SYMPLECTIC EXPRESSION FOR CHROMATICITY

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What is symplectic expression for chromaticity?

This means Hamiltonian expression which use expanded chromaticity. By using Hamiltonian, we can treat not only symplectic beam but also resonance.

Explanation of the expanded chromaticity is in first chapter.

＜Outline＞

First : method of making Hamiltonian expression

Second : derivation of Hamiltonian from mesurement

Third : tracking using Hamiltonian of our model

Motivation

Calculated optics parameter using SAD is often different from mesurement data. This is because many kinds of factor cause error, so we can't include such error exactly in SAD.

↓

<Purpose>

Make model which realize beam motion included machine error from mesurement data.

1.Definition of Chromaticity

$$\nu = \nu_0 + \xi\delta \qquad \delta = \frac{\Delta p}{p}$$

p : beam momentum

ξ : Linear Chromaticity

Generally, momentum change cause
transfer matrix change.

↓

Twiss parameter depends on momentum.

Definition of Chromaticity

We consider storage ring like KEKB.

Beam momentum (p) is large.

➡ Twiss parameter (α,β) can be also expanded by δ.

high order expantion

$$\alpha = \alpha_0 + \alpha_1\delta + \alpha_2\delta^2 + \alpha_3\delta^3$$
$$\beta = \beta_0 + \beta_1\delta + \beta_2\delta^2 + \beta_3\delta^3$$
$$\nu = \nu_0 + \nu_1\delta + \nu_2\delta^2 + \nu_3\delta^3$$

$\alpha_{1,2,3}, \beta_{1,2,3}, \nu_{1,2,3}$ are defined as expanded chromaticity.

Synchrotron oscillation is very small.

➡ **δ can be treated as constant during 1 turn.**

How to determine the model

In general, 1 turn matrix is written by

$$M_{1turn} = R_{6\times6}^{-1}(\delta)M(\delta)_{6\times6}R_{6\times6}(\delta)$$

$$M(\delta)_{6\times6} = \begin{pmatrix} M_X & 0 & 0 \\ 0 & M_Y & 0 \\ 0 & 0 & M_Z \end{pmatrix}$$

R(δ) matrix diagonalize M_{1turn}.

$$M_i = \begin{pmatrix} \cos\nu_i + \alpha_i \sin\nu_i & \beta_i \sin\nu_i \\ -\gamma_i \sin\nu_i & \cos\nu_i - \alpha_i \sin\nu_i \end{pmatrix}$$

Since δ does not change very mach, we assume orbit change which come from dispersion is constant.

→ **Consider only betatron oscillation.**

Since longitudinal direction (z) is function of time, it is independent of other components (x, p_x, y, p_y).

→ **After that, consider only 4×4 matrix.**

Symplectic Matrix

Generally, symplectic matrix is written by

$$M_{4\times4} = M_0 \exp(- \underline{:H:}\, s)$$

Symmetrical matrix

Since 4×4 symplectic matrix,
the number of independent optics parameter is 10.

＜our model＞

$$M_{4\times4} = R^{-1}(\delta) M(\delta) R(\delta)$$

$M(\delta)$ include $\alpha_X, \beta_X, \nu_X, \alpha_Y, \beta_Y, \nu_Y$

$R(\delta)$ includes 4 independent parameter.

Definition of Chromaticity

conditions to determine $R_{4\times4}(\delta)$ matrix

- 4 optics parameters
- Simplify inverse R matrix
- R matrix is symplectic

R(δ) matrix can be derived by these three condition.

$$R(\delta) = \begin{pmatrix} R_0 I_2 & -J_2 R_2^t J_2 \\ -R_2 & R_0 I_2 \end{pmatrix} \qquad R_2 = \begin{pmatrix} r_1 & r_2 \\ r_3 & r_4 \end{pmatrix} \quad J_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$R_0 = \sqrt{1 - \det(R_2)}$$

We can also write r_1,r_2,r_3 and r_4 as expansion of δ as well as tune and twiss parameter.

$$r_i = r_{i0} + r_{i1}\delta + r_{i2}\delta^2 + r_{i3}\delta^3$$

r_{i1},r_{i2},and r_{i3} are defined as R-chromaticity.

2.Symplectic map of Hamiltonian

$$M_{4\times 4} = \underline{R(\delta)M(\delta)R(\delta)^{-1}}$$

$$M(\delta) = \begin{pmatrix} \cos\mu_X + \alpha_X \sin\mu_X & \beta_X \sin\mu_X & 0 & 0 \\ -\gamma_X \sin\mu_X & \cos\mu_X - \alpha_X \sin\mu_X & 0 & 0 \\ 0 & 0 & \cos\mu_Y + \alpha_Y \sin\mu_Y & \beta_Y \sin\mu_Y \\ 0 & 0 & -\gamma_Y \sin\mu_Y & \cos\mu_Y - \alpha_Y \sin\mu_Y \end{pmatrix}$$

To apply our model to Hamiltonian expression, following gererating function ($G_{function}$) is defined as purtubation of Hamiltonian.

$$\begin{aligned} M_{4\times 4} &= M(0)\exp(-:G_{function}:) \\ &= M(0)\underline{Mh(\delta)} \end{aligned}$$

Mh(δ) is defined as $G_{function}$ expanded by δ

(M(0) means M(δ) at δ=0)

Mh(δ) is calculated by comparing their coefficients about each δ dimension.

$G_{function}$ is derived.

Generating Function of infinitesimal canonical transformation.

F_2 is generating function which canonical transformation hold symplectic.

$$F_2(q,\bar{p},s) = q_i \bar{p}_i + G_{function}(q,\bar{p})$$

Upper bar means the quantitiy after beam move infinitesimal distant .

$$K = H + \frac{\partial F_2}{\partial s} = H + G_{function}\delta_p(s)$$

$G_{function}$ is determined by following three conditions

- Longitudinal direction（z） is independent of other conponents.

  ⇒ $G_{function}$ is function of x, p_x, y, p_y, δ

- $G_{function}$ have 10 independent parameter.
- $x, p_x, y, p_y, z, \delta$ are much smaller than 1.

Symplectic map of Hamiltonian

From these three condition, $G_{function}$ is written by

$$G_{function} = \sum_{n=1} (a_n x^2 + b_n x\overline{p}_x + c_n \overline{p}_x^2 + d_n xy + e_n x\overline{p}_y + f_n \overline{p}_x y + g_n \overline{p}_x \overline{p}_y + u_n y^2 + v_n y\overline{p}_y + w_n \overline{p}_y^2)\overline{\delta}^n$$

Define A,B,…,W as following

$$A = \sum_{n=1} a_n \overline{\delta}^n, B = \sum_{n=1} b_n \overline{\delta}^n, C = \sum_{n=1} c_n \overline{\delta}^n, D = \sum_{n=1} d_n \overline{\delta}^n, E = \sum_{n=1} e_n \overline{\delta}^n$$

$$F = \sum_{n=1} f_n \overline{\delta}^n, G = \sum_{n=1} g_n \overline{\delta}^n, U = \sum_{n=1} u_n \overline{\delta}^n, V = \sum_{n=1} v_n \overline{\delta}^n, W = \sum_{n=1} w_n \overline{\delta}^n$$

$G_{function}$ is rewritten by

$$G_{function} = Ax^2 + Bx\overline{p}_x + C\overline{p}_x^2 + Dxy + Ex\overline{p}_y + F\overline{p}_x y + G\overline{p}_x \overline{p}_y + Uy^2 + Vy\overline{p}_y + W\overline{p}_y^2$$

Symplectic map of Hamiltonian

$$F_2(q,\overline{p},s) = q_i \overline{p}_i + G_{function}(q,\overline{p})$$

infinitesimal canonical transformation

$$\overline{x} = \frac{\partial F_2}{\partial \overline{p}_x} = x + \frac{\partial G_{function}}{\partial \overline{p}_x} = x + Bx + 2C\overline{p}_x + Fy + G\overline{p}_y$$

$$p_x = \frac{\partial F_2}{\partial x} = \overline{p}_x + \frac{\partial G_{function}}{\partial x} = \overline{p}_x + 2Ax + B\overline{p}_x + Dy + E\overline{p}_y$$

$$\overline{y} = \frac{\partial F_2}{\partial \overline{p}_y} = y + \frac{\partial G_{function}}{\partial \overline{p}_y} = y + Vy + 2W\overline{p}_y + Ex + G\overline{p}_x$$

$$p_y = \frac{\partial F_2}{\partial y} = \overline{p}_y + \frac{\partial G_{function}}{\partial y} = \overline{p}_y + 2Uy + V\overline{p}_y + Dx + F\overline{p}_x$$

→ $\overline{x}, \overline{y}, \overline{p}_x, \overline{p}_y$ is derived

→ Mh(δ) is derived theorically

$$\begin{pmatrix} \overline{q}_i \\ \vdots \\ \overline{p}_j \end{pmatrix} = Mh(\delta) \begin{pmatrix} q_i \\ \vdots \\ p_j \end{pmatrix}$$

$$\overline{z} = z + \frac{\partial G_{function}}{\partial \overline{\delta}} = z + \sum_{n=1} (a_n x^2 + b_n x\overline{p}_x + c_n \overline{p}_x^2$$

$$+ d_n xy + e_n x\overline{p}_y + f_n \overline{p}_x y + g_n \overline{p}_x \overline{p}_y + u_n y^2 + v_n y\overline{p}_y + w_n \overline{p}_y^2) n\overline{\delta}^{n-1}$$

$$\overline{\delta} = \delta$$

Symplectic map of Hamiltonian

matrix elements of theorical Mh(δ)

$$Mh11 = 1 + B + 2C \cdot Mh21 + G \cdot Mh41$$

$$Mh12 = \frac{-FG + 2C(1+V)}{1 + B - EF + V + BV}$$

$$Mh13 = 2C \cdot Mh23 + F + G \cdot Mh43$$

$$Mh14 = \frac{-2\,CE + G + BG}{1 + B - EF + V + BV}$$

$$Mh21 = \frac{DE - 2A(1+V)}{1 + B - EF + V + BV}$$

$$Mh22 = \frac{1+V}{1 + B - EF + V + BV}$$

$$Mh23 = -\frac{D - 2EU + DV}{1 + B - EF + V + BV}$$

$$Mh24 = -\frac{E}{1 + B - EF + V + BV}$$

$$Mh31 = 2W \cdot Mh41 + E + G \cdot Mh21$$

$$Mh32 = \frac{G + GV - 2FW}{1 + B - EF + V + BV}$$

$$Mh33 = 1 + V + 2W \cdot Mh43 + G \cdot Mh23$$

$$Mh34 = \frac{-EG + 2(1+b)W}{1 + B - EF + V + BV}$$

$$Mh41 = -\frac{D + BD - 2AF}{1 + B - EF + V + BV}$$

$$Mh42 = -\frac{F}{1 + B - EF + V + BV}$$

$$Mh43 = \frac{DF - 2(1+B)U}{1 + B - EF + V + BV}$$

$$Mh44 = \frac{1+B}{1 + B - EF + V + BV}$$

3.How to calculate Hamiltonian

Using off-momentum measurement data, momentum depencence of α, β, ν, r are derived. Then, use δ as horizontal axis, plot these data and fit them. The fitting data is chromaticity.

Then, $M_{4\times4} = R(\delta)M(\delta)R(\delta)^{-1}$ is calculated numerically.

Using the relationship to $M_{4\times4} = M(0)Mh(\delta)$,

$Mh(\delta) = M(0)^{-1}R(\delta)M(\delta)R(\delta)^{-1}$ is derived numerically.

