text{ eV}$

👉 (2) Need dynamical mechanism to generate baryon asymmetry of the Universe η_B :
Primordial abundances of light elements + CMB Anisotropies

$$\propto \eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.21 \pm 0.16) \times 10^{-10}$$

👉 AIM: find models that are simultaneously consistent with (1) & (2)

➔ BAU from Leptogenesis ➔ complex Yukawa couplings

- Favourite option: new physics at high scale M
- Heavy fields manifest in the low energy effective theory (SM) via higher dimensional operators $\delta\mathcal{L}^{d=5}, \delta\mathcal{L}^{d=6}, \dots$

$m_\nu \neq 0 \Rightarrow$ New Physics

Standard Model

- ν_L and no $\nu_R \implies$ No Dirac mass term: $\mathcal{L}_{m_D} = m_D (\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$
- No Higgs triplet \implies No Majorana mass term: $\mathcal{L}_{m_M} = \frac{1}{2} M \bar{\nu}_L^c \nu_L + h.c.$
- Lepton symmetry is accidental \implies Non-renormalisable operators dim 5, 6 ...

SM \equiv Effective theory of a larger one valid at a scale Λ



$$\delta\mathcal{L}^{d=5} = c^{d=5} \mathcal{O}^{d=5}, \quad \mathcal{O}^{d=5} = \frac{1}{\Lambda} \left\{ (\phi\ell)^T (\phi\ell) + h.c. \right\} \xrightarrow{\langle\phi\rangle=v} m_\nu \sim v^2/\Lambda$$

$$m_\nu \sim \sqrt{\Delta m_{\text{atm}}^2} \sim \sqrt{2 \times 10^{-3} \text{eV}^2} \Rightarrow \Lambda \sim 10^{15} \text{ GeV} \text{ (Remarkably near } \Lambda_{\text{GUT}} \text{ !)}$$

Beyond the Standard Model

Typically 3 possible ways to generate $m_\nu \neq 0$:

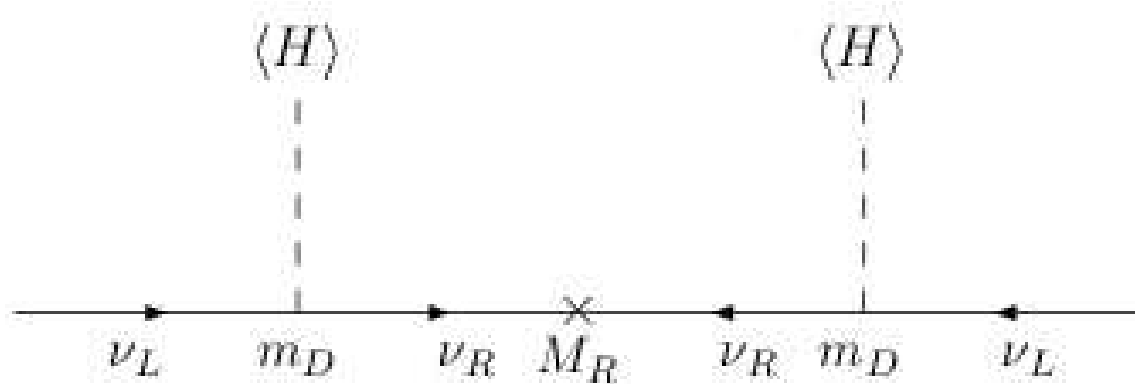
- Seesaw mechanism can be achieved via
 1. type I with RH neutrino exchange
 2. type II with scalar triplet exchange
 3. type III with fermionic triplet exchange
- Radiative corrections \Rightarrow MSSM or various extensions $+ R_p$, Zee model, \dots
- Extra dimensions \Rightarrow alternative to the seesaw

Seesaw Type I, SM + ν_R

$$\mathcal{L} = \mathcal{L}_{SM} + \lambda_{Jk}^\nu \bar{L}_k \nu_{R_J} H - \frac{1}{2} \bar{\nu}_{R_J} M_{R_J} \nu_{R_J}^c + \lambda_\alpha H^c \bar{e}_{R_\alpha} \ell_\alpha, \quad m_D = h^\nu v$$