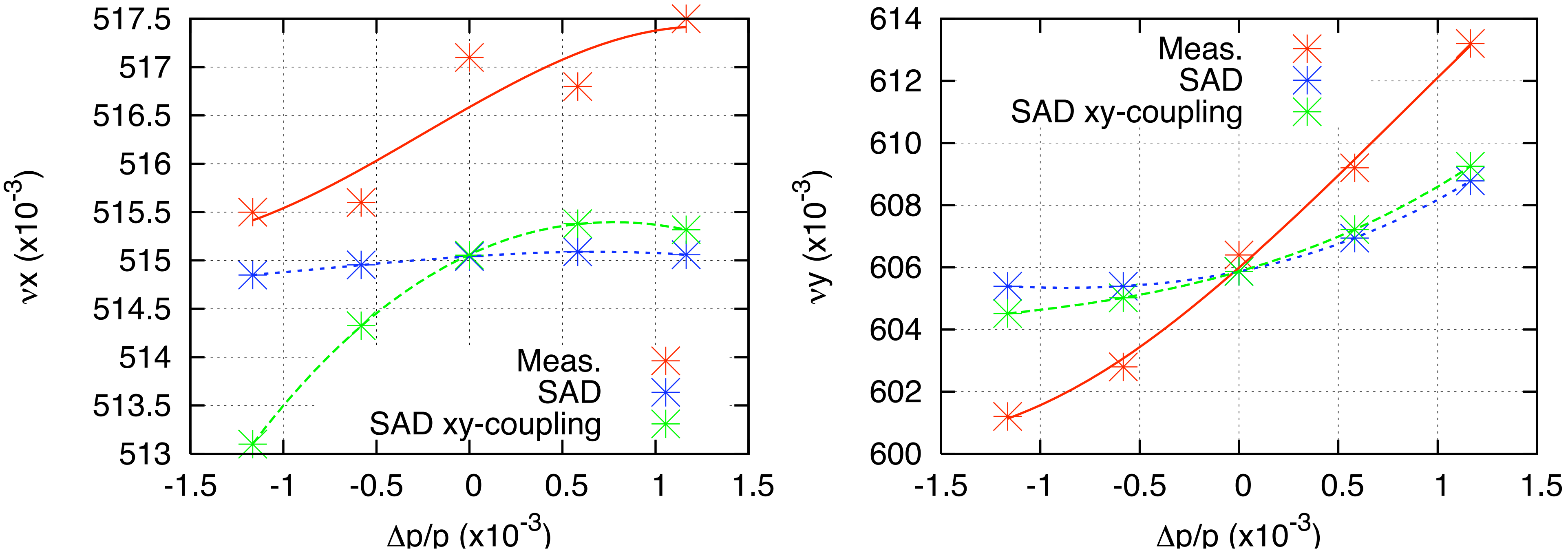
If this model succeed,

The research which suits the accelerator becomes possible, since this model includes machine error.

How to calculate Hamiltonian

Figure fitted up to 3rd order

$$\nu_i = \nu_{i0} + \nu_{i1}\delta + \nu_{i2}\delta^2 + \nu_{i3}\delta^3$$



SAD xy-coupling :

This means design optics with xy-coupling which make about 1% ratio of vertical and horizontal emittance.

This is because measrement emittance raito of them is 1%.

How to calculate Hamiltonian

Meas.

$$\begin{pmatrix} vx \\ vy \end{pmatrix} = \begin{pmatrix} 44.5 \\ 41.6 \end{pmatrix} + \begin{pmatrix} 1.09 \\ 5.61 \end{pmatrix}\delta + \begin{pmatrix} -126 \\ 842 \end{pmatrix}\delta^2 + \begin{pmatrix} -1.69\times 10^5 \\ -3.37\times 10^5 \end{pmatrix}\delta^3$$

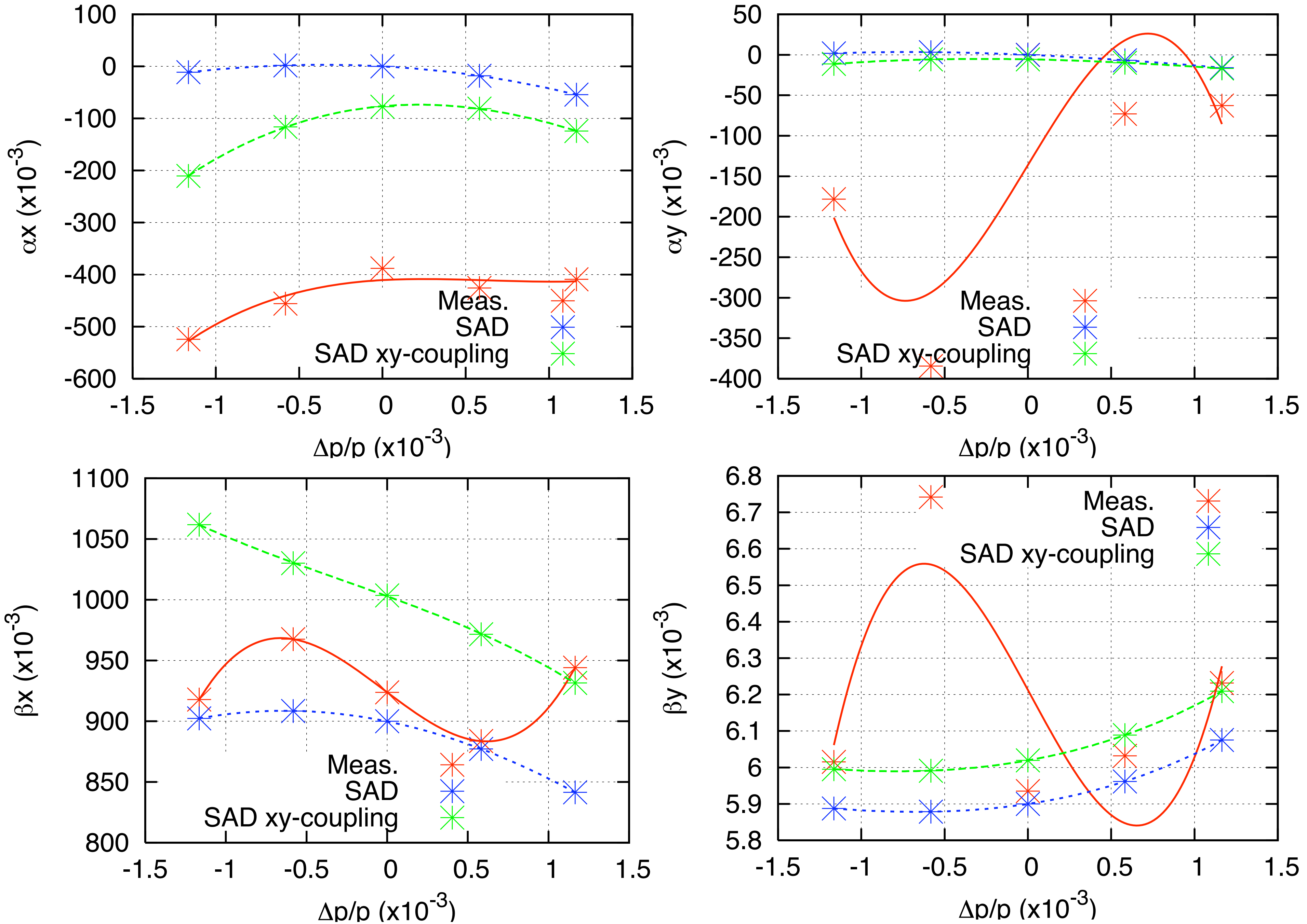
SAD

$$\begin{pmatrix} vx \\ vy \end{pmatrix} = \begin{pmatrix} 44.5 \\ 41.6 \end{pmatrix} + \begin{pmatrix} 0.126 \\ 1.29 \end{pmatrix}\delta + \begin{pmatrix} -64.6 \\ 898 \end{pmatrix}\delta^2 + \begin{pmatrix} -2.7\times 10^4 \\ 1.22\times 10^5 \end{pmatrix}\delta^3$$

SAD xy

$$\begin{pmatrix} vx \\ vy \end{pmatrix} = \begin{pmatrix} 44.5 \\ 41.6 \end{pmatrix} + \begin{pmatrix} 0.892 \\ 1.84 \end{pmatrix}\delta + \begin{pmatrix} -627 \\ 749 \end{pmatrix}\delta^2 + \begin{pmatrix} 4.41\times 10^4 \\ 1.45\times 10^5 \end{pmatrix}\delta^3$$

α_y's and β_y's shapes of difference between measurement and SAD is large.



I don't know why α_y and β_y are so large, but off momentun mesurement range cause such problem.

This is because previous off momentum mesurement data which was measured in larger range than measurement at this time don't have such problem.

Meas.

$$\begin{pmatrix} \alpha x \\ \beta x \\ \alpha y \\ \beta y \end{pmatrix} = \begin{pmatrix} -0.411 \\ 0.924 \\ -0.136 \\ 0.00621 \end{pmatrix} + \begin{pmatrix} 18 \\ -99.8 \\ 340 \\ -0.843 \end{pmatrix} \delta + \begin{pmatrix} -4.4\times 10^4 \\ 5240 \\ -5250 \\ -31.8 \end{pmatrix} \delta^2 + \begin{pmatrix} 2.31\times 10^7 \\ 8.19\times 10^7 \\ -2.14\times 10^8 \\ 6.9\times 10^5 \end{pmatrix} \delta^3$$

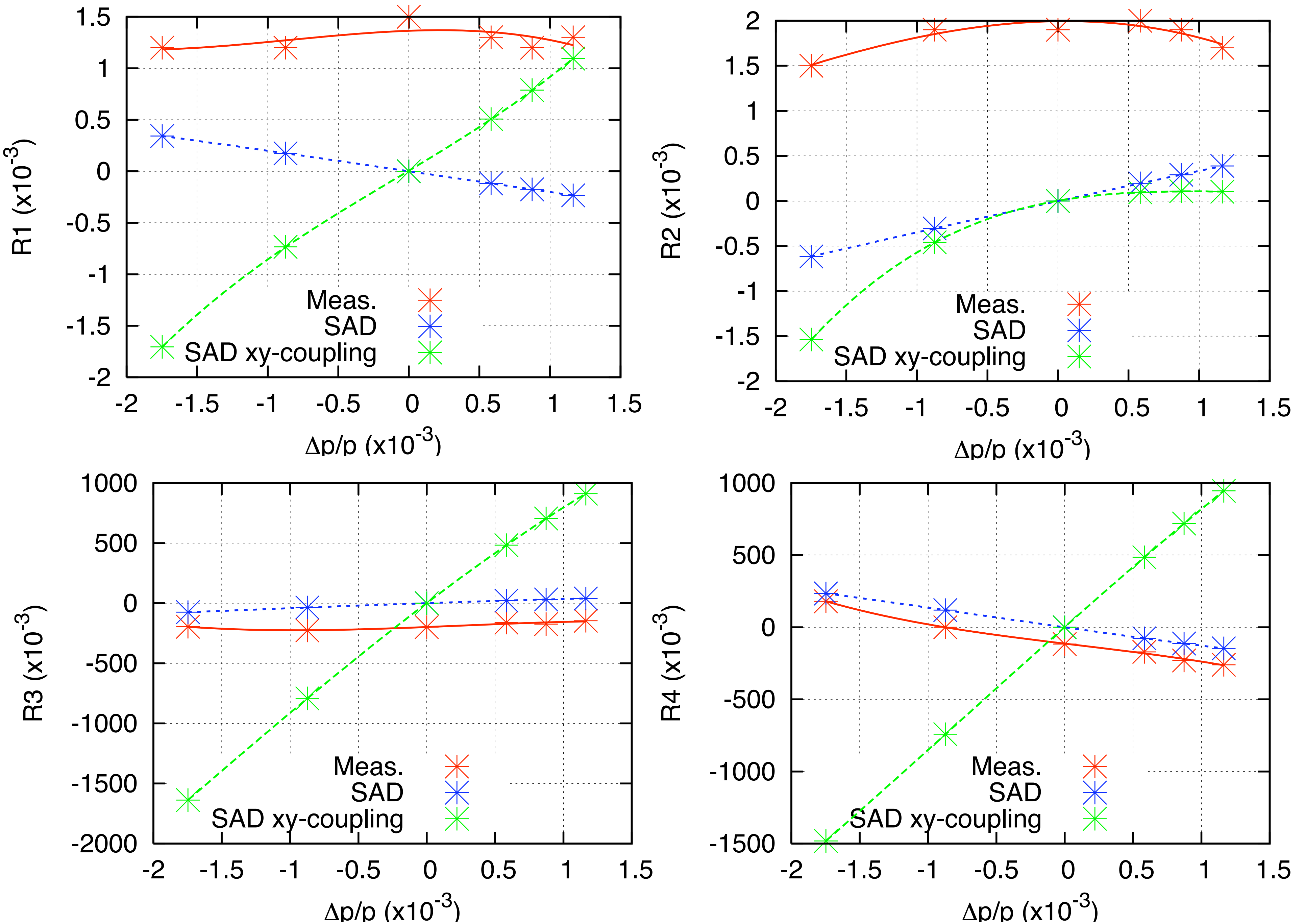
SAD

$$\begin{pmatrix} \alpha x \\ \beta x \\ \alpha y \\ \beta y \end{pmatrix} = \begin{pmatrix} 1.7\times 10^{-5} \\ 0.9 \\ -8.88\times 10^{-6} \\ 0.0059 \end{pmatrix} + \begin{pmatrix} -16.9 \\ -27.1 \\ -9.01 \\ 0.0682 \end{pmatrix} \delta + \begin{pmatrix} -2.43\times 10^4 \\ -2.06\times 10^4 \\ -5207 \\ 60 \end{pmatrix} \delta^2 + \begin{pmatrix} -1.22\times 10^6 \\ 6.76\times 10^5 \\ 1.03\times 10^6 \\ 9240 \end{pmatrix} \delta^3$$

SAD xy

$$\begin{pmatrix} \alpha x \\ \beta x \\ \alpha y \\ \beta y \end{pmatrix} = \begin{pmatrix} -0.0766 \\ 1 \\ -0.00565 \\ 0.00602 \end{pmatrix} + \begin{pmatrix} 27.9 \\ -48.3 \\ -3.8 \\ 0.0812 \end{pmatrix} \delta + \begin{pmatrix} -6.68\times 10^4 \\ -4707 \\ -6224 \\ 61.1 \end{pmatrix} \delta^2 + \begin{pmatrix} 6.61\times 10^6 \\ -5.65\times 10^6 \\ 1.05\times 10^6 \\ 7915 \end{pmatrix} \delta^3$$

How to calculate Hamiltonian



How to calculate Hamiltonian

Meas.

$$\begin{pmatrix} r1 \\ r2 \\ r3 \\ r4 \end{pmatrix} = \begin{pmatrix} 0.00136 \\ 0.002 \\ -0.198 \\ -0.114 \end{pmatrix} + \begin{pmatrix} 0.0495 \\ 0.019 \\ 45.7 \\ -114 \end{pmatrix}\delta + \begin{pmatrix} -99.5 \\ -182 \\ 7860 \\ 4440 \end{pmatrix}\delta^2 + \begin{pmatrix} -3.94\times 10^4 \\ -1.95\times 10^4 \\ -1.09\times 10^7 \\ -1.52\times 10^7 \end{pmatrix}\delta^3$$

SAD

$$\begin{pmatrix} r1 \\ r2 \\ r3 \\ r4 \end{pmatrix} = \begin{pmatrix} 1.44\times 10^{-8} \\ -5.66\times 10^{-7} \\ 2.84\times 10^{-5} \\ -0.00015 \end{pmatrix} + \begin{pmatrix} -0.2 \\ 0.342 \\ 38.8 \\ -134 \end{pmatrix}\delta + \begin{pmatrix} -0.894 \\ -6.41 \\ -3469 \\ 4156 \end{pmatrix}\delta^2 + \begin{pmatrix} 1578 \\ -601 \\ -6.34\times 10^5 \\ 2.3\times 10^6 \end{pmatrix}\delta^3$$

SAD xy

$$\begin{pmatrix} r1 \\ r2 \\ r3 \\ r4 \end{pmatrix} = \begin{pmatrix} 5.55\times 10^{-6} \\ 3.37\times 10^{-6} \\ 0.000817 \\ 0.00126 \end{pmatrix} + \begin{pmatrix} 0.82 \\ 0.275 \\ 864 \\ 843 \end{pmatrix}\delta + \begin{pmatrix} 24 \\ -235 \\ -5.91\times 10^4 \\ -1.74\times 10^4 \end{pmatrix}\delta^2 + \begin{pmatrix} 6.55\times 10^4 \\ 6.4\times 10^4 \\ -9.5\times 10^6 \\ -8.18\times 10^6 \end{pmatrix}\delta^3$$

Numerical Mh(δ)

$$Mh(\delta) = M(0)^{-1}R(\delta)M(\delta)R(\delta)^{-1} = \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 1. & 0 & 0 \\ 0 & 0 & 1. & 0 \\ 0 & 0 & 0 & 1. \end{pmatrix} + \begin{pmatrix} 0.285 & -4.14 & -9.47 & -0.106 \\ -20.6 & -0.285 & 53.5 & -0.428 \\ 0.0428 & -0.110 & 196 & 0.572 \\ 53.5 & 9.47 & 1.23\times 10^{4} & -196 \end{pmatrix}\delta$$

$$+\begin{pmatrix} -3.87\times 10^{3} & -1.39\times 10^{3} & -2.93\times 10^{4} & 65.8 \\ -4.86\times 10^{3} & 3.94\times 10^{3} & -6.73\times 10^{3} & -1.64 \\ 118 & 48.6 & 5.39\times 10^{4} & 19.3 \\ -1.21\times 10^{4} & 2.59\times 10^{4} & -9.37\times 10^{6} & -8.63\times 10^{3} \end{pmatrix}\delta^{2} + \begin{pmatrix} 1.46\times 10^{6} & 8.30\times 10^{6} & 3.37\times 10^{7} & -3.68\times 10^{5} \\ 1.42\times 10^{6} & -1.42\times 10^{6} & -4.08\times 10^{7} & 2.48\times 10^{5} \\ -2.29\times 10^{5} & -3.46\times 10^{5} & -1.27\times 10^{8} & -1.49\times 10^{5} \\ -4.47\times 10^{7} & -3.77\times 10^{7} & -1.05\times 10^{10} & 1.34\times 10^{8} \end{pmatrix}\delta^{3}$$

Comparing their coefficients about each δ dimension, we can derive $G_{function}$.

Theorical Mh(δ)

$$Mh11 = 1 + B + 2C\cdot Mh21 + G\cdot Mh41$$

$$Mh12 = \frac{-FG + 2C(1+V)}{1+B-EF+V+BV}$$

$$Mh13 = 2C\cdot Mh23 + F + G\cdot Mh43$$

$$Mh14 = \frac{-2\,CE + G + BG}{1+B-EF+V+BV}$$

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$$Mh43 = \frac{DF - 2(1+B)U}{1+B-EF+V+BV}$$

$$Mh44 = \frac{1+B}{1+B-EF+V+BV}$$

How to calculate Hamiltonian

$$G_{function} = \sum_{n=1} (a_n x^2 + b_n x\overline{p}_x + c_n \overline{p}_x^2$$
$$+ d_n xy + e_n x\overline{p}_y + f_n \overline{p}_x y + g_n \overline{p}_x \overline{p}_y + u_n y^2 + v_n y\overline{p}_y + w_n \overline{p}_y^2)\delta^n$$

meas.	SAD	SAD-xy
$a_1 = 10.1$	$a_1 = 2.10$	$a_1 = 4.74$
$b_1 = 0.285$	$b_1 = -1.31$	$b_1 = 2.62$
$c_1 = -2.07$	$c_1 = -0.984$	$c_1 = 0.651$
$d_1 = -53.5$	$d_1 = -22.1$	$d_1 = -177$
$e_1 = 0.428$	$e_1 = 0.176$	$e_1 = 3.07$
$f_1 = -9.47$	$f_1 = 65.5$	$f_1 = -80.5$
$g_1 = -0.106$	$g_1 = -0.386$	$g_1 = 3.44$
$u_1 = -6140$	$u_1 = 502$	$u_1 = 539$
$v_1 = 196$	$v_1 = -8.78$	$v_1 = -7.04$
$w_1 = 0.286$	$w_1 = 0.0303$	$w_1 = 0.0501$

How to calculate Hamiltonian

$$G_{function} = \sum_{n=1} (a_n x^2 + b_n x\overline{p}_x + c_n \overline{p}_x^2$$
$$+ d_n xy + e_n x\overline{p}_y + f_n \overline{p}_x y + g_n \overline{p}_x \overline{p}_y + u_n y^2 + v_n y\overline{p}_y + w_n \overline{p}_y^2)\delta^n$$

meas.	SAD	SAD-xy
$a_2 = 2420$	$a_2 = 1160$	$a_2 = 264$
$b_2 = -3950$	$b_2 = -2100$	$b_2 = -3390$
$c_2 = -696$	$c_2 = -1230$	$c_2 = -1320$
$d_2 = 1450$	$d_2 = 577$	$d_2 = 9210$
$e_2 = 85.5$	$e_2 = -11.2$	$e_2 = -152$
$f_2 = -27700$	$f_2 = -1070$	$f_2 = -31000$
$g_2 = 43.2$	$g_2 = 4.30$	$g_2 = -58.0$
$u_2 = 3.48 \times 10^6$	$u_2 = 229000$	$u_2 = 228000$
$v_2 = 46900$	$v_2 = -6360$	$v_2 = -7120$
$w_2 = 65.6$	$w_2 = 25.1$	$w_2 = 26.1$

How to calculate Hamiltonian

$$G_{function} = \sum_{n=1}(a_n x^2 + b_n x\overline{p}_x + c_n \overline{p}_x^2$$
$$+d_n xy + e_n x\overline{p}_y + f_n \overline{p}_x y + g_n \overline{p}_x \overline{p}_y + u_n y^2 + v_n y\overline{p}_y + w_n \overline{p}_y^2)\delta^n$$

meas.	SAD	SAD-xy
$a_3 = -7.13\times 10^6$	$a_3 = -12600$	$a_3 = 50700$
$b_3 = 1.40\times 10^6$	$b_3 = -127000$	$b_3 = -94800$
$c_3 = 4.16\times 10^6$	$c_3 = -49800$	$c_3 = -138000$
$d_3 = 4.40\times 10^7$	$d_3 = 212000$	$d_3 = 1.88\times 10^6$
$e_3 = -229000$	$e_3 = -955$	$e_3 = -2240$
$f_3 = 3.22\times 10^7$	$f_3 = -332000$	$f_3 = 4.16\times 10^6$
$g_3 = -361000$	$g_3 = 840$	$g_3 = -3600$
$u_3 = 5.88\times 10^9$	$u_3 = -2.83\times 10^7$	$u_3 = -2.31\times 10^7$
$v_3 = -1.23\times 10^8$	$v_3 = -66800$	$v_3 = 902$
$w_3 = -59200$	$w_3 = 5390$	$w_3 = 4820$

4.Synchro-beta resonance due to chromaticities

- one turn map

Beam-beam (weak-strong) + Chromaticity +
Linear 1 turn transformation +radiaton damping and quantum diffusion

(dispersion isn't included in the traching)

- one turn matrix

$$M_{1turn} = M_{radiaton} \cdot M_{Linear} \cdot M_{chrom.} \cdot M_{beam-beam}$$

Synchro-beta resonance

Using Ohmi-san's program, simulated by D.M.Zhou

- Basic parameters (KEKB)

	Weak beam	Strong beam
υ_x / υ_y / υ_z	44.51-44.71 /41.58-41.64 /0.024	44.515 /41.606 /0.024
$\beta_x/\beta_y/\beta_z$ (m)	0.9/0.006/10.0	0.9/0.006/10.0
$\varepsilon_x/\varepsilon_y$ ε_z	1.8E-8/1.8E-10 /4.8E-6	1.8E-8/1.8E-10 /4.8E-6
Damping rate (x/y/z)	2.5E-4/2.5E-4 /5.0E-4	2.5E-4/2.5E-4 /5.0E-4
Number of particles	10.0E10	4.37E10

Synchro-beta resonance

difference between left and right fig. is y range.