Majorana Eigenstates (3×3):

$$\begin{cases} \nu = L + L^c = \nu^c & \rightarrow \tilde{m}_L \sim -m_D \frac{1}{M_R} m_D^T \\ N = R + R^c = N^c & \rightarrow \tilde{M}_R \sim M_R \end{cases}$$



$$\begin{cases} m_D \sim 200 \text{ GeV} \\ M_R \sim 10^{15} \text{ GeV} \end{cases} \rightarrow m_\nu \propto \sqrt{\Delta m_{\text{atm}}^2} \sim (10^{-2} - 10^{-1}) \text{ eV} \leftarrow \lambda^\nu \sim \mathcal{O}(1)$$

$$\lambda^\nu \sim h_e \leftarrow M_R \sim \text{few TeV}$$

Dimension 5 operator

- Neutrino masses require **new heavy fields**
- Effects at low energy: **effective theory approach**
Effective operators obtained when expanding the heavy field propagators in $\frac{1}{M}$

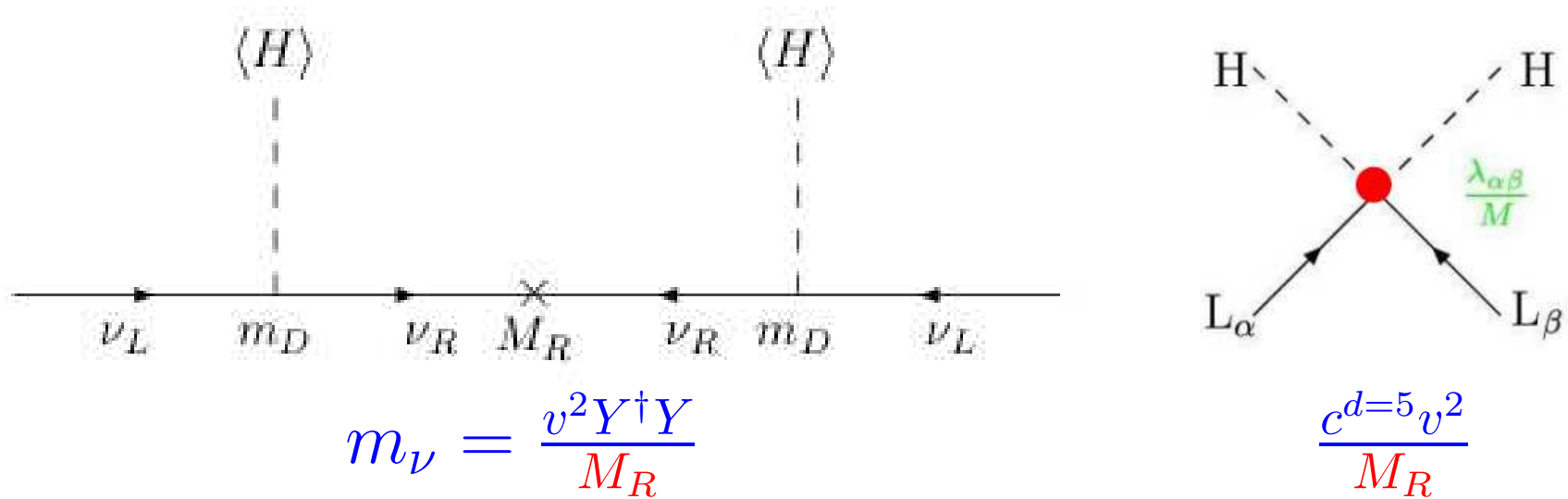
☞ heavy fermion: $\frac{1}{\not{D}-M} \sim -\frac{1}{M} - \frac{1}{M} \not{D} \frac{1}{M} + \dots$

☞ heavy scalar : $\frac{1}{D^2-M^2} \sim -\frac{1}{M^2} - \frac{D^2}{M^4} + \dots$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + c^{d=5} \mathcal{O}^{d=5} + c^{d=6} \mathcal{O}^{d=6} + \dots$$

$$c^{d=5} \propto \frac{1}{M}; \quad c^{d=6} \propto \frac{1}{M^2} \dots;$$

Dimension 5 :
$$\delta\mathcal{L}^{d=5} = \frac{1}{2} c_{\alpha\beta}^{d=5} \left(\overline{\ell_{L\alpha}^c} \tilde{\phi}^* \right) \left(\tilde{\phi}^\dagger \ell_{L\beta} \right) + \text{h.c.},$$



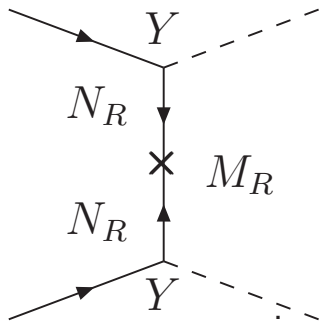
☞ $Y \sim 1 \rightarrow M_R \sim M_{\text{GUT}}$

☞ $Y \sim 10^{-6} \rightarrow M_R \sim \text{TeV}$

Mass operator $\mathcal{O}^{d=5}$ violates lepton number L
 \rightarrow Majorana neutrinos

☞ $\mathcal{O}^{d=5}$ is common to all models of Majorana neutrinos

3 ways to generate tree level $\mathcal{O}^{d=5}$: Seesaw I, II, III



type I (fermionic singlet)

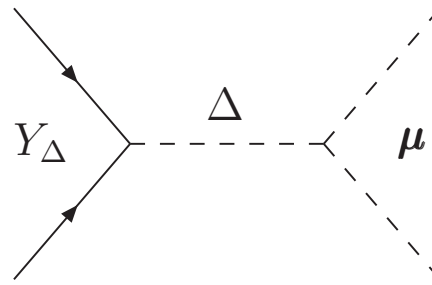
$$\mathbf{m}_\nu = -\frac{1}{2} Y_N^T \frac{v^2}{M_N} Y_N$$

Minkowski, Gell-Man,

Ramond, Slansky

Yanagida, Glashow

Mohapatra, Senjanovic



type II (scalar triplet)

$$\mathbf{m}_\nu = -2Y_\Delta v^2 \frac{\mu_\Delta}{M_\Delta^2}$$

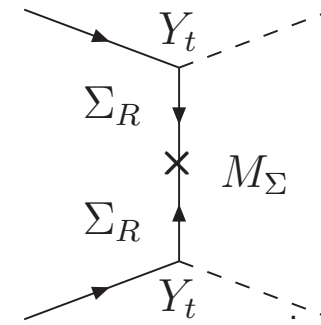
Magg, Wetterich,

Nussinov

Mohapatra, Senjanovic

Schechter, Valle

Ma, Sarkar



type III (fermionic triplet)

$$\mathbf{m}_\nu = -\frac{v^2}{2} Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$$

Ma, Hambye et al.

Bajc, Senjanovic, Lin

A.A., Biggio, Bonnet, Gavela,

Notari, Strumia, Papucci, Dorsner

Fileviez-Perez, Foot, Lew...

Distinguishing among the 3 types of seesaw

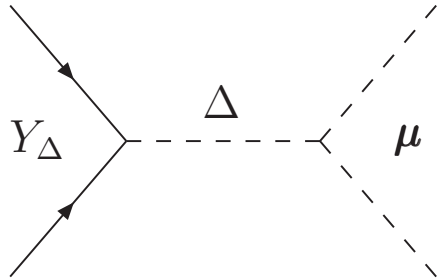
- The $\mathcal{O}^{d=5}$ operator is the same for all extensions of SM
responsible for neutrino masses and mixing (ν data, $(\beta\beta)_{0\nu}$, ...)
- To distinguish the several seesaw mechanisms:
 - ☞ produce the heavy states
 - ☞ or use $\mathcal{O}^{d=6}$ operators

Dimension 6 operators

Model	Effective Lagrangian $\mathcal{L}_{eff} = c_i \mathcal{O}_i$		
	$c^{d=5}$	$c_i^{d=6}$	$\mathcal{O}_i^{d=6}$
Fermionic Singlet	$Y_N^T \frac{1}{M_N} Y_N$	$\left(Y_N^\dagger \frac{1}{M_N^\dagger} \frac{1}{M_N} Y_N \right)_{\alpha\beta}$	$\left(\overline{\ell_{L\alpha}} \tilde{\phi} \right) i \not{\partial} \left(\tilde{\phi}^\dagger \ell_{L\beta} \right)$
Scalar Triplet	$4Y_\Delta \frac{\mu_\Delta}{M_\Delta^2}$	$\frac{1}{M_\Delta^2} Y_{\Delta\alpha\beta} Y_{\Delta\gamma\delta}^\dagger$	$\left(\overline{\ell_{L\alpha}} \vec{\tau} \ell_{L\beta} \right) \left(\overline{\ell_{L\gamma}} \vec{\tau} \ell_{L\delta} \right)$
		$\frac{ \mu_\Delta ^2}{M_\Delta^4}$	$\left(\phi^\dagger \vec{\tau} \tilde{\phi} \right) \left(\overleftarrow{D}_\mu \overrightarrow{D}^\mu \right) \left(\tilde{\phi}^\dagger \vec{\tau} \phi \right)$
		$-2(\lambda_3 + \lambda_5) \frac{ \mu_\Delta ^2}{M_\Delta^4}$	$(\phi^\dagger \phi)^3$
Fermionic Triplet	$Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$	$\left(Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma \right)_{\alpha\beta}$	$\left(\overline{\ell_{L\alpha}} \vec{\tau} \tilde{\phi} \right) i \not{D} \left(\tilde{\phi}^\dagger \vec{\tau} \ell_{L\beta} \right)$