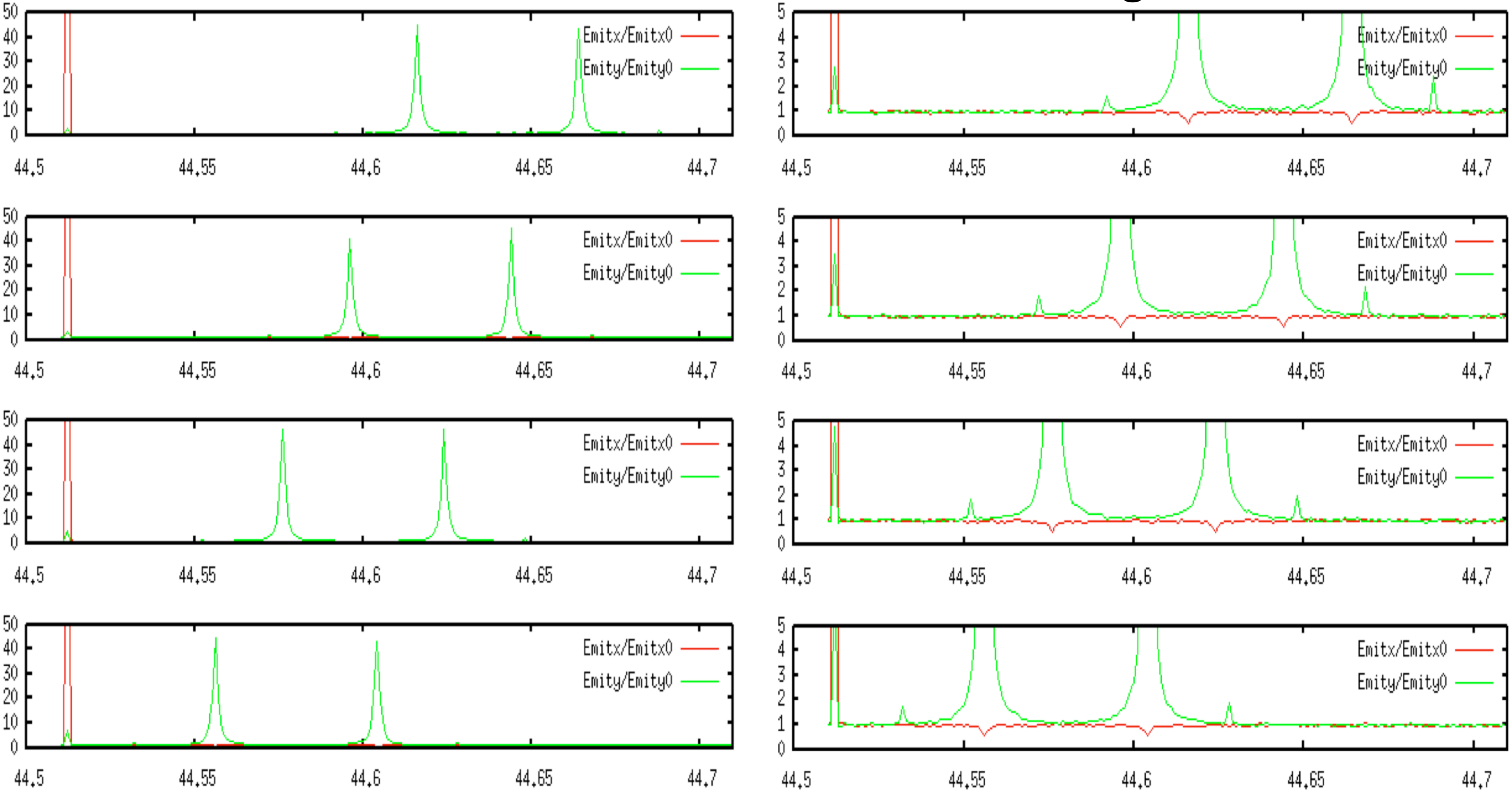
SAD-data (beam-beam effect switched off)

comparison hor. with ver. at only first order chromaticities added in tracking

red is hor. emit.

green is ver. emit.

hor. axis is Nux



$$\nu_x - \nu_y + k\nu_s = N \quad k \in Z \quad \nu_y = 41.58, 41.60, 41.62, 41.64$$

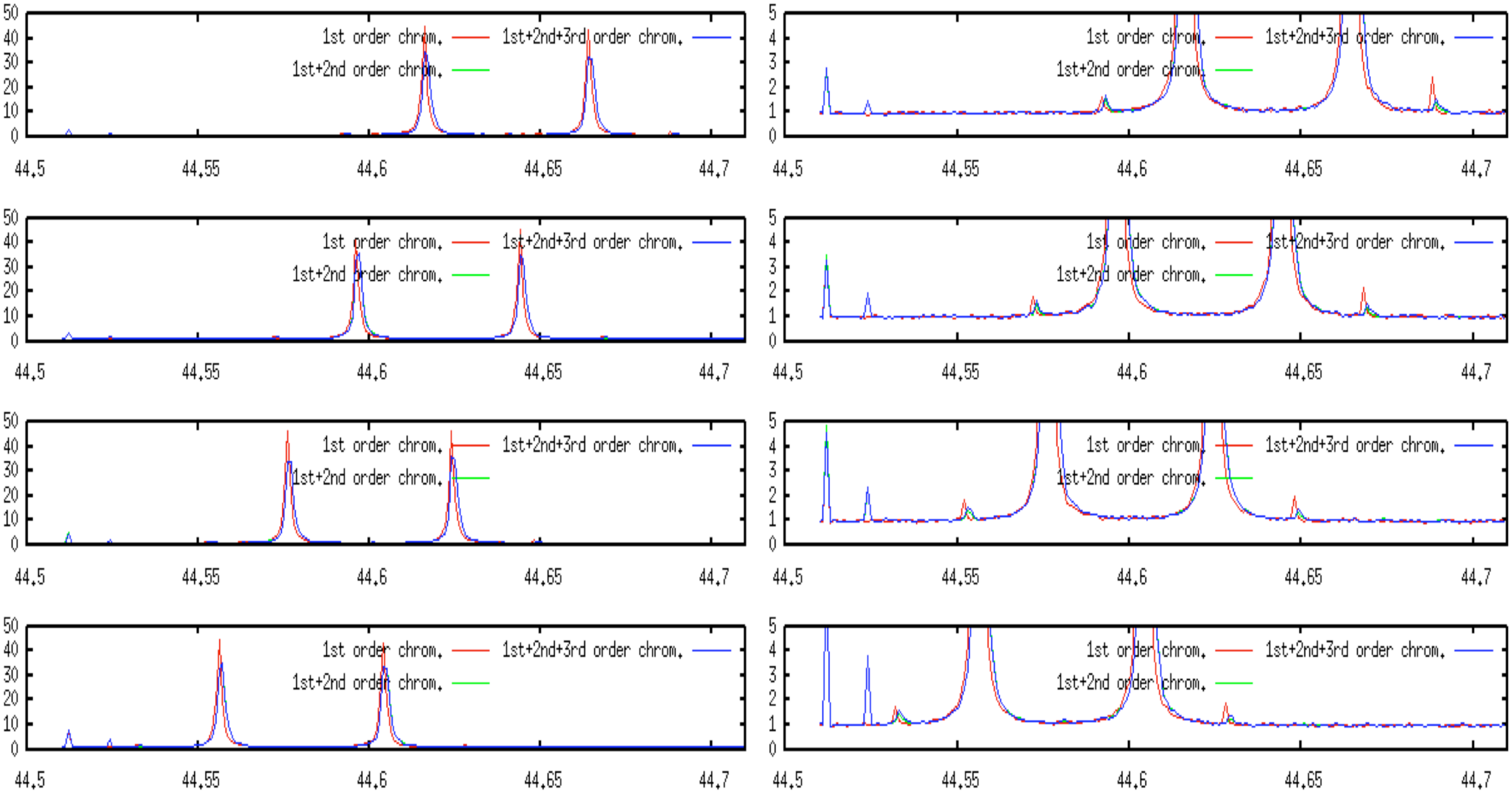
Synchro-beta resonance

difference between left and right fig. is y range.

SAD-data (beam-beam effect switched off)

red is 1st emit.
green is 2nd
blue is 3rd
hor. axis is Nux

different set of chromaticities



$$\nu_x - \nu_y + k\nu_s = N \quad k \in Z \quad \nu_y = 41.58, 41.60, 41.62, 41.64$$

Synchro-beta resonance

difference between left and right fig. is y range.

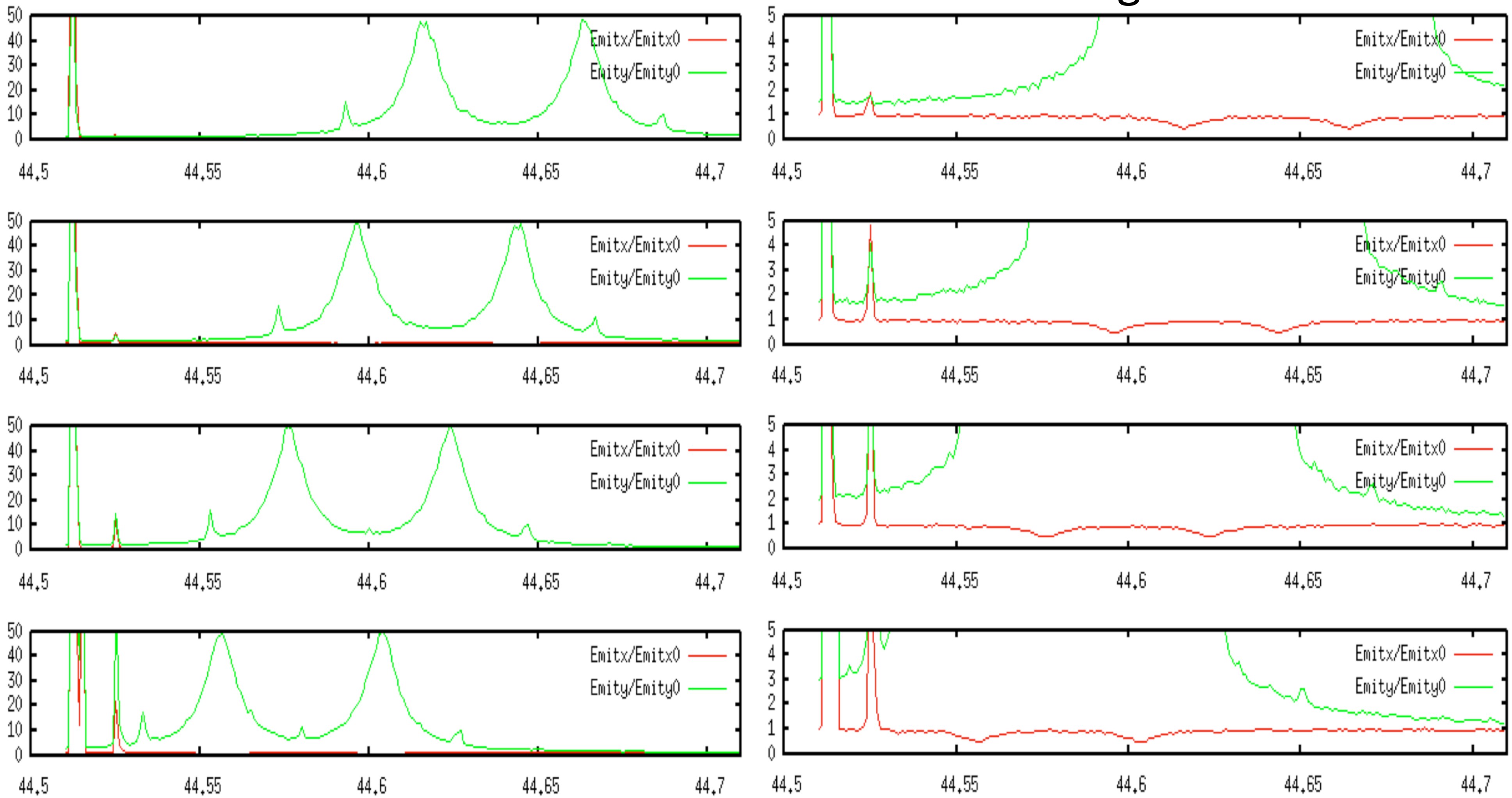
SADxy-data (beam-beam effect switched off)

red is hor. emit.