Fermions: if $Y \sim \mathcal{O}(1)$, $c^{d=6} \sim (c^{d=5})^2$ and the smallness m_ν would preclude observable effects from $\mathcal{O}_i^{d=6}$. Not the case for scalars!

Scalar triplet (type II)



$$\Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \end{pmatrix} \sim (1, 3, 2), \quad L_\Delta = -2$$

Yukawa coupling:

$$Y_{\Delta ij} \overline{(l_L)^c}_{ia} (l_L)_{jb} (i\tau_2 \tau_\alpha)_{ab} \Delta^\alpha + h.c.$$

Scalar coupling:

$$\mu \phi_a^t \phi_b (i\tau_2 \tau_\alpha) (\Delta^\dagger)^\alpha + h.c.$$

$$-M_\Delta^2 \Delta^\dagger \Delta - \frac{1}{2} \lambda_2 (\Delta^\dagger \Delta)^2 \\ -\lambda_3 (\phi^\dagger \phi) (\Delta^\dagger \Delta) + \dots$$

d=5 Operator: Mass

$$m_\nu = v^2 Y_\Delta \frac{\mu}{M_\Delta^2} \quad \rightarrow \text{2 different scales } \mu, M_\Delta$$

possible to have $Y_\Delta \sim \mathcal{O}(1)$ $M_\Delta \sim 1 \text{ TeV}$ ($\mu \sim 100 \text{ eV}$)

Low energy effects of dimension 6 operators:

$$\frac{1}{2M_\Delta^2} Y_{\Delta ij} Y_{\Delta kl}^\dagger (\overline{l_{Li}} \gamma^\mu l_{Lk}) (\overline{l_{Lj}} \gamma_\mu l_{Ll}) \rightarrow \text{LFV, constraints not suppressed by } \mu$$

$$\left. \begin{aligned} & -2 \frac{\mu^2}{M_\Delta^4} \partial_\mu (\phi^\dagger \phi) \partial^\mu (\phi^\dagger \phi) \\ & 2\lambda_3 \frac{\mu^2}{M_\Delta^4} (\phi^\dagger \phi)^3 \\ & 4 \frac{\mu^2}{M_\Delta^4} [\phi^\dagger D_\mu \phi]^\dagger [\phi^\dagger D_\mu \phi] \end{aligned} \right\} \rightarrow \text{EW prec. data, couplings to gauge bosons}$$

$$-2 \frac{\mu^2}{M_\Delta^4} (\phi^\dagger \phi) \{ Y_e \bar{l} e_R \phi + Y_d \bar{q} d \phi - Y_u \bar{q} i \tau_2 u \phi + h.c. \} \rightarrow \text{top physics.....}$$

Direct LFV: $d = 5$ operator suppressed by small scale μ ;
 $d = 6$ operator not suppressed

Testing seesaws

□ Scalar seesaw (Type II): Lepton Flavour Violation

$$\mu \rightarrow eee, \tau \rightarrow lll, \mu \rightarrow e\gamma, \tau \rightarrow e\gamma$$

→ bounds on various combinations of $\frac{Y_\Delta}{M_\Delta}$

□ Fermionic seesaw (Type I and III): non unitarity

$$|NN^\dagger - 1|_{\alpha\beta} = |\epsilon^\Sigma| = \frac{v^2}{2} |c^{d=6}|_{\alpha\beta} = \frac{v^2}{2} |Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma|_{\alpha\beta}$$

▶ Type I: also Lepton Flavour Violation

$$\mu \rightarrow e\gamma, \tau \rightarrow e\gamma, W, Z \text{ (semi) leptonique decays,}$$