green is ver. emit.

hor. axis is Nux

comparison hor. with ver. at only first order chromaticities added in tracking



$$\nu_x - \nu_y + k\nu_s = N \quad k \in Z \quad \nu_y = 41.58, 41.60, 41.62, 41.64$$

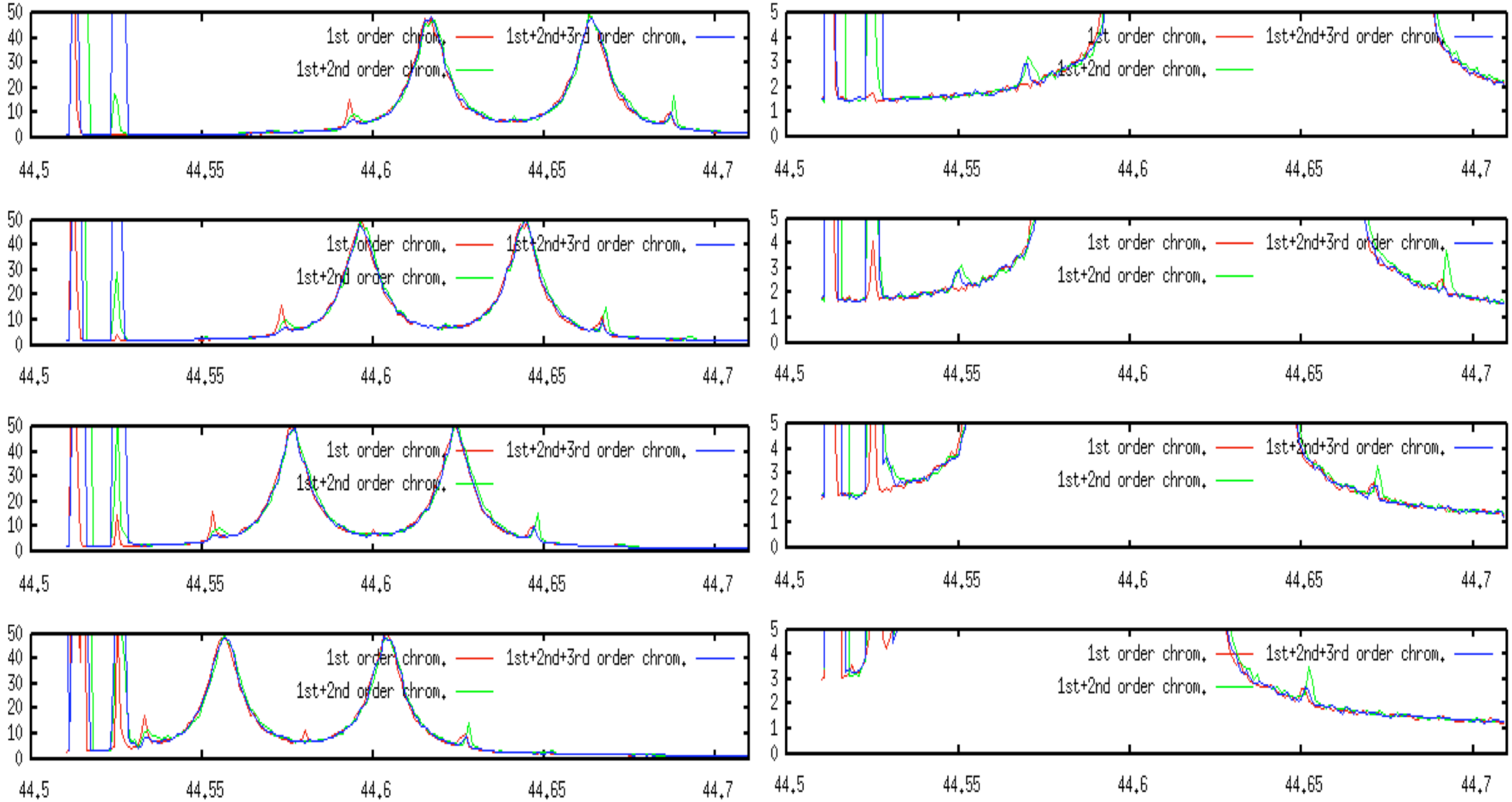
Synchro-beta resonance

difference between left and right fig. is y range.

SADxy-data (beam-beam effect switched off)

red is 1st emit.
green is 2nd
blue is 3rd
hor. axis is Nux

different set of chromaticities



$$\nu_x - \nu_y + k\nu_s = N \quad k \in Z \quad \nu_y = 41.58, 41.60, 41.62, 41.64$$

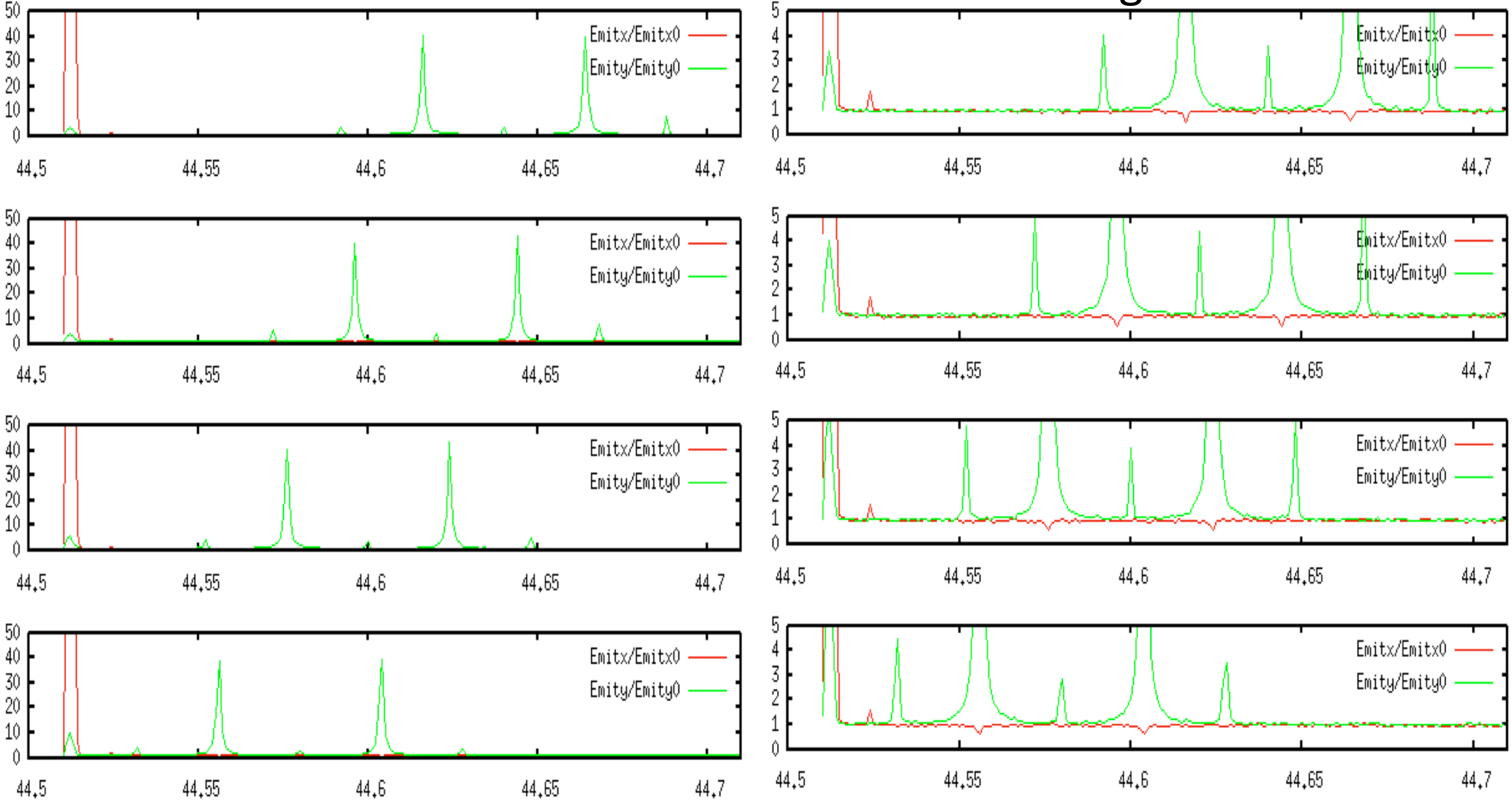
Synchro-beta resonance

difference between left and right fig. is y range.

meas.-data (beam-beam effect switched off)

red is hor. emit.
green is ver. emit.
hor. axis is Nux

comparison hor. with ver. at only first order chromaticities added in tracking



$$\nu_x - \nu_y + k\nu_s = N \quad k \in Z \quad \nu_y = 41.58, 41.60, 41.62, 41.64$$

Synchro-beta resonance

difference between left and right fig. is y range.

meas.-data (beam-beam effect switched off)

red is 1st emit.

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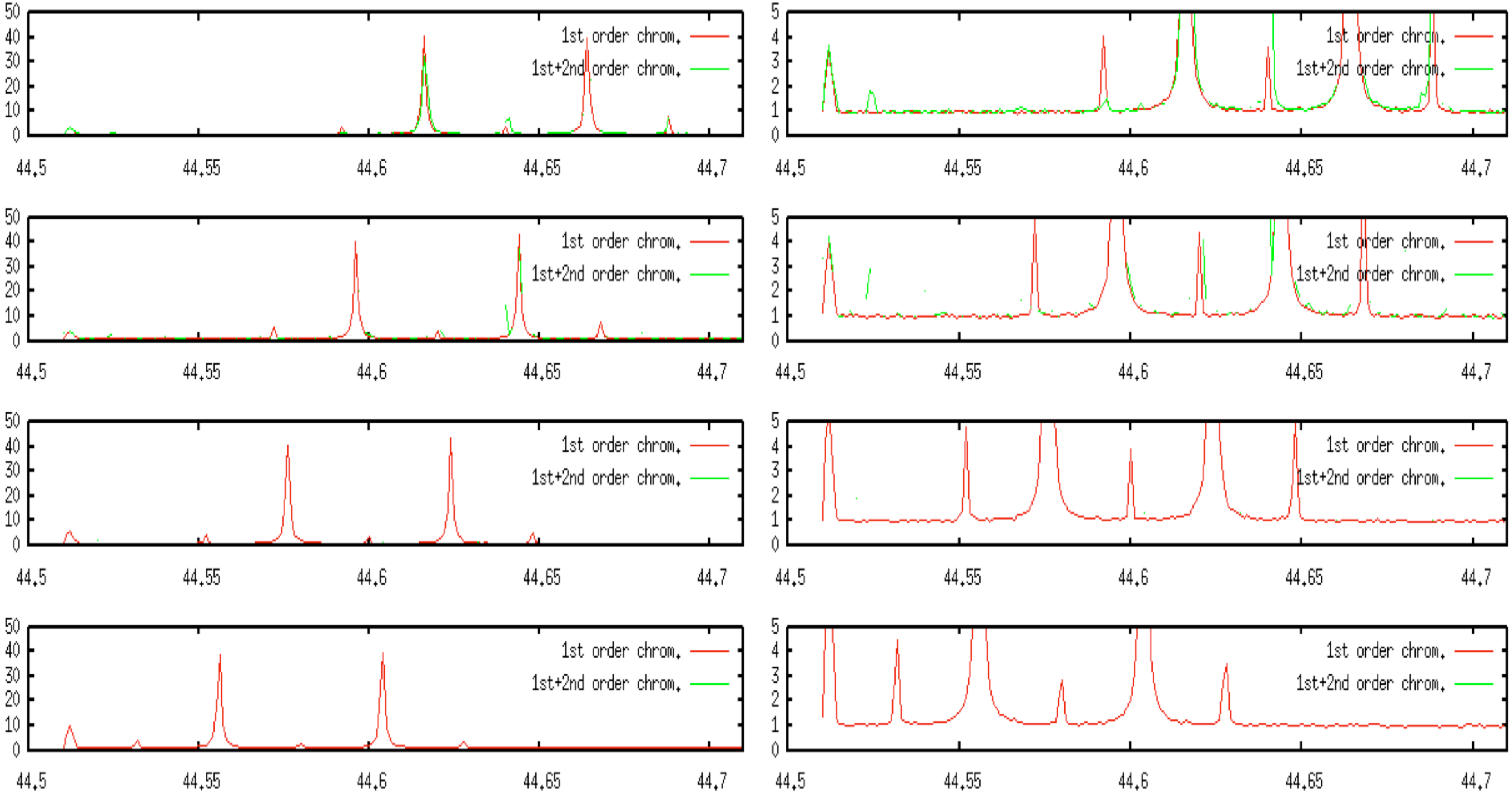
blue is 3rd

hor. axis is Nux

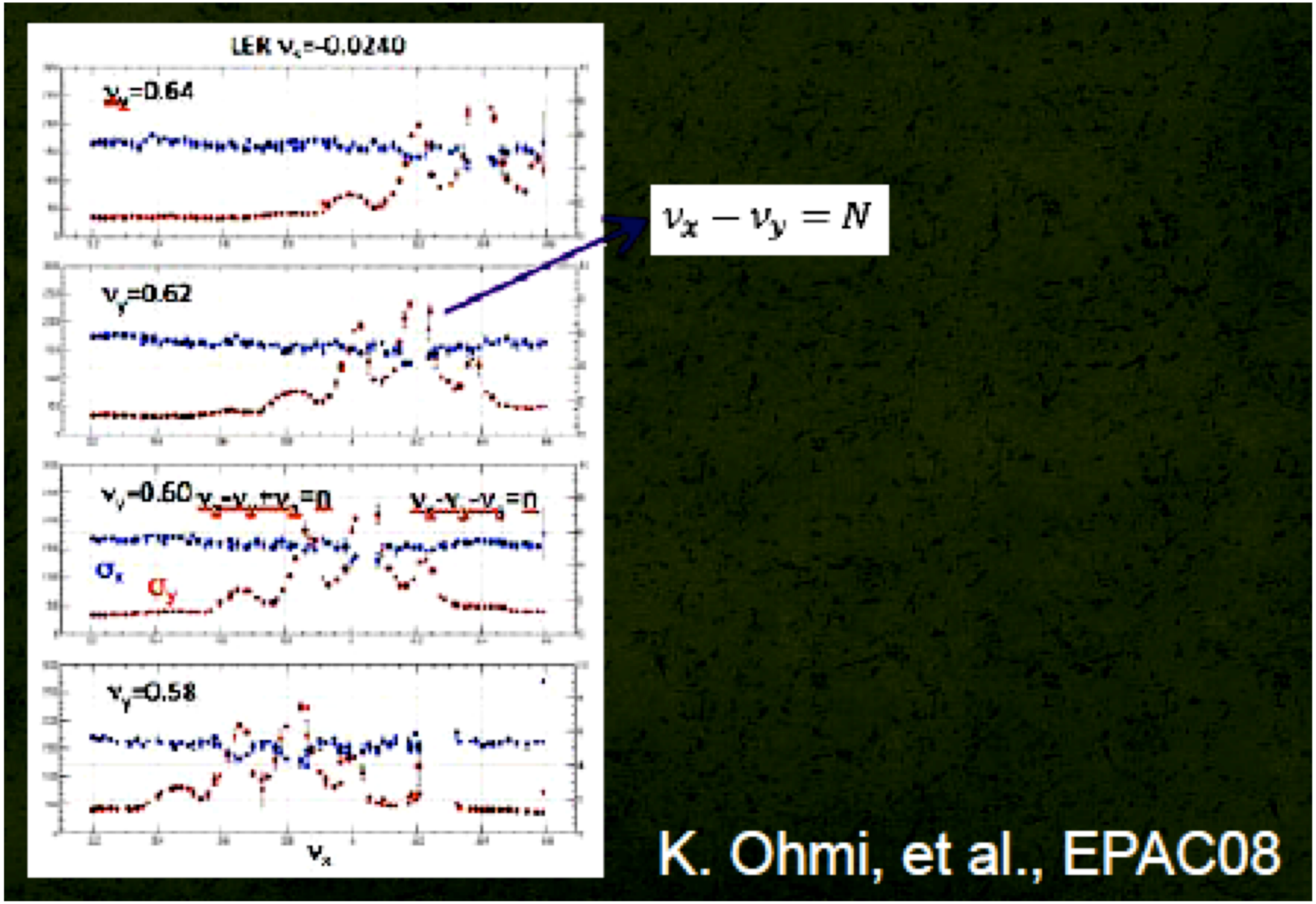
different set of chromaticities



unseen parts of the curves goto NaN(unstable at 2nd and 3rd order)

Synchro-beta resonance

Simulations agree with experimental tune scan of beam size except the resonance of xy coupling



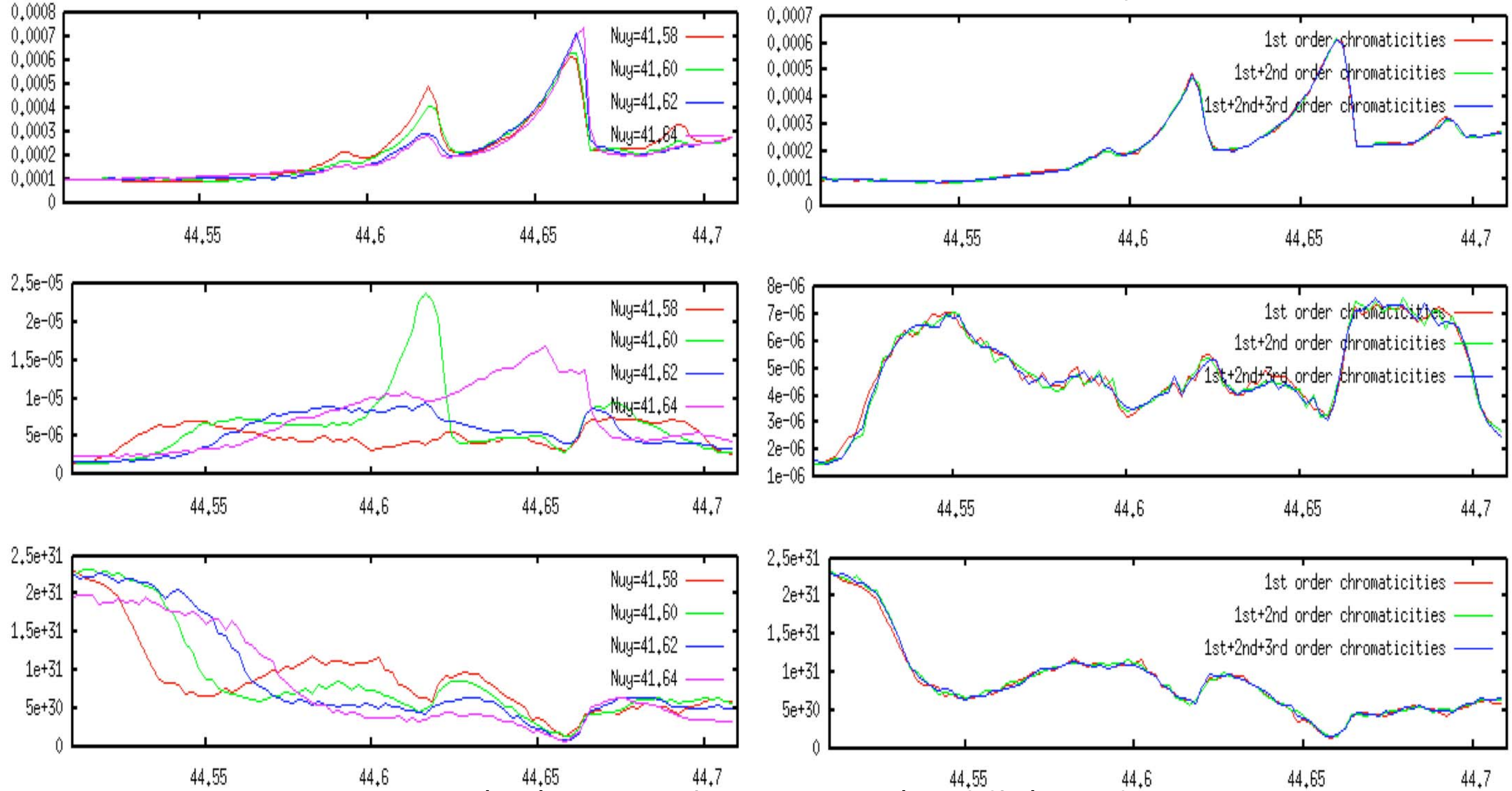
Synchro-beta resonance

(no significant emittance growth or Luminosity decrease from 2nd and 3rd chromaticity.)