▶ Type III: same as Type I + tree level Lepton Flavour Violation

$$(\mu \rightarrow eee, \tau \rightarrow lll) \text{ at tree level}$$

Bounds on Type II seesaw

$\mu \rightarrow eee$ and $\tau \rightarrow 3l$

$$\Gamma(\mu^- \rightarrow e^+ e^- e^-) = \frac{m_\mu^5}{192\pi^3} \frac{1}{M_\Delta^4} |Y_{\Delta_{\mu e}}|^2 |Y_{\Delta_{ee}}|^2$$

which gives

$$\text{Br}(\mu^- \rightarrow e^+ e^- e^-) \simeq \frac{\Gamma(\mu^- \rightarrow e^+ e^- e^-)}{\Gamma(\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e)} = \frac{1}{M_\Delta^4 G_F^2} |Y_{\Delta_{e\mu}}|^2 |Y_{\Delta_{ee}}|^2$$

For τ decays:

$$\Gamma(\tau^- \rightarrow l_i^+ l_j^- l_j^-) = \frac{m_\tau^5}{192\pi^3} \frac{1}{M_\Delta^4} |Y_{\Delta_{\tau i}}|^2 |Y_{\Delta_{jj}}|^2 \quad (\text{for any } i \text{ and } j)$$

$$\Gamma(\tau^- \rightarrow l_i^+ l_j^- l_k^-) = \frac{m_\tau^5}{96\pi^3} \frac{1}{M_\Delta^4} |Y_{\Delta_{\tau i}}|^2 |Y_{\Delta_{jk}}|^2 \quad (\text{for any } i, j, k \text{ with } j \neq k)$$

Using exp. Brs (or upper limits) derive bounds for Yukawa couplings

Process	Constraint on	Bound $\left(\times \left(\frac{M_\Delta}{1\text{TeV}}\right)^2\right)$
$\mu^- \rightarrow e^+ e^- e^-$	$ Y_{\Delta\mu e} Y_{\Delta ee} $	$< 1.2 \times 10^{-5}$
$\tau^- \rightarrow e^+ e^- e^-$	$ Y_{\Delta\tau e} Y_{\Delta ee} $	$< 1.3 \times 10^{-2}$
$\tau^- \rightarrow \mu^+ \mu^- \mu^-$	$ Y_{\Delta\tau\mu} Y_{\Delta\mu\mu} $	$< 1.2 \times 10^{-2}$
$\tau^- \rightarrow \mu^+ e^- e^-$	$ Y_{\Delta\tau\mu} Y_{\Delta ee} $	$< 9.3 \times 10^{-3}$
$\tau^- \rightarrow e^+ \mu^- \mu^-$	$ Y_{\Delta\tau e} Y_{\Delta\mu\mu} $	$< 1.0 \times 10^{-2}$
$\tau^- \rightarrow \mu^+ \mu^- e^-$	$ Y_{\Delta\tau\mu} Y_{\Delta\mu e} $	$< 1.8 \times 10^{-2}$
$\tau^- \rightarrow e^+ e^- \mu^-$	$ Y_{\Delta\tau e} Y_{\Delta\mu e} $	$< 1.7 \times 10^{-2}$

non-observation of these LFV transitions \Rightarrow bounds on M_Δ
most stringent decay: $\mu \rightarrow eee \Rightarrow M_\Delta \geq 294 \text{ TeV}$, for $Y_\Delta \sim \mathcal{O}(1)$

$$\mu \rightarrow e\gamma \text{ and } \tau \rightarrow l\gamma$$

Radiative processes due to exchange between lepton fields of the Δ^{++} and Δ^+ fields

$$\text{Br}(l_1 \rightarrow l_2\gamma) = \frac{\alpha}{48\pi} \frac{25}{16} \frac{\left| \sum_l Y_{\Delta ll_1}^\dagger Y_{\Delta l_2 l} \right|^2}{G_F^2 M_\Delta^4} \text{Br}(l_1 \rightarrow e\nu_1\bar{\nu}_e)$$