SAD-data (beam-beam effect included)

comparison each Nuy
only first order chromaticities added in tracking

different set of chromaticities
(Nuy=41.58)



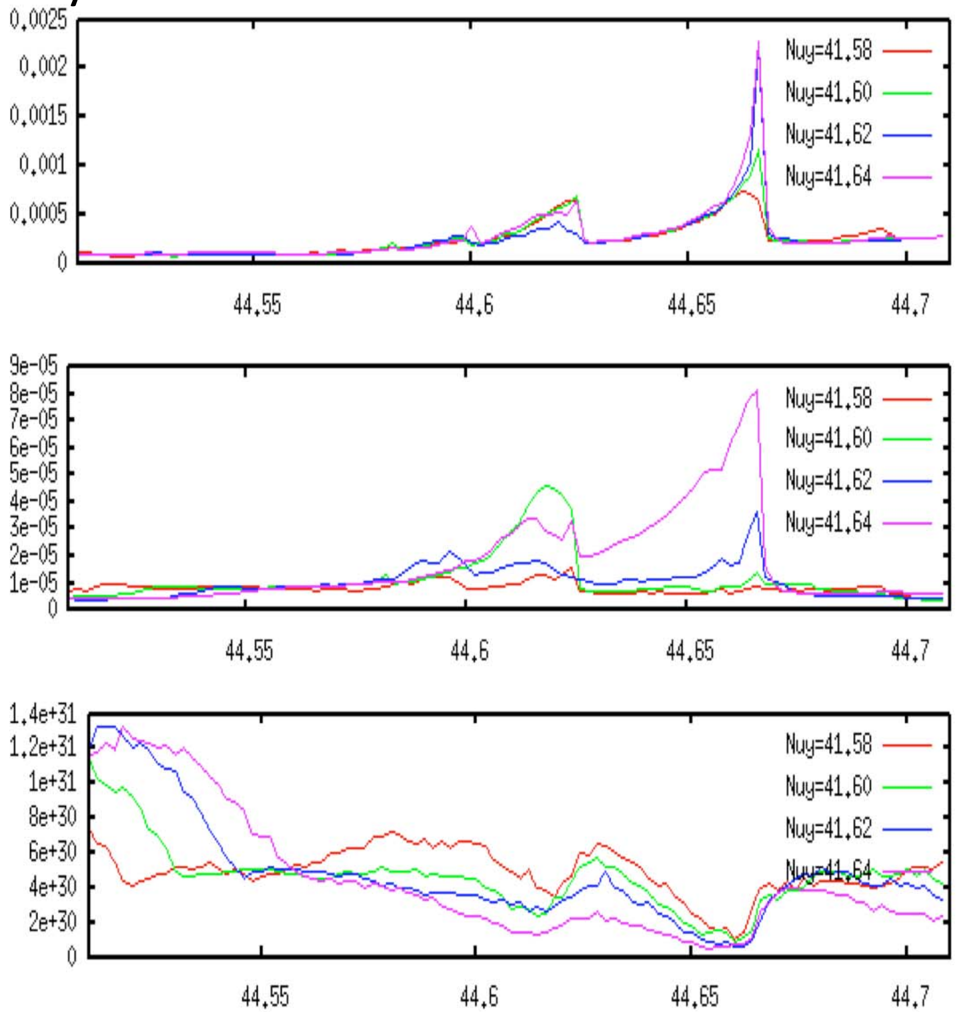
Hor. Beam size (up), Vertical Beam size (middle), and Luminosity (down) at different settings of chromaticities

Synchro-beta resonance

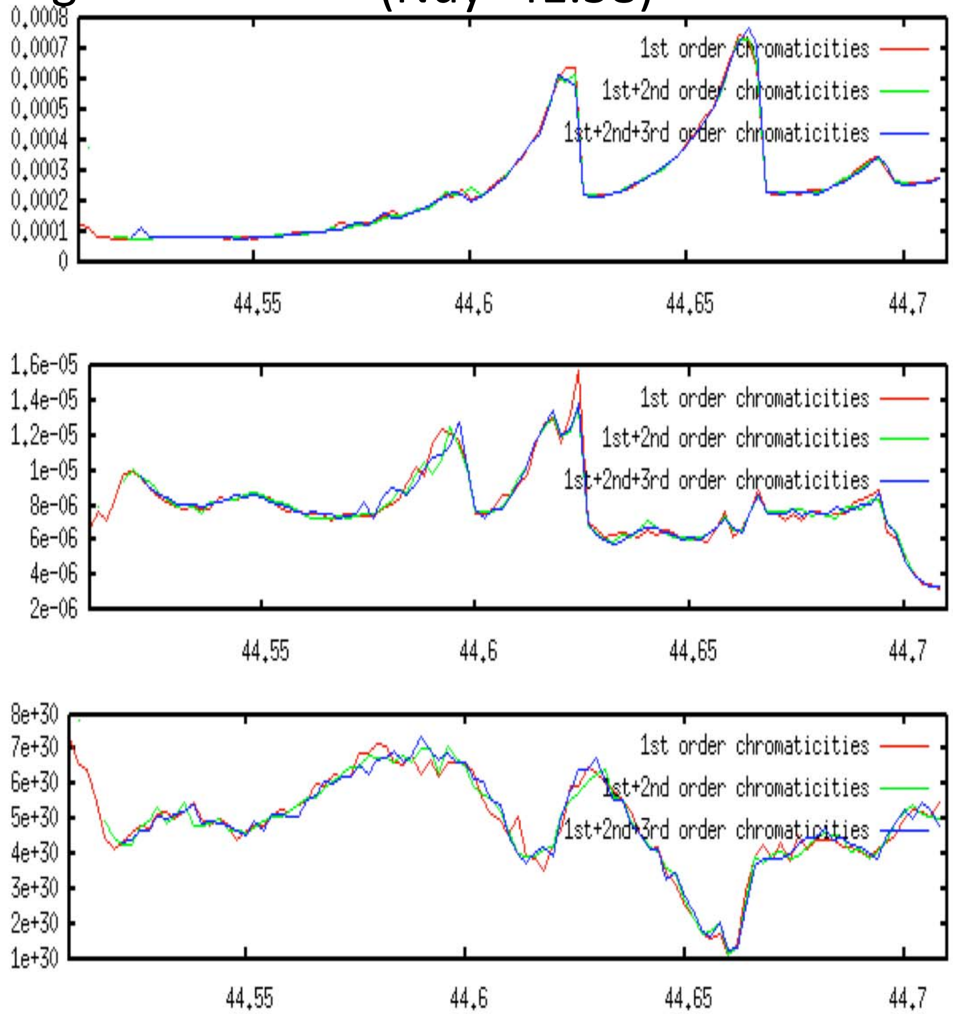
(no significant emittance growth or Luminosity decrease from 2nd and 3rd chromaticity.)

SADxy-data (beam-beam effect included)

comparison each Nuy
only first order chromaticities added in tracking



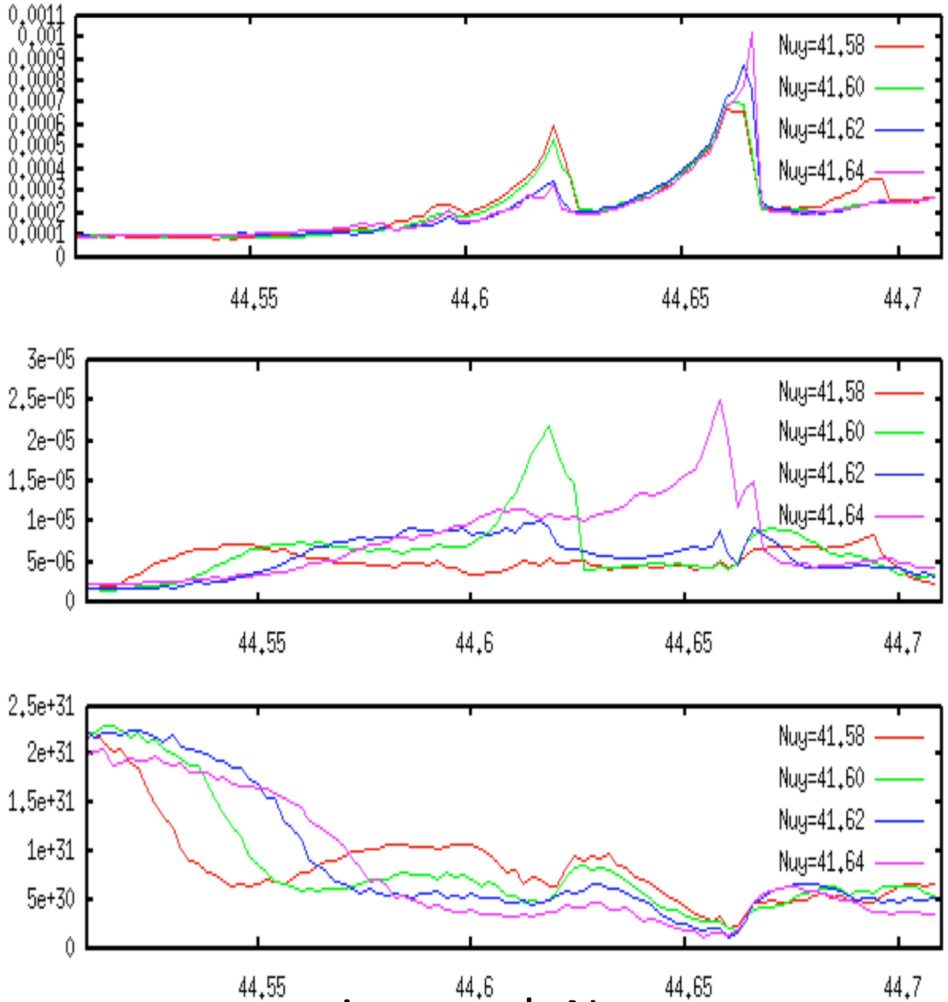
different set of chromaticities
(Nuy=41.58)



Hor. Beam size (up), Vertical Beam size (middle), and Luminosity (down) at different settings of chromaticities

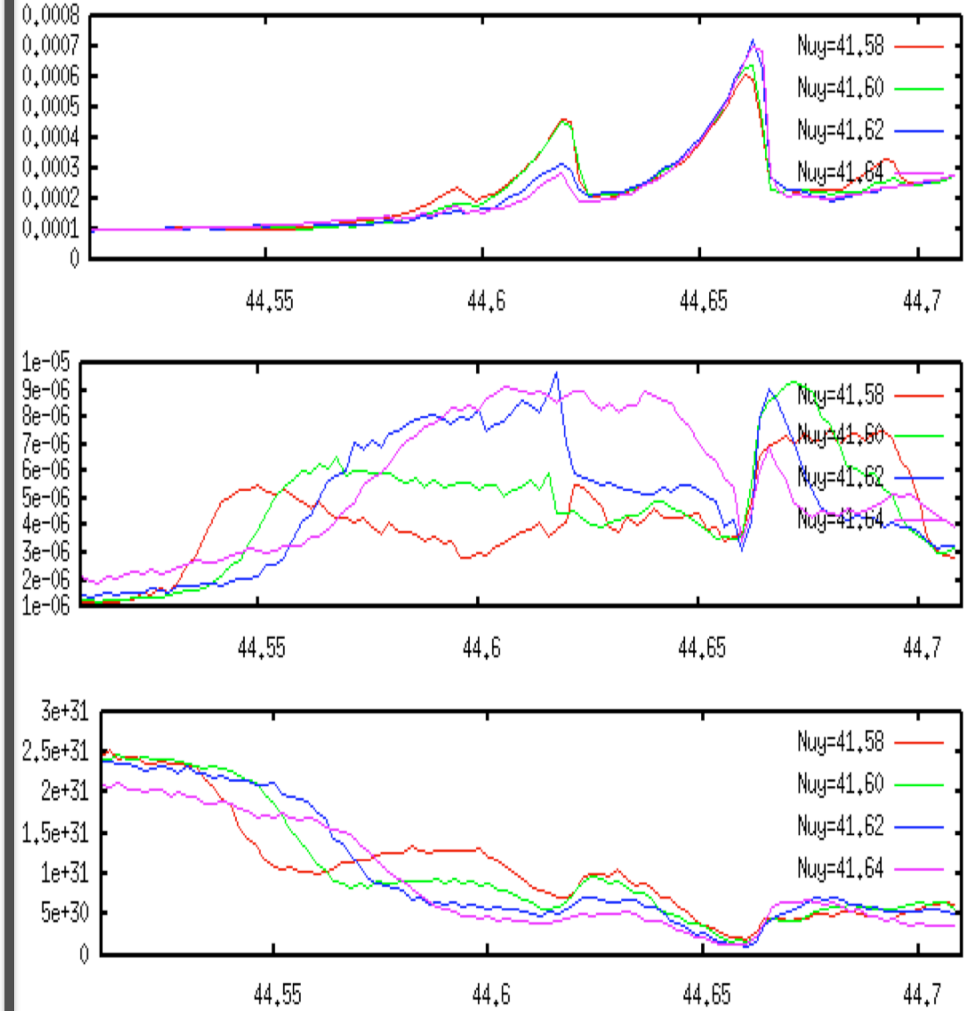
Synchro-beta resonance

meas.-data (beam-beam effect included)



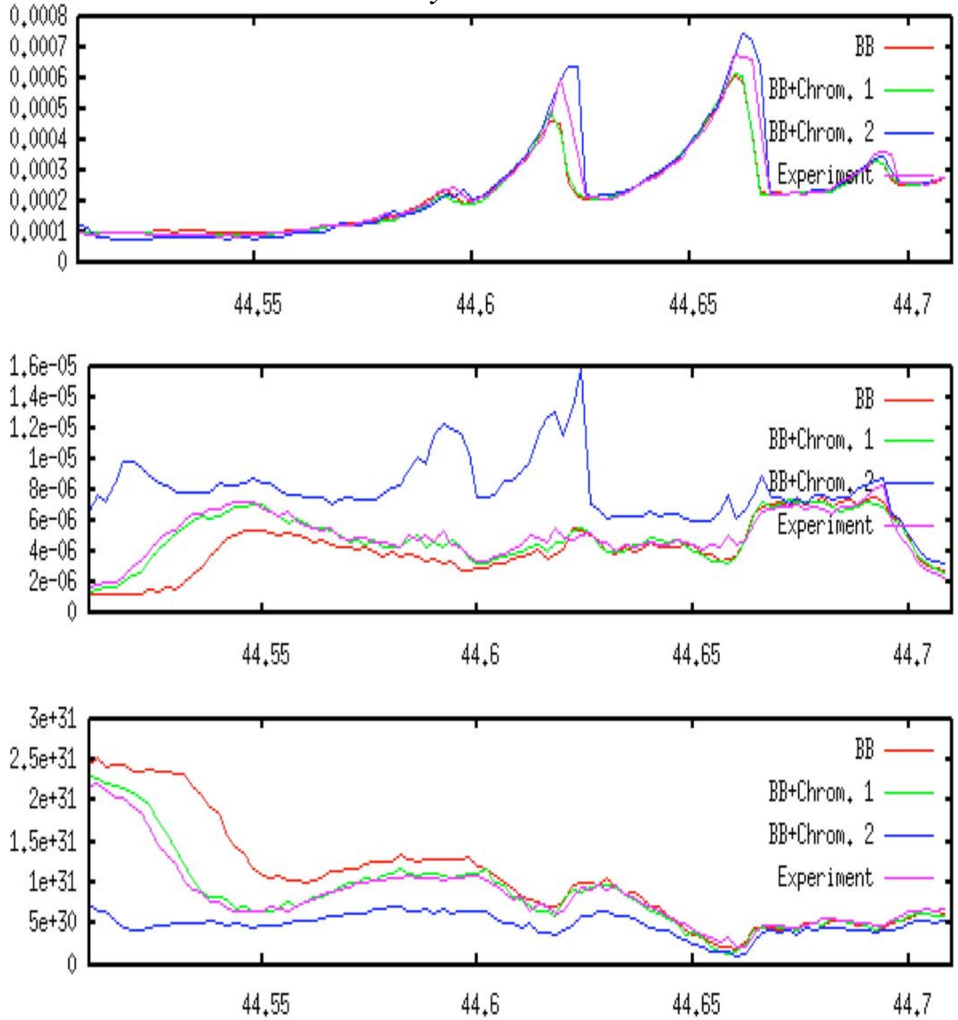
- comparison each Nuy
- only first order chromaticities added in tracking

beam-beam tracking without chromaticities

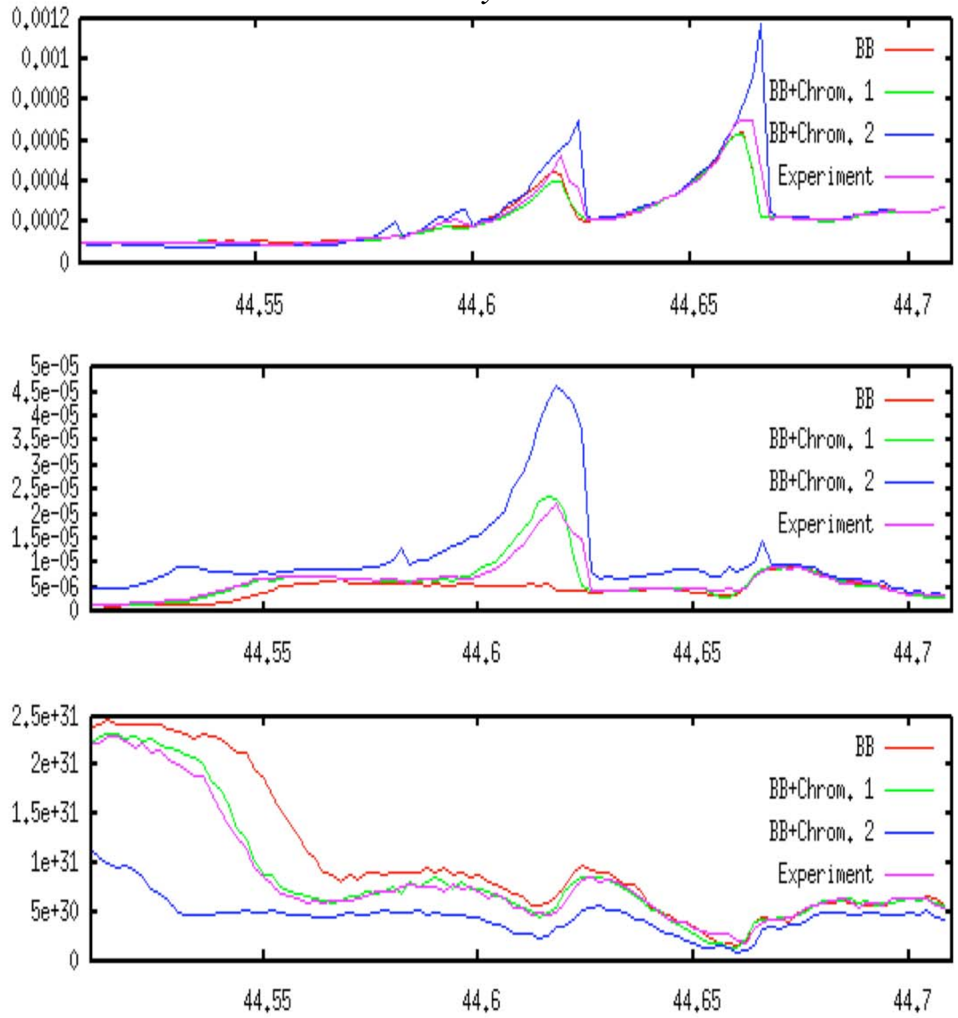


Summary for this simulation

$\nu_y = 41.58$



$\nu_y = 41.60$

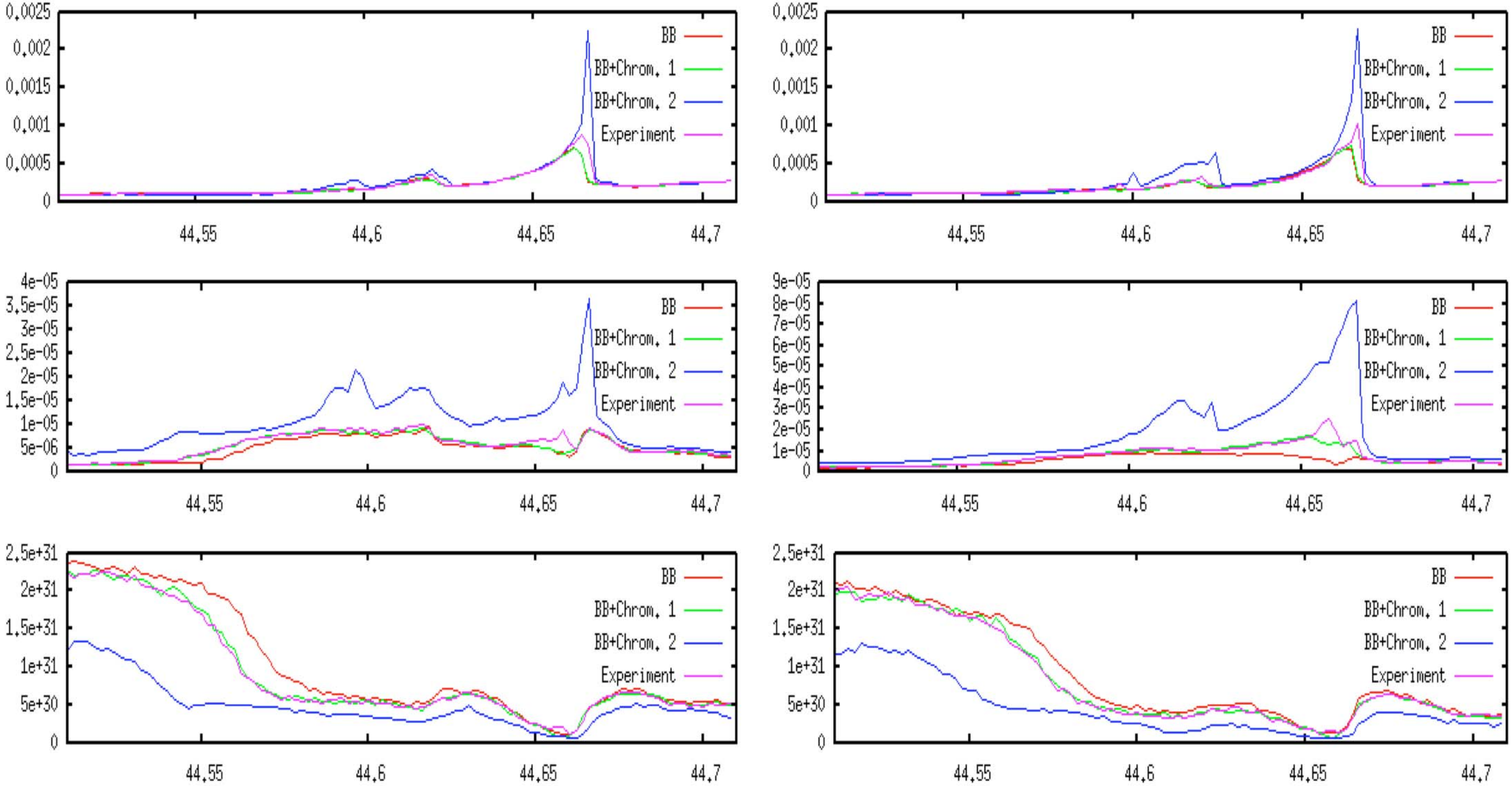


Hor. Beam size (up), Vertical Beam size (middle), and Luminosity (down) at different settings of chromaticities (meas. : only first order chromaticities added in tracking)

red is only beam-beam effect (BB) included green is BB + SAD chrom.

blue is BB + SADxy chrom. pink is BB + meas. only 1st chrom.

$\nu_y = 41.62$ $\nu_y = 41.64$



Hor. Beam size (up), Vertical Beam size (middle), and Luminosity (down) at different settings of chromaticities (meas. : only first order chromaticities added in tracking)

red is only beam-beam effect (BB) included green is BB + SAD chrom.

blue is BB + SADxy chrom. pink is BB + meas. only 1st chrom.

Summary

- SAD and SAD-xy data : no significant emittance growth or Luminosity decrease from 2nd and 3rd chromaticity.
- experimental data : beam very unstable when tracking with 2nd or 3rd chromaticity. Measured chromaticity too large?
- Luminosity is sensitive to vertical beam size
- beam-beam interactions dominate the dynamics, it is hard to characterize the effect of chromaticity from beam-beam collisions.

Future work

- off momentum measurement over large range as much as possible.
- confirm whether beam tracking simulation using this Hamiltonian is valid or not.
  (Using simulation on scanning $d\alpha_{x,y}/d\delta$,$d\beta_{x,y}/d\delta$,$\sigma_{x,y}$, and Luminosity, compare with beam turning experiments.)
- strong-strong simulation with chromaticity
- calculate Hamiltonian used new model which includes despersion.
  (We have already succeeded in including despersion in our model mathematically)