Process	Constraint on	Bound $\left(\times \left(\frac{M_\Delta}{1 \text{ TeV}} \right)^2 \right)$
$\mu \rightarrow e\gamma$	$ \sum_{l=e,\mu,\tau} Y_{\Delta l\mu}^\dagger Y_{\Delta el} $	$< 4.7 \times 10^{-3}$
$\tau \rightarrow e\gamma$	$ \sum_{l=e,\mu,\tau} Y_{\Delta l\tau}^\dagger Y_{\Delta el} $	< 1.05
$\tau \rightarrow \mu\gamma$	$ \sum_{l=e,\mu,\tau} Y_{\Delta l\tau}^\dagger Y_{\Delta \mu l} $	$< 8.4 \times 10^{-1}$

Bounds on combinations of Y_Δ

Combined bounds		
Process	Yukawa	Bound $\left(\times \left(\frac{M_\Delta}{1\text{TeV}}\right)^4\right)$
$\mu \rightarrow e\gamma$	$ Y_{\Delta\mu\mu}^\dagger Y_{\Delta\mu e} + Y_{\Delta\tau\mu}^\dagger Y_{\Delta\tau e} $	$< 4.7 \times 10^{-3}$
$\tau \rightarrow e\gamma$	$ Y_{\Delta\tau\tau}^\dagger Y_{\Delta\tau e} $	< 1.05
$\tau \rightarrow \mu\gamma$	$ Y_{\Delta\tau\tau}^\dagger Y_{\Delta\tau\mu} $	$< 8.4 \times 10^{-1}$

Other bounds on Y_Δ

- Stringent bounds from $M_W \rightarrow |Y_{\Delta\mu e}|^2 < 7.3 \times 10^{-2} \times \left(\frac{M_\Delta}{1\text{TeV}}\right)$
- Additional bounds from Bhabha scattering, muon anomalous magnetic moment,

👉 $Y_{\Delta} \lesssim 10^{-1} \times \left(\frac{M_{\Delta}}{1\text{TeV}}\right)$ or stronger

👉 observation of $\mu \rightarrow e\gamma$ at MEG (sensitivity of 10^{-13} ?)

- for $Y_{\Delta} \sim \mathcal{O}(1)$ \rightarrow $15\text{ TeV} < M_{\Delta} < 50\text{ TeV}$
- for $Y_{\Delta} \sim \mathcal{O}(10^{-2})$ \rightarrow $0.15\text{ TeV} < M_{\Delta} < 0.50\text{ TeV}$

👉 if M_{Δ} turns out to be as low as $\mathcal{O}(\text{TeV})$, possibility of clean signals in hadronic accelerators (Tevatron, LHC)

- The production of Δ^{++} and Δ^{--} , decaying into pairs of same-sign leptons
 \rightarrow striking signals, free from SM backgrounds
- If Δ observed, must verify whether a scalar-mediated Seesaw is at work
 \rightarrow observe in addition at least three LFV processes (to measure and disentangle the individual $Y_{\Delta ij}$ couplings)

Bounds on Yukawas in Type III

- Lepton Flavour Violation at tree level $\mu \rightarrow eee, \tau \rightarrow 3l$
- Lepton Flavour Violation at loop level $\mu \rightarrow e\gamma, \tau \rightarrow e\gamma$
- Z decays ($Z \rightarrow e\mu, Z \rightarrow e\tau, Z \rightarrow \mu\tau$)
- W decays, Universality tests, Invisible Z width...
→ For $M \sim 1 \text{ TeV}, |Y| < 10^{-2}$

$l \rightarrow l'\gamma$ versus $l \rightarrow 3l'$ Generic results:

- ☞ In type I seesaw (fermionic singlet): $\Gamma(l \rightarrow \gamma l') \gg \Gamma(l \rightarrow 3l')$
- ☞ In type III seesaw (fermionic triplet): $\Gamma(l \rightarrow \gamma l') \ll \Gamma(l \rightarrow 3l')$

→ a measurement of a $\Gamma(l \rightarrow 3l')$ in the next generation of expt. would rule out TeV Fermionic triplet seesaw as its origin

Conclusions

- $d=6$ operators allow to discriminate among models of Majorana ν
- $d=5$ operator violates B-L but all $d=6$ operators conserve it
 - ➔ Direct Lepton Violation pattern:
 $d=5$ operator suppressed by a small scale and not the $d=6$ operators
Natural with scalar triplet, not in fermionic case (way out: inverse seesaw)
- Rich phenomenology associated to TeV seesaw : LFV, ...

Detailed analysis - see

JHEP 0712:061, 2007; Phys.Rev.D78:033007, 2008